

## Introduction

## Standar Model is complete

Discovery of the last SM particle


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Black box theorem

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e- energy spectrum, Katrin exp. (hierarchy). There are some bounds from the Tritium $\beta$ decay $\left(m_{v e} \approx 2 \mathrm{eV}\right)$ \& from the Cosmology ( $\sum m_{v} \leqslant 0.23 \mathrm{eV}$ ).

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$\because$ Nature of neutrinos: Dirac or Majorana fermions (can be proved by some proceses with $\boldsymbol{\Delta L}=\mathbf{2}$, as $0 \mathrm{v} \boldsymbol{\beta} \boldsymbol{\beta}$ via blackbox theorem).
 Bounds in solar angle

\%Unknown the absolute mass scale (hierarchy). There are some bounds from the Tritium $\beta$ decay ( $m_{v e} \approx 2 \mathrm{eV}$ ) \& from the Cosmology ( $\sum m_{v} \leqslant 0.23 \mathrm{eV}$ ).
$\therefore$ The existence of neutrino masses are evidence of physics BSM.


## Introduction

## Neutrinos

$\because$ Dirac and Majorana mass terms:

$$
\begin{gathered}
-\mathcal{L}=\frac{1}{2}\left(\begin{array}{ll}
\bar{\nu}_{L} & \bar{\nu}_{L}^{c}
\end{array}\right)\left(\begin{array}{cc}
m_{M} & m_{D} \\
m_{D} & m_{s}
\end{array}\right)\binom{\nu_{R}^{c}}{\nu_{R}}+\text { h.c. } \\
-\mathcal{L}_{D}=m_{D}\left(\bar{\nu}_{L} \nu_{R}+\bar{\nu}_{R} \nu_{L}\right) \quad-\mathcal{L}_{M}=\frac{m_{M}}{2}\left(\bar{\nu}_{L} \nu_{R}^{c}+\bar{\nu}_{R}^{c} \nu_{L}\right) \quad-\mathcal{L}_{S}=\frac{m_{S}}{2}\left(\bar{\nu}_{L}^{c} \nu_{R}+\bar{\nu}_{R} \nu_{L}^{c}\right)
\end{gathered}
$$

$\because$ Lepton mixing matrix (PMNS): From mismatch between mass and interaction (flavour) eigenstates.

$$
U=V_{L}^{l \dagger} V_{L}^{\nu}
$$

For 3 light Dirac neutrinos (3 mixing angles, $\mathbf{1} \mathbf{C P}$ phase)

$$
U^{D}=\left(\begin{array}{ccc}
c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i \delta} \\
-s_{12} c_{23}-c_{12} s_{23} s_{13} e^{i} \delta & c_{12} c_{23}-s_{12} s_{23} s_{13} e^{i} \delta & s_{23} c_{13} \\
s_{12} s_{23}-c_{12} c_{23} s_{13} e^{i} \delta & -c_{12} s_{23}-s_{12} c_{23} s_{13} e^{i} \delta & c_{23} c_{13}
\end{array}\right)
$$

Majorana neutrinos (2 additional CP phases)

$$
U_{M}=U_{D} D_{M} \quad D_{M}=\operatorname{diag}\left(1, e^{i \lambda_{2}}, e^{i \lambda_{3}}\right)
$$

## Introduction

## Neutrinos

In analogy with quarks and charged leptons in the SM.
Mass term for Dirac neutrinos (RH neutrinos $\mathbf{v}_{\mathbf{R}}$ added).

$$
-\mathcal{L}=y_{D} \bar{L} H \nu_{R}+\text { h.c. }
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Assuming Yukawa coupling of quark top of order 1.

$$
y_{D} \sim \mathcal{O}\left(10^{-12}\right)
$$

$$
y_{e} \sim \mathcal{O}\left(10^{-6}\right) \quad \text { For the lightest charged lepton ( } \mathrm{e}^{-} \text {). }
$$

Seems unnatural the Yukawa coupling for a Dirac neutrino

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In analogy with quarks and charged leptons in the SM.
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Assuming Yukawa coupling of quark top of order 1.

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\begin{aligned}
y_{D} & \sim \mathcal{O}\left(10^{-12}\right) \\
y_{e} & \sim \mathcal{O}\left(10^{-6}\right)
\end{aligned} \quad \text { For a Dirac neutrino. }
$$

Seems unnatural the Yukawa coupling for a Dirac neutrino

## Introduction

## Family symmetries

$\%$ They have been used to reduce the $n^{\circ}$ of Yukawa couplings and correlations among observables: masses, mixings \& CP phases.
$\because$ Sometimes gives predictions, as certain mass matrix textures (TM, BM,TBM, BTM).

Family (horizontal or flavour) symmetry


## Introduction

## Family symmetries

Non-Abelian finite groups of order < 32 constructed from direct products of $Z, D, Q, S$ and $T$.

| Frompton ard Kephart, PRD64 (01) |  |
| :---: | :---: |
| order | groups |
| 6 | $S_{3} \equiv D_{5}$ |
| 8 | $D_{i}, Q=Q_{4}$ |
| 10 | $D_{5}$ |
| 12 | $D_{6},\left(, T \equiv h_{4}\right.$ |
| 14 | $D_{7}$ |
| 16 | $D_{8}: Q_{8}, Z_{2} \times D_{4}, Z_{2} \times Q$ |
| 18 | $D_{4}, Z_{3} \times D_{3}$ |
| 20 | $D_{10}, Q_{10}$ |
| 22 | $D_{11}$ |
| 24 | Z $Z_{2} \times Q_{6,}, Z_{2} \times T, Z_{3} \times D_{4}, Z_{3} \times Q, Z_{4} \times D_{3,} S_{3}$ |
| 26 | $D_{13}$ |
| 28 | $D_{14}, Q_{14}$ |
| 30 | $D_{\text {: } 5, ~}, D_{5} \times Z_{3}, D_{3} \times Z_{5}$ |


|  | $\begin{array}{cc} -23{\mathrm{Mev} / \mathrm{c}^{3}}_{2 / 2}^{2 / 2} & \text { U } \\ & \text { up } \end{array}$ |  | $\begin{array}{cc} 1 / 23 \\ & \dagger \\ & \text { top } \end{array}$ | $\begin{array}{rr} \text { gluon } \end{array}$ | $\underbrace{* 125 \text { Gevkes }}_{\substack{\text { Higgs } \\ \text { boson }}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \frac{y}{y} \\ & \underline{a} \\ & \frac{1}{2} \end{aligned}$ |  |  | $$ | $\begin{array}{lr} 0 & \\ & \text { photon } \end{array}$ |  |
|  | $$ | $\underbrace{105.7 \mathrm{MeVloz}^{2}}_{\text {muon }}$ | $\begin{array}{cc} 1.777 \mathrm{Gev} / \mathrm{/c}^{1} \\ { }_{1}^{-1} & \mathrm{~T} \\ & \\ & \text { tau } \end{array}$ |  | $\begin{aligned} & n \\ & \mathbf{Z} \\ & \mathbf{0} \end{aligned}$ |
| $\begin{aligned} & n \\ & 0 \\ & 0 \\ & 0 \\ & \hline 1 \end{aligned}$ |  | $\underbrace{\cos ^{20.17 \mathrm{MeV} / \mathrm{c}^{2}}}_{\substack{1 / 2 \\ \text { muon } \\ \text { neutrino }}}$ | $\begin{array}{cc} \text { <1E.5 Mevic } \\ 0 & \text { D. } \\ \substack{\text { tau } \\ \text { neutrino }} \end{array}$ | $\begin{aligned} & { }_{1}^{80.4} \mathrm{GeV} / \mathrm{c}^{2} \\ & \text { W boson } \end{aligned}$ | $\begin{aligned} & 0 \\ & \infty \\ & \omega \\ & \vdots \\ & 0 \\ & \hline \mathbf{c} \end{aligned}$ |

Alternating group $\left(\mathrm{A}_{4}\right)$ : Flavour symmetry group. [E. Ma, etal. '01]
Non-abelian, discrete group. It has:
Tree 1-dim. irreps.: $\mathbf{1}_{\mathbf{1}}, \mathbf{1}_{\mathbf{2}}, \mathbb{1}_{3}$.
One 3-dim. irrep.: 3.

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Product rule: One 3-dim. irrep.: 3.

$$
\begin{aligned}
& \mathbf{1}_{1} \otimes \mathbf{1}_{\mathbf{1}}=\mathbf{1}_{1}, \mathbf{1}_{2} \otimes 1_{2}=1_{3}, 1_{3} \otimes 1_{3}=\mathbf{1}_{2}, \\
& \mathbf{1}_{\mathbf{1}} \otimes \mathbf{1}_{\mathbf{2}}=\mathbf{1}_{2}, \mathbf{1}_{\mathbf{1}} \otimes 1_{3}=1_{3}, \mathbf{1}_{2} \otimes 1_{3}=\mathbf{1}_{\mathbf{1}},
\end{aligned}
$$

$$
3 \otimes 3=\mathbf{1}_{1} \oplus \mathbf{1}_{2} \oplus 1_{3} \oplus 3 \oplus 3
$$

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$\mathbf{1}_{1} \otimes \mathbf{1}_{1}=\mathbf{1}_{1}, 1_{2} \otimes 1_{2}=1_{3}, 1_{3} \otimes 1_{3}=1_{2}$,
$\mathbf{1}_{1} \otimes \mathbf{1}_{2}=\mathbf{1}_{2}, \mathbf{1}_{\mathbf{1}} \otimes 1_{3}=1_{3}, \mathbf{1}_{2} \otimes 1_{3}=\mathbf{1}_{\mathbf{1}}$,
$3 \otimes 3=\mathbf{1}_{1} \oplus \mathbf{1}_{2} \oplus \mathbb{1}_{3} \oplus 3 \oplus 3$.
$A_{4}$ has two subgroups $Z_{2}, Z_{3}$.

Two generators: $S, T$.

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$$
S=\left(\begin{array}{ccc}
1 & 0 & 0 \\
0 & -1 & 0 \\
0 & 0 & -1
\end{array}\right)
$$

A4 has two subgroups $Z_{2}, Z_{3}$.

Two generators: $S, T$.

Generators in the a 3 dim. rep. (with $S$ real and diagonal).

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$A_{4}$ has two subgroups $Z_{2}, Z_{3}$.


Two generators: $S, T$.

$$
S=\left(\begin{array}{ccc}
1 & 0 & 0 \\
0 & -1 & 0 \\
0 & 0 & -1
\end{array}\right) \quad T=\left(\begin{array}{ccc}
0 & 1 & 0 \\
0 & 0 & 1 \\
1 & 0 & 0
\end{array}\right) \begin{aligned}
& \text { Generators in the a } \\
& 3 \text { dim. rep. (with } S \\
& \text { real and diagonal). }
\end{aligned}
$$

## Model

## Enhancement of the SM symmetry

 SM -> SM× $\mathbf{A}_{4} \times \mathbf{Z}_{3} \times \mathbf{Z}_{2}$|  | $\bar{L}$ | $\ell_{R}$ | $\nu_{R}$ | $H^{d}$ | $\phi$ | $\sigma$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $S U(2)_{L} \otimes U(1)_{Y}$ | $(2,-1 / 2)$ | $(1,-1)$ | $(1,0)$ | $(2,1 / 2)$ | $(2,-1 / 2)$ | $(1,0)$ |
| $A_{4}$ | $\mathbf{3}$ | $\mathbf{3}$ | $\mathbf{3}$ | $\mathbf{3}$ | $\mathbf{3}$ | $\mathbf{3}$ or $\mathbf{1}_{\mathbf{i}}$ |
| $\mathbb{Z}_{3}$ | $\omega^{2}$ | $\omega$ | $\omega$ | 1 | 1 | 1 |
| $\mathbb{Z}_{2}$ | + | + | - | + | - | - |

$$
\begin{array}{cc}
H^{d}=\left(H_{1}^{d}, H_{2}^{d}, H_{3}^{d}\right) & \phi=\left(\phi_{1}, \phi_{2}, \phi_{3}\right) \\
H_{i}^{d}=\binom{h_{i}^{d+}}{h_{i}^{d 0}} & \phi_{i}=\binom{\phi_{i}^{0}}{\phi_{i}^{-}}
\end{array}
$$

Higgs doublets


$$
\nu_{R}=\left(\nu_{R_{1}}, \nu_{R_{2}}, \nu_{R_{3}}\right)
$$

RH Neutrinos

Flavon fields $\boldsymbol{\sigma}_{\mathbf{i}}$

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| $\mathbb{Z}_{2}$ | + | + | - | + | - | - |

$$
H^{d}=\left(H_{1}^{d}, H_{2}^{d}, H_{3}^{d}\right)
$$

$\phi=\left(\phi_{1}, \phi_{2}, \phi_{3}\right)$


$$
\nu_{R}=\left(\nu_{R_{1}}, \nu_{R_{2}}, \nu_{R_{3}}\right)
$$

$\left\langle H^{d}\right\rangle=\left(v_{h_{1}^{d}}, v_{h_{2}^{d}}, v_{h_{3}^{d}}\right)$
$\langle\phi\rangle=\left(v_{\phi_{1}}, v_{\phi_{2}}, v_{\phi_{3}}\right)$.
$A_{4}$ and $Z_{2}$ are broken.

## Model

Enhancement of the SM symmetry SM -> SM× $\mathbf{A}_{4} \times \mathbf{Z}_{3} \times \mathbf{Z}_{2}$

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| $\mathbb{Z}_{2}$ | + | + | - | + | - | - |

$\mathbf{A}_{4}$ gives the flavour structure


## Model

Enhancement of the SM symmetry SM -> SM $\times \mathbf{A}_{4} \times \mathbf{Z}_{3} \times \mathbf{Z}_{2}$

|  | $\bar{L}$ | $\ell_{R}$ | $\nu_{R}$ | $H^{d}$ | $\phi$ | $\sigma$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $S U(2)_{L} \otimes U(1)_{Y}$ | $(2,-1 / 2)$ | $(1,-1)$ | $(1,0)$ | $(2,1 / 2)$ | $(2,-1 / 2)$ | $(1,0)$ |
| $\Lambda_{\mathbf{4}}$ | $\mathbf{3}$ | $\mathbf{3}_{2}$ | $\mathbf{3}$ | $\mathbf{3}$ | $\mathbf{3}$ | $\mathbf{3}$ or $\mathbf{1}_{\mathbf{i}}$ |
| $\mathbb{Z}_{3}$ | $\omega^{2}$ | $\omega$ | $\omega$ | 1 | 1 | 1 |
| $\mathbb{Z}_{2}$ | + | + | - | + |  | - |

$\mathbf{A}_{4}$ gives the flavour structure

$\mathbf{Z}_{3}$
forbids $M_{M_{R} \nu_{R} \nu_{R}}$

Maj. mass term RH

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$A_{4}$ gives the flavour structure


$$
L H^{d} L H^{d}
$$

$\mathbb{Z}_{3}$
forbids $\begin{array}{cc}M_{R} \nu_{R} \nu_{R} & L \tilde{\phi} L \tilde{\phi} \\ & \\ & L H^{d} L \tilde{\phi}\end{array}$
Maj. mass term RH
dim-5 Op.

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| $\mathbb{Z}_{2}$ | + | + | - | + |  |  |

$A_{4}$ gives the flavour structure


$$
L H^{d} L H^{d} \quad \nu_{R} \nu_{R} \sigma^{n}
$$

$\begin{array}{cccc}\mathbf{Z}_{3} & M_{R} \nu_{R} \nu_{R} & L \tilde{\phi} L \tilde{\phi} & \left(H^{d \dagger} H^{d}\right)^{n} \\ \text { forbids } & & L H^{d} L \tilde{\phi} & \left(\phi^{\dagger} \phi\right)^{n}\end{array}$

Maj. mass term RH
loop level Maj. masses

## Model

Enhancement of the SM symmetry SM -> SM $\times \mathbf{A}_{4} \times \mathbf{Z}_{3} \times \mathbf{Z}_{2}$

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| $\mathbb{Z a}_{9}$ | $\mathbb{Z}_{2}^{2}$ | $\omega$ | $\omega$ | 1 | 1 | 1 |
|  | + | + | - | + | - | - |

$\mathbf{A}_{4}$ gives the flavour structure

$$
L H^{d} L H^{d} \quad \nu_{R} \nu_{R} \sigma^{n}
$$

$Z_{3}$
forbids

$$
\begin{array}{ll}
M_{R} \nu_{R} \nu_{R} & L \tilde{\phi} L \tilde{\phi} \\
& L H^{d} L \tilde{\phi}
\end{array}
$$

$$
\left(H^{d \dagger} H^{d}\right)^{n}
$$

$\left(\phi^{\dagger} \phi\right)^{n}$

$\begin{array}{cc}\mathbf{Z}_{\mathbf{2}} & \bar{L} \tilde{\phi} \ell_{R} \\ \text { forbids } & \bar{L} \tilde{H}^{d} \nu_{R}\end{array}$

Maj. mass term RH
dim-5 Op. Ioop level Maj. masses

## Model

## Enhancement of the SM symmetry

 SM -> SM× $\mathbf{A}_{4} \times \mathbf{Z}_{3} \times \mathbf{Z}_{2}$|  | $\bar{L}$ | $\ell_{R}$ | $\nu_{R}$ | $H^{d}$ | $\phi$ | $\sigma$ |
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| $\mathbb{Z}_{3}$ | $\omega^{2}$ | $\omega$ | $\omega$ | 1 | 1 | 1 |
| $\mathbb{Z}_{2}$ | + | + | - | + | - | - |

$$
\mathcal{L}_{Y} \supset Y_{\ell}^{i}\left[\bar{L}, H^{d}\right]_{3_{i}} \ell_{R}+Y_{\nu}^{i}[\bar{L}, \phi]_{3_{i}} \nu_{R}+\text { h.c. }
$$


$\phi$ acquires a small induced vev. [c. Bonilla et. al., 2016]

## Model

|  | $\bar{L}$ | $\ell_{R}$ | $\nu_{R}$ | $H^{d}$ | $\phi$ | $\sigma$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $S U(2)_{L} \otimes U(1)_{Y}$ | $(2,-1 / 2)$ | $(1,-1)$ | $(1,0)$ | $(2,1 / 2)$ | $(2,-1 / 2)$ | $(1,0)$ |
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| $\mathbb{Z}_{3}$ | $\omega^{2}$ | $\omega$ | $\omega$ | 1 | 1 | 1 |
| $\mathbb{Z}_{2}$ | + | + | - | + | - | - |

Complete model (inspired in [S. King, et al. (2013)] )


|  | $\bar{Q}$ | $\bar{L}$ | $u_{R_{i}}$ | $d_{R}$ | $\ell_{R}$ | $\nu_{R}$ | $H^{u}$ | $H^{d}$ | $\phi$ | $\sigma$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $S U(2)_{L} \otimes U(1)_{Y}$ | $(2,1 / 6)$ | $(2,-1 / 2)$ | $(1,2 / 3)$ | $(1,-1 / 3)$ | $(1,-1)$ | $(1,0)$ | $(2,-1 / 2)$ | $(2,1 / 2)$ | $(2,-1 / 2)$ | $(1,0)$ |
| $A_{4}$ | $\mathbf{3}$ | $\mathbf{3}$ | $\mathbf{1}_{\mathbf{i}}$ | $\mathbf{3}$ | $\mathbf{3}$ | $\mathbf{3}$ | $\mathbf{3}$ | $\mathbf{3}$ | $\mathbf{3}$ | $\mathbf{3} / \mathbf{1}_{\mathbf{i}}$ |
| $Z_{3}$ | 1 | $\omega^{2}$ | 1 | 1 | $\omega$ | $\omega$ | 1 | 1 | 1 | 1 |
| $Z_{2}$ | + | + | + | + | + | - | + | + | - | - |
| $Z_{2}^{d}$ | + | + | + | - | + | + | + | - | + | + |

Accounts for the masses and mixings of quarks and leptons

## Model

Flavour structure provides the generalised bottom-tau mass relation.

$$
\mathcal{L}_{Y} \supset Y_{\ell}^{i}\left[\bar{L}, H^{d}\right]_{3_{i}} \ell_{R}+\text { h.c. }
$$

## Mass matrix for charged leptons and down type quarks.

$$
m_{\ell}=\left(\begin{array}{ccc}
0 & a_{\ell} \alpha_{\ell} e^{i \theta_{\ell}} & b_{\ell} \\
b_{\ell} \alpha_{\ell} & 0 & e^{i \theta_{\ell}} a_{\ell} \rho_{\ell} \\
a_{\ell} e^{i \theta_{\ell}} & b_{\ell} \rho_{\ell} & 0
\end{array}\right)
$$

with

$$
\begin{gathered}
a_{\ell}=v_{h_{2}^{d}}\left(Y_{\ell}^{1}+Y_{\ell}^{3}\right) \quad b_{\ell}=v_{h_{2}^{d}}\left(Y_{\ell}^{2}+Y_{\ell}^{4}\right) \\
\alpha_{\ell}=v_{h_{3}^{d}} / v_{h_{2}^{d}} \quad \rho_{\ell}=v_{h_{1}^{d}} / v_{h_{2}^{d}} \\
\left\langle H^{d}\right\rangle=\left(v_{h_{1}^{d}}, v_{h_{2}^{d}}, v_{h_{3}^{d}}\right)=v_{h_{2}^{d}}\left(\rho_{\ell}, 1, \alpha_{\ell}\right)
\end{gathered}
$$

## Model

Flavour structure provides the generalised bottom-tau mass relation.

$$
\mathcal{L}_{Y} \supset Y_{\ell}^{i}\left[\bar{L}, H^{d}\right]_{3_{i}} \ell_{R}+\text { h.c. }
$$

Mass matrix for charged leptons and down type quarks.

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0 & a_{\ell} \alpha_{\ell} e^{i \theta_{\ell}} & b_{\ell} \\
b_{\ell} \alpha_{\ell} & 0 & e^{i \theta_{\ell}} a_{\ell} \rho_{\ell} \\
a_{\ell} e^{i \theta_{\ell}} & b_{\ell} \rho_{\ell} & 0
\end{array}\right)
$$

Bi-unitary invariants of the mass squared matrix $M_{\ell}^{2}=m_{\ell} m_{\ell}^{\dagger}$

$$
\begin{aligned}
\operatorname{Tr} M_{\ell}^{2} & =m_{1}^{2}+m_{2}^{2}+m_{3}^{2} \\
\operatorname{det} M_{\ell}^{2} & =m_{1}^{2} m_{2}^{2} m_{3}^{2} \\
\left(\operatorname{Tr} M_{\ell}^{2}\right)^{2}-\operatorname{Tr}\left(M_{\ell}^{2}\right)^{2} & =2 m_{1}^{2} m_{2}^{2}+2 m_{2}^{2} m_{3}^{2}+2 m_{1}^{2} m_{3}^{2}
\end{aligned}
$$

## Model

Flavour structure provides the generalised bottom-tau mass relation.

$$
\mathcal{L}_{Y} \supset Y_{\ell}^{i}\left[\bar{L}, H^{d}\right]_{3_{i}} \ell_{R}+\text { h.c. }
$$

Mass matrix for charged leptons and down type quarks.

$$
m_{\ell}=\left(\begin{array}{ccc}
0 & a_{\ell} \alpha_{\ell} e^{i \theta_{\ell}} & b_{\ell} \\
b_{\ell} \alpha_{\ell} & 0 & e^{i \theta_{\ell}} a_{\ell} \rho_{\ell} \\
a_{\ell} e^{i \theta_{\ell}} & b_{\ell} \rho_{\ell} & 0
\end{array}\right)
$$

Bi-unitary invariants of the mass squared matrix $M_{\ell}^{2}=m_{\ell} m_{\ell}^{\dagger}$

$$
\begin{aligned}
\left(b_{\ell} \rho_{\ell}\right)^{2} & \approx m_{3}^{2} \\
\left(b_{\ell}^{3} \rho_{\ell} \alpha_{\ell}\right)^{2} & \approx m_{1}^{2} m_{2}^{2} m_{3}^{2} \\
\left(a_{\ell} b_{\ell} \rho_{\ell}^{2}\right)^{2} & \approx m_{2}^{2} m_{3}^{2}
\end{aligned}
$$

$$
\begin{aligned}
& \rho_{\ell} \gg \alpha_{\ell} \\
& \rho_{\ell} \gg 1 \\
& b_{\ell}>a_{\ell} \\
& \rho_{\ell} \gg \frac{b_{\ell}}{a_{\ell}}
\end{aligned}
$$

## Model

## Flavour structure provides the generalised bottom-tau mass

 relation.$$
\begin{aligned}
\left(b_{\ell} \rho_{\ell}\right)^{2} & \approx m_{3}^{2} \\
\left(b_{\ell}^{3} \rho_{\ell} \alpha_{\ell}\right)^{2} & \approx m_{1}^{2} m_{2}^{2} m_{3}^{2} \\
\left(a_{\ell} b_{\ell} \rho_{\ell}^{2}\right)^{2} & \approx m_{2}^{2} m_{3}^{2} \\
& \text { Leading to }
\end{aligned}
$$

Solving

$$
a_{\ell} \approx \frac{m_{2}}{m_{3}} \sqrt{\frac{m_{1} m_{2}}{\alpha_{\ell}}}
$$

$$
\rho_{\ell} \gg \alpha_{\ell}
$$

$$
\rho_{\ell} \gg 1
$$

$$
b_{\ell}>a_{\ell}
$$

$$
\rho_{\ell} \gg \frac{b_{\ell}}{a_{\ell}}
$$

$$
\begin{aligned}
& b_{\ell} \approx \sqrt{\frac{m_{1} m_{2}}{\alpha_{\ell}}} \\
& \frac{\rho_{\ell}}{\sqrt{\alpha_{\ell}}} \approx \frac{m_{3}}{\sqrt{m_{1} m_{2}}}
\end{aligned}
$$

$$
\frac{m_{\tau}}{\sqrt{m_{e} m_{\mu}}}=\frac{m_{b}}{\sqrt{m_{d} m_{s}}}
$$

as

$$
\begin{aligned}
\rho_{\ell} & =\rho_{d} \\
\alpha_{\ell} & =\alpha_{d}
\end{aligned}
$$

## Results

No actual perdition for the CKM, but it can be fitted in the model. However there are predictions for the LMM (PMNS).

$$
\mathcal{L}_{Y} \supset \cdot Y_{\nu}^{i}[\bar{L}, \phi]_{3_{i}} \nu_{R}+\text { h.c. }
$$

Neutrino mass matrix

$$
m_{\nu}=\left(\begin{array}{ccc}
0 & a_{\nu} \alpha_{\nu} & b_{\nu} e^{i \theta_{\nu}} \\
b_{\nu} e^{i \theta_{\nu}} \alpha_{\nu} & 0 & a_{\nu} \rho_{\nu} \\
a_{\nu} & b_{\nu} e^{i \theta_{\nu}} \rho_{\nu} & 0
\end{array}\right)
$$

real Yukawas $\quad a_{\nu}=v_{\phi_{2}}\left(Y_{\nu}^{1}+Y_{\nu}^{3}\right) \quad b_{\nu}=v_{\phi_{2}}\left(Y_{\nu}^{2}+Y_{\nu}^{4}\right)$

$$
\alpha_{\nu}=v_{\phi 3} / v_{\phi 2}
$$

$$
\rho_{\nu}=v_{\phi 1} / v_{\phi 2}
$$

$\theta_{\nu}$ CPV phase lepton sector

## Results

$$
m_{\nu}=\left(\begin{array}{ccc}
0 & a_{\nu} \alpha_{\nu} & b_{\nu} e^{i \theta_{\nu}} \\
b_{\nu} e^{i \theta_{\nu}} \alpha_{\nu} & 0 & a_{\nu} \rho_{\nu} \\
a_{\nu} & b_{\nu} e^{i \theta_{\nu}} & 0
\end{array}\right)
$$

Invariants of the squared neutrino $\quad M_{\nu}^{2}=m_{\nu} m_{\nu}^{\dagger}$ matrix

$$
\begin{array}{ll}
\left(a_{\nu}^{2}+b_{\nu}^{2}\right)\left(1+\alpha_{\nu}^{2}+\rho_{\nu}^{2}\right), & =m_{1}^{2}+m_{2}^{2}+m_{3}^{2} \\
\left(a_{\nu}^{6}+b_{\nu}^{6}+2 a_{\nu}^{3} b_{\nu}^{3} \cos \left(3 \theta_{\nu}\right)\right) \alpha_{\nu}^{2} \rho_{\nu}^{2}, & =m_{1}^{2} m_{2}^{2} m_{3}^{2} \\
a_{\nu}^{2} b_{\nu}^{2}\left(1+\alpha_{\nu}^{4}+\rho_{\nu}^{4}\right)+\left(a_{\nu}^{4}+b_{\nu}^{4}\right)\left(\rho_{\nu}^{2}+\alpha_{\nu}^{2}\left(1+\rho_{\nu}^{2}\right)\right) & =2 m_{1}^{2} m_{2}^{2}+2 m_{2}^{2} m_{3}^{2}+2 m_{1}^{2} m_{3}^{2}
\end{array}
$$

Fixed by CKM

Lepton mixing matrix (PMNS) $\quad V=U_{l}^{\dagger} U_{\nu}$

$$
\begin{aligned}
U_{\nu}^{\dagger} m_{\nu} V_{\nu} & =D_{\nu} \\
U_{\ell}^{\dagger} m_{\ell} V_{\ell} & =D_{\ell}
\end{aligned}
$$

$$
D=\operatorname{diag}\left(m_{\nu_{1}}, m_{\nu_{2}}, m_{\nu_{3}}\right)
$$

fitting.
Close to a diagonal

## Results

## Global Neutrino Fit [P. F. de Salas, et al., 2017]

| parameter | best fit $\pm 1 \sigma$ | $2 \sigma$ range | $3 \sigma$ range |
| :--- | :---: | :---: | :---: |
| $\Delta m_{21}^{2}\left[10^{-5} \mathrm{eV}^{2}\right]$ | $7.56 \pm 0.19$ | $7.20-7.95$ | $7.05-8.14$ |
| $\left\|\Delta m_{31}^{2}\right\|\left[10^{-3} \mathrm{eV}^{2}\right](\mathrm{NO})$ | $2.55 \pm 0.04$ | $2.47-2.63$ | $2.43-2.67$ |
| $\left\|\Delta m_{31}^{2}\right\|\left[10^{-3} \mathrm{eV}^{2}\right](\mathrm{IO})$ | $2.47_{-0.05}^{+0.04}$ | $2.39-2.55$ | $2.34-2.59$ |
| $\sin ^{2} \theta_{12} / 10^{-1}$ | $3.21_{-0.16}^{+0.18}$ | $2.89-3.59$ | $2.73-3.79$ |
| $\theta_{12} /{ }^{\circ}$ | $34.5_{-1.0}^{+1.1}$ | $32.5-36.8$ | $31.5-38.0$ |
| $\sin ^{2} \theta_{23} / 10^{-1}(\mathrm{NO})$ | $4.30_{-0.18}^{+0.20}{ }^{\circ}$ | $3.98-4.78 \& 5.60-6.17$ | $3.84-6.35$ |
| $\theta_{23} /{ }^{\circ}$ | $41.0 \pm 1.1$ | $39.1-43.7 \& 48.4-51.8$ | $38.3-52.8$ |
| $\sin ^{2} \theta_{23} / 10^{-1}(\mathrm{IO})$ | $5.98_{-0.15}^{+0.17}{ }^{6}$ | $4.09-4.42 \& 5.61-6.27$ | $3.89-4.88 \& 5.22-6.41$ |
| $\theta_{23} /{ }^{\circ}$ | $50.7_{-0.9}^{+1.0}$ | $39.8-41.7 \& 48.5-52.3$ | $38.6-44.3 \& 46.2-53.2$ |
| $\sin ^{2} \theta_{13} / 10^{-2}(\mathrm{NO})$ | $2.155_{-0.075}^{+0.090}$ | $1.98-2.31$ | $1.89-2.39$ |
| $\theta_{13} /{ }^{\circ}$ | $8.44_{-0.15}^{+0.18}$ | $8.1-8.7$ | $7.9-8.9$ |
| $\sin ^{2} \theta_{13} / 10^{-2}(\mathrm{IO})$ | $2.155_{-0.092}^{+0.076}$ | $1.98-2.31$ | $1.90-2.39$ |
| $\theta_{13} /{ }^{\circ}$ | $8.44_{-0.18}^{+0.15}$ | $8.1-8.7$ | $7.9-8.9$ |
| $\delta / \pi(\mathrm{NO})$ | $1.40_{-0.20}^{+0.31}$ | $0.85-1.95$ |  |
| $\delta /{ }^{\circ}$ | $252_{-36}^{+56}$ | $153-351$ | $0.00-2.00$ |
| $\delta / \pi(\mathrm{IO})$ | $1.56_{-0.26}^{+0.22}$ | $1.07-1.97$ | $0-360$ |
| $\delta /{ }^{\circ}$ | $281_{-47}^{+39}$ | $193-355$ | $0.00-0.17 \& 0.83-2.00$ |

${ }^{a}$ There is a local minimum in the second octant, at $\sin ^{2} \theta_{23}=0.596$ with $\Delta \chi^{2}=2.1$ with respect to the global minimum.
${ }^{b}$ There is a local minimum in the first octant, at $\sin ^{2} \theta_{23}=0.426$ with $\Delta \chi^{2}=3.0$ with respect to the global minimum for IO.
TABLE I: Neutrino oscillation parameters summary determined from this global analysis. The ranges for inverted ordering refer to the local minimum of this neutrino mass ordering.

## Results

## Numerical scan in the parameter region taking as inputs the $\mathbf{3 \sigma}$ values of the neutrino oscillation parameters.



Figure 2. The regions in the atmospheric mixing angle $\theta_{23}$ and the lightest neutrino mass $m_{3}$ plane allowed by current oscillation data are the shaded (green) areas, see text. The horizontal dashed line represents the best-fit value for $\sin ^{2} \theta_{23}$, whereas the horizontal shaded region corresponds to the $1 \sigma$ allowed region from Ref. [1].

## Results

# Numerical scan in the parameter region taking as inputs the $\mathbf{3 \sigma}$ values of the neutrino oscillation parameters. 

Data from [P. F. de Salas, et al., 2017]

IO



Figure 3. Correlation between the CP violation and the lightest neutrino mass. Left: correlation between the Jarlskog invatiant and the lightest neutrino mass $m_{3}$ allowed by the current oscillation data from Ref. [1]. Right: We plot also the allowed region for the correlation between the the Dirac CP phase $\delta_{C P}$ and the lightest neutrino mass $m_{3}$.

## Results

## Numerical scan in the parameter region taking as inputs the $\mathbf{3 \sigma}$ values of the neutrino oscillation parameters.

Data from [P. F. de Salas, et al., 2017]



Figure 4. The allowed regions of the atmospheric mixing angle and $\delta_{C P}$ are indicated in shaded (green). They result from a numerical scan keeping only those choices that lie within $3 \sigma$ of their preferred best fit values Ref. [1] The unshaded regions are 90 and $99 \%$ CL regions obtained directly in the unconstrained three-neutrino oscillation global fit [1].

## Summary and Conclusions

We have propose a SM extension with underlying A4 flavour symmetry.

The model addresses both aspects of the flavour problem: the explanation of mass hierarchies of quark and leptons, as well as restricting the structure of the lepton mixing matrix.

The model predicts the golden flavour-dependent bottom-tau mass relation.

## Summary and Conclusions

Requires an IO and non-maximal atmospheric mixing angle (Neither the preference for normal ordering nor the indication for an octant are currently statistically significant).

The residual flavour symmetry forbids the Majorana mass terms at any order and provides a natural realisation of a type-II Dirac seesaw mechanism.

The CKM matrix, although no definite predictions are made, the required CKM matrix elements can be adequately described. Then the contribution to the neutrino mixing matrix that comes from the charged lepton sector is fixed.

## Gracias!

