CRITICAL CHIRAL HYPERSURFACE OF THE MAGNETIZED NJL MODEL

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INTRODUCTION

The dynamical mass generation mechanism is the reason why the actual mass of the hadrons is various orders of magnitude bigger than the constituent quarks. This mechanism comes from the QCD theory, nevertheless it is known that extracting information from QCD at low energies is really HARD.

To explore this low energy regions, there exist various tools that have been developed over the course of time, among them are LQCD, Schwinger-Dyson equation, effective models, etc…

In this work we explore the dynamical mass generation mechanism using the Nambu-Jona-Lasinio (NJL) model applied to quarks.

Using the NJL model we were able to find a critical condition on the free parameters (coupling, external magnetic field and temperature) for the dynamical generation of mass to be possible.

INTRODUCTION

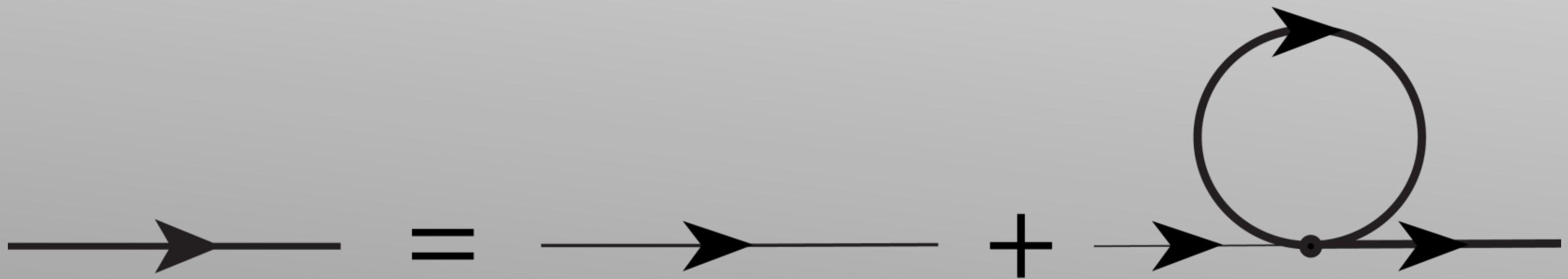
The NJL model is an QCD effective model at low energies, simpler than QCD but with dynamical mass generation, unfortunately it doesn't include confinement and it can´t be renormalized.

The lagrangian for the NJL model is given by

$$\mathcal{L} = \overline{\psi}(i\partial\!\!\!/ - m_q)\psi + G\left\{\left(\overline{\psi}\psi\right)^2 + \left(\overline{\psi}i\gamma^5\vec{\tau}\psi\right)^2\right\}$$

GAP EQUATION

Using the Schwinger-Dyson equations for the self energy



We arrive at the gap equation for the NJL model

$$m = m_q - 2G \langle \bar{\psi}\psi \rangle$$

where $\langle \bar{\psi}\psi \rangle$ is the chiral condensate, defined as:

$$\langle \bar{\psi}\psi \rangle = -\int \frac{d^4k}{(2\pi)^4} Tr[iS(k)]$$

GAP EQUATION

Since the NJL model isn´t renormalizable, it is necessary to use a regularization scheme. In our work we use proper time regularization, and after taking the chiral limit $m_q = 0$, the gap equation in the vacuum becomes:

$$m = \frac{GN_f N_c}{2\pi^2} m \int_{\Lambda_{TP}}^{\infty} ds \frac{e^{-m^2 s}}{s^2}$$

CRITICAL COUPLING

To get the critical coupling, we apply the condition:

$$\frac{d}{dm}m\bigg|_{m=0} = G_c \frac{d}{dm} m I(m)\bigg|_{m=0}$$

Where G_c is the critical coupling and $I(m) = \frac{3}{2\pi^2}\int_{\Lambda_{TP}}^{\infty} ds \frac{e^{-m^2 s}}{s^2}$

Then, for the vacuum, the critical coupling from where the mass start to generate is

$$1 = \frac{3}{2\pi^2} G_c \frac{1}{\Lambda_{TP}} \longrightarrow G_c = \frac{2\pi^2}{3}\Lambda_{TP}$$

CRITICAL CONDITION FOR A THERMOMAGNETIC MEDIUM

In this case, we will need the Green function for a magnetic medium

$$G(x,x') = -(4\pi)^2 \int_0^\infty \frac{ds}{s^2} \left[m + 12\gamma \cdot \sinh^{-1}(esF) e^{-esF} eF(x-x')\right]$$
$$\cdot exp\left\{-im^2 s - \frac{i}{2} es\, tr(\sigma F)\right\}$$
$$\cdot exp\left\{-\frac{1}{2} tr\, \ln\left[(esF)^{-1}\sinh^{-1}(esF)\right] - \frac{i}{4}(x-x')eF\coth(esF)(x-x')\right\}$$
$$\cdot exp\left\{ie \int_x^{x'} dz \left[A_\mu(z) - \frac{1}{2}F_{\mu\nu}(z-x')^\nu\right]\right\}$$

Setting the magnetic field in the z direction

$$G(x) = -(4\pi)^{-2} \int_0^\infty \frac{ds}{s^2} \frac{eBs}{\sin(eBs)} exp(-im^2 s + ieBs\sigma_3)$$
$$\cdot exp\left\{-\frac{i}{4s}\left(x_\parallel^2 - eBs\, \cot(eBs)\, x_\perp^2\right)\right\}$$
$$\cdot \left[m + \frac{1}{2s}\left(\gamma \cdot x_\parallel - \frac{eBs}{\sin(eBs)} exp(-ieBs\sigma_3)\gamma \cdot x_\perp\right)\right]$$

$$G(p) = -i \int_0^\infty \frac{ds}{\cos(eBs)} exp\left[-is\left(m^2 - p_\parallel^2 + \frac{\tan(eBs)}{eBs} p_\perp^2\right)\right]$$
$$\cdot \left[\left[\cos(eBs) + \gamma_1\gamma_2 \sin(eBs)\right](m + \gamma \cdot p_\parallel) - \frac{\gamma \cdot p_\perp}{\cos(eBs)}\right].$$

$$-\langle\bar{\psi}\psi\rangle = \frac{3meB}{\pi} \int \frac{d^2 p_\parallel}{(2\pi)^2} \int_\Lambda^\infty ds\, \frac{e^{-is(p_\parallel^2 + m^2)}}{\tan(eBs)}$$

CRITICAL CONDITION FOR A THERMOMAGNETIC MEDIUM

To include the thermal effects, we will use the Matsubara formalism

$$\int \frac{dp_0}{2\pi} f(p_0) \quad \longrightarrow \quad T \sum_{n=-\infty}^{\infty} f(\omega_n)$$

$$\omega_n = 2(n + \frac{1}{2})\pi T$$

$$-\langle\psi\psi\rangle = \frac{3meB}{2\pi^{\frac{3}{2}}} \int \frac{ds}{s^{\frac{1}{2}}} \frac{e^{-sm^2}}{\tanh(eBs)} \times T \sum_{-\infty}^{\infty} e^{-s\omega_n^2}$$

The sum can be identified with the Jacobi function θ_3

$$\theta_3(z, \tau) = 1 + 2\sum_{n=1}^{\infty} q^{n^2} \cos(2nz)$$

$$\sum_{n=-\infty}^{\infty} e^{-s\omega_n^2} = e^{-\pi^2 T^2 s} \theta_3(i2\pi^2 T^2 s, i4\pi T^2 s)$$

$$= (4\pi T^2 s)^{-\frac{1}{2}} \theta_3 \left(-\frac{\pi}{2}, \frac{i}{4\pi T^2 s}\right)$$

$$-\langle\psi\psi\rangle = \frac{3}{m} eB 2\pi^{\frac{3}{2}} \frac{1}{(4\pi)^{\frac{1}{2}}} \int_{\Lambda}^{\infty} \frac{ds}{s} \frac{e^{-sm^2}}{\tanh(eBs)} \times \theta_3 \left(-\frac{\pi}{2}, \frac{i}{4\pi T^2 s}\right)$$

CRITICAL CONDITION FOR A THERMOMAGNETIC MEDIUM

Then the gap equation for a thermomagnetic medium is given by

$$m = Gm\frac{3eB}{2\pi^2}\int_{\Lambda}^{\infty}\frac{ds}{s}\frac{e^{-sm^2}}{\tanh(eBs)} \times \theta_3\left(-\frac{\pi}{2}, \frac{i}{4\pi T^2 s}\right)$$

$$m = Gm\frac{3}{2\pi^2}\left\{\int_{\Lambda}^{\infty}\frac{ds}{s^2}e^{-sm^2} + \int_{0}^{\infty}\frac{ds}{s^2}e^{-sm^2}\left(\frac{eBs}{\tanh(eBs)} - 1\right) + 2eB\int_{0}^{\infty}\frac{ds}{s}\frac{e^{-sm^2}}{\tanh(eBs)}\sum_{1}^{\infty}(-1)^n e^{-\frac{n^2}{4T^2 s}}\right\}$$

Vacuum — Magnetic contribution — Magnetic + Thermal contribution

CRITICAL CONDITION FOR A THERMOMAGNETIC MEDIUM

As before we apply the critical condition to the gap equation and setting $B \to 0$ and $T \to 0$ we get the critical condition for each medium

$B \to 0$

$$1 = \frac{3}{2\pi^2} G_c^T \left(1 - \frac{2}{3}\pi^2 T^2\right) \Longrightarrow G_c^T = \frac{2\pi^2}{3} \frac{1}{(1 - \frac{2}{3}\pi^2 T^2)}$$

$T \to 0$

$$1 = \frac{3}{2\pi^2} G_c^M \left[1 + \int_0^\infty \frac{ds}{s^2} \left(\frac{eBs}{\tanh(eBs)} - 1\right)\right]$$

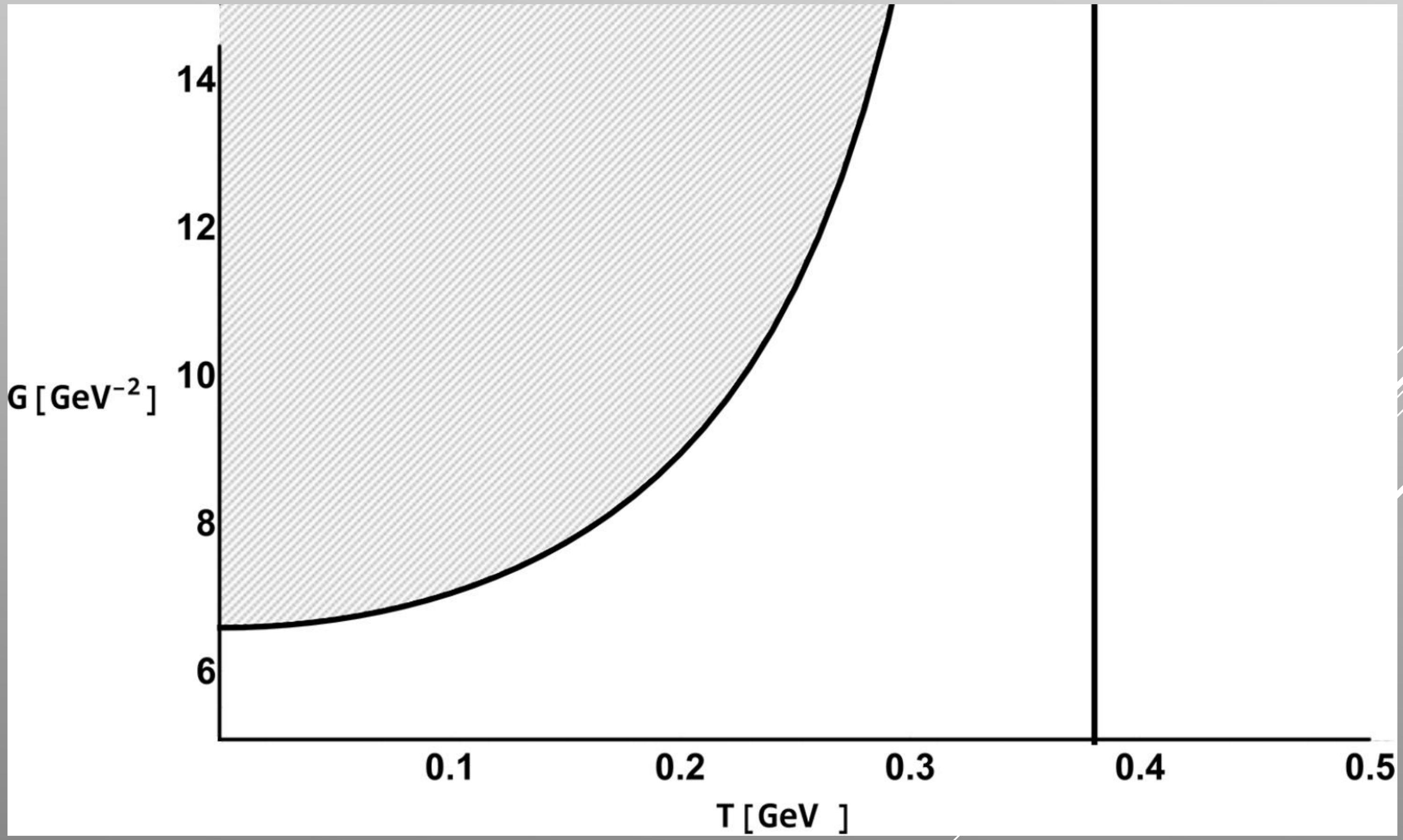
$B, T \neq 0$

$$1 = \frac{3}{2\pi^2} G_c^{TM} \left[1 + \int_0^\infty \frac{ds}{s^2} \left(\frac{eBs}{\tanh(eBs)} - 1\right) + 2eB \sum_{n=1}^{\infty} \int_0^\infty \frac{ds}{s} \frac{(-1)^n \exp(-sn^2)}{\tanh(\frac{eB}{4T^2 s})}\right]$$

CRITICAL CONDITION: THERMAL CASE

After setting $B \to 0$ we got a condition for the critical coupling under a thermal bath.
In the plot, the gray area give us the combination of values for the coupling and the temperature that can generate mass

$$G_c^T = \frac{2\pi^2}{3} \frac{1}{(1 - \frac{2}{3}\pi^2 T^2)}$$

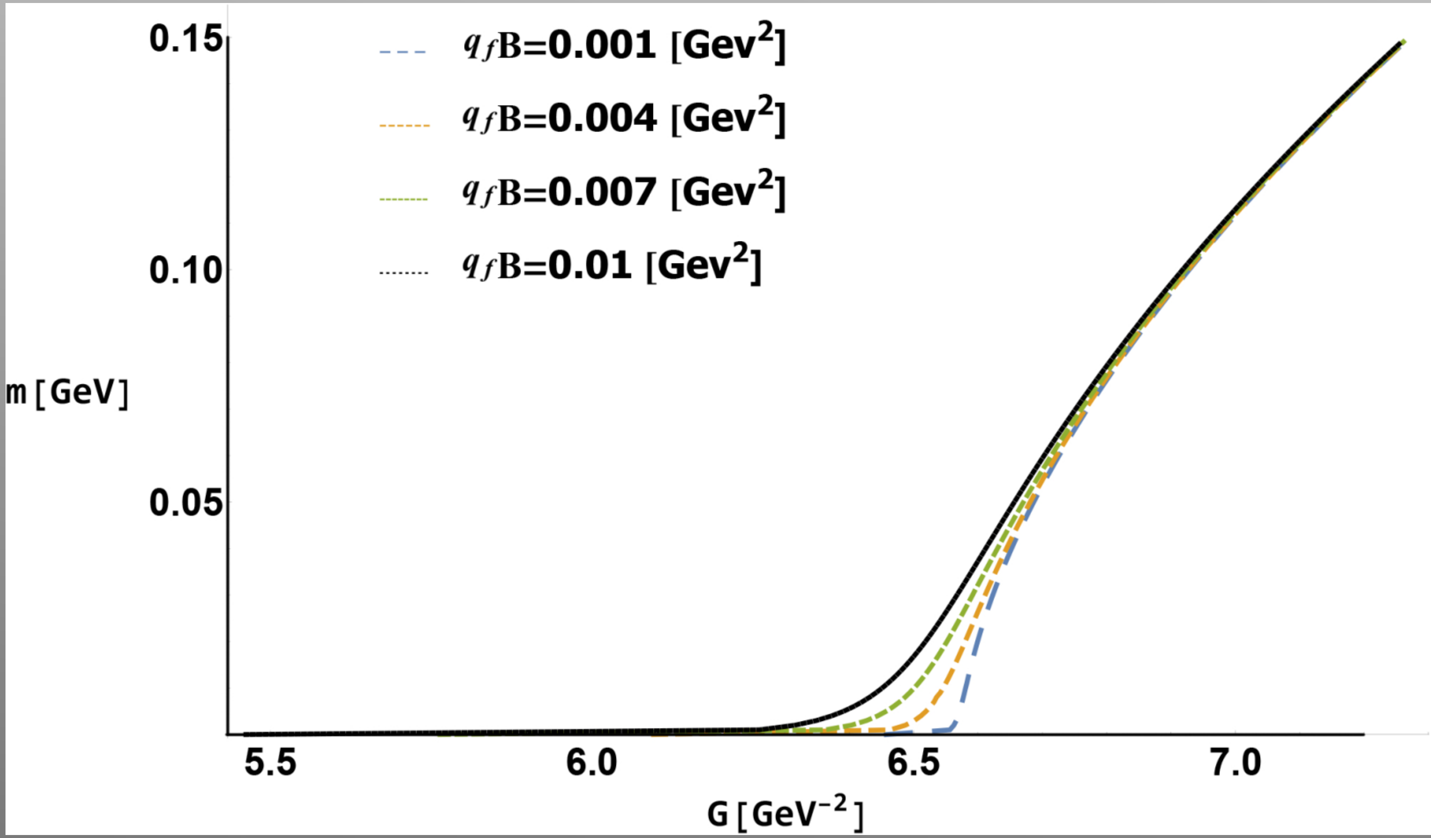


We get a divergence in the critical curve, this give us a critical temperature

$$T = \sqrt{\frac{3}{2\pi^2}}$$

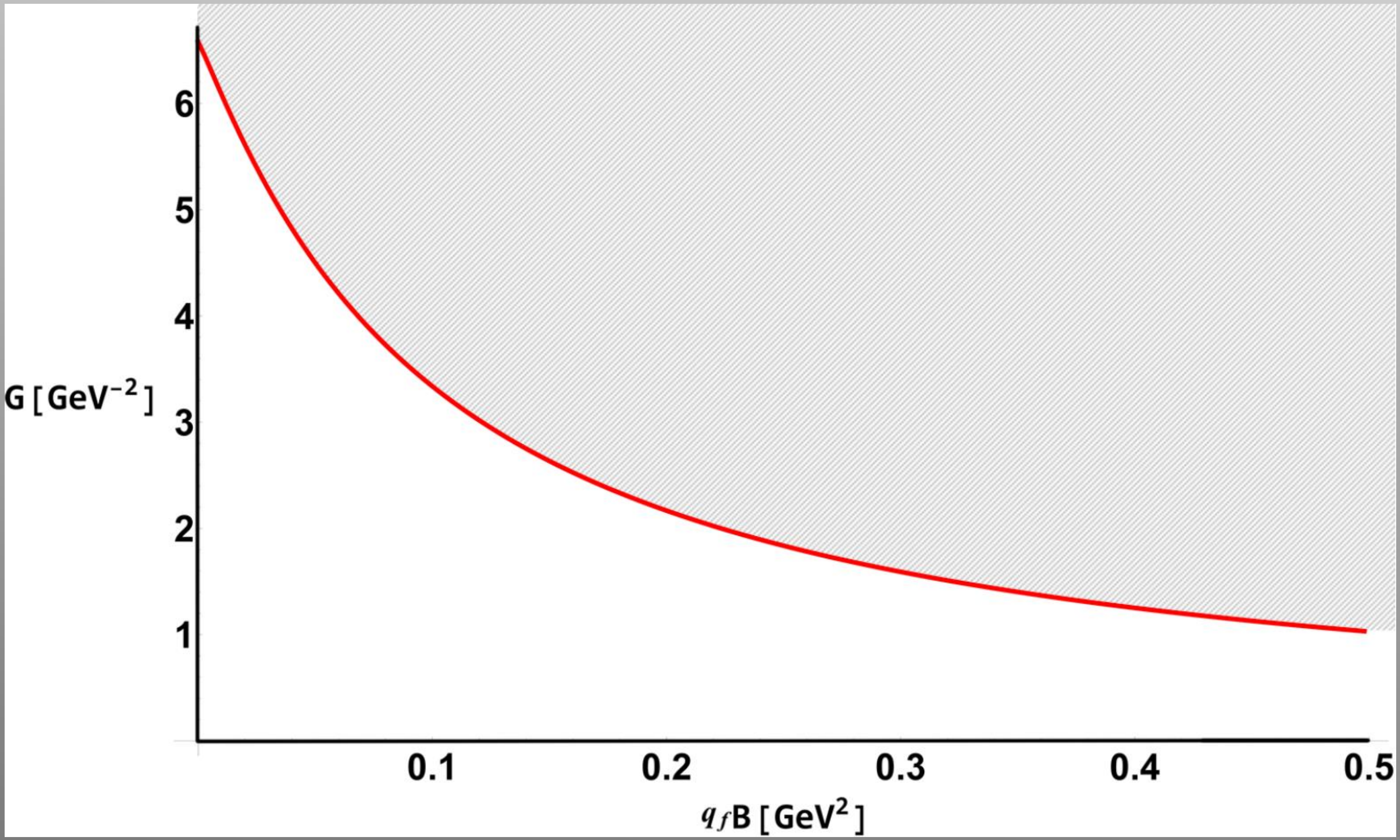
CRITICAL CONDITION: MAGNETIC CASE

In the magnetic case, the magnetic contribution diverge $\int_0^\infty \frac{ds}{s^2}\left(\frac{eBs}{\tanh(eBs)}-1\right)\to\infty$.
This tell us that in the presence of a magnetic field, any coupling will generate mass.



Nevertheless, as we can see, the mass that is generated is really small so we can define a “soft“ critical coupling, in such a way that we want masses bigger than $10^{-4} GeV$.

CRITICAL CONDITION: MAGNETIC CASE

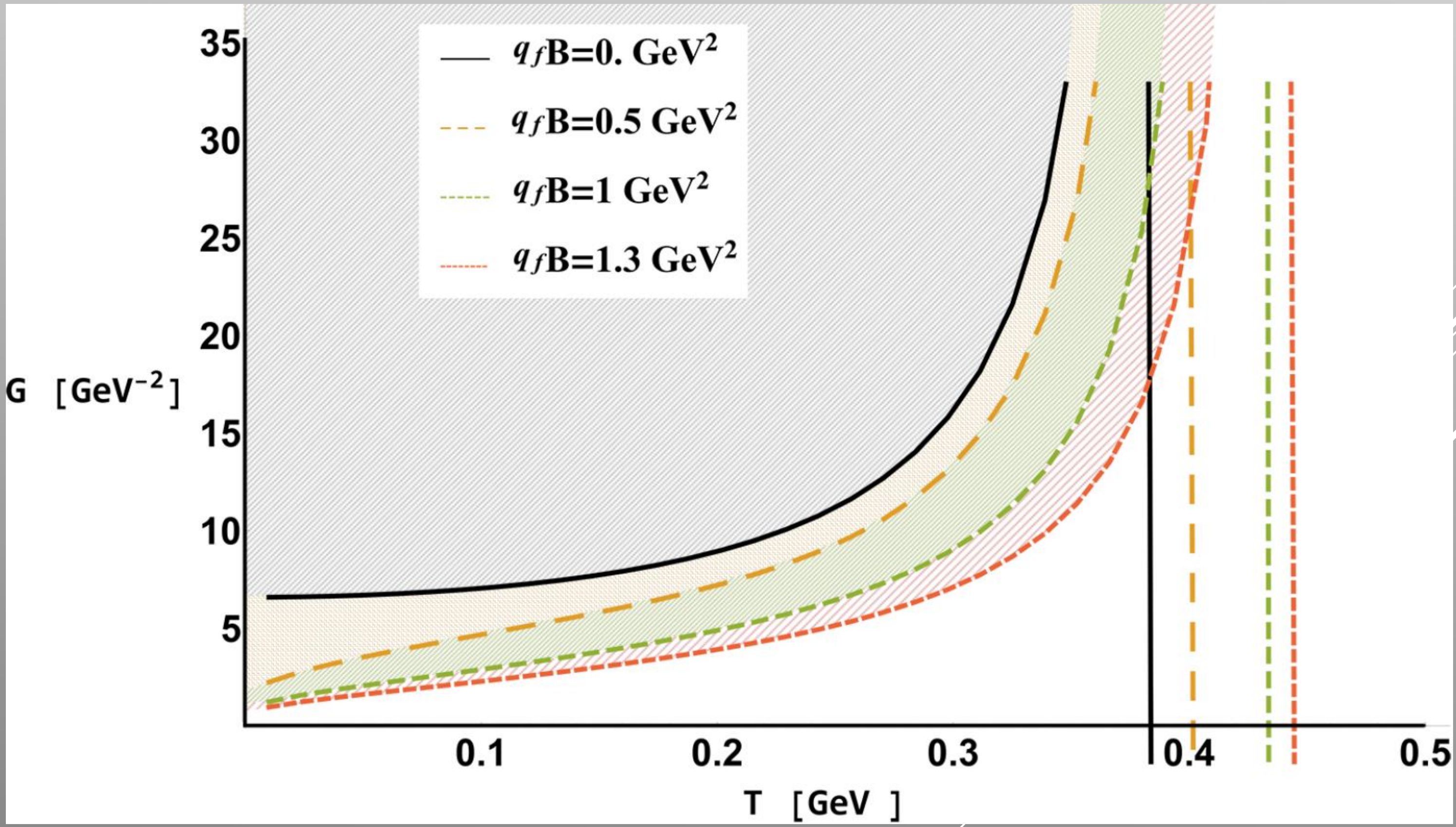


As the magnetic field increases, the coupling needed to generate mass tends to cero.

CRITICAL CONDITION: FULL THERMOMAGNETIC CASE, FIXED MAGNETIC FIELD

From the plot, we can see that the critical temperature is still present in the curve, nevertheless the magnetic field delays the appearance of the critical temperature.
In the other hand, for low temperatures, the coupling needed to generate mass is really small.

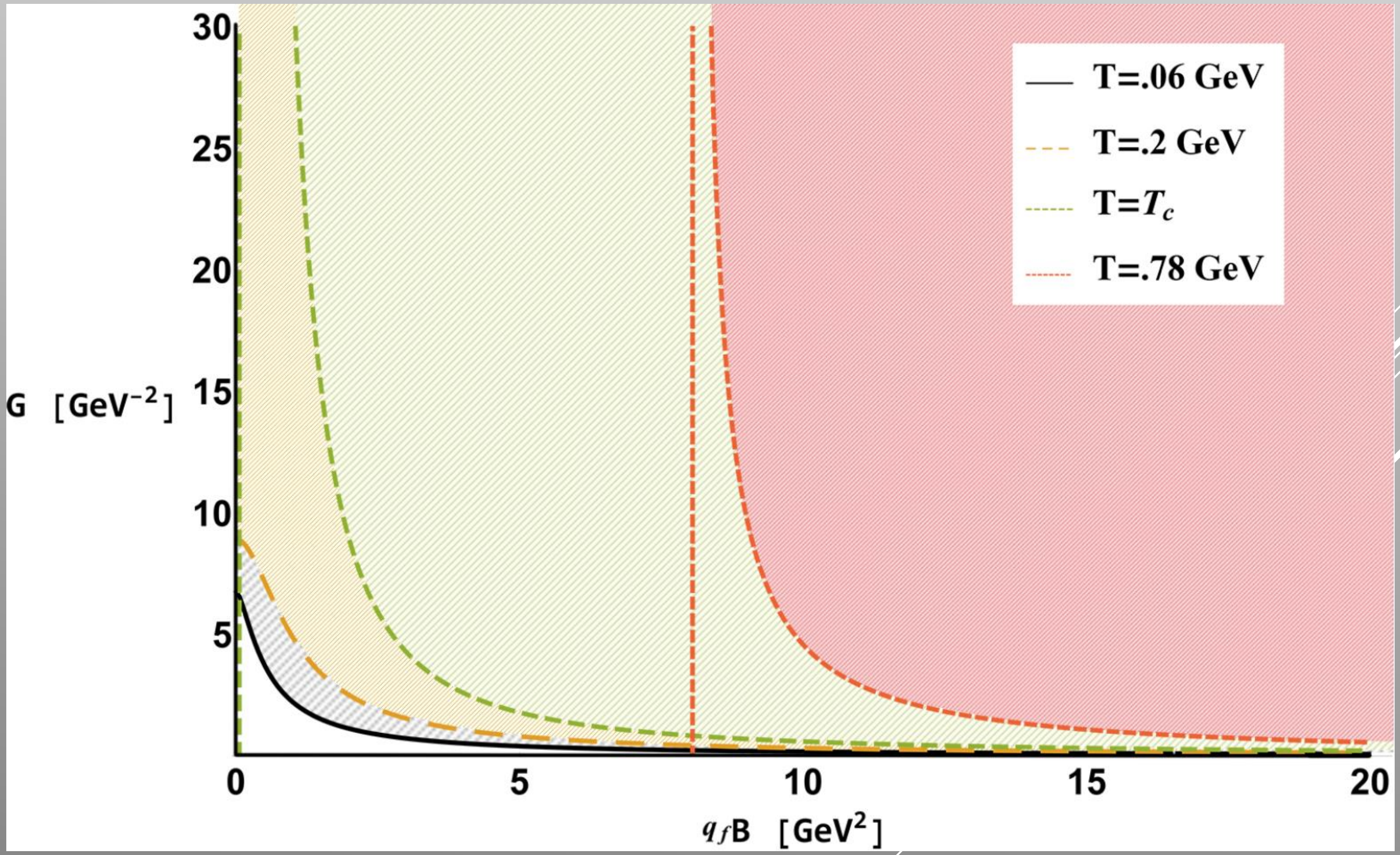
$$1 = \frac{3}{2\pi^2} G_c^{TM} \left[1 + \int_0^\infty \frac{ds}{s^2} \left(\frac{eBs}{\tanh(eBs)} - 1 \right) + 2eB \sum_{n=1}^{\infty} \int_0^{\infty} \frac{ds}{s} \frac{(-1)^n \exp(-sn^2)}{\tanh(\frac{eB}{4T^2 s})} \right]$$



CRITICAL CONDITION: FULL THERMOMAGNETIC CASE, FIXED TEMPERATURE

$$1 = \frac{3}{2\pi^2} G_c^{TM} \left[1 + \int_0^\infty \frac{ds}{s^2} \left(\frac{eBs}{\tanh(eBs)} - 1 \right) + 2eB \sum_{n=1}^{\infty} \int_0^{\infty} \frac{ds}{s} \frac{(-1)^n \exp(-sn^2)}{\tanh(\frac{eB}{4T^2 s})} \right]$$

For $T > T_c$ we find a critical magnetic field too, but this time is in such a way that for fields before it, no matter how strong the coupling constant is, there is not dynamical mass.
On the other hand, for $T < T_c$ we can find a coupling strong enough to generate mass.



CRITICAL CONDITION: MODELS FOR THE COUPLING

We can use the critical condition to obtain the critical curves for various models of the coupling constant, some of them, describe the inverse magnetic catalysis phenomenon. In particular we consider the following models:

- Mean field, that is, the coupling constant is independent of the magnetic field.

$$G^{TM} \equiv G^0$$

- The running coupling of QCD in a background magnetic field. (1)

$$G^{TM} = \frac{G^0}{\ln\left(e + \frac{|q_f B|}{\Lambda_{QCD}^2}\right)}$$

- A Padé fit. (2,3,4)

$$G^{TM}(q_f B) = G^0 \left(\frac{1 + a\zeta^2 + b\zeta^3}{1 + c\zeta^2 + d\zeta^4}\right)$$

- A non trivial fit. (5)

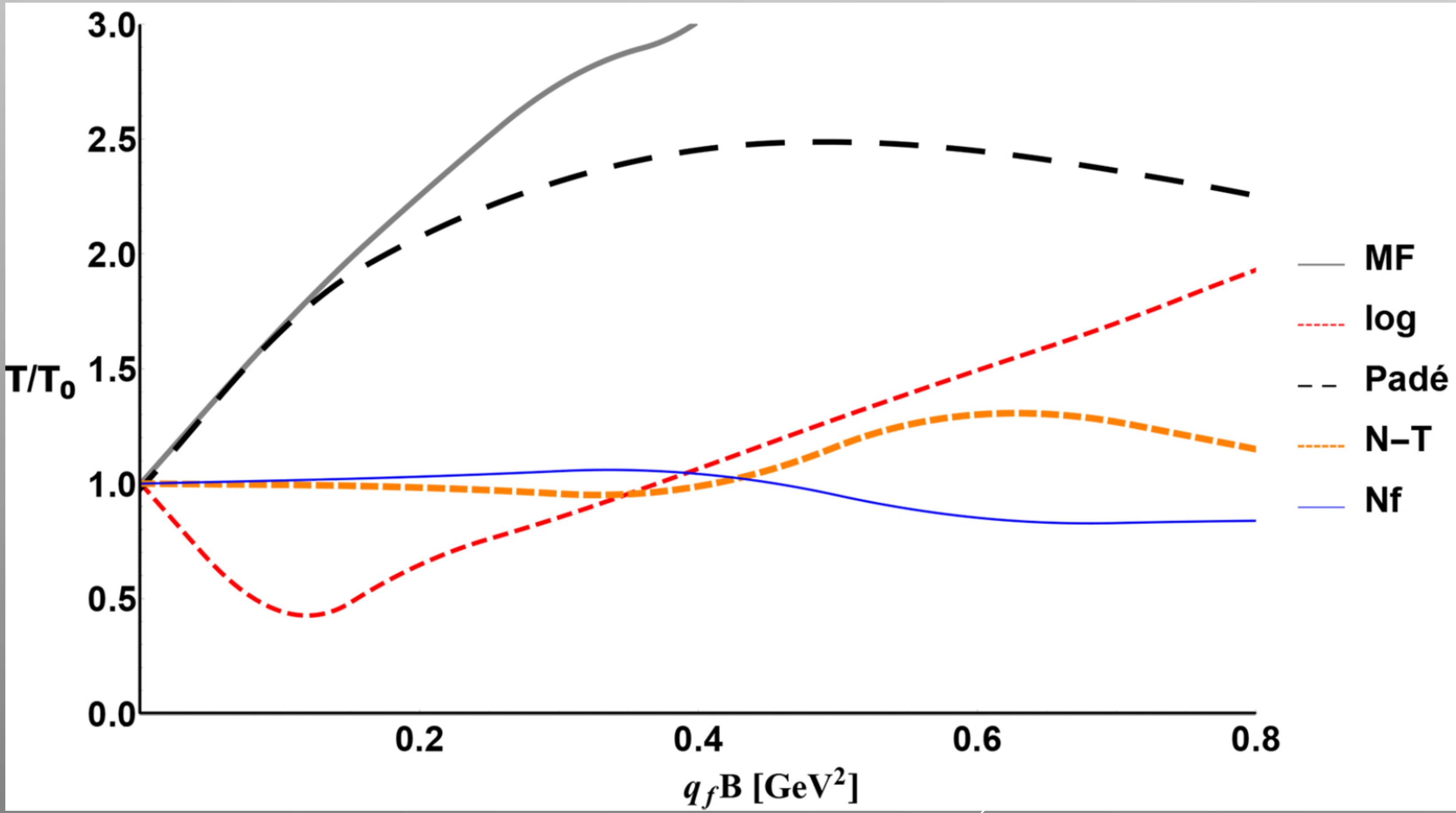
$$G^{TM} = c(B)\left[1 - \frac{1}{1 + \exp\left[\beta(B)(T_a(B) - T)\right]}\right] + s(B)$$

- Finally a numerical fit. (6)

$$G_N^{TM}$$

CRITICAL CONDITION: MODELS FOR THE COUPLING

As we can see, for mean field and running coupling of QCD, the growth of the temperature is monotone in accordance with magnetic catalysis, but for the other three models we notice a diminish and then a growth of the temperature, signals of inverse magnetic catalysis.



CONCLUSIONS

- With this critical condition we were able to obtain the critical surface for the dynamical mass generation phenomenon in the NJL model for the non-medium, magnetic, thermal and thermomagnetic cases.

- Furthermore, using the same critical condition, we test various models for the coupling constant, some of them describe the inverse magnetic phenomenon and derive the critical surface for each one.

Thank you

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EXTRA SLIDE: CRITICAL CONDITION