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Transverse momentum dependent parton evolution – a possibility for precision physics at the LHC?

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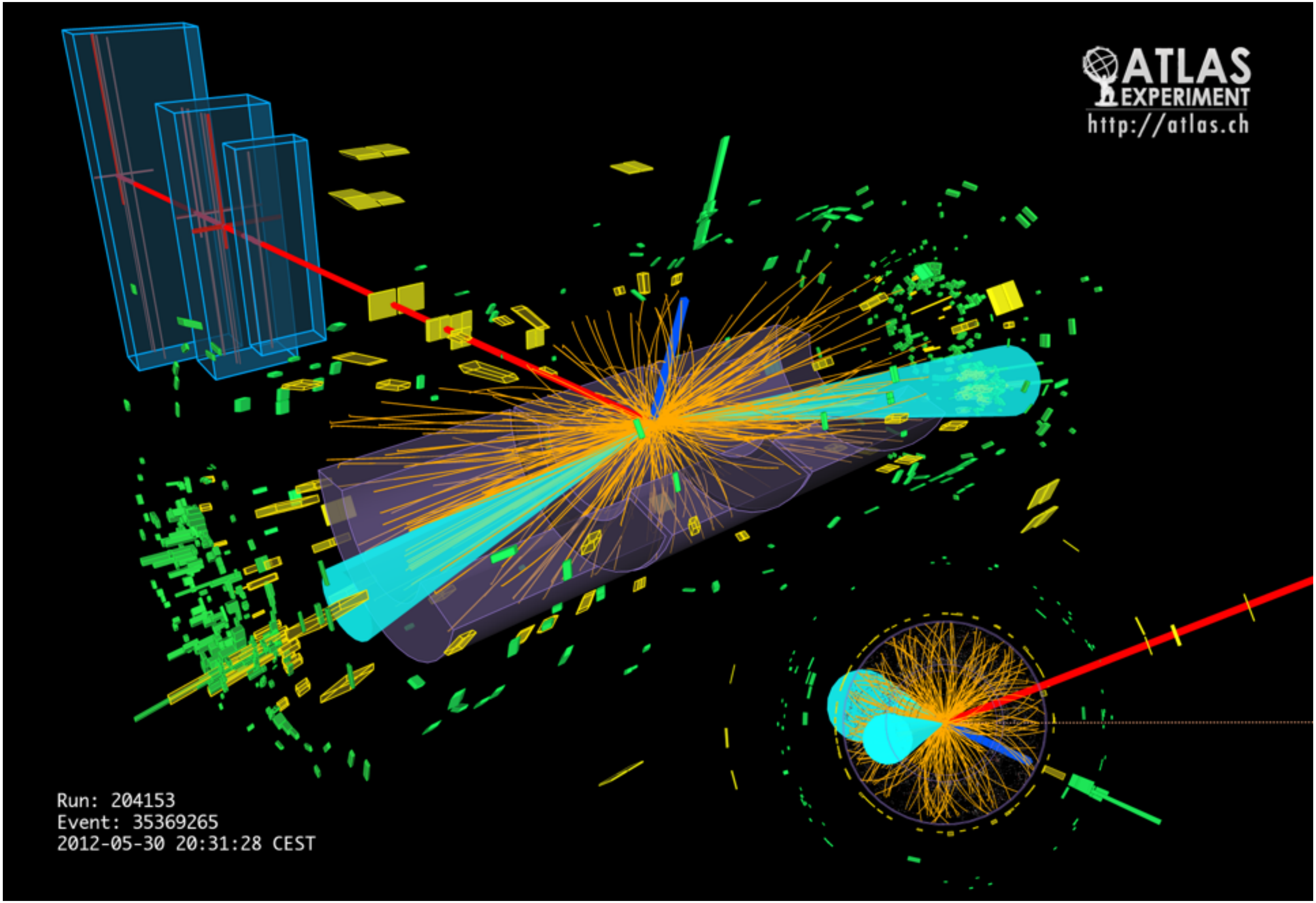
based on
O. Gituliar, MH, K. Kutak, arXiv:1511.08439, JHEP 1601 (2016) 181.
MH, A. Kusina, K. Kutak, arXiv:1607.0150, PRD 94 (2016) no.11, 114013.
A. Kusina, K. Kutak, M. Serino, arXiv:1711.0458, EPJC 78 (2018) no.3, 174.

XXXII Reunión Anual de la División de Partículas y Campos de la SMF, ICN-UNAM, CDMX, Mexico, 28-30 of May 2018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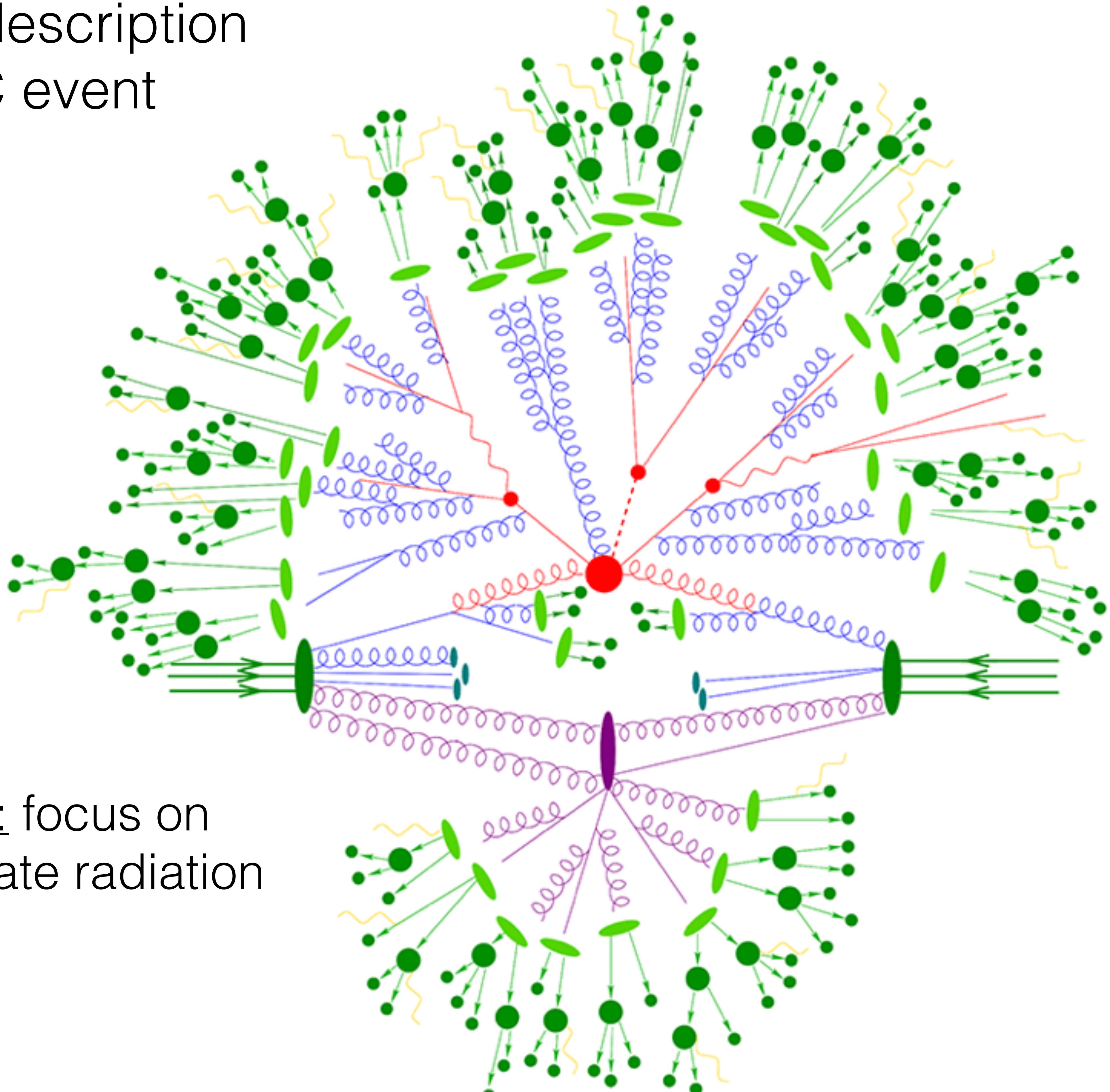
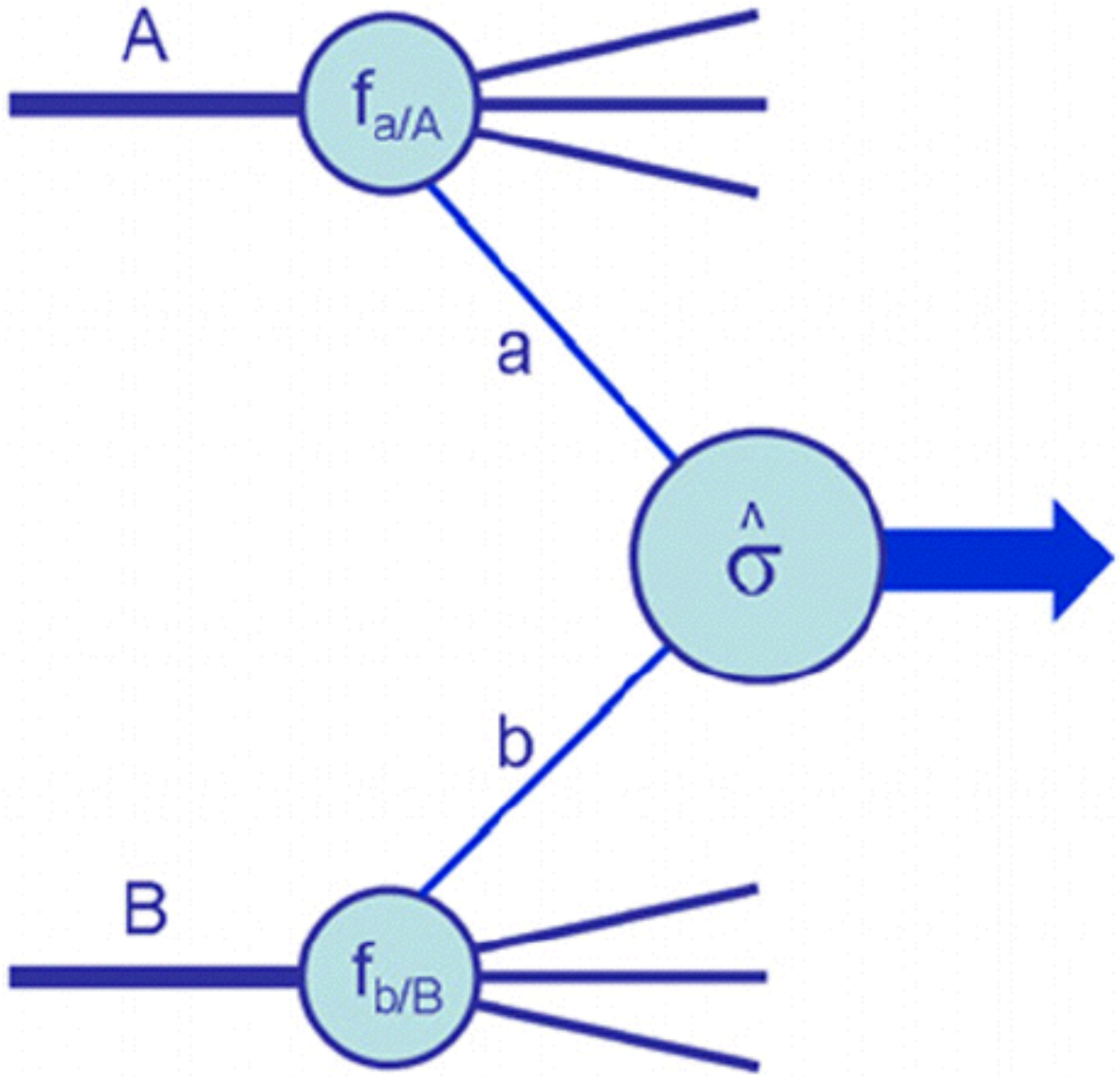
QCD description of LHC event



<u>this talk:</u> focus on initial state radiation

Underlying framework: collinear factorization

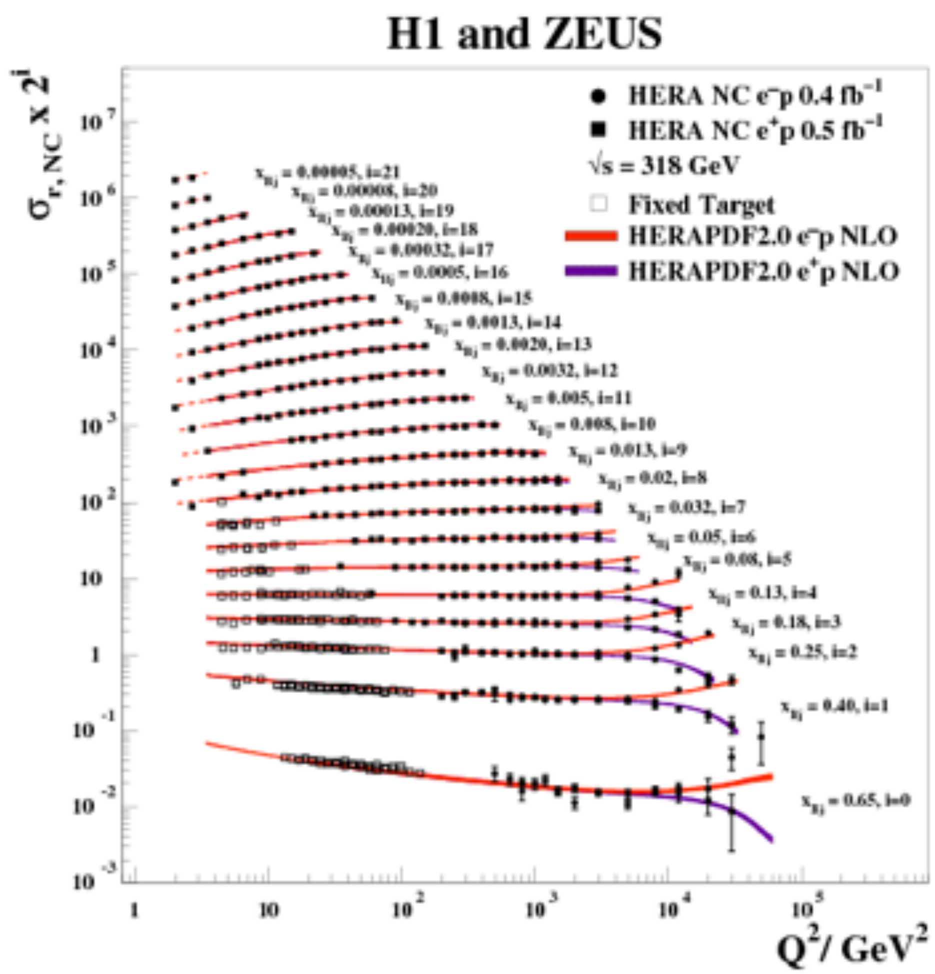


- hard scale $M \gg \Lambda_{QCD}$ → process factorizes into
- partonic process (= scattering of quarks and gluons + production of 'hard' final state)
- parton distribution functions (probability to find parton with momentum fraction x inside hadron)
- Everything else suppressed by powers of Q_0/M

$$\sigma(s, M^2) = \sum_{a,b=q,g} \int_0^1 dx_a \int_0^1 dx_b\, f_a(x_a, M^2) f_b(x_b, M^2) \hat{\sigma}_{ab}(x_a x_b s, M^2)$$

- x-dependence of pdfs from fit to HERA, Tevatron, LHC, …. data

- M^2-dependence: theory prediction: DGLAP evolution



$$\frac{d}{d\ln M^2} f_a(x, M^2) = \sum_{b=q,g} \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P_{ab}(z) f_b\left(\frac{x}{z}, M^2\right)$$

- $P_{ab}(z)$: splitting kernel

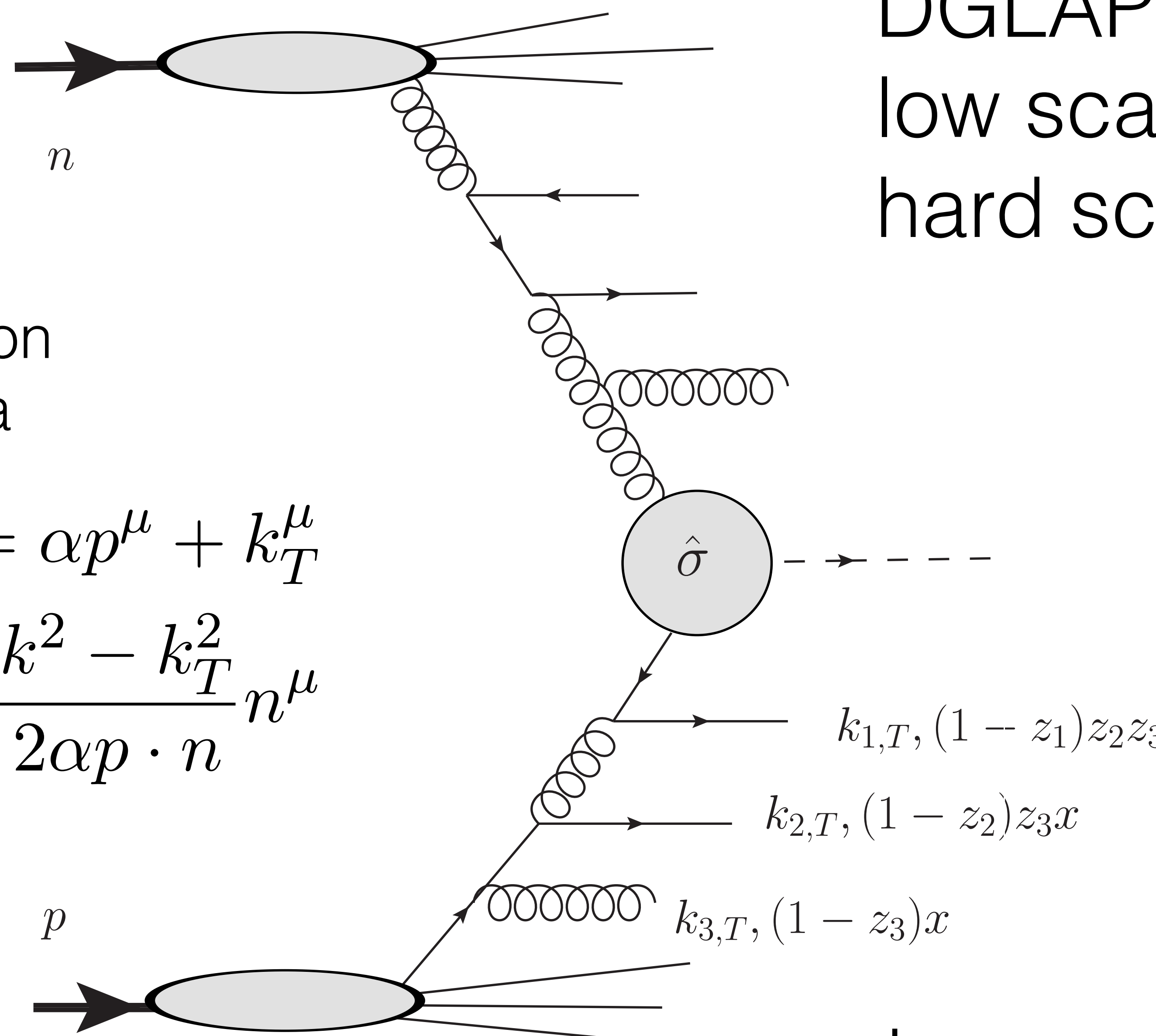
$$\begin{aligned} P_{qq}(z) &= \hat{P}_{qq}(z)_+ = C_F \left(\frac{1+z^2}{1-z}\right)_+ \\ P_{qg}(z) &= \hat{P}_{qg}(z) = T_R\,[z^2 + (1-z)^2] \\ P_{gq}(z) &= C_F \frac{1+(1-z)^2}{z}\,. \end{aligned}$$

$$\begin{aligned} P_{gg}(z) &= 2C_A \left[\left(\frac{z}{1-z} + \tfrac{1}{2}z(1-z)\right)_+ + \frac{1-z}{z} \right. \\ &\quad \left. + \tfrac{1}{2}z(1-z)\right] - \frac{2}{3} N_f T_R\,\delta(1-z)\,, \end{aligned}$$

DGLAP= evolution from low scale (hadron) to hard scale (process)

decomposition of momenta

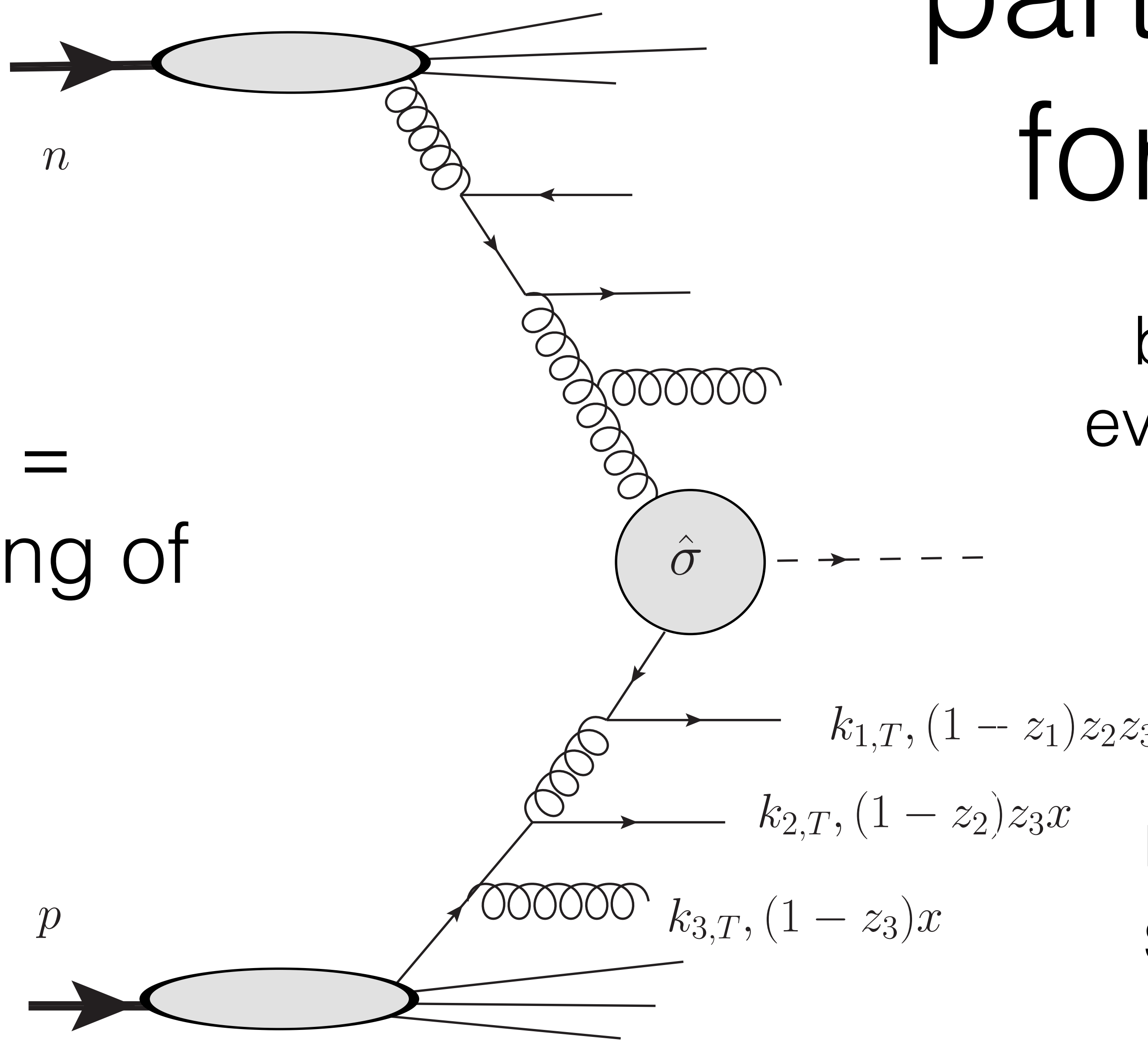
$$k^\mu = \alpha p^\mu + k_T^\mu + \frac{k^2 - k_T^2}{2\alpha p \cdot n} n^\mu$$



transverse momenta strongly ordered $\boldsymbol{k}_{i,T} \gg \boldsymbol{k}_{i+1,T}$ (=neglect information on kT⟷ isolate logarithmic enhanced term ~ collinear factorization)

proton momentum fraction $\boldsymbol{\alpha}$ treated exactly (no approximation)

parton shower formulation



basis of Monte Carlo event generator (Pythia, Herwig, …)

DGLAP = branching of partons

real part of splitting kernels = emission probability

virtual parts: exponentiated into non-emission probability

$$\Delta_q(t) \simeq \exp\left[-\int_{2t_0}^{t} \frac{dt'}{t'} \int_{t_0/t'}^{1-t_0/t'} dz \frac{\alpha_S}{2\pi} \hat{P}_{qq}(z)\right]$$

Monte Carlo principle (simplified)

- throw dices (=random numbers) to obtain a certain momentum configuration for *n* final state particles (exact momentum conservation!)

- order momentum in k_T and assign them their place in the evolution chain

- use parton shower formulation of DGLAP to calculate probability weight of this configuration ….

- For inclusive event: must agree with QCD result→ theoretical basis

- Obtain ***exclusive*** information about the event → essential for detector simulation, unfolding, ….

Note: there's a fine mismatch

- DGLAP (theory): transverse momenta strongly ordered $\boldsymbol{k}_{i,T} \gg \boldsymbol{k}_{i+1,T}$ (=neglect information on kT⟷ isolate logarithmic enhanced term ~ collinear factorization)

- Monte Carlo: momentum configuration which obeys exact momentum conservation + order them (no <u>strong ordering</u>) to assign probability weight

- Does it matter?

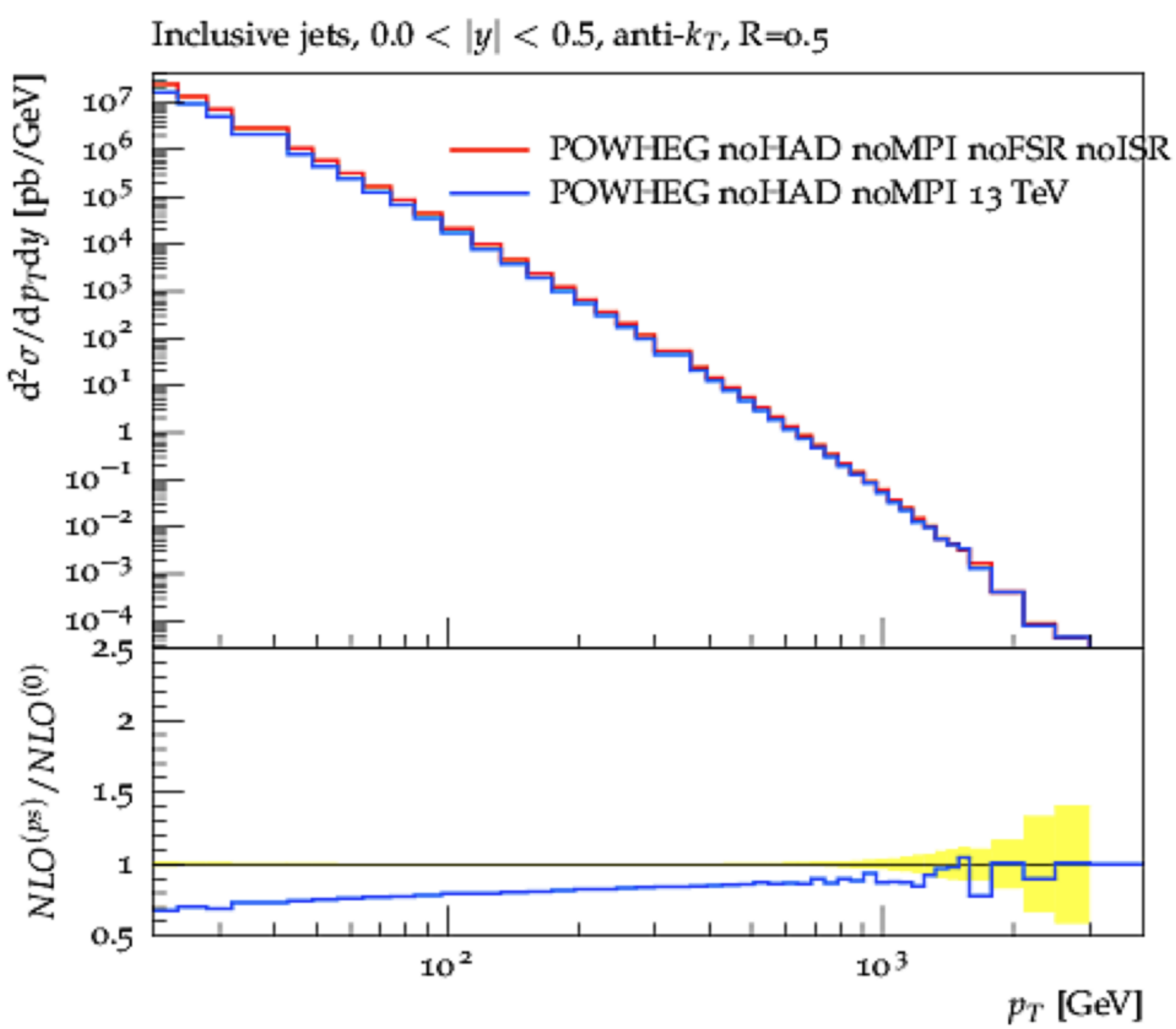
Hannes Jung at RBRC workshop: compare parton shower (MC) against pure NLO for single inclusive jet

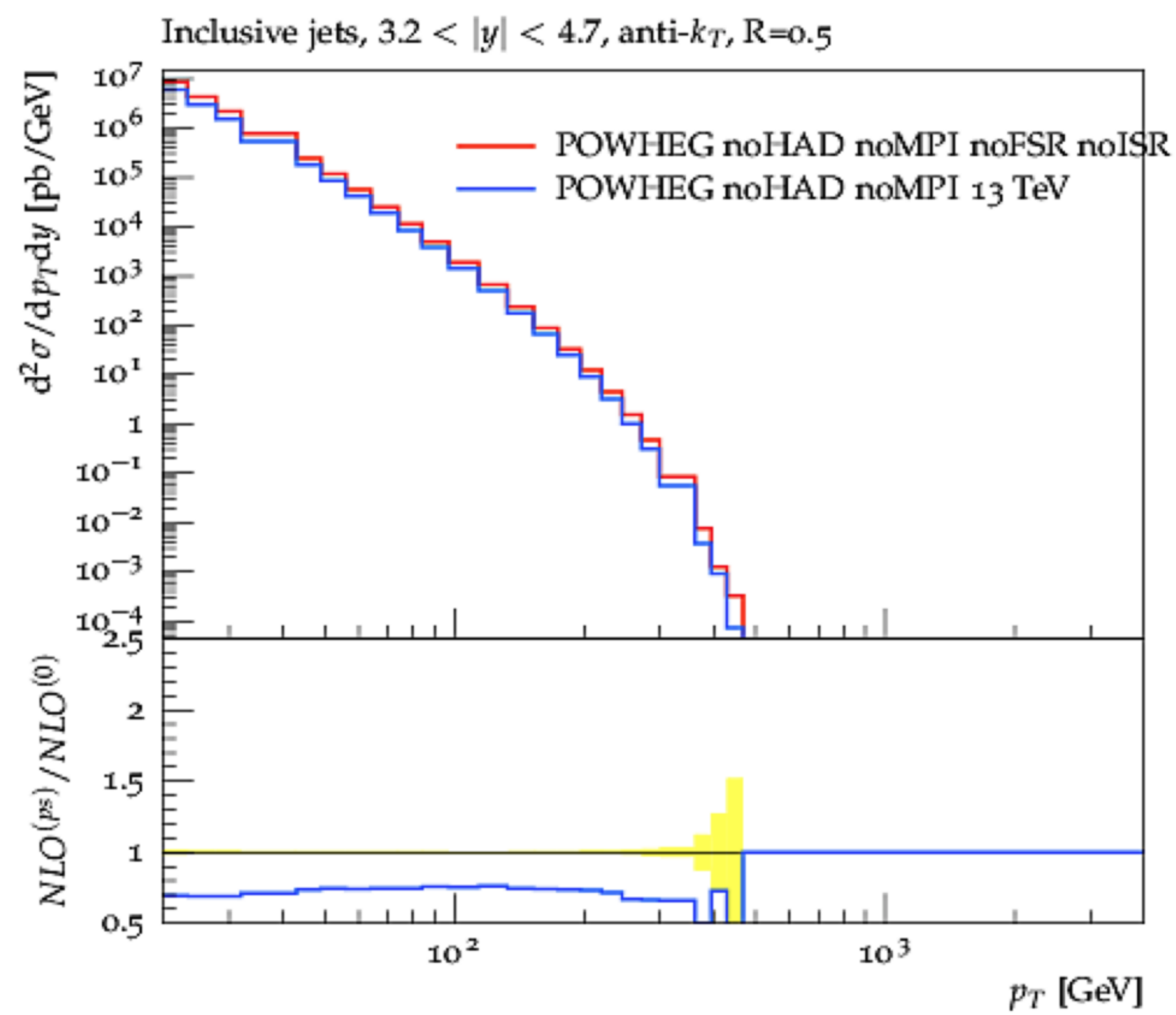
Approach described in: S. Dooling et al Phys.Rev., D87:094009, 2013.

- use NLO+PS to calculate:

$$K^{PS} = \frac{N^{(ps)}_{NLO-MC}}{N^{(0)}_{NLO-MC}}$$

- Corrections to be applied to fixed order NLO calculations:
  - kinematic effects: TMDs !
  - radiation outside of jet-cone





- mismatch due to “exact kinematics” (parton shower in MC) vs. “strong ordering”(pure NLO)
- not a problem for an approximate description & lower energies — a challenge for higher precision at high center of mass energies (LHC) → higher pT
- large in low x region (forward rapidities), but also sizable at mid-rapidities →equal or larger than uncertainties of current (N)NLO calculations
- collinear factorization: “exact kinematics” enters only through higher order corrections → one reason why (N)NLO corrections can be large

Transverse momentum dependent (TMD) factorization

- proposed solution: start with exact kinematics from the very beginning — at least keep momentum fractions & transverse momentum
- a wide field, see R. Angeles-Martinez et al.., “Transverse Momentum Dependent (TMD) parton distribution functions: status and prospects”, arXiv: 1507.05267, Acta Phys.Polon. B46 (2015) no.12, 2501-2534. for detailed overview
- Here: extend DGLAP to exact transverse momentum, guided by ***high energy factorization***
- central: desire a QCD description

In addition: (less ambiguous goals …)

- practical need for low x phenomenologist: many (forward) observables require integration over gluon x → sensitivity to large x region (e.g. fragmentation function, not completely exclusive final state, applications to MPI …)
- need to model BFKL/BK gluon in large x region (error!) or introduce matching scheme (how?)
- BEST: low x pdf that works for all x

2 versions of partonic evolution

- DGLAP: ordering in kT↔ kT not conserved →resum $\ln(M^2)$
- BFKL: ordering in momentum fraction α_i

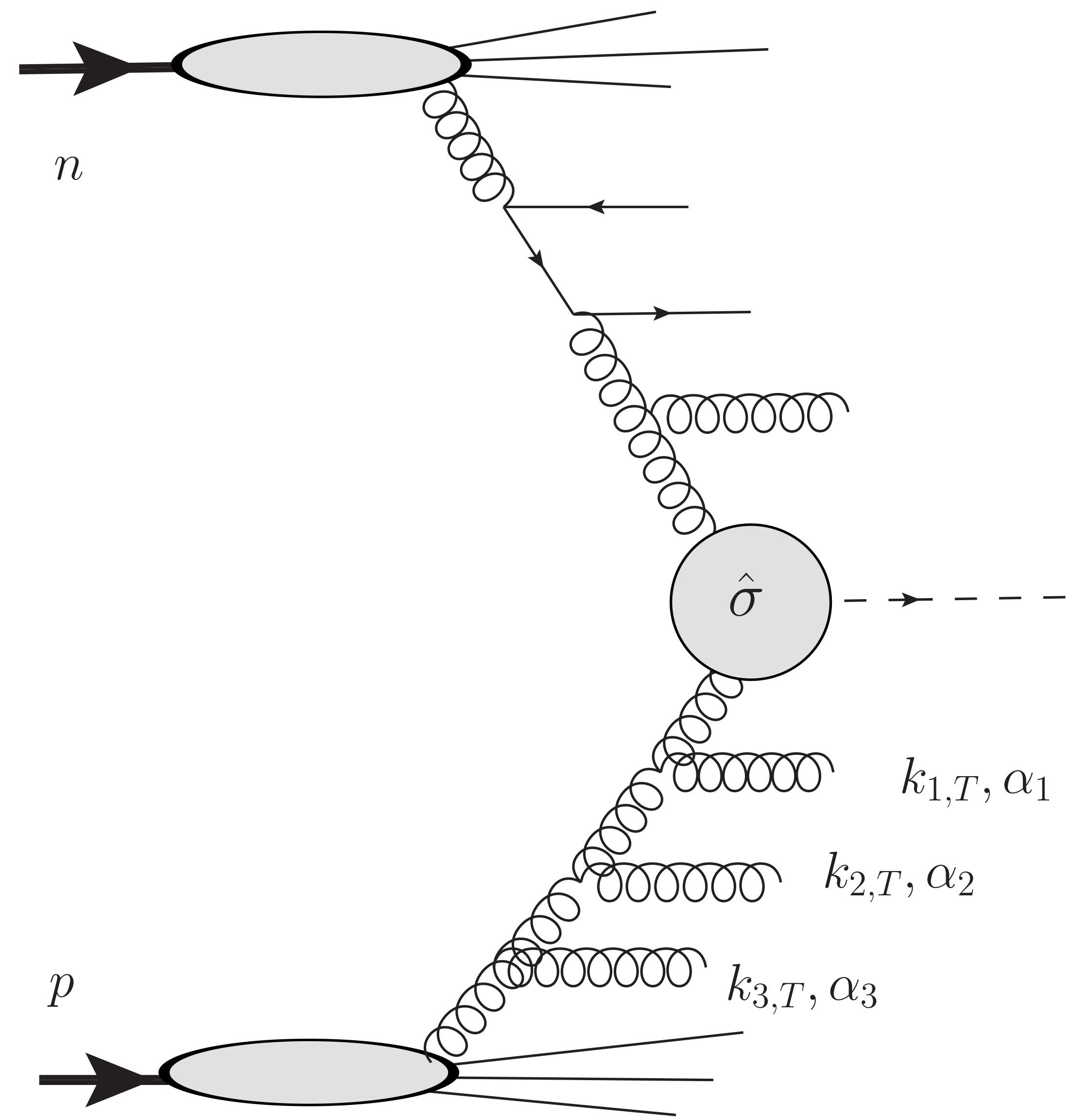
  → α/“energy” not conserved

  → resum ln(s)

  +gluons only at leading order
- both agree in the *double* logarithmic limit

  $\alpha_n \gg \alpha_{n-1} \gg \ldots \gg \alpha_1$ and

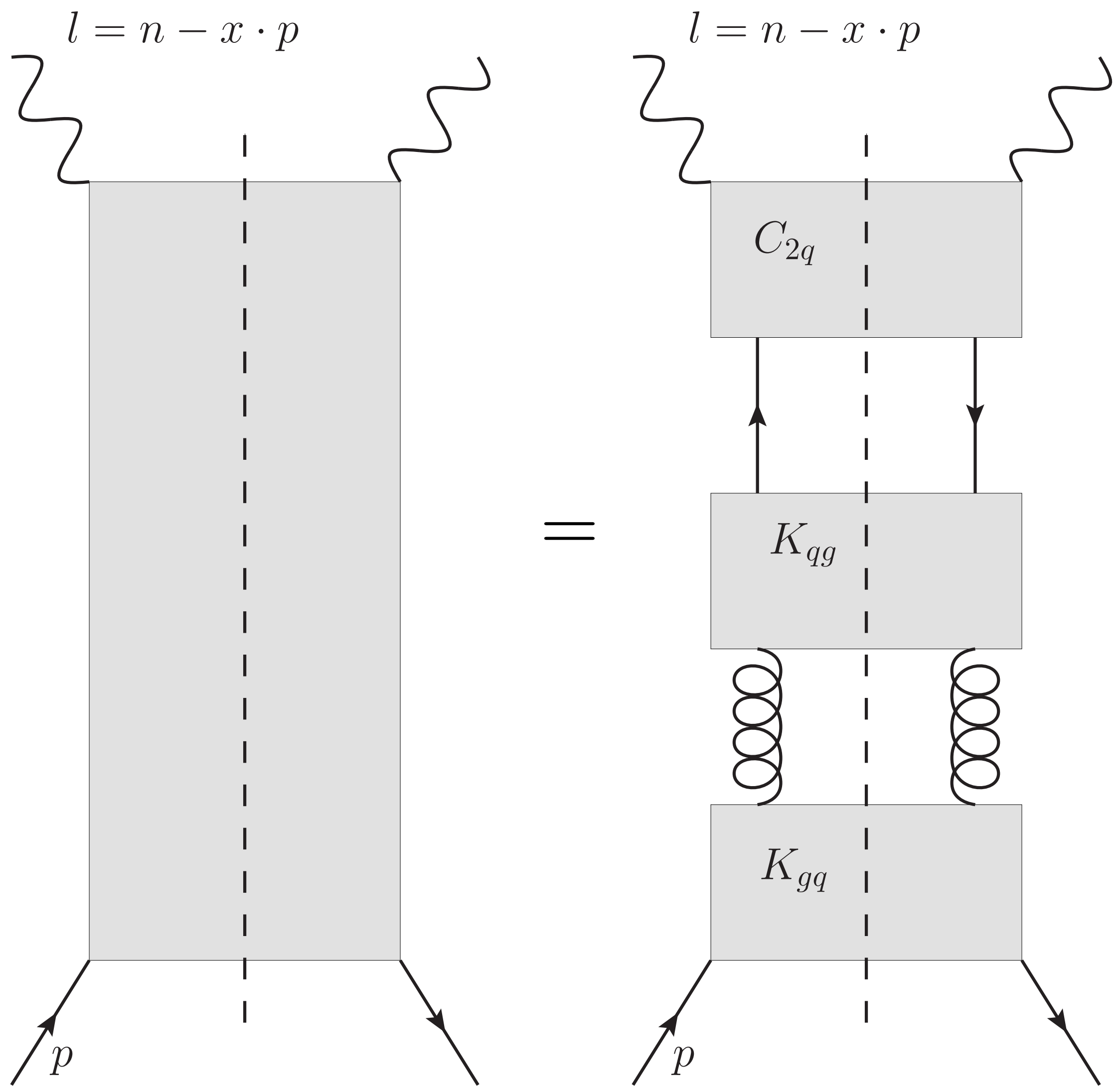
  $k_{1,T} \gg k_{2,T} \gg \ldots \gg k_{n,T}$



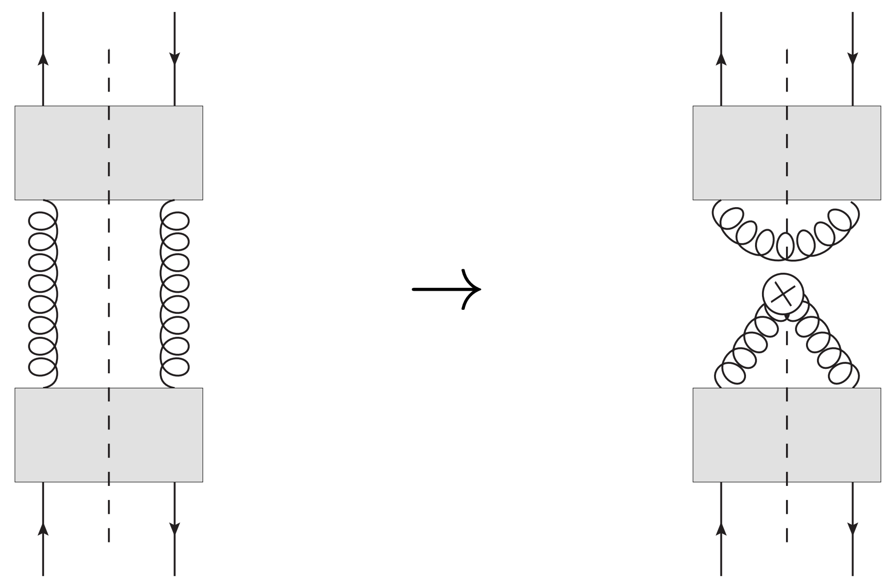
essential: extension requires new underlying matrix elements; NOT simply generalize kinematics

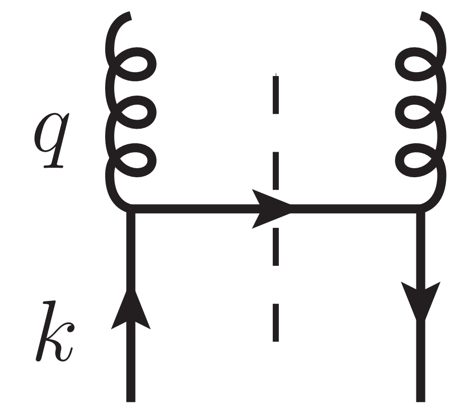
our approach: start with diagrammatic definition of collinear factorization

[Curci, Furmanski, Petronzio, Nucl.Phys. B 175 (1980) 27]



- axial, light-cone gauge: collinear singularities only form propagator which connect sub-amplitudes
- to isolate coefficient of collinear singularities use projectors in spinor/Lorentz space
- calculate DGLAP splitting functions as expansion in α_s





“upper” (outgoing) projectors:

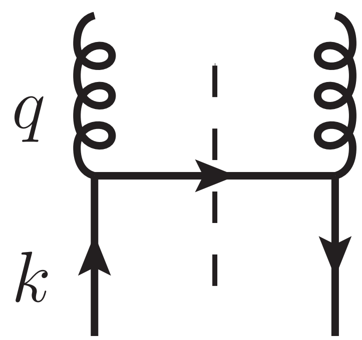
$$\mathbb{P}^{\mu\nu}_{\text{gluon, out}} = -g^{\mu\nu}, \qquad \mathbb{P}_{\text{quark, out}} = \frac{\not{n}}{2q\cdot n}$$

“lower” (incoming) projectors:

$$\mathbb{P}^{\mu\nu}_{\text{gluon, in}} = \frac{1}{d-2}\left(-g^{\mu\nu} + \frac{k^\mu n^\nu + n^\mu k^\nu}{k\cdot n}\right), \qquad \mathbb{P}_{\text{quark, in}} = \frac{\not{k}}{2}$$

Calculating a splitting function ...

$$\hat{K}_{gq}^{(0)}(q,k) \equiv$$



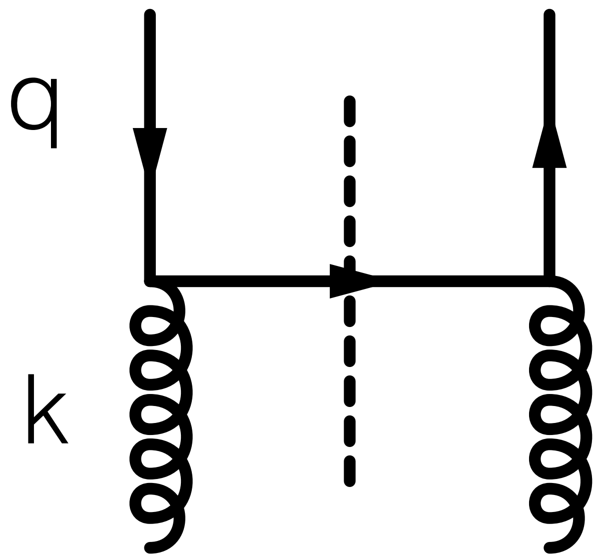
- contains propagator of out-going parton
- incoming propagators amputated + incoming on-shell $k^2 = 0$

$$\hat{K}_{gq}\left(z, \frac{\boldsymbol{k}^2}{\mu^2}, \epsilon, \alpha_s\right) = \int \frac{dq^2 d^{2+2\epsilon}\boldsymbol{q}}{2(2\pi)^{4+2\epsilon}} \Theta(\mu_F^2 - q^2)\mathbb{P}_{q,\,\mathsf{in}} \otimes \hat{K}_{gq}^{(0)}(q,k) \otimes \mathbb{P}_{g,\,\mathsf{out}}$$

$$= \frac{\alpha_s}{2\pi\Gamma(1+\epsilon)}\, z \int_0^{\mu_F^2} \frac{d\boldsymbol{q}^2}{\boldsymbol{q}^2} \left(\frac{e^{-\gamma_E}\boldsymbol{q}^2}{\mu^2}\right)^{\epsilon} P_{gq}^{(0)}(z;\epsilon)$$

allows to extract splitting function $P_{gq}^{(0)}(z;\epsilon)$

first TMD splitting: P_{gq} by Catani-Hautmann (low x resummed splitting kernels) [Catani, Hautmann, NPB427 (1994)]



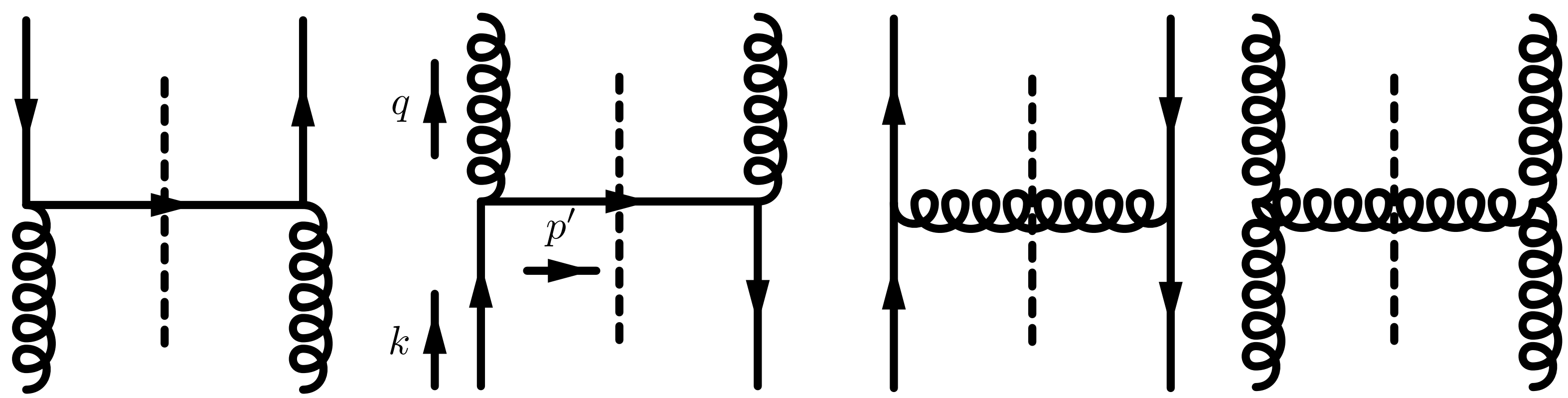
$$P_{qg}^{(0)}\left(z, \frac{\boldsymbol{k}^2}{\tilde{\boldsymbol{q}}^2}, \epsilon\right) = \mathrm{Tr}\left(\frac{\boldsymbol{\Delta}^2}{\boldsymbol{\Delta}^2 + z(1-z)\,\boldsymbol{k}^2}\right)^2 \cdot \left[z^2 + (1-z)^2 + 4z^2(1-z)^2\frac{\boldsymbol{k}^2}{\boldsymbol{\Delta}^2}\right]$$

$$\boldsymbol{\Delta} = \boldsymbol{q} - z\boldsymbol{k}$$

generalization to off-shell outgoing momentum *q*, using effective vertices (reggeized quarks) adapted to finite momentum fraction [Hautmann, MH, Jung; 1205.1759]

advantage: well defined as coefficient of high energy resumed splitting kernels

The challenge:



(a) $\tilde{P}_{qg}$ (b) $\tilde{P}_{gq}$ (c) $\tilde{P}_{qq}$ (d) $\tilde{P}_{gg}$

- standard collinear factorization: incoming parton (k) on-shell →gauge invariance/current conservation with light-cone gauge + standard QCD vertices
- extension to off-shell: require generalized production vertices → help from high energy factorization

Quark splittings — easier

$$\Gamma^{\mu}_{q^*g^*q}(q,k,p') = i\,g\,t^a\left(\gamma^\mu - \frac{n^\mu}{k\cdot n}\,\not{q}\right),$$
$$\Gamma^{\mu}_{g^*q^*q}(q,k,p') = i\,g\,t^a\left(\gamma^\mu - \frac{p^\mu}{p\cdot q}\,\not{k}\right),$$
$$\Gamma^{\mu}_{q^*q^*g}(q,k,p') = i\,g\,t^a\left(\gamma^\mu - \frac{p^\mu}{p\cdot p'}\,\not{k} + \frac{n^\mu}{n\cdot p'}\,\not{q}\right)$$

- production vertices of high energy factorization generalize relatively easy **[Lipatov, Vyazovski, hep-ph/0009340]**
- guarantees current conservation for off-shell legs (close relation to Wilson lines) **[Gituliar, MH, Kutak, 1511.08439]**

Gluon significantly more complicated

$$\Gamma^{\mu_1\mu_2\mu_3}_{g^*g^*g}(q,k,p') = \mathcal{V}^{\lambda\kappa\mu_3}(-q,k,-p')\, d^{\mu_1}{}_{\lambda}(q)\, d^{\mu_2}{}_{\kappa}(k) + d^{\mu_1\mu_2}(k)\,\frac{q^2 n^{\mu_3}}{n\cdot p'} - d^{\mu_1\mu_2}(q)\,\frac{k^2 p^{\mu_3}}{p\cdot p'}$$

3-gluon vertex

pol. tensor of light-cone gauge

- turns out: Lipatov high energy effective action in light-cone does the job, but does not allow to directly verify current conservation **Lipatov, hep-ph/9502308**

- complete expression: analysis of helicity spinor amplitudes in high energy limit in light-cone gauge

[MH, Kusina, Kutak, Serino; 1711.04587]

use effective vertices + CFP projectors adapted to off-shell incoming particle: TMD splitting kernels (real part)

$$\begin{aligned}
\bar{P}_{qg}^{(0)} &= T_R \left(\frac{\tilde{\boldsymbol{q}}^2}{\tilde{\boldsymbol{q}}^2 + z(1-z)\boldsymbol{k}^2} \right)^2 \left[z^2 + (1-z)^2 + 4z^2(1-z)^2 \frac{\boldsymbol{k}^2}{\tilde{\boldsymbol{q}}^2} \right], \\
\bar{P}_{gq}^{(0)} &= C_F \left[\frac{2\tilde{\boldsymbol{q}}^2}{z|\tilde{\boldsymbol{q}}^2 - (1-z)^2\boldsymbol{k}^2|} - \frac{(2-z)\tilde{\boldsymbol{q}}^4 + z(1-z^2)\boldsymbol{k}^2\tilde{\boldsymbol{q}}^2}{\left(\tilde{\boldsymbol{q}}^2 + z(1-z)\boldsymbol{k}^2\right)^2} \right], \\
\bar{P}_{qq}^{(0)} &= C_F \frac{\tilde{\boldsymbol{q}}^2}{\tilde{\boldsymbol{q}}^2 + z(1-z)\boldsymbol{k}^2} \\
&\times \left[\frac{\tilde{\boldsymbol{q}}^2 + (1-z^2)\boldsymbol{k}^2}{(1-z)|\tilde{\boldsymbol{q}}^2 - (1-z)^2\boldsymbol{k}^2|} + \frac{z^2\tilde{\boldsymbol{q}}^2 - z(1-z)(1-3z+z^2)\boldsymbol{k}^2}{(1-z)(\tilde{\boldsymbol{q}}^2 + z(1-z)\boldsymbol{k}^2)} \right].
\end{aligned}$$

[Gituliar, MH, Kutak, 1511.08439]

reduced to DGLAP splittings in collinear limit ✓

$$\begin{aligned}
\bar{P}_{gg}^{(0)}\left(z, \frac{\boldsymbol{k}^2}{\tilde{\boldsymbol{q}}^2}\right) &= C_A \frac{\tilde{\boldsymbol{q}}^2}{\tilde{\boldsymbol{q}}^2 + z(1-z)\boldsymbol{k}^2} \left[\frac{(2-z)\tilde{\boldsymbol{q}}^2 + (z^3 - 4z^2 + 3z)\boldsymbol{k}^2}{z(1-z)\left|\tilde{\boldsymbol{q}}^2 - (1-z)^2\boldsymbol{k}^2\right|} \right. \\
&+ \left. \frac{(2z^3 - 4z^2 + 6z - 3)\tilde{\boldsymbol{q}}^2 + z(4z^4 - 12z^3 + 9z^2 + z - 2)\boldsymbol{k}^2}{(1-z)(\tilde{\boldsymbol{q}}^2 + z(1-z)\boldsymbol{k}^2)} \right]
\end{aligned}$$

[MH, Kusina, Kutak, Serino; 1711.04587]

P_{gg} satisfies important constrains

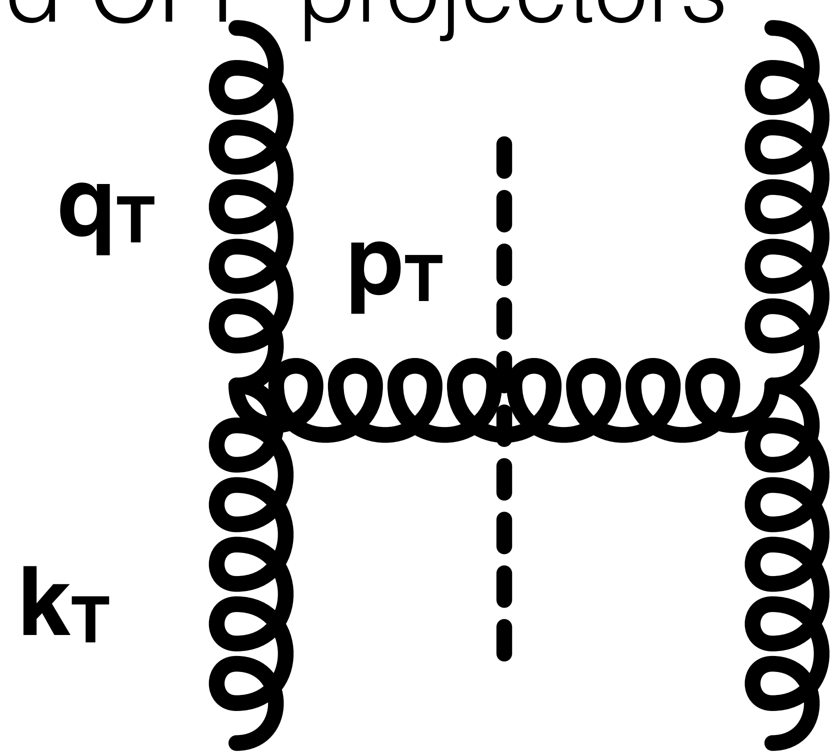
✓ from 2→3 scattering amplitude or Lipatov's action in light-cone gauge + generalized CFP projectors

✓ current conservation

✓ collinear limit: DGLAP splitting

✓ low x limit: BFKL kernel

✓ soft limit $p_T \to 0$: CCFM kernel
byproduct from requesting the first 3 points



just the beginning not the end …

- complete set of 4 ***real*** TMD splitting kernels
→satisfies all necessary constraints so far
- partial evolution equation already formulated
[MH, Kusina, Kutak; 1607.01507]
- virtual corrections = work in progress
- in general: need to properly develop the whole framework → what are we actually doing?
- at the very least: a consistent way to combine DGLAP and BFKL;
- valuable to extend low x distributions to large x
- hope: get a handle on kinematic corrections → precision