NEUTRINO OSCILLATIONS FOR THE STUDY OF THE EARTH'S INTERIOR

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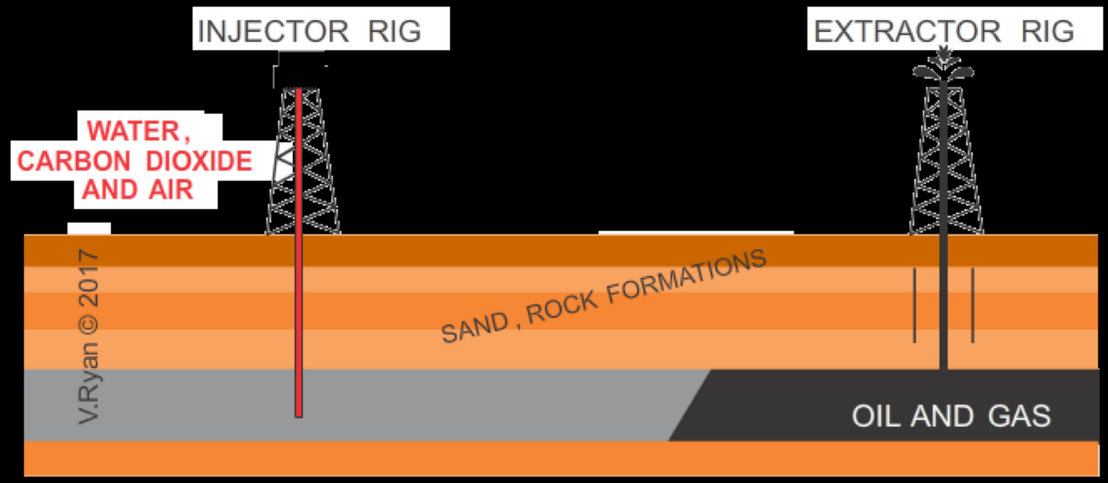
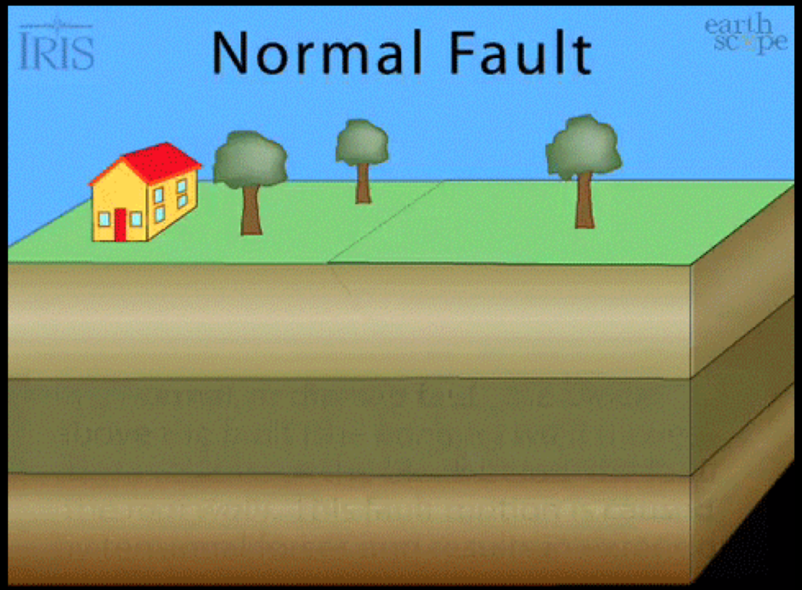
INTRODUCTION

Why to study the Earth's interior?

The knowledge of the composition and inner structure is essential for the understanding of the basic geological phenomena, such as volcanology, earthquakes and mountain formation.

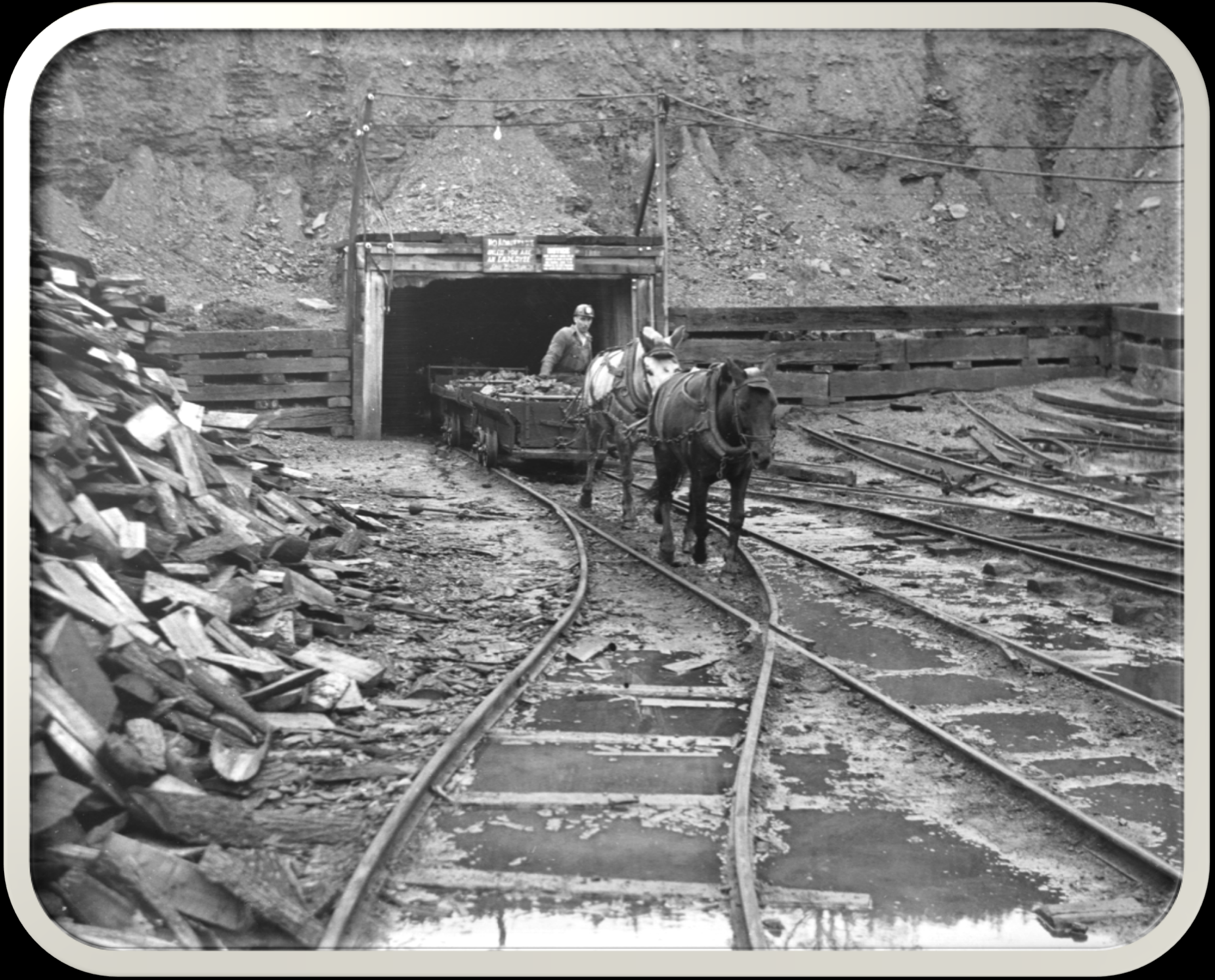
Among other applications, are also the extraction of minerals and the location of oil fields.

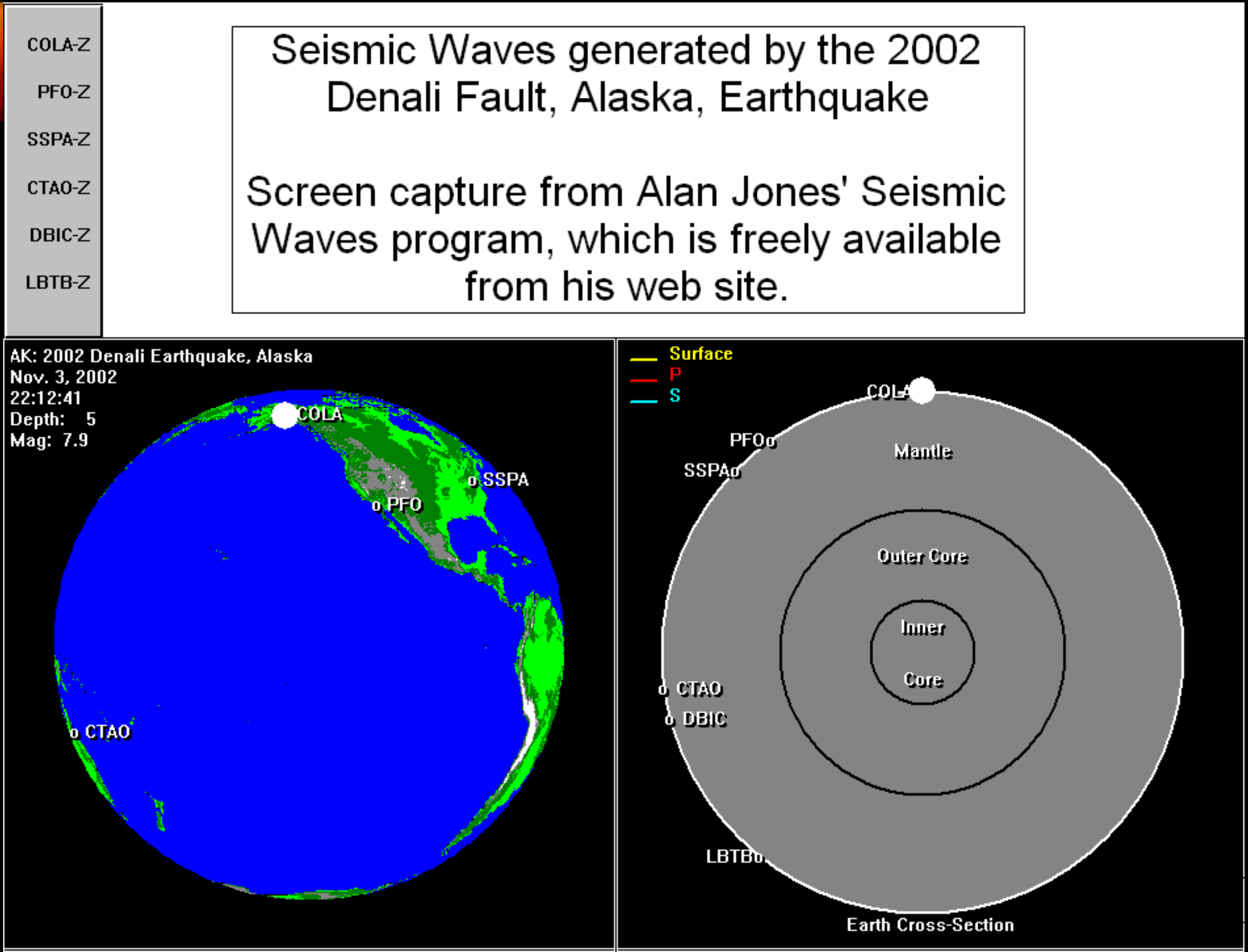


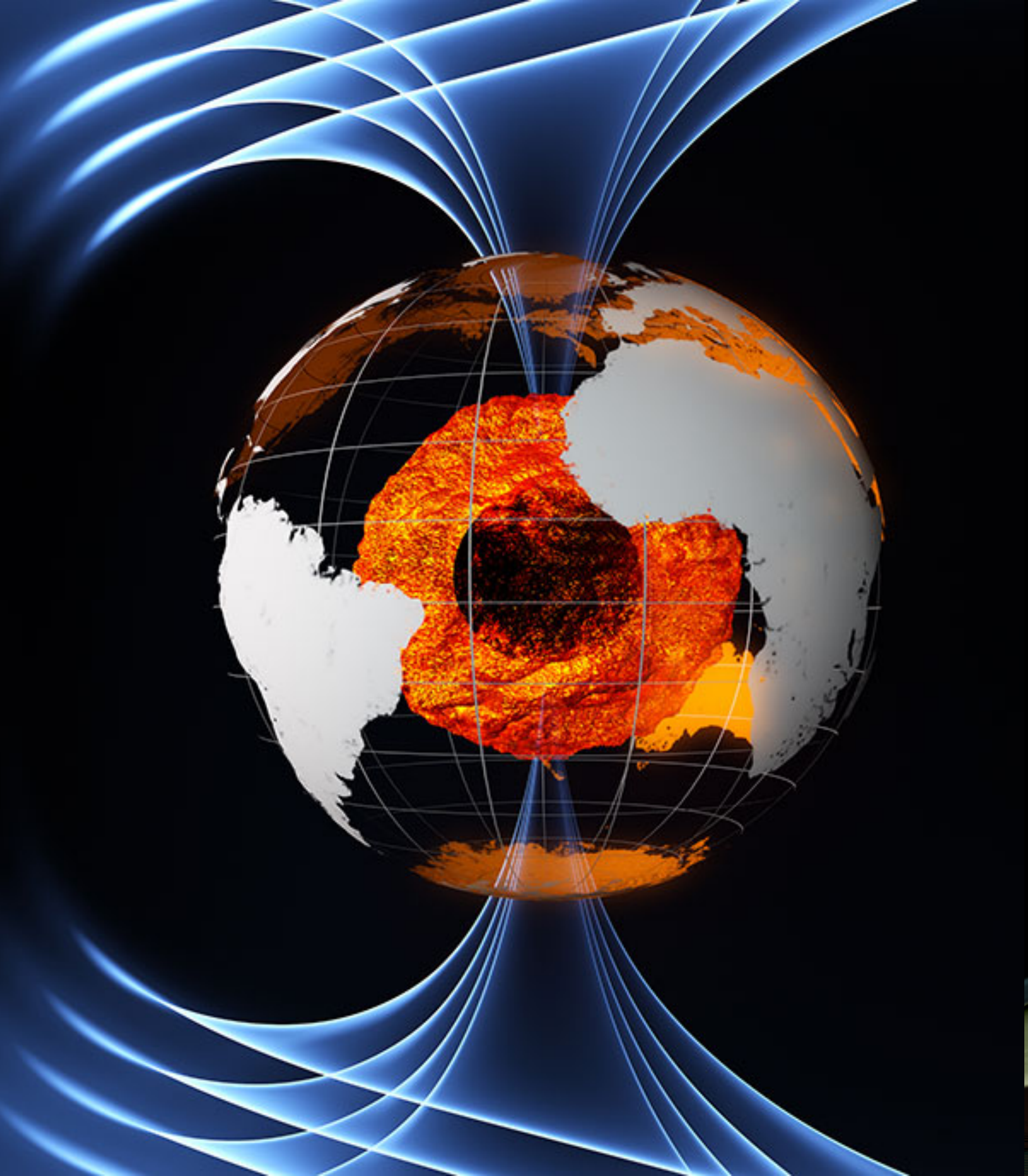


WHAT METHODS ARE THERE TO STUDY THE INTERIOR OF THE EARTH?

The two main ways to study the internal composition of our planet are the **direct extraction of minerals and rocks** from the surface and the **study of the propagation of seismic waves**.




COLA-Z
PFO-Z
SSPA-Z
CTAO-Z
DBIC-Z
LBTB-Z
Seismic Waves generated by the 2002 Denali Fault, Alaska, Earthquake
Screen capture from Alan Jones' Seismic Waves program, which is freely available from his web site.
AK: 2002 Denali Earthquake, Alaska
Nov. 3, 2002
22:12:41
Depth: 5
Mag: 7.9
COLA
o SSPA
o PFO
o CTAO
Surface
P
S
COLA
PFOo
SSPAo
Mantle
Outer Core
Inner
Core
o CTAO
o DBIC
LBTB
Earth Cross-Section



Outer core

Fundamental roles

- Convection currents
- Tectonic plates
- Earth magnetic field

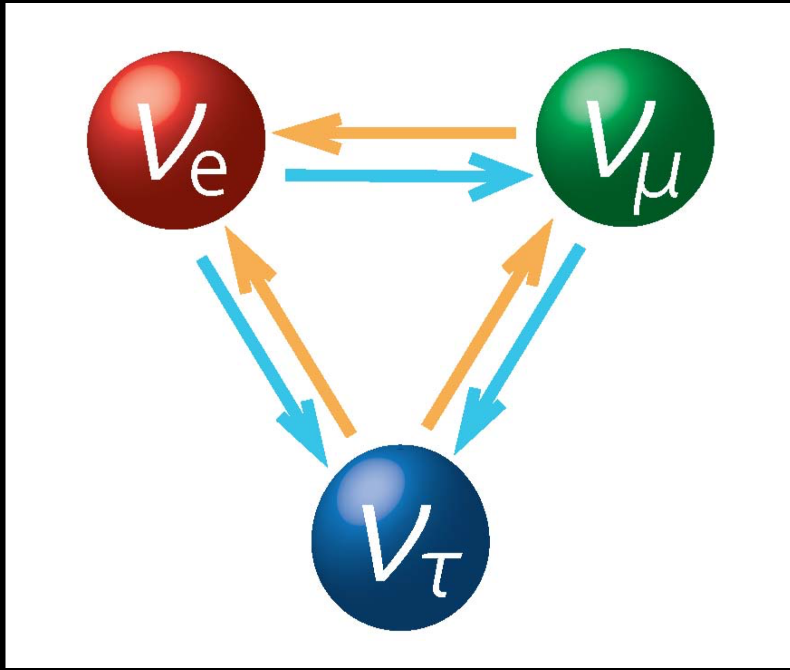
Composition

Composed mainly of **iron** together with other elements not yet determined. These could be: nickel, oxygen, carbon, silicon, hydrogen and sulfur.

NEUTRINO OSCILLATIONS

A way to determine the presence of the elements that constitute the interior of the Earth is to use the phenomenon of neutrino oscillations.

An interesting feature of neutrino oscillations is that the conversion of one flavor to another is affected by the density of electrons that exist in the medium where they propagate. The main motivation of this work was to try to show that the phenomenon of oscillations can help to determine the structural composition of the Earth, in particular that of the outer core.



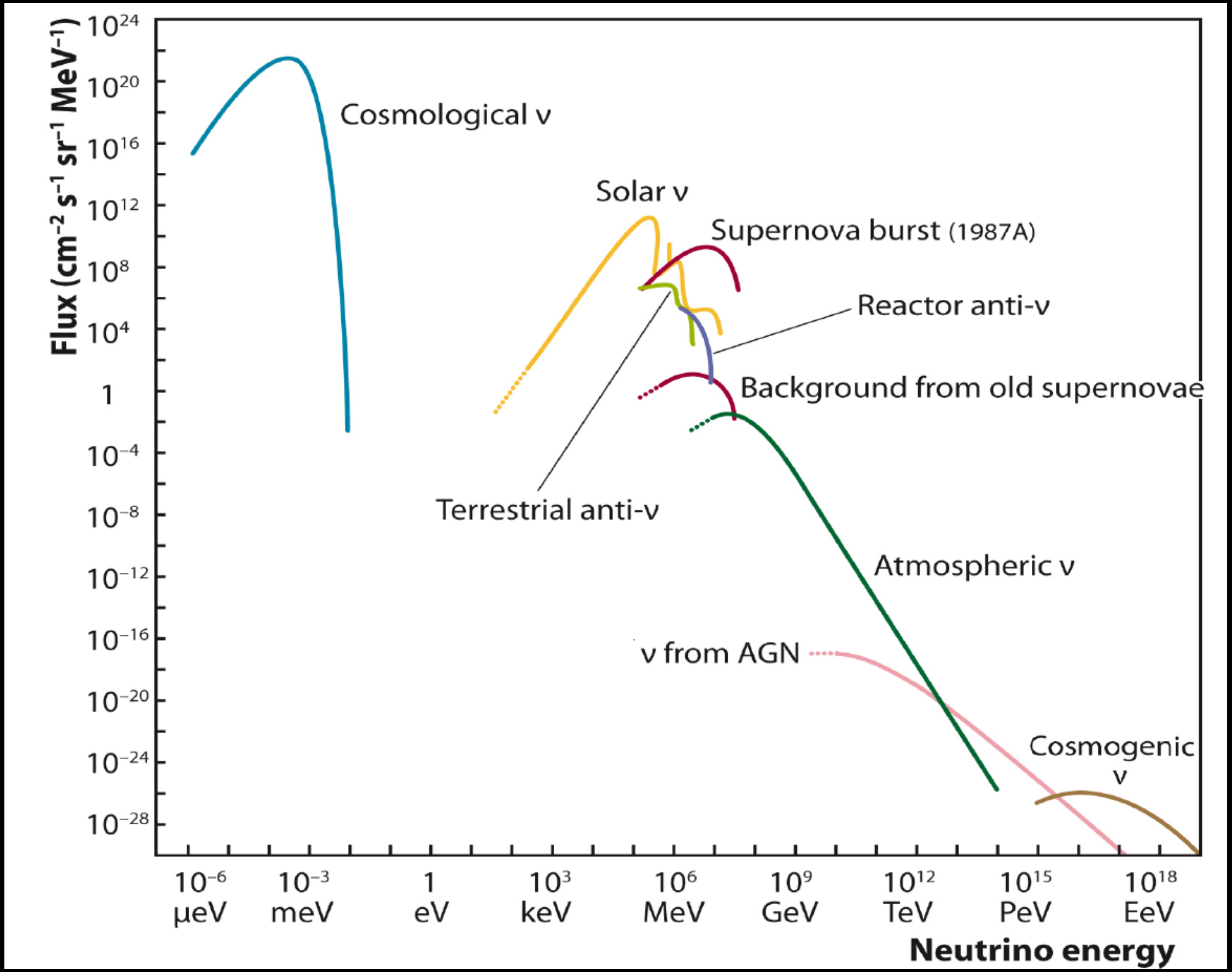
NEUTRINO SOURCES

Natural sources:

- Sun
- Supernova
- Big Bang
- Atmospheric

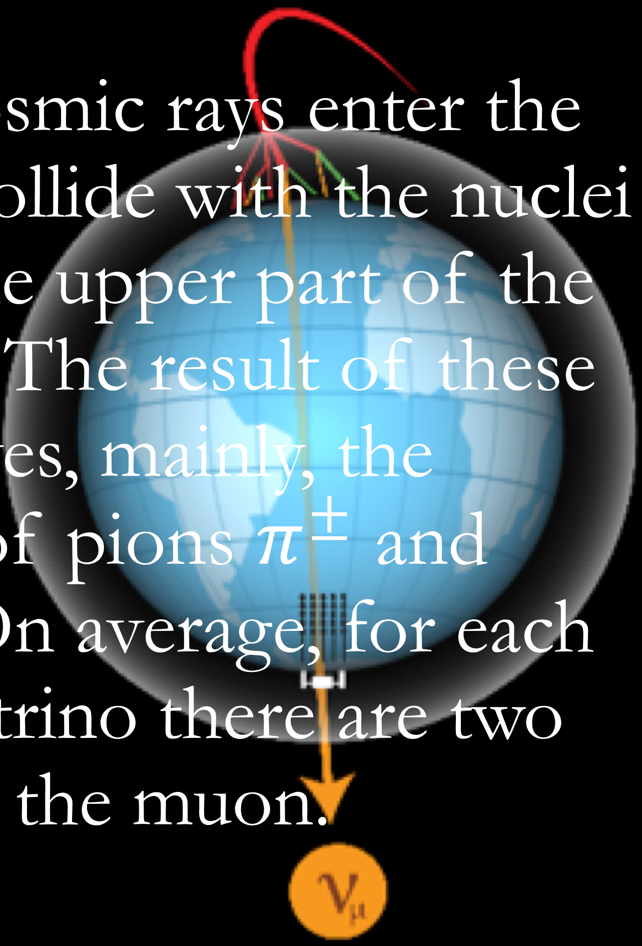
Artificial sources:

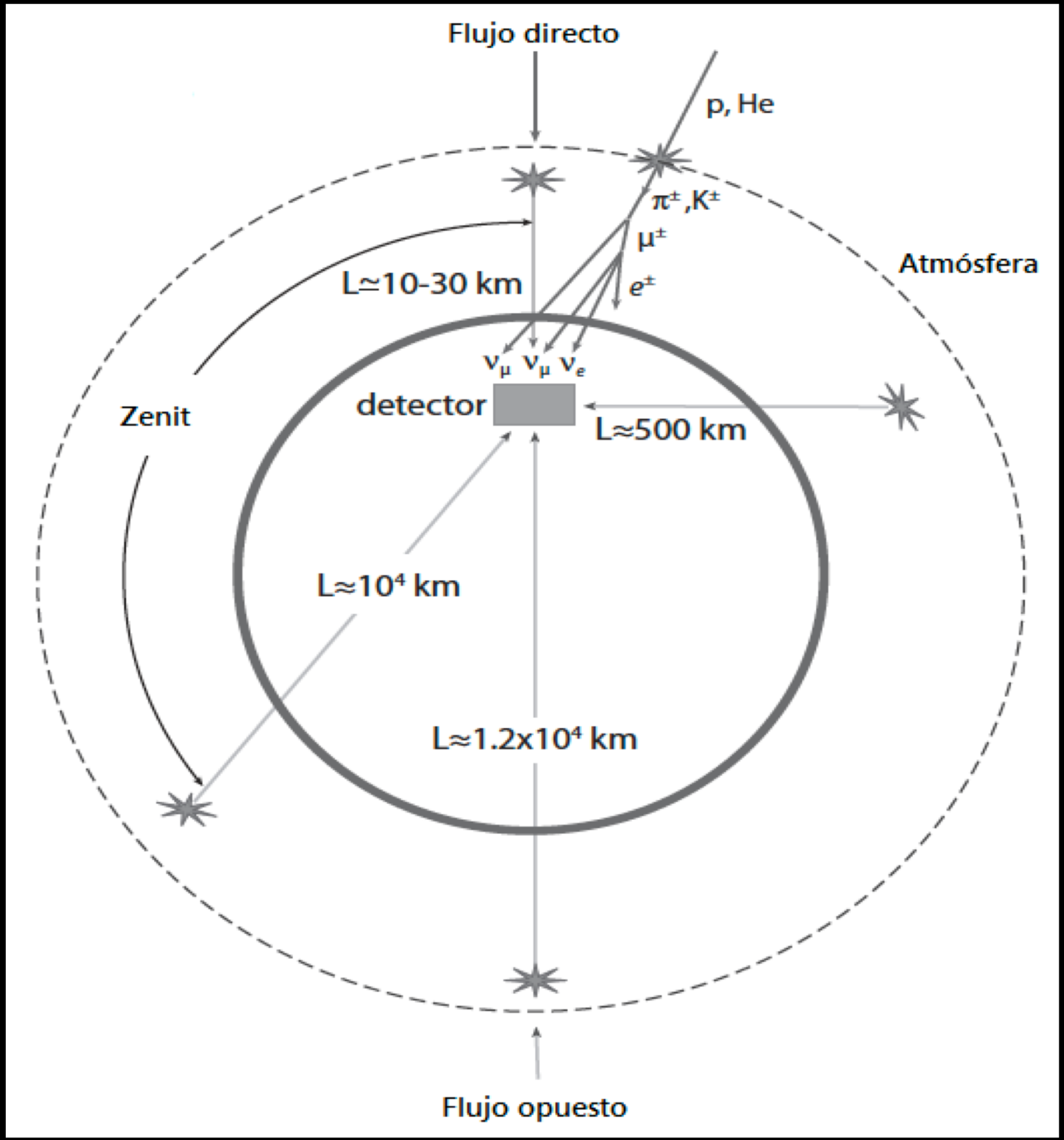
- Nuclear reactors
- Accelerators



ATMOSPHERIC NEUTRINOS

When the cosmic rays enter the Earth they collide with the nuclei that are in the upper part of the atmosphere. The result of these collisions gives, mainly, the production of pions $\pi^{\pm}$ and kaons $K^{\pm}$. On average, for each electron neutrino there are two neutrinos of the muon.





NEUTRINO OSCILLATIONS

Oscillations in vacuum

$$\mathcal{P}_{\nu_\alpha \to \nu_\beta}(t) = \sum_j |U_{\beta j}|^2 |U_{\alpha j}|^2 + 2 \sum_{j<k} \mathrm{Re}\left(U^*_{\beta k}\, U_{\alpha k}\, U_{\beta j}\, U^*_{\alpha j}\, e^{-\frac{i}{\hbar}(t-t_0)\Delta_{kj}} \right) \qquad \Delta_{kj} \equiv \frac{(m_k^2 - m_j^2)c^4}{2E}$$

Principal Oscillations

$$\mathcal{P}_{\nu_e \to \nu_e}(r) \simeq 1 - \sin^2 2\theta_{13} \sin^2 \phi_{31}$$

$$\mathcal{P}_{\nu_\mu \to \nu_\mu}(r) \simeq 1 - (\mathrm{c}_{13}^4 \sin^2 2\theta_{23} + \mathrm{s}_{23}^2 \sin^2 2\theta_{13}) \sin^2 \phi_{31}$$

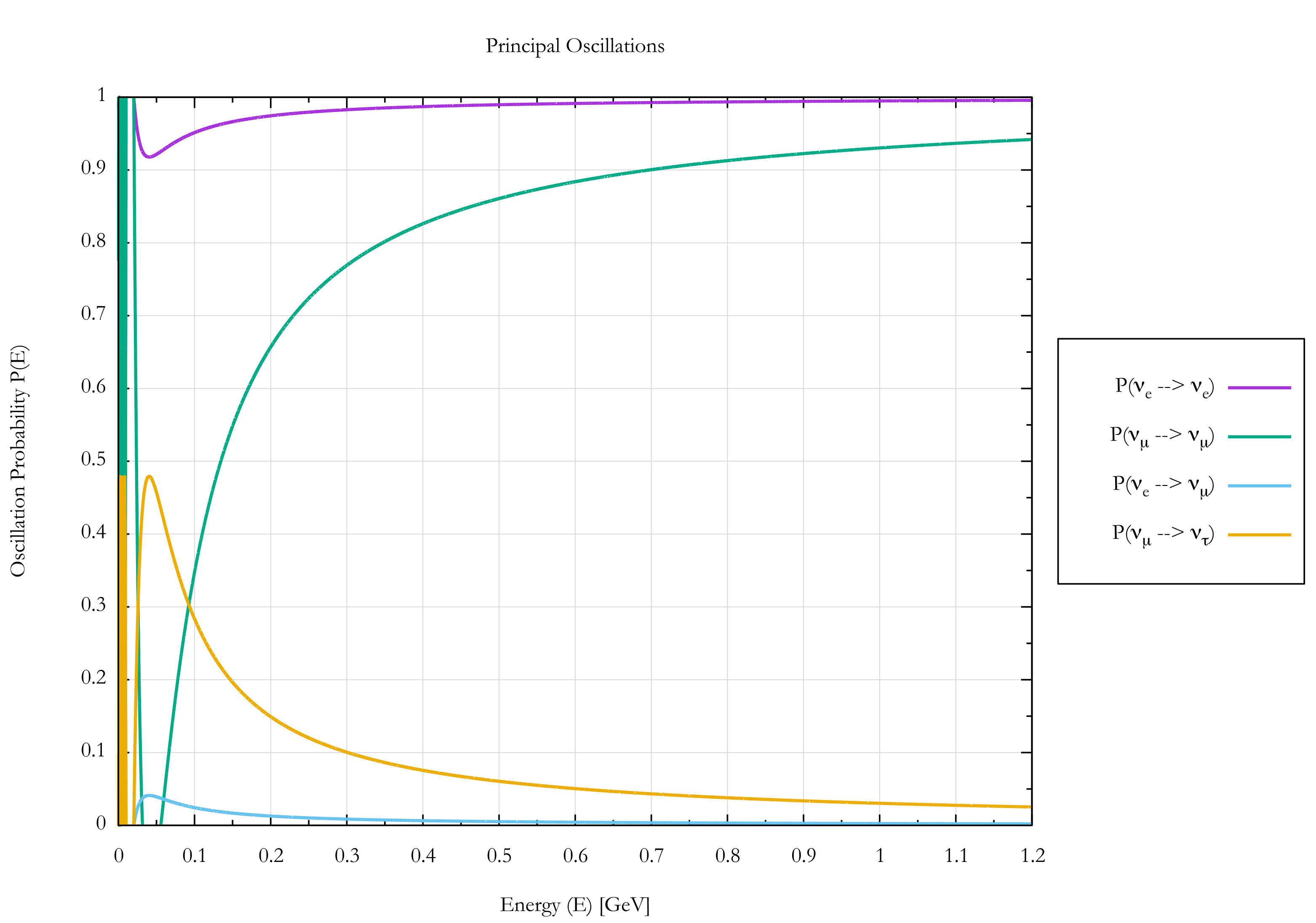
$$\mathcal{P}_{\nu_e \to \nu_\mu}(r) \simeq \mathrm{s}_{23}^2 \sin^2 2\theta_{13} \sin^2 \phi_{31}$$

$$\mathcal{P}_{\nu_\mu \to \nu_\tau}(r) \simeq \mathrm{c}_{13}^4 \sin^2 2\theta_{23} \sin^2 \phi_{31}$$

$$\phi_{31} = \frac{(m_3^2 - m_1^2)}{4E} \frac{c^3}{\hbar}(r - r_0)$$

NEUTRINO OSCILLATIONS

Oscillations in vacuum



NEUTRINO OSCILLATIONS

Oscillations in matter

The presence of matter in the trajectory of neutrinos can affect the transition probabilities. The result of this interaction is the addition of a potential to the Hamiltonian in the Schrödinger equation.

$$H_{\text{eff}}(r) = H_{\text{vac}} + V_m(r)$$

$$V_m(r) = \begin{pmatrix} V_{\text{e}}(r) & 0 & 0 \\ 0 & V_\mu(r) & 0 \\ 0 & 0 & V_\tau(r) \end{pmatrix} = \begin{pmatrix} V_{\text{CC}}(r) + V_{\text{CN}}(r) & 0 & 0 \\ 0 & V_{\text{CN}}(r) & 0 \\ 0 & 0 & V_{\text{CN}}(r) \end{pmatrix}$$

$$V_{\text{CC}}(r) = \begin{cases} +\sqrt{2}G_{\text{F}}N_{\text{e}}(r) & \text{for } \nu_{\text{e}} \\ -\sqrt{2}G_{\text{F}}N_{\text{e}}(r) & \text{for } \bar{\nu}_{\text{e}} \end{cases}$$

$$V_{\text{CN}}(r) = \begin{cases} -\dfrac{1}{\sqrt{2}}G_{\text{F}}N_{\text{n}}(r) & \text{for } \nu \\ +\dfrac{1}{\sqrt{2}}G_{\text{F}}N_{\text{n}}(r) & \text{for } \bar{\nu} \end{cases}$$

NEUTRINO OSCILLATIONS

Oscillations in matter

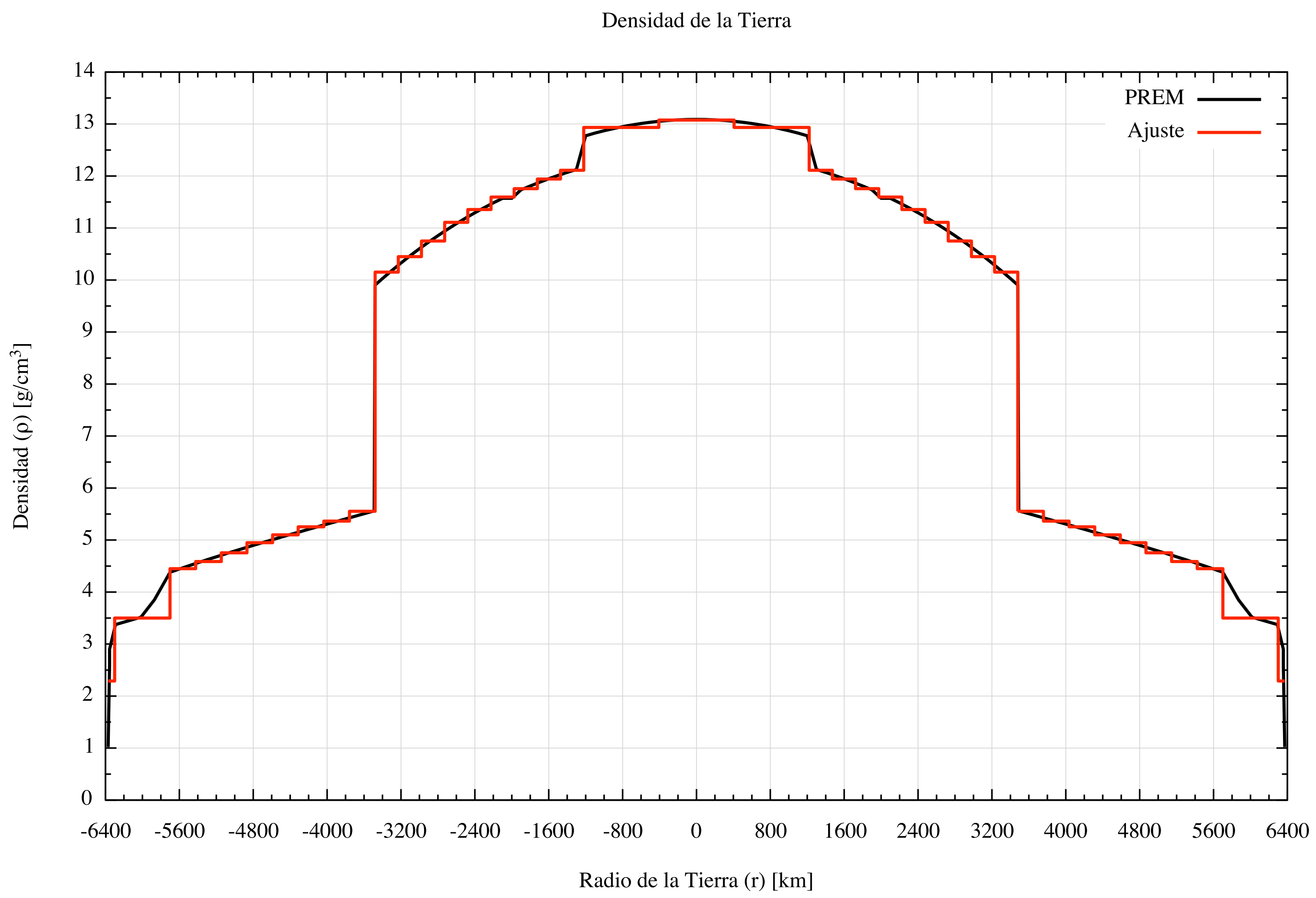
By means of a phase transformation in the Schrödinger equation and a few algebraic manipulations, it is possible to get rid of the neutral current term V_{CC} to obtain the Hamiltonian:

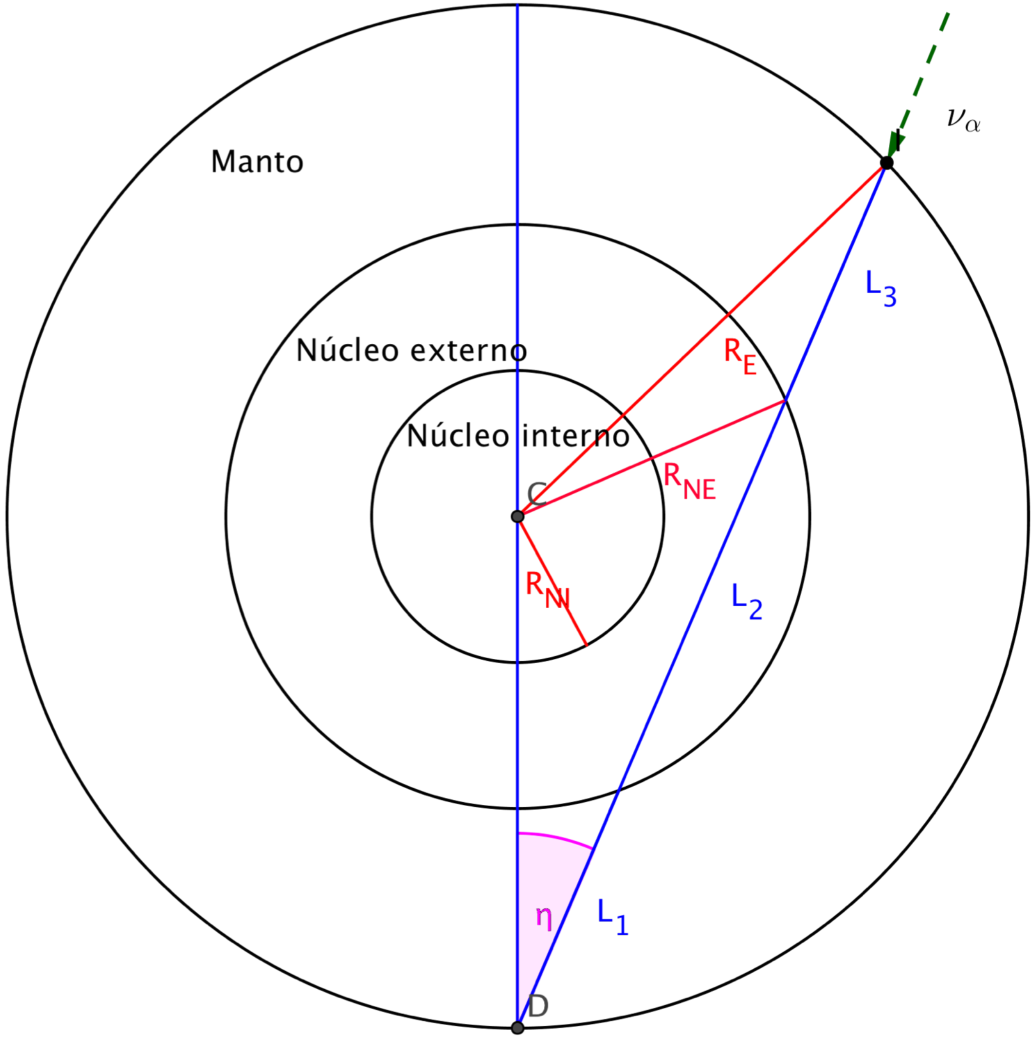
$$H(r) = U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta_{21} & 0 \\ 0 & 0 & \Delta_{31} \end{pmatrix} U^\dagger + \begin{pmatrix} V_{\mathrm{CC}}(r) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \text{with} \quad V_{\mathrm{CC}}(r) = \pm\sqrt{2} G_{\mathrm{F}} \frac{Z}{A} \frac{\rho(r)}{m_{\mathrm{N}}} \approx \pm 7.5721 \times 10^{-14} \frac{Z}{A} \rho \left[\frac{\mathrm{g}}{\mathrm{cm}^3}\right](r) \ \mathrm{eV}$$

$$i\hbar c \frac{\mathrm{d}\Psi(r)}{\mathrm{d}r} = H(r)\,\Psi(r) \qquad \Psi(r) = \hat{\mathcal{U}}(r, r_0)\Psi(r_0) \qquad \hat{\mathcal{U}}(r, r_0) = e^{-\frac{i}{\hbar c}(r - r_0)H}$$

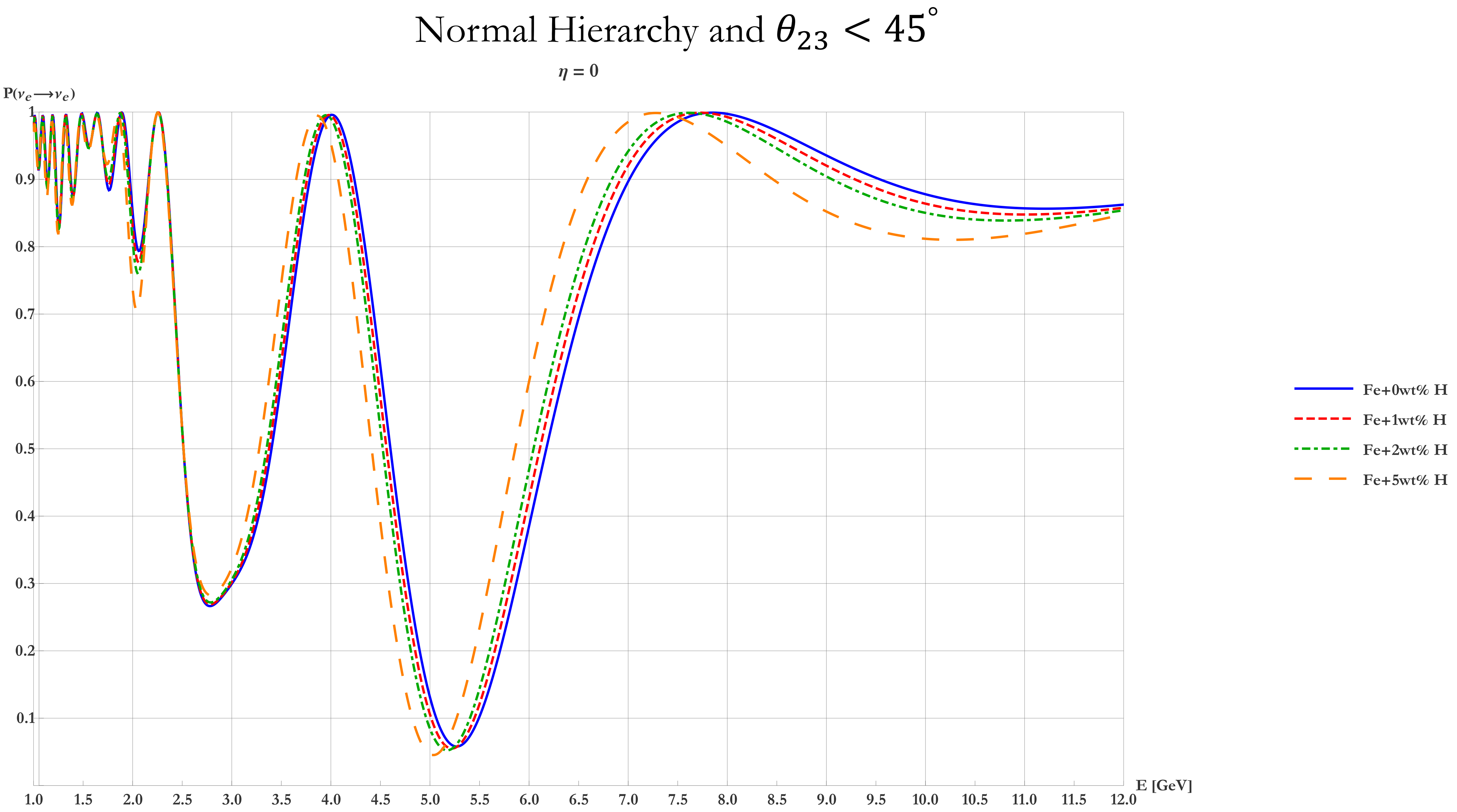
Matrix element

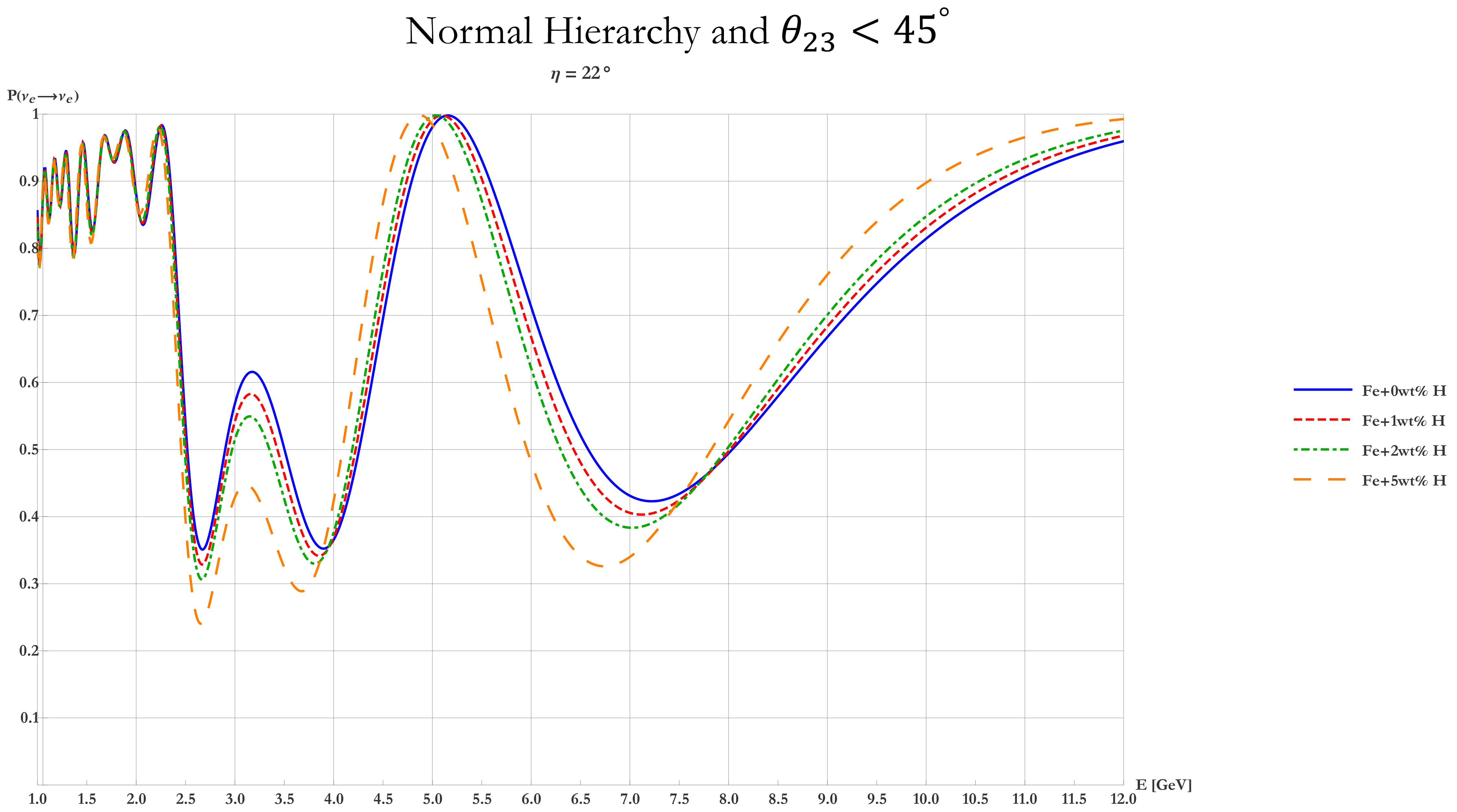
$$\boxed{P(\nu_\alpha \to \nu_\beta) = |\hat{\mathcal{U}}_{\alpha\beta}(r_2, r_1)|^2}$$


Densidad de la Tierra
PREM
Ajuste
Densidad (ρ) [g/cm3]
Radio de la Tierra (r) [km]
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
-6400
-5600
-4800
-4000
-3200
-2400
-1600
-800
0
800
1600
2400
3200
4000
4800
5600
6400

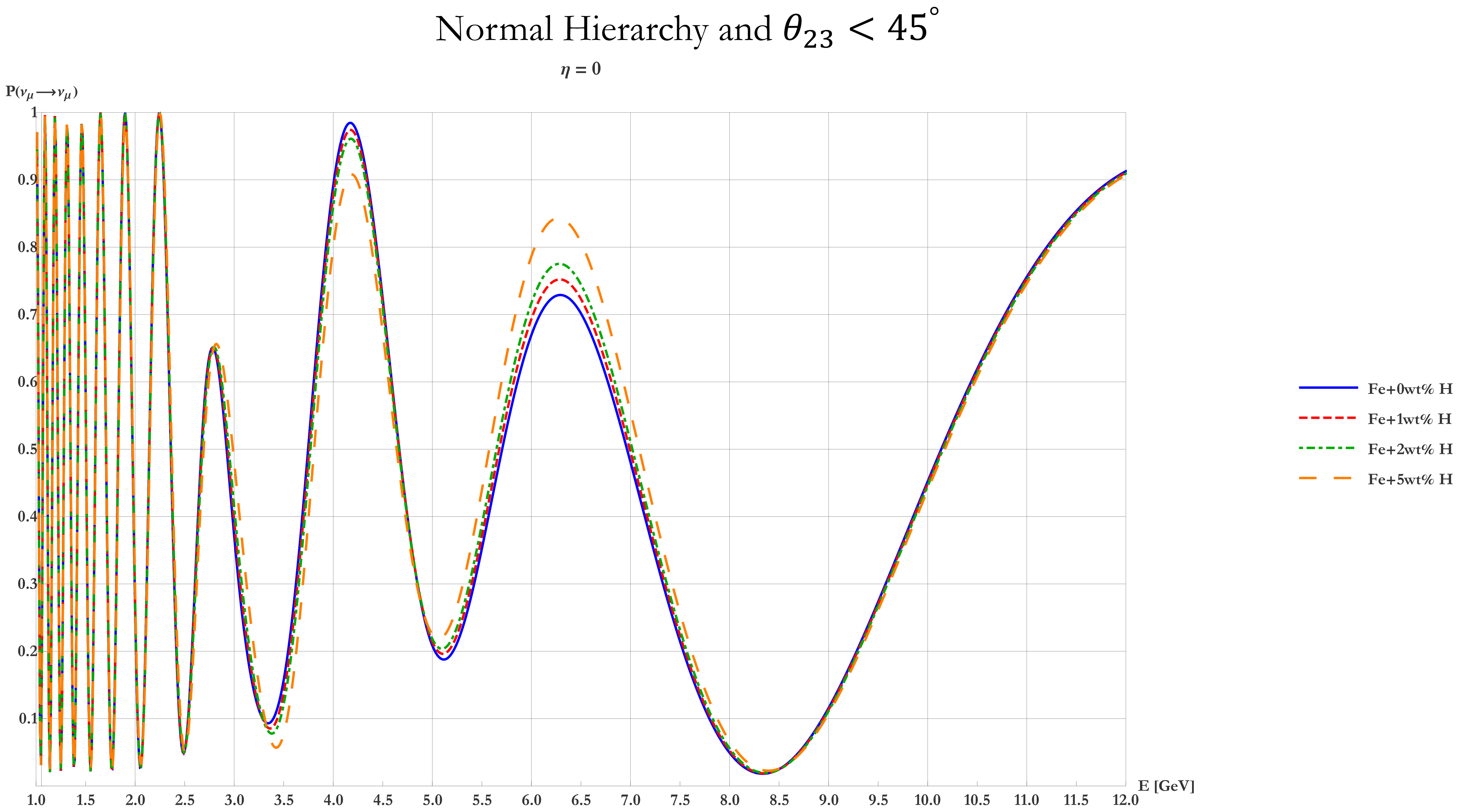


Volumen, masa y densidad de la Tierra							
	Profundidad	**Radio**	**Volumen**		**Masa**		**Densidad**
	(km)	(km)	10^{18} m^3	(%)	10^{21} kg	(%)	(g/cm^3)
Corteza	0 - Moho[4]	Moho - 6371	10	0.9	28	0.5	2.60 - 2.90
Manto superior	Moho - 670	5701 - Moho	297	27.4	1064	17.8	3.38 - 3.99
Manto inferior	670 - 2891	3480 - 5701	600	55.4	2940	49.2	4.38 - 5.56
Núcleo externo	2891 - 5150	1221 - 3480	169	15.6	1841	30.8	9.90 - 12.16
Núcleo interno	5150 - 6371	0 - 1221	8	0.7	102	1.7	12.76 - 13.08
Tierra	0 - 6371	6371 - 0	1083	100	5975	100	

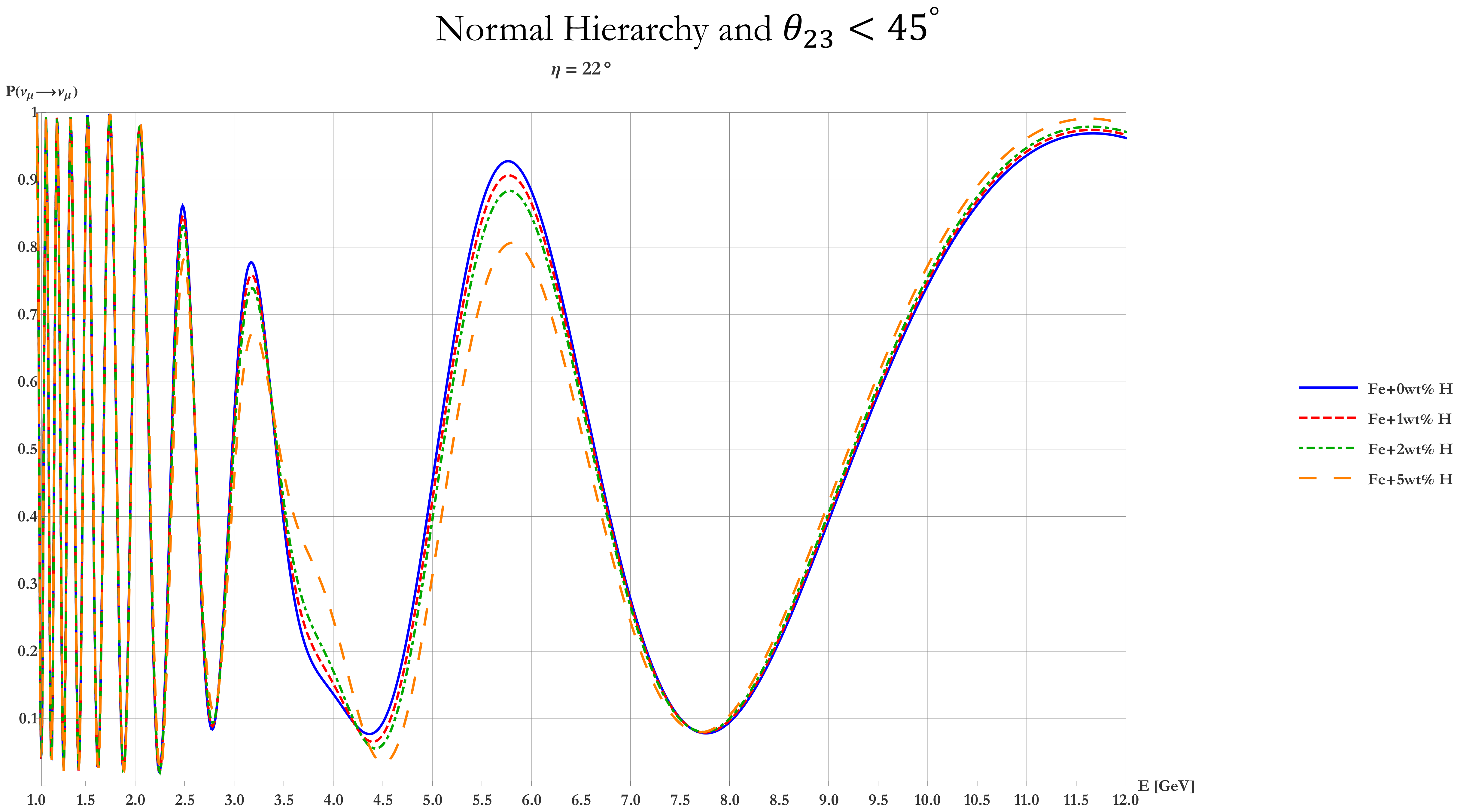

Normal Hierarchy and $\theta_{23} < 45^{\circ}$
$\eta = 0$
$P(\nu_e \longrightarrow \nu_e)$
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
5.5
6.0
6.5
7.0
7.5
8.0
8.5
9.0
9.5
10.0
10.5
11.0
11.5
12.0
E [GeV]
Fe+0wt% H
Fe+1wt% H
Fe+2wt% H
Fe+5wt% H



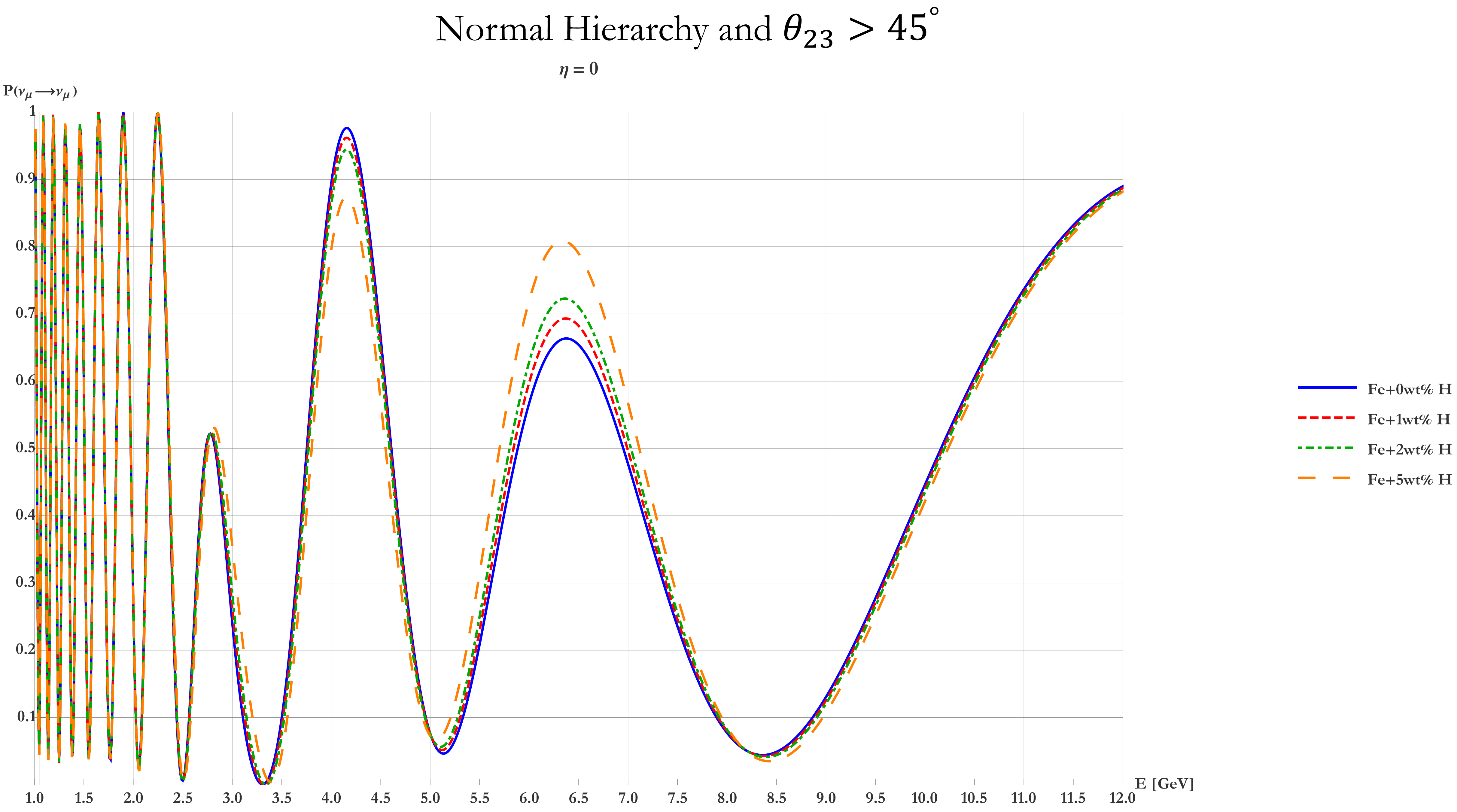
Normal Hierarchy and $\theta_{23} < 45^\circ$
$\eta = 22°$
$P(\nu_e \longrightarrow \nu_e)$
E [GeV]
Fe+0wt% H
Fe+1wt% H
Fe+2wt% H
Fe+5wt% H



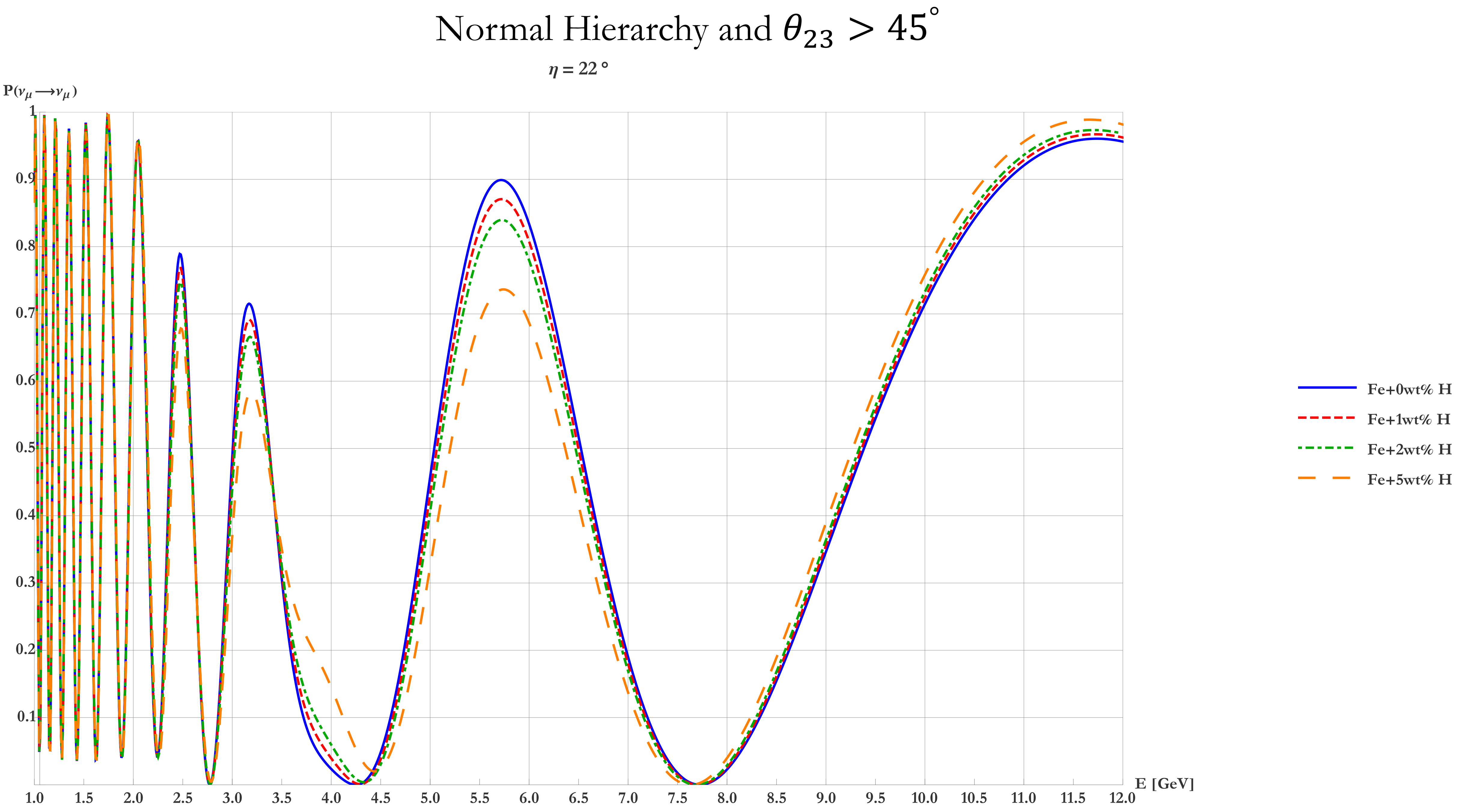
Normal Hierarchy and $\theta_{23} < 45^{\circ}$
$\eta = 0$
$P(\nu_\mu \longrightarrow \nu_\mu)$
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
5.5
6.0
6.5
7.0
7.5
8.0
8.5
9.0
9.5
10.0
10.5
11.0
11.5
12.0
E [GeV]
Fe+0wt% H
Fe+1wt% H
Fe+2wt% H
Fe+5wt% H



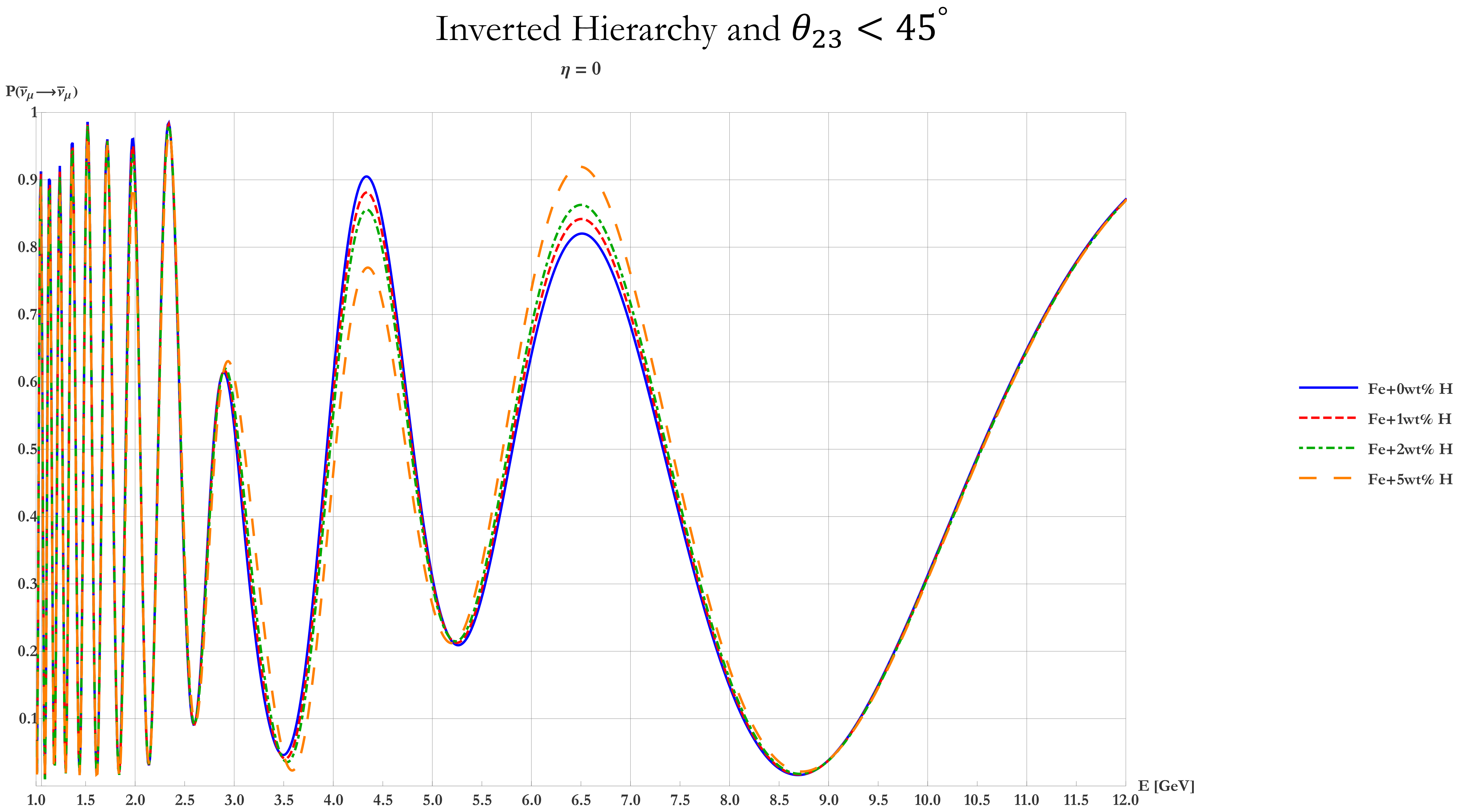
Normal Hierarchy and $\theta_{23} < 45^{\circ}$
$\eta = 22°$
$P(\nu_\mu \longrightarrow \nu_\mu)$
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0 5.5 6.0 6.5 7.0 7.5 8.0 8.5 9.0 9.5 10.0 10.5 11.0 11.5 12.0
E [GeV]
Fe+0wt% H
Fe+1wt% H
Fe+2wt% H
Fe+5wt% H



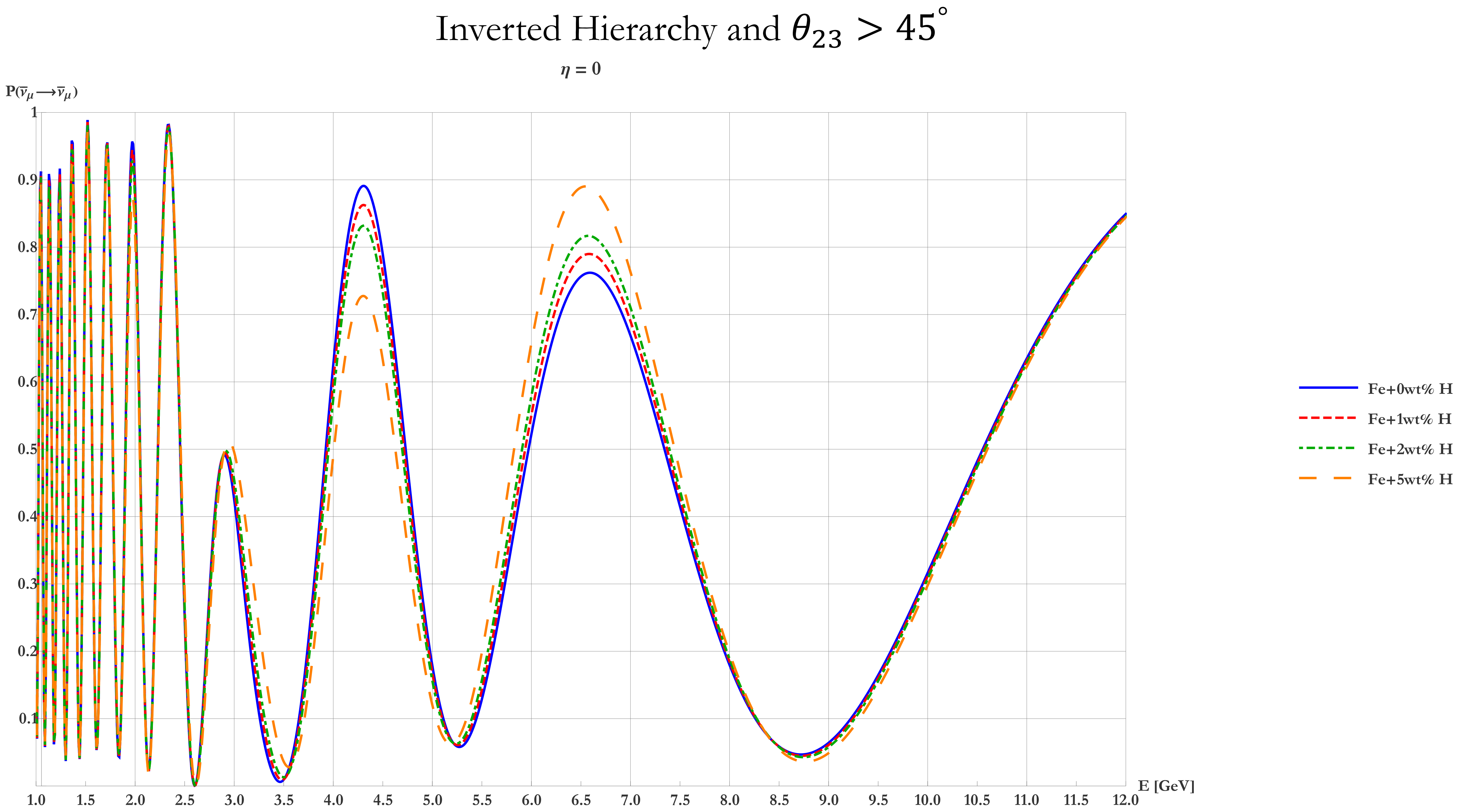
Normal Hierarchy and $\theta_{23} > 45^{\circ}$
$\eta = 0$
$P(\nu_\mu \longrightarrow \nu_\mu)$
E [GeV]
Fe+0wt% H
Fe+1wt% H
Fe+2wt% H
Fe+5wt% H



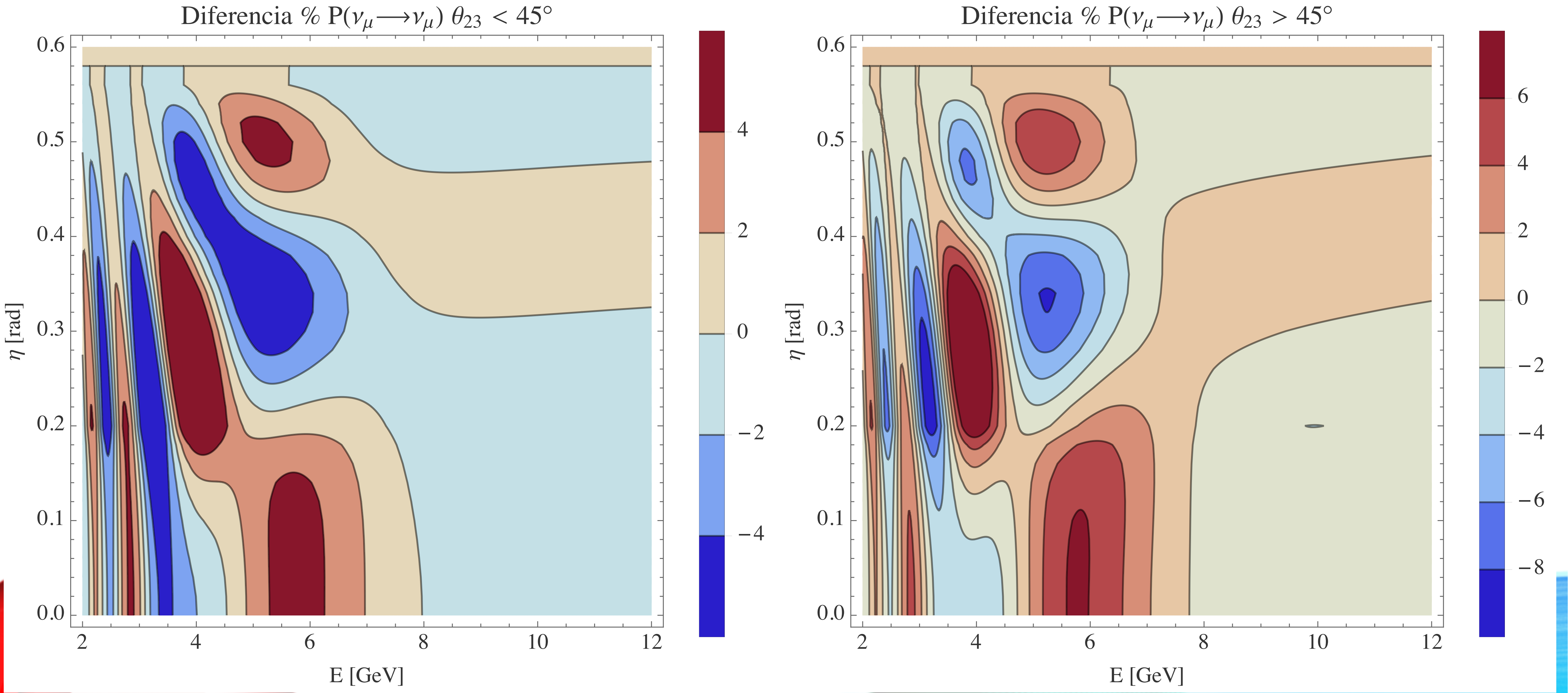
Normal Hierarchy and $\theta_{23} > 45^{\circ}$
$\eta = 22°$
$P(\nu_\mu \longrightarrow \nu_\mu)$
E [GeV]
Fe+0wt% H
Fe+1wt% H
Fe+2wt% H
Fe+5wt% H



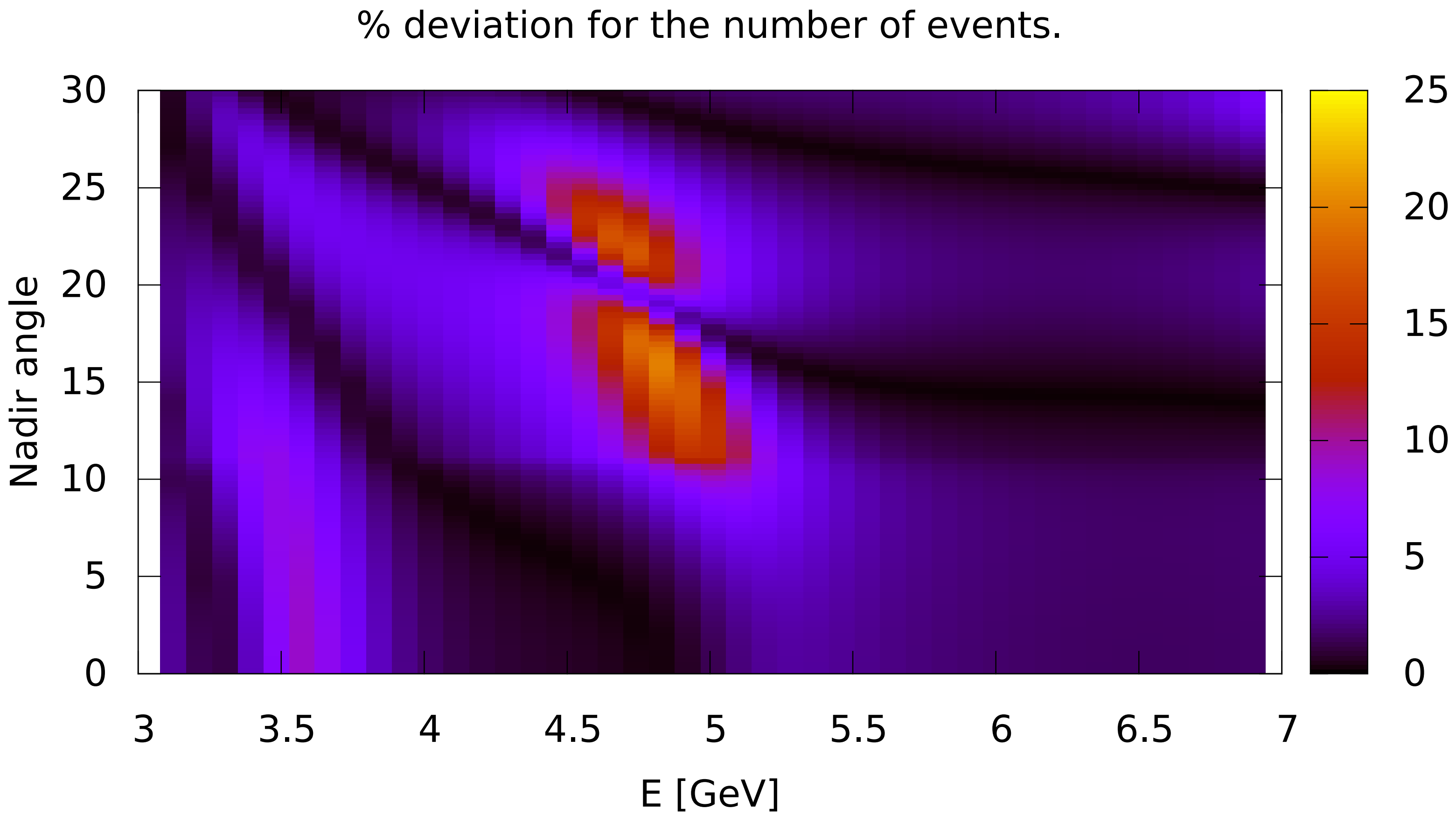
Inverted Hierarchy and $\theta_{23} < 45^{\circ}$
$\eta = 0$
$P(\bar{\nu}_\mu \longrightarrow \bar{\nu}_\mu)$
E [GeV]
Fe+0wt% H
Fe+1wt% H
Fe+2wt% H
Fe+5wt% H



Inverted Hierarchy and $\theta_{23} > 45^{\circ}$
$\eta = 0$
$P(\bar{\nu}_\mu \longrightarrow \bar{\nu}_\mu)$
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0 5.5 6.0 6.5 7.0 7.5 8.0 8.5 9.0 9.5 10.0 10.5 11.0 11.5 12.0
E [GeV]
Fe+0wt% H
Fe+1wt% H
Fe+2wt% H
Fe+5wt% H



Diferencia % P($\nu_\mu \longrightarrow \nu_\mu$) θ_{23} < 45°
Diferencia % P($\nu_\mu \longrightarrow \nu_\mu$) θ_{23} > 45°
η [rad]
E [GeV]
0.0
0.1
0.2
0.3
0.4
0.5
0.6
2
4
6
8
10
12
4
2
0
−2
−4
6
4
2
0
−2
−4
−6
−8


% deviation for the number of events.
Nadir angle
E [GeV]
30
25
20
15
10
5
0
3
3.5
4
4.5
5
5.5
6
6.5
7
25
20
15
10
5
0