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What is the right formalism to search for resonances?

César Fernández-Ramírez

Instituto de Ciencias Nucleares
Universidad Nacional Autónoma de México

and *Joint Physics Analysis Center*

References

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Special Article - Tools for Experiment and Theory

Mikhasenko et al.,
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arXiv:1712.02815 [hep-ph]

What is the right formalism to search for resonances?

Joint Physics Analysis Center

M. Mikhasenko[1,a], A. Pilloni[2,b], J. Nys[2,3,4,5], M. Albaladejo[6], C. Fernández-Ramírez[7], A. Jackura[4,5], V. Mathieu[2], N. Sherrill[4,5], T. Skwarnicki[8], A. P. Szczepaniak[2,4,5]

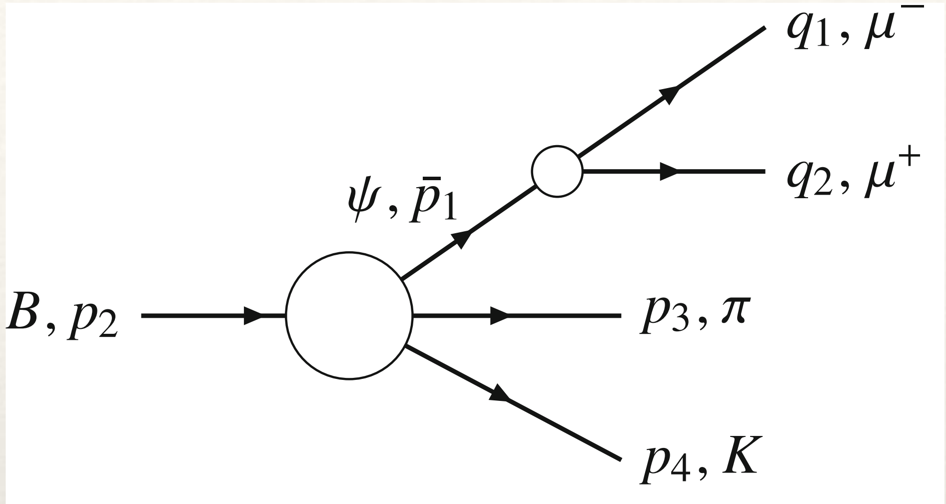
JLAB-THY-18-2700

Pilloni et al.
arXiv:1805.02113 [hep-ph]

What is the right formalism to search for resonances?
II. The pentaquark chain

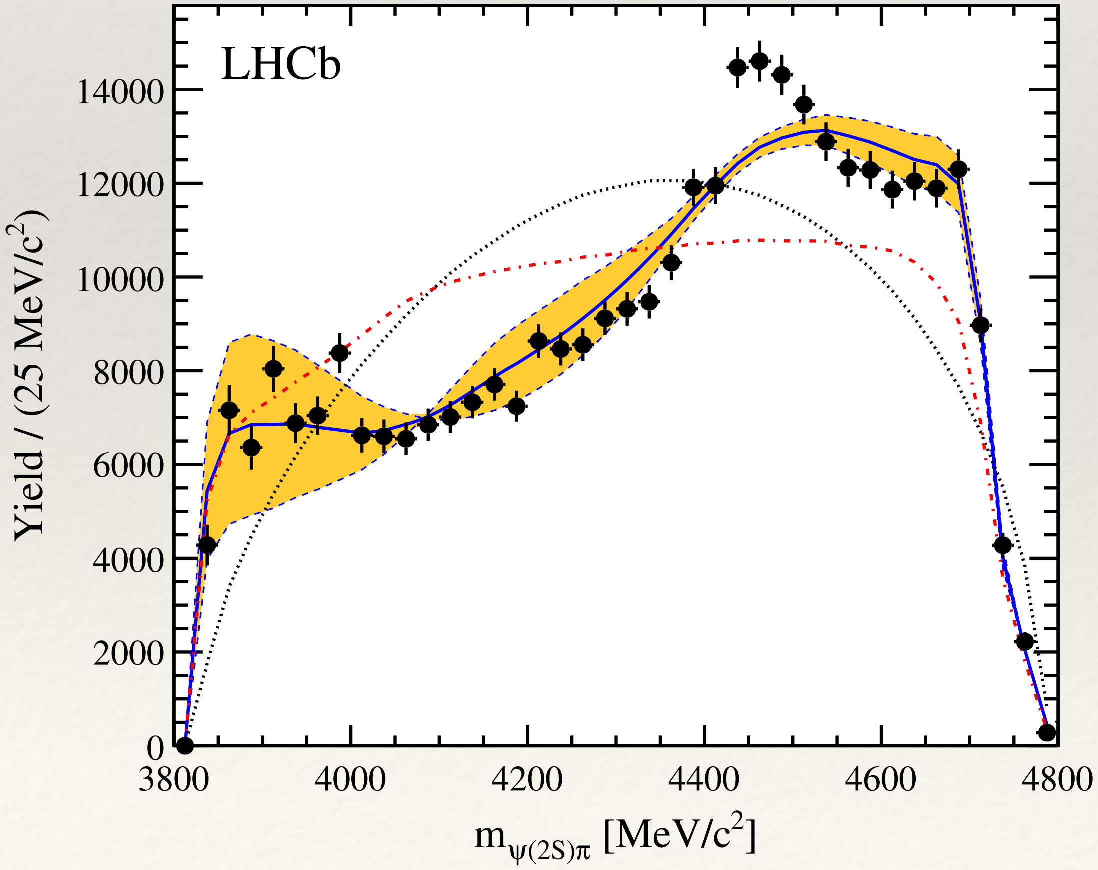
A. Pilloni,[1,*] J. Nys,[1,2,3,4,†] M. Mikhasenko,[5,‡] M. Albaladejo,[6] C. Fernández-Ramírez,[7] A. Jackura,[3,4] V. Mathieu,[1] N. Sherrill,[3,4] T. Skwarnicki,[8] and A. P. Szczepaniak[1,3,4]
(Joint Physics Analysis Center)

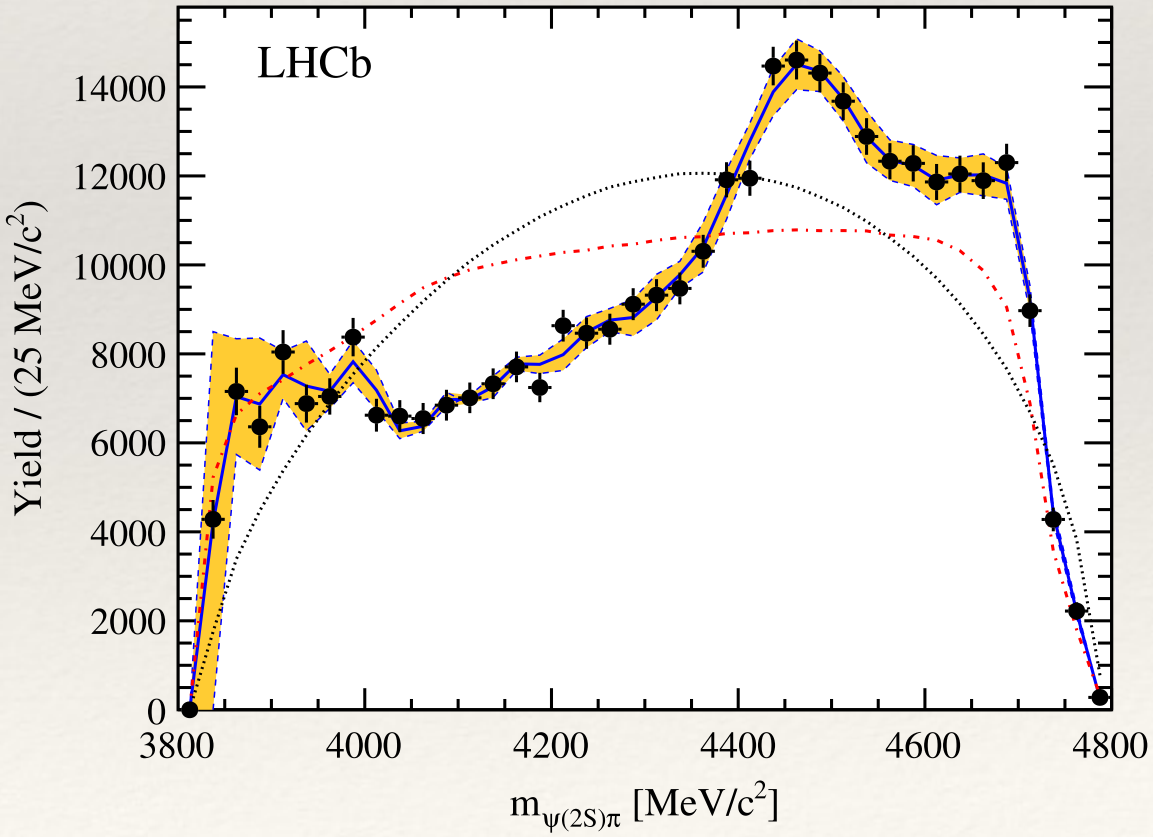
Three body decays



Z(4430)⁻

LHCb, PRD 92, 112009 (2015)





S-matrix principles

1. Something must happen
2. We can exchange particles and antiparticles
3. Causes precede effects

$$\Downarrow$$

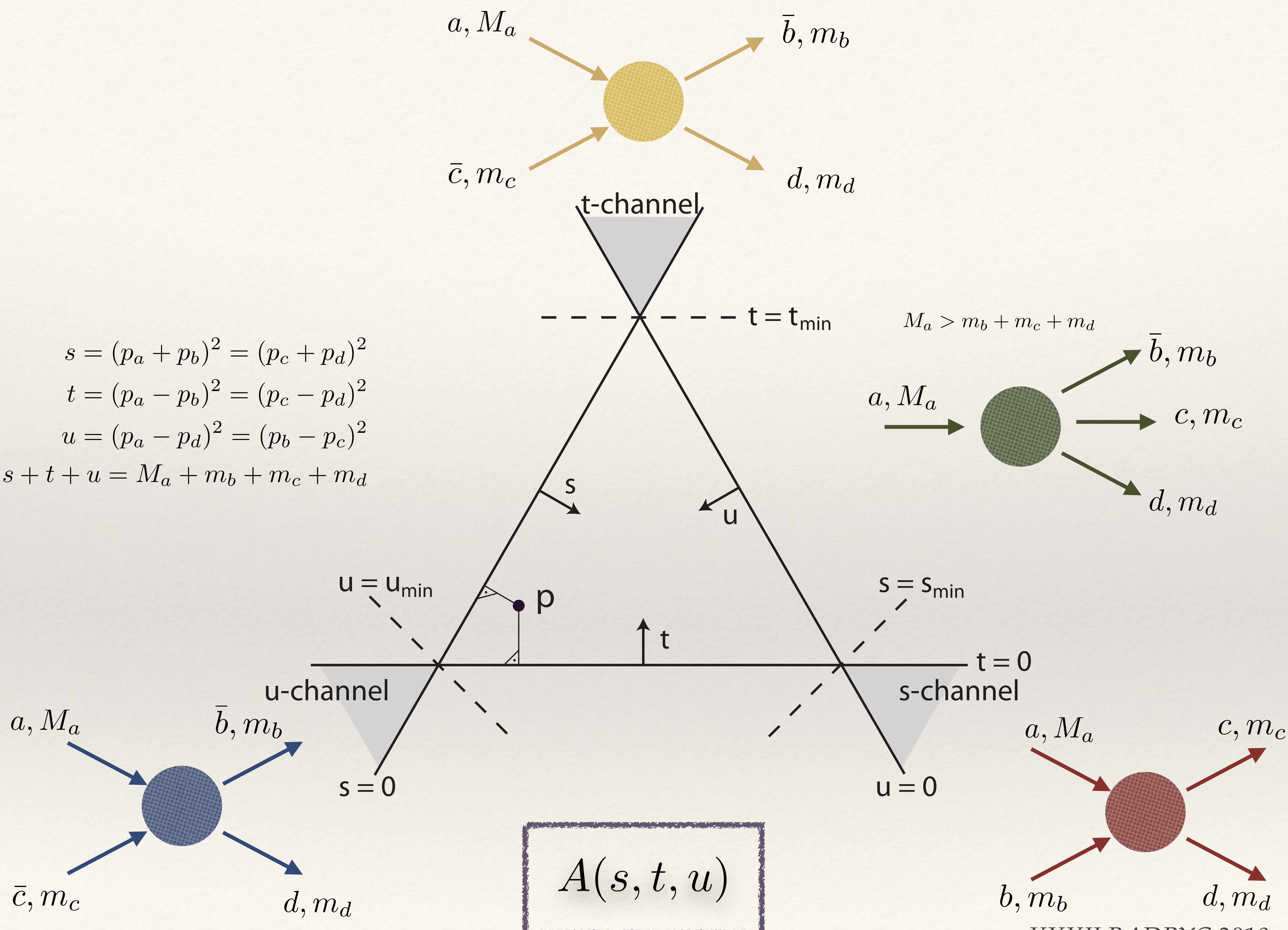
1. Unitarity
2. Crossing symmetry
3. Causality $\Rightarrow$ Analyticity

Singularities

- We want to study scattering
- We need to build amplitudes according to S-matrix theory

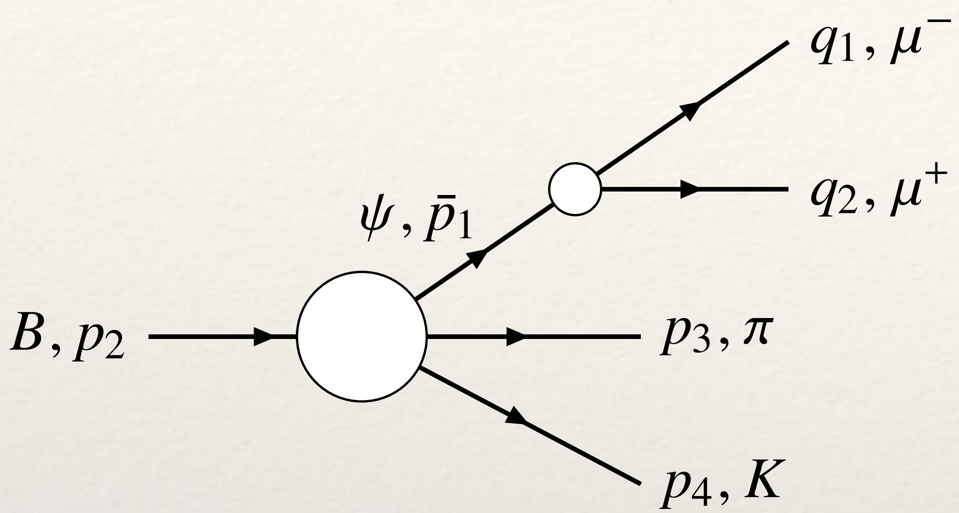
$$S = I + 2i\,A$$

- That means to understand the singularities of the amplitude
  - **Kinematical ⇒ From external momenta and spins**
  - Dynamical ⇒ The physics we are after: resonances, QCD, BSM, etc.

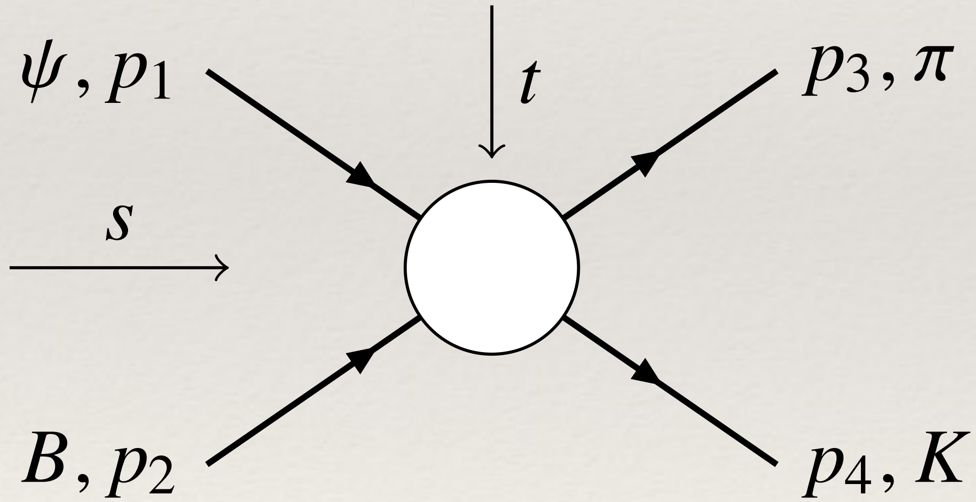


$$s = (p_a + p_b)^2 = (p_c + p_d)^2$$
$$t = (p_a - p_b)^2 = (p_c - p_d)^2$$
$$u = (p_a - p_d)^2 = (p_b - p_c)^2$$
$$s + t + u = M_a + m_b + m_c + m_d$$

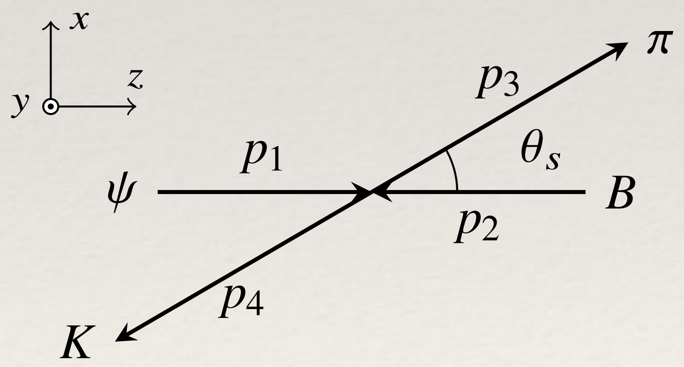
$B^0 \rightarrow \psi\pi^- K^+$ amplitude



- B^0 decays weakly ⇒ PC and **PV** amplitudes
- We can use crossing symmetry to treat the decay channel
- The *s* channel is K* dominated
- Once we have the s channel, the *t* channel can be built similarly (ψπ resonances)



$$
\begin{aligned}
s &= (p_3 + p_4)^2, \\
t &= (\bar{p}_1 + p_3)^2 \\
u &= (\bar{p}_1 + p_4)^2 \\
s + t + u &= \sum_i m_i^2
\end{aligned}
$$



$$\langle \psi\pi K, \text{out} | B, \text{in} \rangle = (2\pi)^4 \delta^4 (p_2 - \bar{p}_1 - p_3 - p_4) \mathcal{A}_\lambda$$

Non-PW expanded amplitude

$$A_\lambda(s,t) = \epsilon_\mu(\lambda, p_1)\left[(p_3 - p_4)^\mu - \frac{m_3^2 - m_4^2}{s}(p_3 + p_4)^\mu\right] C(s,t) + \epsilon_\mu(\lambda, p_1)(p_3 + p_4)^\mu B(s,t)$$

This is a choice for the tensors, there are others and provide the same results

***C(s,t)* and *B(s,t)* are scalar functions that are kinematical singularity free**

Fine, but if we are going to search for resonances we are going to need this **PW expanded,** and that is where the **headache starts**

PW expanded amplitude

To incorporate resonances in the πK system with certain spin j, we expand the amplitude in partial waves

$$\mathcal{A}_\lambda(s,t,u) = \frac{1}{4\pi}\sum_{j=|\lambda|}^{\infty}(2j+1)\,A_\lambda^j(s)\,d^j_{\lambda 0}(z_s)$$

The analysis of kinematical singularities has general validity, and may be applied to the original untruncated series

Kinematical singularities

$d^j_{\lambda 0}(z_s) = \hat{d}^j_{\lambda 0}(z_s)\xi_{\lambda 0}(z_s)$, where $\xi_{\lambda 0}(z_s) = \left(\sqrt{1-z_s^2}\right)^{|\lambda|} = \sin^{|\lambda|}\theta_s$ is the so-called half angle factor that contains all the kinematical singularities in t.

$\hat{d}^j_{\lambda 0}(z_s)$ is a polynomial in s and t of order $j - |\lambda|$ divided by the factor $\lambda_{12}^{(j-|\lambda|)/2}\lambda_{34}^{(j-|\lambda|)/2}$

The helicity partial waves $A^j_\lambda(s)$ have singularities in s. These have both dynamical and kinematical origin

First, the term $(pq)^{j-|\lambda|}$ is factorized out from the helicity amplitude $A^j_\lambda(s)$. This factor is there to cancel the threshold and pseudothreshold singularities in s that appear in $\hat{d}^j_{\lambda 0}(z_s)$

We introduce the kinematic factor $K_{\lambda 0}$ ('$\pm$' is short for $\lambda = \pm 1$), required to account for a mismatch between the j and L dependence in the angular momentum barrier factors in presence of particles with spin.

Kinematically singularity-free helicity partial waves

$$A_0^j(s) = K_{00}\ (pq)^j\ \hat{A}_0^j(s) \qquad \text{for } j \geq 1,$$

$$A_\pm^j(s) = K_{\pm 0}\ (pq)^{j-1}\ \hat{A}_\pm^j(s) \qquad \text{for } j \geq 1,$$

$$A_0^0(s) = \frac{1}{K_{00}} \hat{A}_0^0(s) \qquad \text{for } j = 0,$$

with K_{00} and $K_{\pm 0}$ given by

$$K_{00} = \frac{m_1}{p\sqrt{s}} = \frac{2m_1}{\lambda_{12}^{1/2}},$$

$$K_{\pm 0} = q = \frac{\lambda_{34}^{1/2}}{2\sqrt{s}}.$$

$A_\lambda^j(s) \sim p^{L_1} q^{L_2}$ at threshold, where L_1 and L_2 are the lowest possible orbital angular momenta in the given helicity and parity combination

The K-factors have powers of $\sqrt{s}$ as required to ensure factorization of the vertices of Regge poles.

We match the PW and the non-PW expanded amplitudes

$$-C(s,t)\frac{n(s,t)(s+m_1^2-m_2^2)}{4m_1^2 s}+B(s,t)\frac{\lambda_{12}}{4m_1^2}=\frac{A_0(s)}{K_{00}\,\xi_{00}(z_s)}=\frac{1}{4\pi}\left(\sum_{j>0}(2j+1)(pq)^j\hat{A}_0^j(s)\hat{d}_{00}^j(z_s)+\frac{\lambda_{12}}{4m_1^2}\hat{A}_0^0(s)\right)$$

$$\pm\sqrt{2}C(s,t)=\frac{A_\pm(s)}{K_{\pm0}\,\xi_{10}(z_s)}=\pm\frac{1}{4\pi}\sum_{j>0}(2j+1)(pq)^{j-1}\hat{A}_\pm^j(s)\,\hat{d}_{10}^j(z_s)$$

Combining them we obtain

$$4\pi B(s,t)=\hat{A}_0^0(s)+\frac{4m_1^2}{\lambda_{12}}\sum_{j>0}(2j+1)(pq)^j\left[\hat{A}_0^j(s)\hat{d}_{00}^j(z_s)+\frac{s+m_1^2-m_2^2}{\sqrt{2}m_1^2}\hat{A}_+^j(s)\,z_s\hat{d}_{10}^j(z_s)\right]$$

Two poles at $s_\pm=(m_1\pm m_2)^2$

Unless this guy cancel them out

The fact that ***C(s,t)*** and ***B(s,t)*** cannot have kinematical singularities imposes **constrains** in the PW expanded amplitudes

$s_\pm$ poles

Consequence: the $\hat{A}^j{}_\lambda(s)$ for different λ cannot be independent at pseudo(threshold)

In the $s \to s_\pm$ limit at fixed t, $z_s \to \infty$ so

$$\hat{d}^j_{\lambda 0}(z_s) \xrightarrow{z_s \to \infty} (-1)^{\frac{\lambda+|\lambda|}{2}} \frac{(2J)!\,[J(2J-1)]^{1/2}}{2^J J\,[(1+\lambda)!(1-\lambda)!]^{1/2}} \frac{z_s^{J-|\lambda|}}{\langle j-1,0;1,\lambda|j,\lambda\rangle} \qquad \text{for } |\lambda| \leq 1$$

$$\hat{A}^j_0(s) \frac{(z_s)^j}{\langle j-1,0;1,0|j,0\rangle} - \frac{s+m_1^2-m_2^2}{\sqrt{2}\,m_1^2} \hat{A}^j_+(s) \frac{(z_s)^j}{\sqrt{2}\,\langle j-1,0;1,1|j,1\rangle}$$

And the bracket in previous slide has to vanish, so

$$\hat{A}^j_+(s) = \langle j-1,0;1,1|j,1\rangle\, g_j(s) + \lambda_{12}\, f_j(s)$$

$$\hat{A}^j_0(s) = \langle j-1,0;1,0|j,0\rangle \frac{s+m_1^2-m_2^2}{2m_1^2} g'_j(s) + \lambda_{12}\, f'_j(s)$$

where $g_j(s)$, $f_j(s)$, $g'_j(s)$, and $f'_j(s)$ are regular functions at $s=s_\pm$, and $g_j(s_\pm)=g'_j(s_\pm)$

Final PW amplitudes

$$A_+^j(s) = p^{j-1}q^j \left[\langle j-1,0;1,1|j,1\rangle \, g_j(s) + \lambda_{12}\, f_j(s) \right]$$

$$A_0^j(s) = p^{j-1}q^j \left[\langle j-1,0;1,0|j,0\rangle \, \frac{s+m_1^2-m_2^2}{2m_1\sqrt{s}} \, g_j'(s) + \frac{m_1}{\sqrt{s}}\lambda_{12}\, f_j'(s) \right]$$

and

$$A_0^0(s) = \lambda_{12}^{1/2}/(2m_1)\, \hat{A}_0^0(s)$$

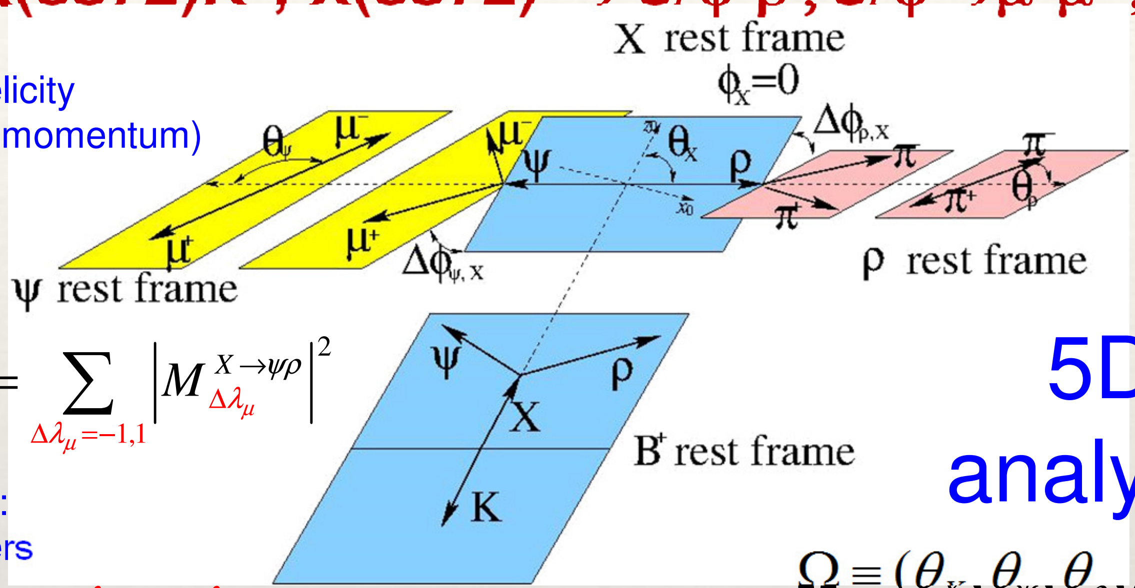
Comparison to LS and CPM

LS PWA

Helicity amplitudes for

$B^+ \to X(3872)K^+$, $X(3872) \to J/\psi\,\rho$, $J/\psi \to \mu^+\mu^-$, $\rho \to \pi^+\pi^-$

λ – particle helicity (spin projection onto its momentum)



$$A \to BC$$

$$D^{J}_{\lambda_A,\lambda_B-\lambda_C}(\phi,\theta,0)^* = e^{i\phi} d^{J}_{\lambda_A,\lambda_B-\lambda_C}(\theta)$$

$$\left|M(\Omega \mid J_X^{P_X}, H^{J_X}_{\lambda_\psi,\lambda_\rho})\right|^2 = \sum_{\Delta\lambda_\mu=-1,1} \left|M^{X\to\psi\rho}_{\Delta\lambda_\mu}\right|^2$$

Helicity couplings: nuisance parameters

5D analysis

$$\Omega \equiv (\theta_X, \theta_\psi, \theta_\rho, \Delta\phi_{\psi,X}, \Delta\phi_{\rho,X})$$

$$\Delta\lambda_\mu = \lambda_{\mu^+} - \lambda_{\mu^-}$$

$$M^{X\to\psi\rho}_{\Delta\lambda_\mu} = \sum_{\lambda_\psi=-1,0,1} \sum_{\lambda_\rho=-1,0,1} H^{J_X}_{\lambda_\psi,\lambda_\rho} D^{J_X}_{0,\lambda_\psi-\lambda_\rho}(0,\theta_X,0)^* D^{1}_{\lambda_\psi,\Delta\lambda_\mu}(\Delta\phi_{\psi,X},\theta_\psi,0)^* D^{1}_{\lambda_\rho,0}(\Delta\phi_{\rho,X},\theta_\rho,0)^*$$

$$H^{J_X}_{\lambda_\psi,\lambda_\rho} = \sum_L \sum_S B^{J_X}_{L,S} \sqrt{\frac{2L+1}{2J_X+1}} \left(\begin{array}{cc|c} J_\psi & J_\rho & S \\ \lambda_\psi & -\lambda_\rho & \lambda_\psi-\lambda_\rho \end{array} \right) \left(\begin{array}{cc|c} L & S & J_X \\ 0 & \lambda_\psi-\lambda_\rho & \lambda_\psi-\lambda_\rho \end{array} \right)$$

Clebsch-Gordan coefficients

$$|J_\psi - J_\rho| \le S \le J_\psi + J_\rho$$
$$|J_X - S| \le L \le J_X + S$$
$$P_X = P_\psi P_\rho (-1)^L = (-1)^L$$

(P-conservation since strong decay)

Number of B_{LS} coupling equals number of independent $H_{\lambda\psi,\lambda\rho}$ couplings (1-5 depending on J_X) – neglecting high L values can reduce number of couplings to fit to the data

Stolen from Tomasz Skwarnicki (LHCb)

LS for $B^0 \to \psi\pi^- K^+$ amplitude

$$|j\Lambda; LS> = \sqrt{\frac{2L+1}{2j+1}} \sum_{\lambda_1\lambda_2} < L, 0; S, \lambda_1 - \lambda_2 | j\Lambda >< j_1, \lambda_1; j_2, -\lambda_2 | S, \lambda_1 - \lambda_2 > |j\Lambda; \lambda_1\lambda_2 >$$

$$G^j_L(s) = \sqrt{\frac{2L+1}{2j+1}} \sum_{\lambda} < L, 0; 1, \lambda | j\lambda > A^j_\lambda(s)$$

We invert it

$$A^j_\lambda(s) = \underline{p^{j-1}q^j} \left(\sqrt{\frac{2j-1}{2j+1}} \langle j-1, 0; 1, \lambda | j, \lambda \rangle \hat{G}^j_{j-1}(s) + \sqrt{\frac{2j+3}{2j+1}} \langle j+1, 0; 1, \lambda | j, \lambda \rangle p^2 \hat{G}^j_{j+1}(s) \right)$$

Note: relativistic but not covariant

Matching

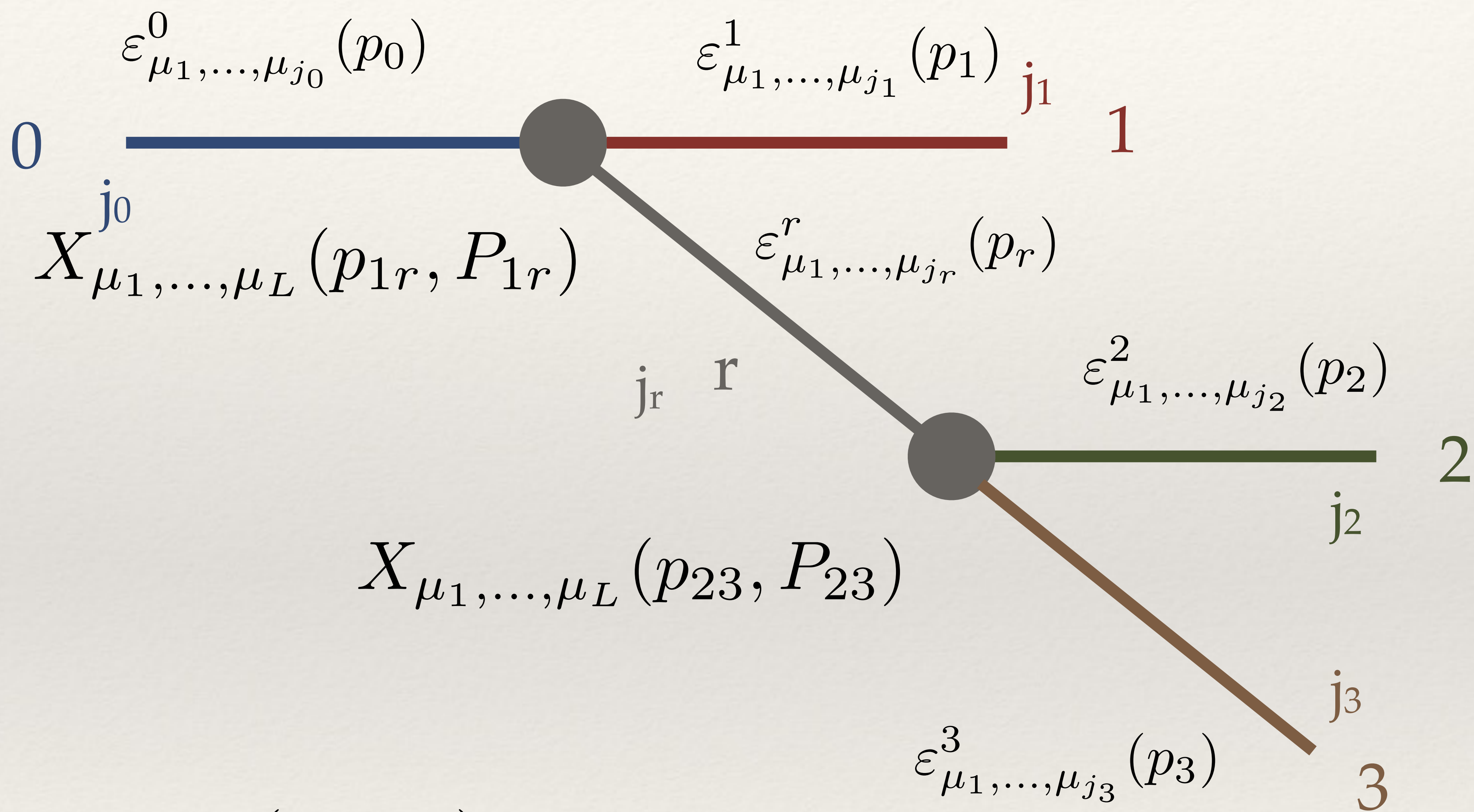
$$g_j(s) = \sqrt{\frac{2j-1}{2j+1}} \hat{G}^j_{j-1}(s)$$

$$f_j(s) = \frac{1}{4s}\sqrt{\frac{2j+3}{2j+1}} \langle j+1, 0; 1, 1 | j, 1\rangle \, \hat{G}^j_{j+1}(s)$$

$$g'_j(s) = \frac{2m_1\sqrt{s}}{s + m_1^2 - m_2^2}\sqrt{\frac{2j-1}{2j+1}} \hat{G}^j_{j-1}(s)$$

$$f'_j(s) = \frac{1}{4m_1\sqrt{s}}\sqrt{\frac{2j+3}{2j+1}} \langle j+1, 0; 1, 0 | j, 0\rangle \, \hat{G}^j_{j+1}(s)$$

Covariant Projection Method



$0 \to 1\, r\, (\to 2\, 3)$

Can be used both for scattering and decay

Scattering *vs* decay for CPM

CPM explicitly violates crossing symmetry

$$B \to \bar{D}\pi\pi \qquad\qquad DB \to \pi\pi$$

in P wave

$$\mathcal{A}_{[BD\to\pi\pi]} = pq\cos\theta_s\, g_P(s), \quad \mathcal{A}_{[B\to\bar{D}\pi\pi]} = \gamma(s)\frac{\sqrt{s}}{m_2}pq\cos\theta_s\, g_P(s)$$

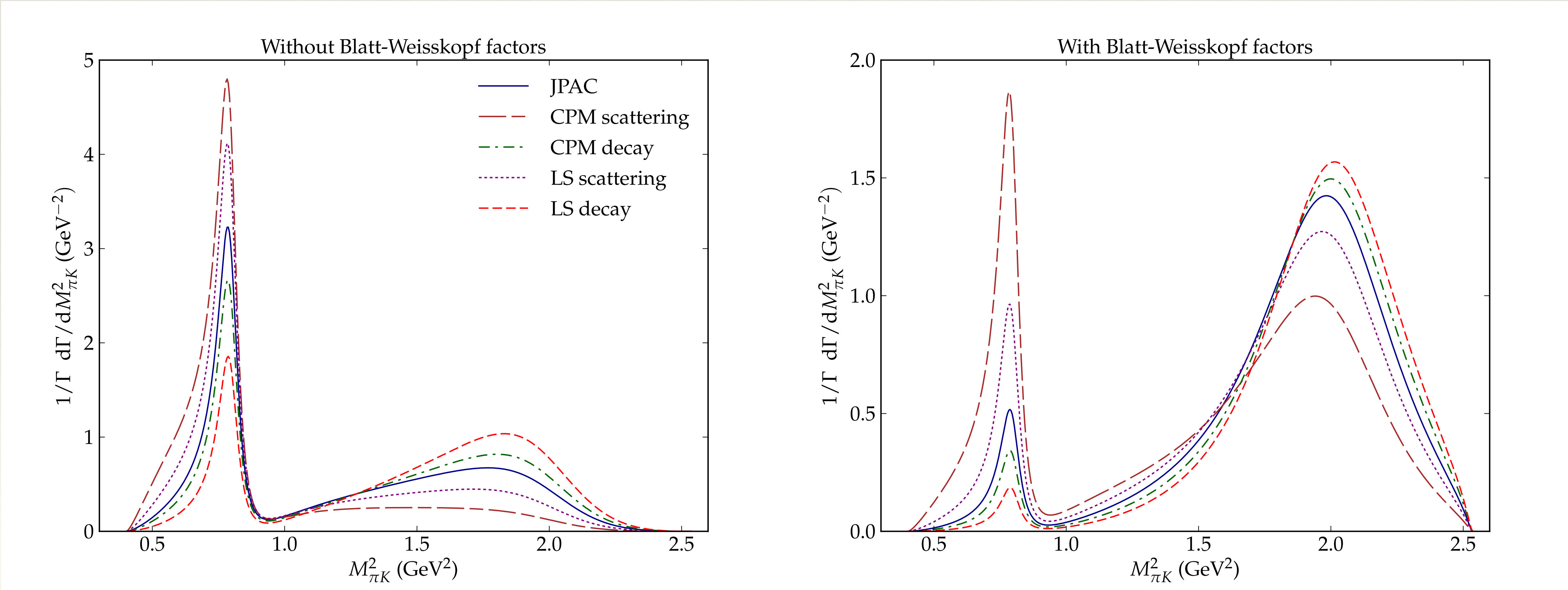
$$\gamma(s)\sqrt{s}/m_2 = (s - m_1^2 + m_2^2)/(2m_2^2)$$

Simple model

$$g_S(s) = \hat{G}_0^1(s) = F_0^1(s) = 0 \text{ and } g_D(s) = \hat{G}_2^1(s) = F_2^1(s) = T_{K^*}(s) B_1(q) B_2(p)$$

$$T_{K^*}(s) \equiv \frac{0.1}{M^2_{K^*(892)} - s - i M_{K^*(892)} \Gamma_{K^*(892)}} + \frac{1}{M^2_{K^*(1410)} - s - i M_{K^*(1410)} \Gamma_{K^*(1410)}}$$

$$B_1(q) = \sqrt{\frac{1}{1 + q^2 R^2}}; \quad B_2(q) = \sqrt{\frac{1}{9 + 3q^2R^2 + q^4R^4}} \quad \frac{d\Gamma}{ds} = \sum_j N_j \left(\left| A_0^j(s) \right|^2 + 2 \left| A_+^j(s) \right|^2 \right) \rho(s)$$



Conclusions

- Kinematical singularities matter **A LOT**
- **Kinematical and dynamical singularities are entangled**
- **Careful** with the formalism, it introduces model dependencies
- **Careful** when you write your hadron model (BW?), you might be careful with the singularities
- Compare apples to apples
- Doing it properly is a nightmare
- **Growing spins ⇒ Growing pains**