$L^- \to \ell^- \ell'^- \ell'^+$ LFV decays in the SM with massive neutrinos

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XXXII Reunión anual de la división de partículas y campos de la SMF

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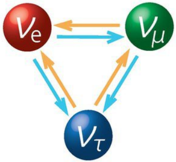
1. Motivation ($L^- \to \ell^-\ell'^-\ell'^+$ in the SM with massive neutrinos)
2. Previous calculations
   - $\mu^- \to e^-e^+e^-$ ⋆ S. T. Petcov, Sov. J. Nucl. Phys. 25, 340 (1977).
   - $\tau^- \to \mu^-\ell^+\ell^-$ † X. Y. Pham, Eur. Phys. J. C 8, 513 (1999).
3. Our computation and results
4. Summary and Conclusions

- In the original formulation of the SM the flavour sector is defined by the Yukawa lagrangian given by

$$-\mathcal{L}_Y = (Y_u)_{ij}\bar{Q}_{L_i}u_{R_j}\tilde{\Phi} + (Y_d)_{ij}\bar{Q}_{L_i}d_{R_j}\Phi + (Y_e)_{ij}\bar{L}_{L_i}e_{R_j}\Phi + h.c \qquad (1)$$

The minimality of construction of eq. (1) implies massless neutrinos and the fact that lepton flavor numbers are individually conserved at any order.

However, experimental evidence of neutrino oscillation $\Rightarrow$ LF numbers are not conserved, and claims for an extended model with tiny neutrino mass.



- The mixing of three light neutrinos could be described through U_{PMNS} matrix, which connects flavour eigenstates with mass eigestates

$$\begin{bmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{bmatrix} = \begin{bmatrix}U_{e1} & U_{e2} & U_{e3}\\ U_{\mu1} & U_{\mu2} & U_{\mu3}\\ U_{\tau1} & U_{\tau2} & U_{\tau3}\end{bmatrix}\begin{bmatrix}\nu_1\\ \nu_2\\ \nu_3\end{bmatrix} \qquad (2)$$

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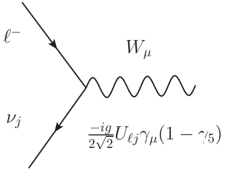
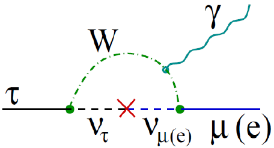
Motivation

- *SM + neutrino oscillation*



- The U_{PMNS} matrix can also give rise, at one loop level, to charged Lepton Flavor Violations (cLFV) processes as $l_i^\pm \to l_j^\pm \gamma$, $Z \to \ell_i^\pm \ell_j^\mp$, $l_i^\pm \to \ell_j^\pm \ell_k^\pm \ell_k^\mp$.



- However negligible rates are expected due to a GIM-like mechanism. *Except for* $\tau^\pm \to \mu^\pm \ell^\pm \ell^\mp$ *??????*

Experimental limits

So far, no evidence of cLFV!

Reaction	Present limit	C.L.	Experiment	Year
$\mu^+ \to e^+\gamma$	$< 4.2 \times 10^{-13}$	90%	MEG at PSI	2016
$\mu^+ \to e^+e^-e^+$	$< 1.0 \times 10^{-12}$	90%	SINDRUM	1988
$\mu^-\mathrm{Ti} \to e^-\mathrm{Ti}$ †	$< 6.1 \times 10^{-13}$	90%	SINDRUM II	1998
$\mu^-\mathrm{Pb} \to e^-\mathrm{Pb}$ †	$< 4.6 \times 10^{-11}$	90%	SINDRUM II	1996
$\mu^-\mathrm{Au} \to e^-\mathrm{Au}$ †	$< 7.0 \times 10^{-13}$	90%	SINDRUM II	2006
$\mu^-\mathrm{Ti} \to e^+\mathrm{Ca}^*$ †	$< 3.6 \times 10^{-11}$	90%	SINDRUM II	1998
$\mu^+e^- \to \mu^-e^+$	$< 8.3 \times 10^{-11}$	90%	SINDRUM	1999
$\tau \to e\gamma$	$< 3.3 \times 10^{-8}$	90%	BaBar	2010
$\tau \to \mu\gamma$	$< 4.4 \times 10^{-8}$	90%	BaBar	2010
$\tau \to eee$	$< 2.7 \times 10^{-8}$	90%	Belle	2010
$\tau \to \mu\mu\mu$	$< 2.1 \times 10^{-8}$	90%	Belle	2010
$\tau \to \pi^0 e$	$< 8.0 \times 10^{-8}$	90%	Belle	2007
$\tau \to \pi^0 \mu$	$< 1.1 \times 10^{-7}$	90%	BaBar	2007
$\tau \to \rho^0 e$	$< 1.8 \times 10^{-8}$	90%	Belle	2011
$\tau \to \rho^0 \mu$	$< 1.2 \times 10^{-8}$	90%	Belle	2011

Limits for some branching ratio of cLFV processes. Presently the best limit is on the $\mu^+ \to e^+\gamma$ decay set by the MEG experiment.

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So far, no evidence of cLFV!

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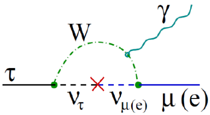
Belle-II shall be able to set limits on the $\tau \to 3\mu$ decay at the level of $3 \cdot 10^{-10}$ with their full data set $(50\, ab^{-1})$.

Belle-II Physics Book, Belle-II Collaboration and B2TIP-Community

Theoretical predictions

Lepton-flavor-violating (LFV) decays of τ



Model	Reference	τ→μγ	τ→μμμ
SM+ ν oscillations	EPJ C8 (1999) 513	10^{-40}	10^{-14}
SM+ heavy Maj ν_R	PRD 66 (2002) 034008	10^{-9}	10^{-10}
Non-universal Z'	PLB 547 (2002) 252	10^{-9}	10^{-8}
SUSY SO(10)	PRD 68 (2003) 033012	10^{-8}	10^{-10}
mSUGRA+seesaw	PRD 66 (2002) 115013	10^{-7}	10^{-9}
SUSY Higgs	PLB 566 (2003) 217	10^{-10}	10^{-7}

- Probabililty of LFV decays of charged leptons is extremely small in the Standard Model, $\mathcal{B}(\tau \to l\gamma) = \frac{3\alpha}{32\pi} |\sum_i U^*_{\tau i} U_{\mu i} \frac{\triangle^2_{3i}}{m^2_W}|^2 \leq 10^{-53} \sim 10^{-49}$
- Many models beyond the SM predict LFV decays with the branching fractions up to $\lesssim 10^{-8}$. As a result observation of LFV is a clear signature of New Physics (NP).
- τ lepton is an excellent laboratory to search for the LFV decays due to the enhanced couplings to the new particles as well as large number of LFV decay modes
- Study of the different τ LFV decay modes allows us to test various NP models.

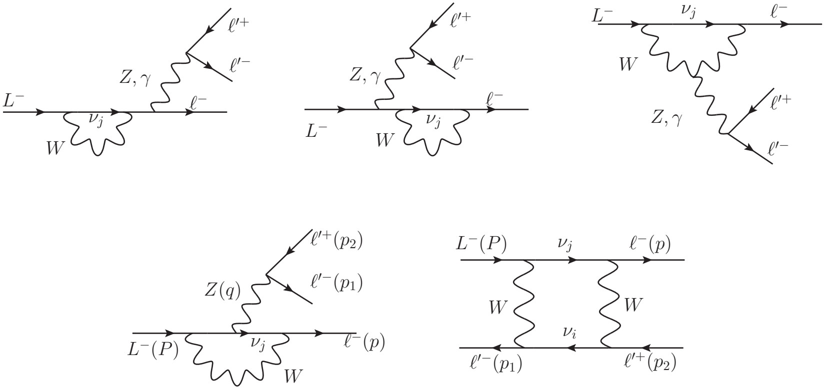
- $\mu \to e\gamma$ T. P. Cheng and L. F. Li, Gauge Theory Of Elementary Particle Physics

$$BR(\mu \to e\gamma) \simeq \frac{\Gamma(\mu \to e\gamma)}{\Gamma(\mu \to e\nu\bar{\nu})} = \frac{3\alpha}{32\pi} \left| \sum_{k=1,3} \frac{U_{\mu k} U^*_{ek} m^2_{\nu k}}{m^2_W} \right|^2 \sim 10^{-54}.$$

At this point it is very important to note that:

- *There is an unexpected difference of more than 30 orders of magnitude between the predictions for the* $\tau^{\pm} \to \mu^{\pm}\gamma$ *decay and* $\tau \to \mu^{\pm}\mu^{\pm}\mu^{\mp}$ *channel.*

Contributions to $L^- \to \ell^- \ell'^- \ell'^+$ LFV decays



Feynman diagrams for the $L^- \to \ell^- \ell'^- \ell'^+$ decays, in the presence of lepton mixing.

- $\mu^{\pm} \to e^{\pm} e^{\pm} e^{\mp}$

⋆ S. T. Petcov, Sov. J. Nucl. Phys. 25, 340 (1977).

Some relevant aspects of this calculation are

- Masses and momenta of the external particles were neglected for the penguin with two neutrino propagators and box diagrams
- Getting analytical expressions for the loop integrals is possible
- The dominant amplitudes come from the penguin with two neutrino propagators and box diagrams, and both are proportional to $\Delta_{ij} = m_i^2 - m_j^2$.

- $\tau^{\pm} \to \mu^{\pm} \ell^{\pm} \ell^{\mp}$

† X. Y. Pham, Eur. Phys. J. C 8, 513 (1999).

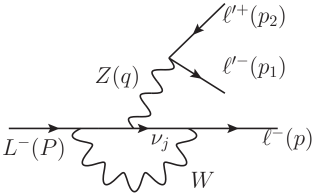
Some relevant aspects of this calculation are

- The momentum transfer q^2 by the Z boson is neglected in the denominator of the Feynman parameters integrals
- The dominant amplitude comes from the penguin diagram proportional to $\log\left(m_i^2/m_j^2\right)$.

point out that in these neutrinoless modes, the GIM cancelation is much milder with only a logarithmic behavior $\log(m_j/m_k)$ where $m_{j,k}$ are the neutrino masses. This is in sharp contrast with the vanishingly small amplitude $\tau^{\pm} \to \mu^{\pm} + \gamma$ strongly suppressed by the quadratic power $(m_j^2 - m_k^2)/M_W^2$. In comparison with the hopelessly small branching ratio $B(\tau^{\pm} \to \mu^{\pm} + \gamma) \approx 10^{-40}$, the $B(\tau^{\pm} \to \mu^{\pm} + \ell^{+} + \ell^{-})$ could be larger than 10^{-14}. The latter mode, if measurable, could give one more constraint to the lepton mixing angle $\sin 2\theta_{jk}$ and the neutrino mass ratio m_j/m_k, and therefore is complementary to neutrino oscillation

It is worth to note that:

- *In the limit of massless neutrino the behavior for the* $\mu^\pm \to e^\pm e^\pm e^\mp$ *prediction respects exactly the GIM mechanism, such as it is expected. This is not the case for* $\tau \to \mu^\pm \mu^\pm \mu^\mp$ *channel.*
- *Even worse, if the prediction for the* $\tau \to \mu^\pm \mu^\pm \mu^\mp$ *channel were right, there would be no way to cure such infrared behavior.*



The amplitude is given by

$$\mathcal{M}_1 \sim -\frac{i}{m_Z^2} l^\lambda_{L\ell} \ell_{\ell'\ell'\lambda}, \quad (3)$$

where the effective $ZL\ell$ vertex is defined by

$$l^\lambda_{L\ell} = \left(\frac{-ig}{4c_W}\right)\left(\frac{-ig}{2\sqrt{2}}\right)^2 \sum_{j=1}^{3} U^*_{\ell j} U_{Lj} \bar{u}(p) \Gamma^\lambda_j u(P), \quad (4)$$

and the loop integral is given by

$$\Gamma^\lambda_j = \int \frac{d^4k}{(2\pi)^4} \frac{\gamma_\rho(1-\gamma_5) i \left[(\not{p}+\not{k}) + m_j\right] \gamma^\lambda (1-\gamma_5) i \left[(\not{P}+\not{k}) + m_j\right] \gamma_\sigma (1-\gamma_5)(-ig^{\rho\sigma})}{\left[(p+k)^2 - m_j^2\right]\left[(P+k)^2 - m_j^2\right]\left[k^2 - m_W^2\right]}. \quad (5)$$

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Penguin Contribution

- After making the loop integration

$$\Gamma^\lambda(q^2, m_j^2) = F_a\gamma^\lambda(1-\gamma^5) + F_b\gamma^\lambda(1+\gamma^5) + F_c(P+p)^\lambda(1+\gamma^5) + F_d(P+p)^\lambda(1-\gamma^5) + F_e q^\lambda(1+\gamma^5) + F_f q^\lambda(1-\gamma^5), \quad (6)$$

We have obtained the F_k $(k = a, b...f)$ using both Feynman parametrization and Passarino-Veltman method.

- Feynman parametrization

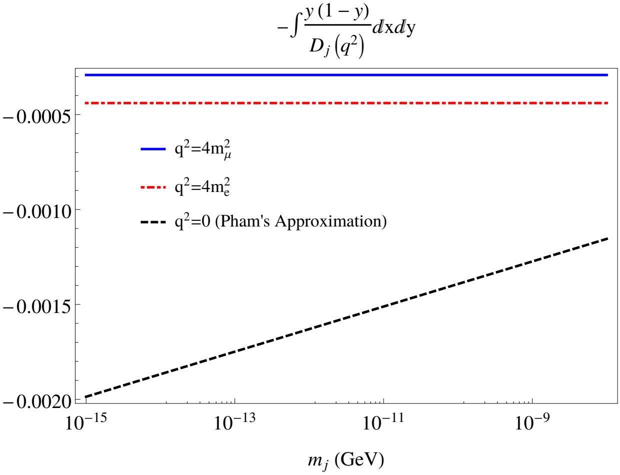
$$F_k(q^2, m_j^2) = \frac{1}{2\pi^2}\int_0^1\int_0^{1-x} f_k(q^2, m_j^2)\,dxdy, \quad (7)$$

where

$$f_a = 2 + \log\left(D_j(q^2)/\mu^2\right) + \frac{(q^2-m^2)x(y-1) + M^2x(x+y) + q^2y(y-1)}{D_j}, \quad (8)$$

and

$$D_j(q^2) = -(x-1)m_j^2 - m^2xy + xm_W^2 + M^2x(x+y-1) - q^2y(1-x-y) \quad (9)$$



$-\int \frac{y(1-y)}{D_j(q^2)} dx dy$
−0.0005
−0.0010
−0.0015
−0.0020
$q^2=4m_\mu^2$
$q^2=4m_e^2$
$q^2=0$ (Pham's Approximation)
10^{-15}
10^{-13}
10^{-11}
10^{-9}
m_j (GeV)

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Penguin Contribution

- Passarino-Veltman

$$F_k(q^2, m_j^2) = \frac{1}{2\pi^2}\frac{N_{PV_k}}{D_{PV_k}}, \quad (10)$$

with

$$\begin{aligned} D_{PV_a} = 2D_{PV_b} &= -\lambda(m^2, M^2, q^2), \\ D_{PV_c} = D_{PV_e} &= \frac{M}{2}D^2_{PV_a} \quad D_{PV_d} = D_{PV_f} = \frac{m}{2}D^2_{PV_a}, \end{aligned} \quad (11)$$

$$\begin{aligned} N_{PV_k} = & \ \xi_{k_1} B_0(m^2, m_j^2, m_W^2) + \xi_{k_2} B_0(M^2, m_j^2, m_W^2) + \xi_{k_3} B_0(q^2, m_j^2, m_j^2) \\ & + \xi_{k_4} B_0(0, m_j^2, m_W^2) + \xi_{k_5} C_0(m^2, M^2, q^2, m_j^2, m_W^2, m_j^2) + \xi_{k_0}. \end{aligned}$$

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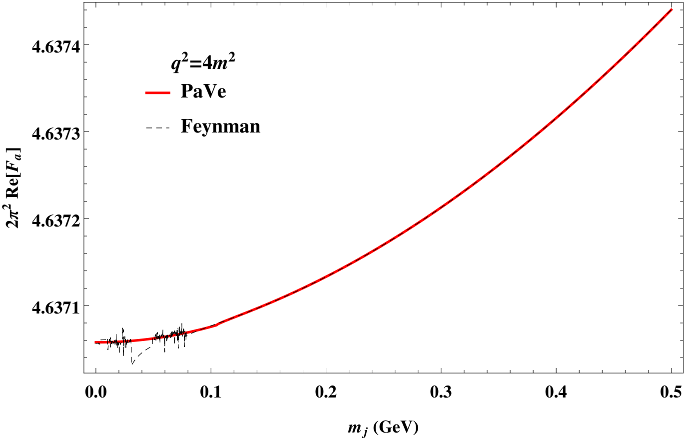

Comparison for the F_a of the effective $Z\tau\mu$ vertex as function of m_j considering a fixed value of $q^2 = 4m_\mu^2$. Black dashed line stands for the numerical evaluation of the Feynman parameters integrals, whereas the red line corresponds to the evaluation of the PaVe functions. Good agreement is found for $m_j \geq 0{,}8$ GeV, while the Feynman parameters integrals do not converge well for lower values of m_j.

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- Owing to the fact that the q^2 minimum in the $L^- \to \ell^- \ell'^+ \ell'^-$ decay is given by $4m_{\ell'}^2$, a better approximation for the F_k functions in the physical region for the neutrinos masses can be obtained fitting the curves for the real and imaginary parts of the F_k functions evaluated in terms of the PaVe functions. We have found a reasonably good fit (excellent for the real parts) of the form

$$
\begin{aligned}
Re[F_k] &= \frac{1}{2\pi^2 u}\left(Q_{R_k} + \frac{m_j^2}{m_W^2} R_{R_k}\right), \\
Im[F_k] &= \frac{1}{2\pi^2 u}\left(Q_{I_k} + \frac{m_j^2}{m_W^2} R_{I_k}\right), \qquad (12)
\end{aligned}
$$

where $u = 1$ for $k = a, b$ and $u = M$ for $k = c, d, e, f$.

$Z\tau\mu$ ($q^2 = 4m_\mu^2$)	Q_{R_k}	R_{R_k}	Q_{I_k}	R_{I_k}
F_a	4,63706	11,5451	$-7,14896 \times 10^{-6}$	3,4098
F_b	$1,38093 \times 10^{-5}$	$-3,31777 \times 10^{-4}$	$9,85094 \times 10^{-11}$	$-6,76208 \times 10^{-5}$
F_c	$-1,49047 \times 10^{-5}$	$3,62348 \times 10^{-3}$	$-7,884 \times 10^{-10}$	$5,4035 \times 10^{-4}$
F_d	$-9,20638 \times 10^{-6}$	$1,2469 \times 10^{-4}$	$-4,9267 \times 10^{-11}$	$3,38191 \times 10^{-5}$
F_e	$2,04592 \times 10^{-3}$	191,959	$4,69628 \times 10^{-4}$	$-126,096$
F_f	$-1,26365 \times 10^{-5}$	$-11,8554$	$-2,95163 \times 10^{-5}$	8,05527

Values for the Q_{R_k} (Q_{I_k}) and R_{R_k} (R_{I_k}) coefficients of the $Z\tau\mu$ vertex for $q^2 = 4m_\mu^2$.

Penguin contribution

Decay channel	Our result	Petcov's approximation
$\mu^- \to e^- e^+ e^-$	$7{,}3 \cdot 10^{-55}$	$1{,}1 \cdot 10^{-53}$
$\tau^- \to e^- e^+ e^-$	$5{,}4 \cdot 10^{-56}$	$1{,}8 \cdot 10^{-54}$
$\tau^- \to \mu^- e^+ e^-$	$7{,}9 \cdot 10^{-55}$	$2{,}6 \cdot 10^{-53}$
$\tau^- \to e^- \mu^+ \mu^-$	$3{,}2 \cdot 10^{-56}$	$1{,}1 \cdot 10^{-54}$
$\tau^- \to \mu^- \mu^+ \mu^-$	$1{,}1 \cdot 10^{-54}$	$3{,}8 \cdot 10^{-53}$

Branching ratios for the $L^- \to \ell^- \ell'^- \ell'^+$ decays (neglecting the box contribution), which are obtained using the current knowledge of the PMNS matrix.

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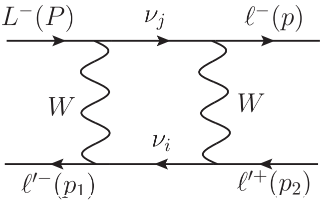
Box contribution



After a lot Dirac algebra, the amplitude for the box diagram is given by

$$\mathcal{M} = \left(\frac{-ig}{2\sqrt{2}}\right)^4 \sum_j \sum_i U_{Lj} U^*_{lj} U_{\ell' i} U^*_{\ell' i} T_{\sigma\sigma'} I^{\sigma\sigma'}$$

where we have defined

$$T_{\sigma\sigma'} = 4\,\bar{u}(p)\gamma_\mu\gamma_\sigma\gamma_\nu(1-\gamma_5)u(P)\bar{u}(p_1)\gamma_\nu\gamma'_\sigma\gamma_\mu(1-\gamma_5)v(p_2)$$

and the relevant loop integral is

$$I^{\sigma\sigma'} = \int \frac{d^4k}{(2\pi)^4} \frac{(P+k)^\sigma (k+p_1)^{\sigma'}}{(k^2-m_W^2)((p_1+p_2+k)^2-m_W^2)((P+k)^2-m_j^2)((k+p_1)^2-m_i^2)}.$$

Box contributions

Now, since the amplitude depends only on P, p_1 and p_2 the integral must be take to the form

$$\begin{aligned} I^{\sigma\sigma'} &= g^{\sigma\sigma'}H_a + P^{\sigma}P^{\sigma'}H_b + P^{\sigma}p_1^{\sigma'}H_c + P^{\sigma}p_2^{\sigma'}H_d + p_1^{\sigma}P^{\sigma'}H_e \\ &+ p_1^{\sigma}p_1^{\sigma'}H_f + p_1^{\sigma}p_2^{\sigma'}H_g + p_2^{\sigma}P^{\sigma'}H_h + p_2^{\sigma}p_1^{\sigma'}H_i + p_2^{\sigma}p_2^{\sigma'}H_j. \end{aligned} \tag{13}$$

Just like the penguin contribution, we have obtained the H_k $(k = a, b...j)$ functions in terms of both Feynman parameters integrals and PaVe functions.

- It is clear that in the simple case, where masses an momenta of the external particles are neglected the only contribution is given by the H_a function.
- Besides, the H_k $(k =, b, c...j)$ are associated to dimension 6 operator, thus they are naturally suppresed.

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Box contributions

- Feynman

$$H_k = \frac{i}{16\pi^2}\int_0^1 dx \int_0^{1-x} dy \int_0^{1-x-y} h_k(s_{12}, s_{13}, m_j^2, m_i^2)dz \quad (14)$$

where

$$h_a = -\frac{1}{2M_F^2}, \quad h_b = \frac{z(z-1)}{M_F^4}, \quad h_c = -\frac{(z-1)(x+z)}{M_F^4}, \quad h_d = \frac{y(z-1)}{M_F^4}$$

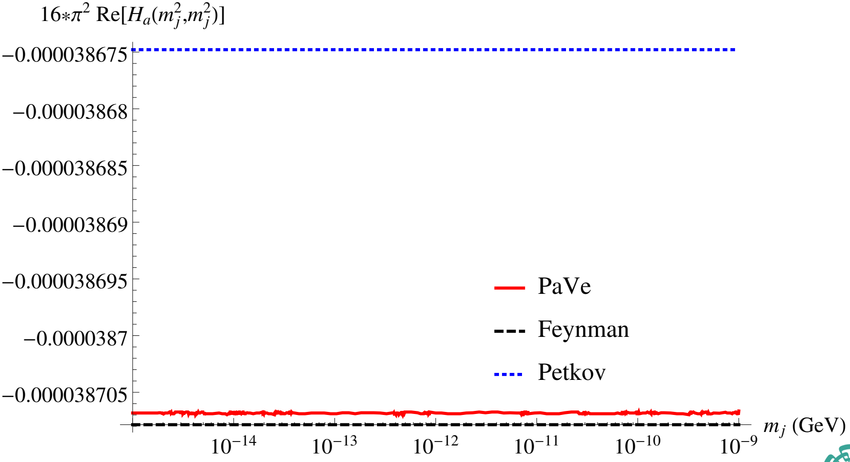
$$h_e = -\frac{z(x+z-1)}{M_F^4}, \quad h_f = \frac{(x+z-1)(x+z)}{M_F^4}, \quad h_g = -\frac{y(x+z-1)}{M_F^4}, \quad h_h = \frac{yz}{M_F^4}$$

$$h_i = -\frac{y(x+z)}{M_F^4}, \quad h_j = \frac{y^2}{M_F^4}.$$

and

$$\begin{aligned} M_F^2 &= -m_j^2(x+y-1) + m_l^2(x+y-1)(x+y) + m_W^2(x+y) - s_{12}xy \\ &+ z\left(m_i^2 - m_j^2 + (x+y)\left(3m_l^2 - s_{12} - s_{13}\right) - 2m_l^2 + m^2(x-1) + M^2(y-1) + s_{12} \right. \\ &+ z^2\left(2m_l^2 + m^2 + M^2 - s_{12} - s_{13}\right). \end{aligned}$$

Box contributions



Summary and Conclusions

We have revisited the $L^- \to \ell^- \ell'^- \ell'^+$ decays in the SM with massive neutrinos. We obtained expressions for the relevant loop integrals in terms of both Feynman parameters and Pasarino-Veltman functions without any approximation. Opposed to the previous calculation reported in †, we found that all the possible amplitudes for these processes are strongly suppressed (proportional to the neutrino mass square). In the particular case of the penguin contribution with two neutrino propagators, we highlighted that it is crucial to maintain the dependence on the momentum transfer in the Feynman integrals in order to evaluate the amplitude in the physical region for the neutrino masses. This fact avoids the incorrect divergent logarithm behavior in the amplitude.

As far as the box contribution concerned, we found that the dominant term comes from H_a function that is associated with a (V-A)×(V-A) operator, and it is in a good agreement with the approximation done in ref ⋆.

We conclude that any signal at Belle-II (or forthcoming facilities) of these modes would be an irrefutable new physics manifestation.

⋆ S. T. Petcov, Sov. J. Nucl. Phys. 25, 340 (1977).

† X. Y. Pham, Eur. Phys. J. C 8, 513 (1999).

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Thank you!

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