Few-hadron systems from QCD

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OLD DOMINION UNIVERSITY

Jefferson Lab

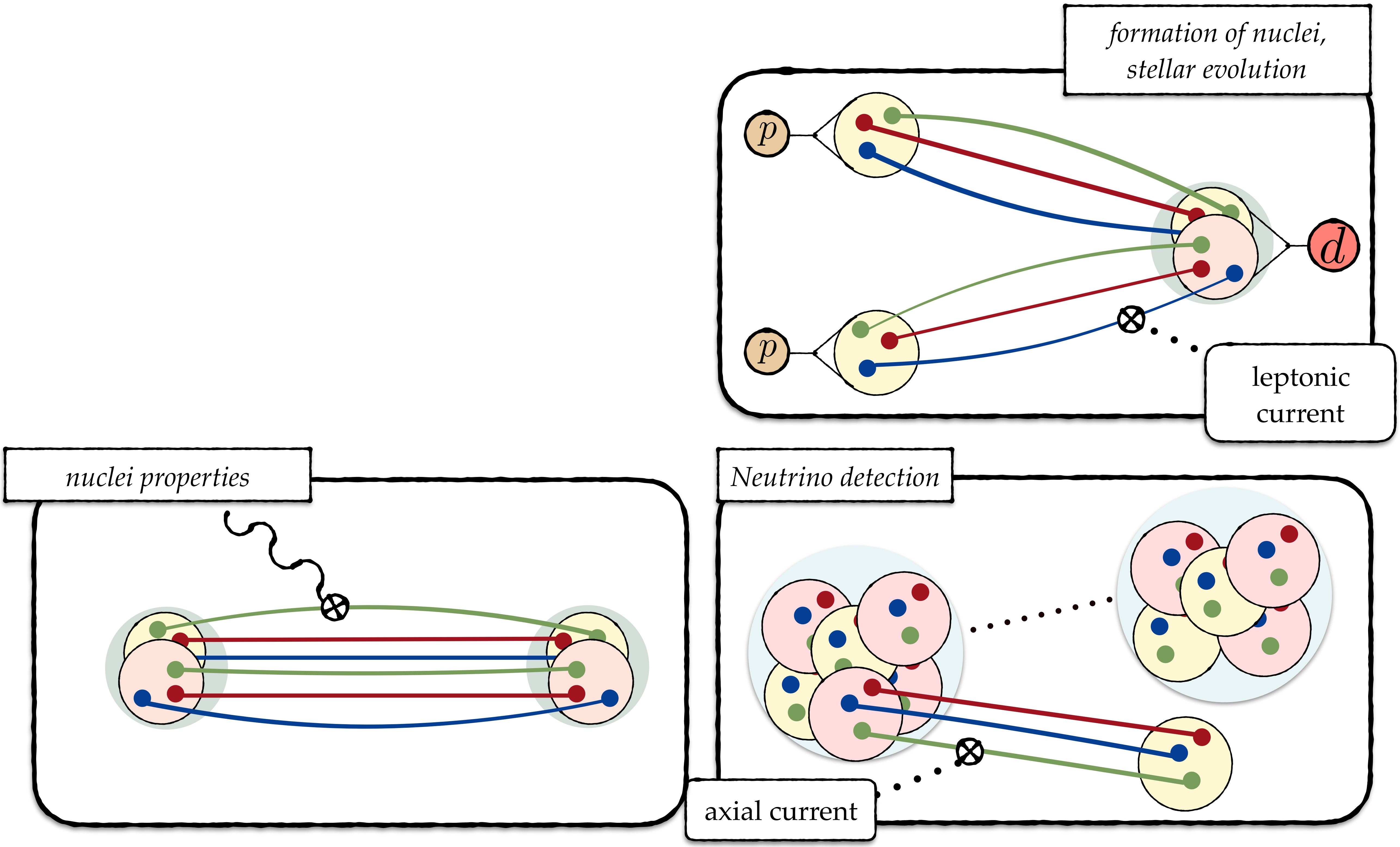
hadspec

UNAM, 2018

JLab, VA

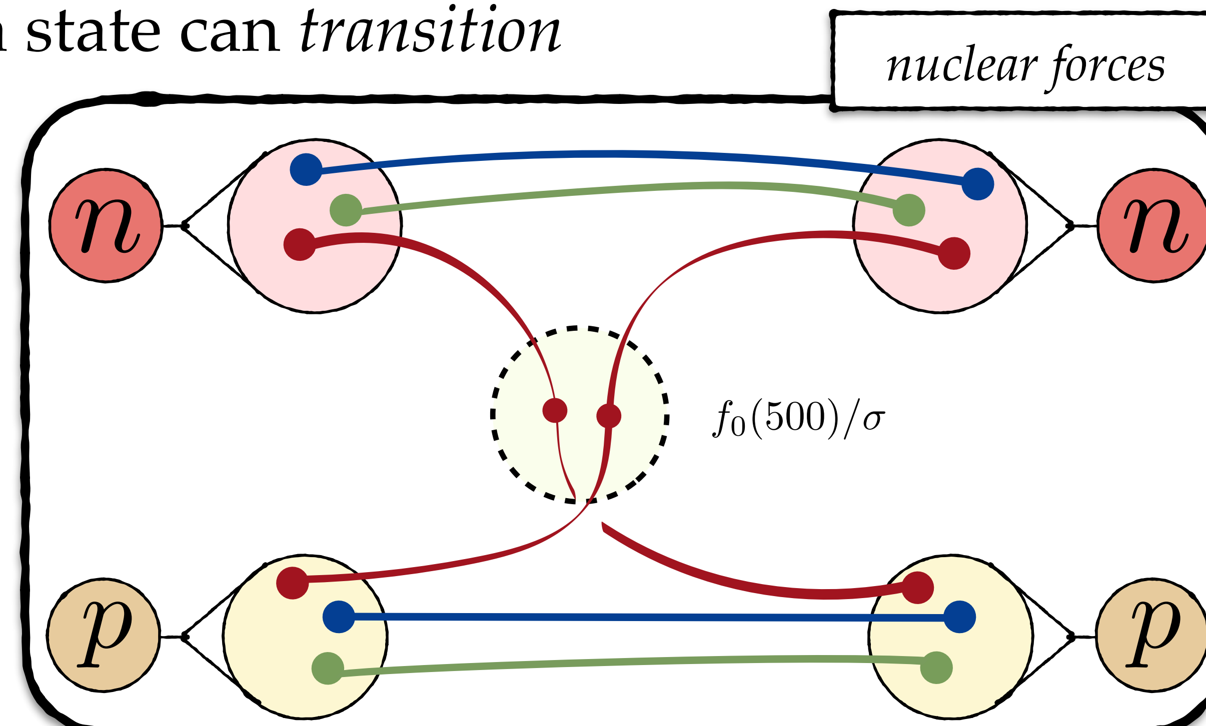
Few-hadron systems in QCD

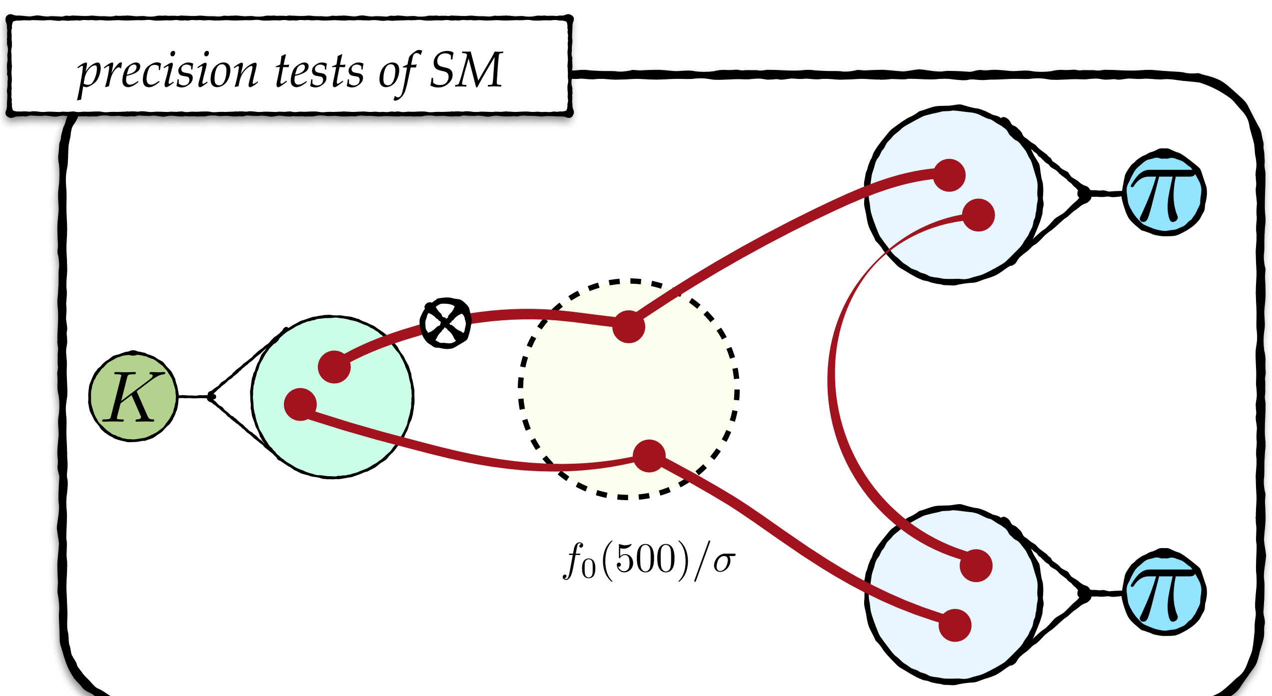
- The vast majority of QCD states are **composite states**, which are either:
  - *accidentally* stable (bound states, e.g. nuclei)

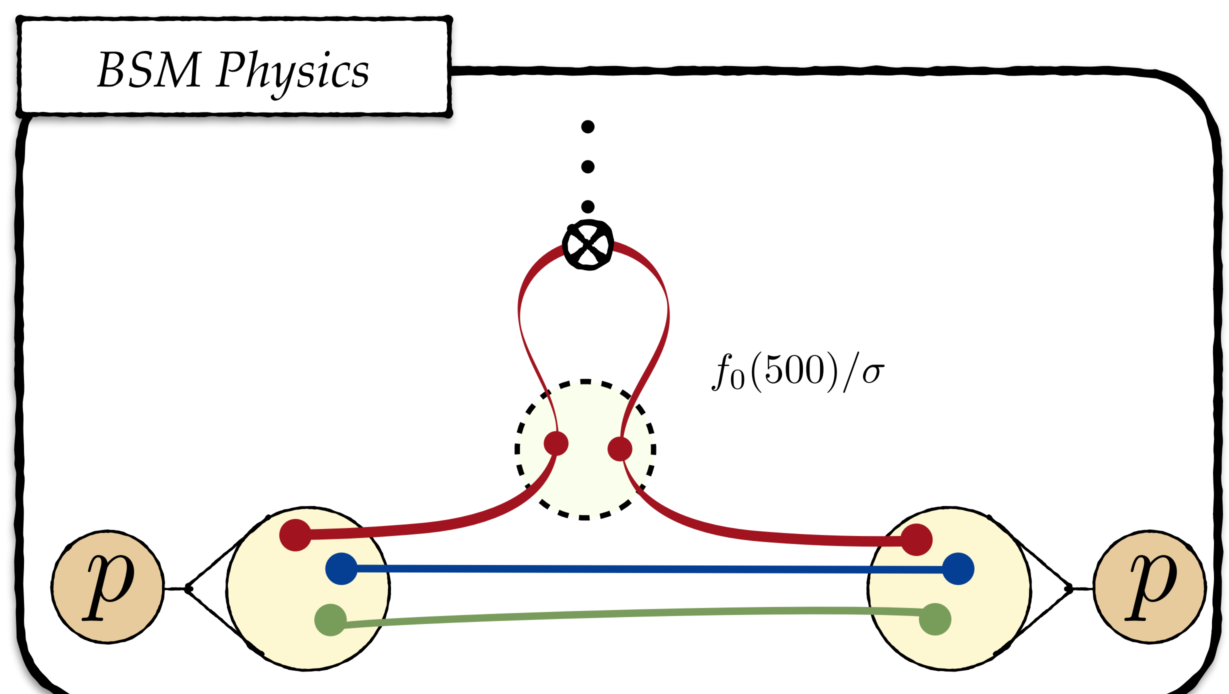


Few-hadron systems in QCD

- The vast majority of QCD states are **composite states**, which are either:
  - *accidentally* stable (bound states, e.g. nuclei)
  - *accidentally* unstable under QCD (resonances)
  - depending on the QCD parameters, a state can *transition*



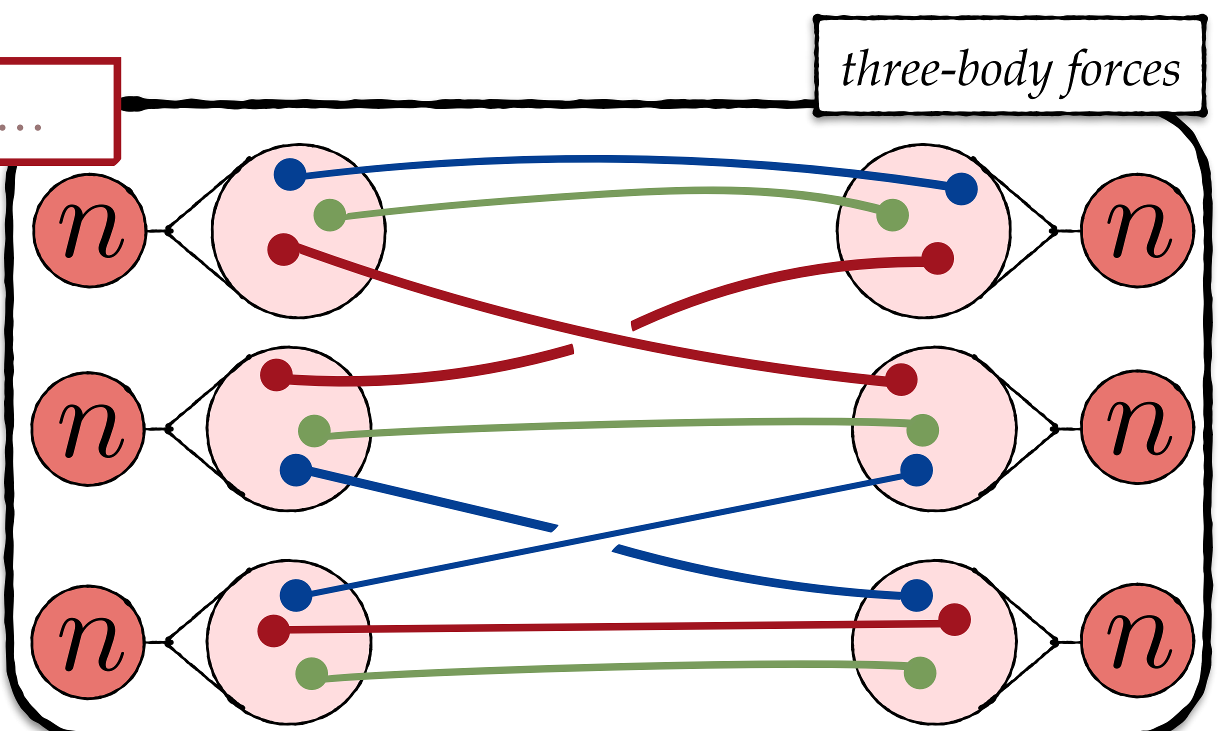




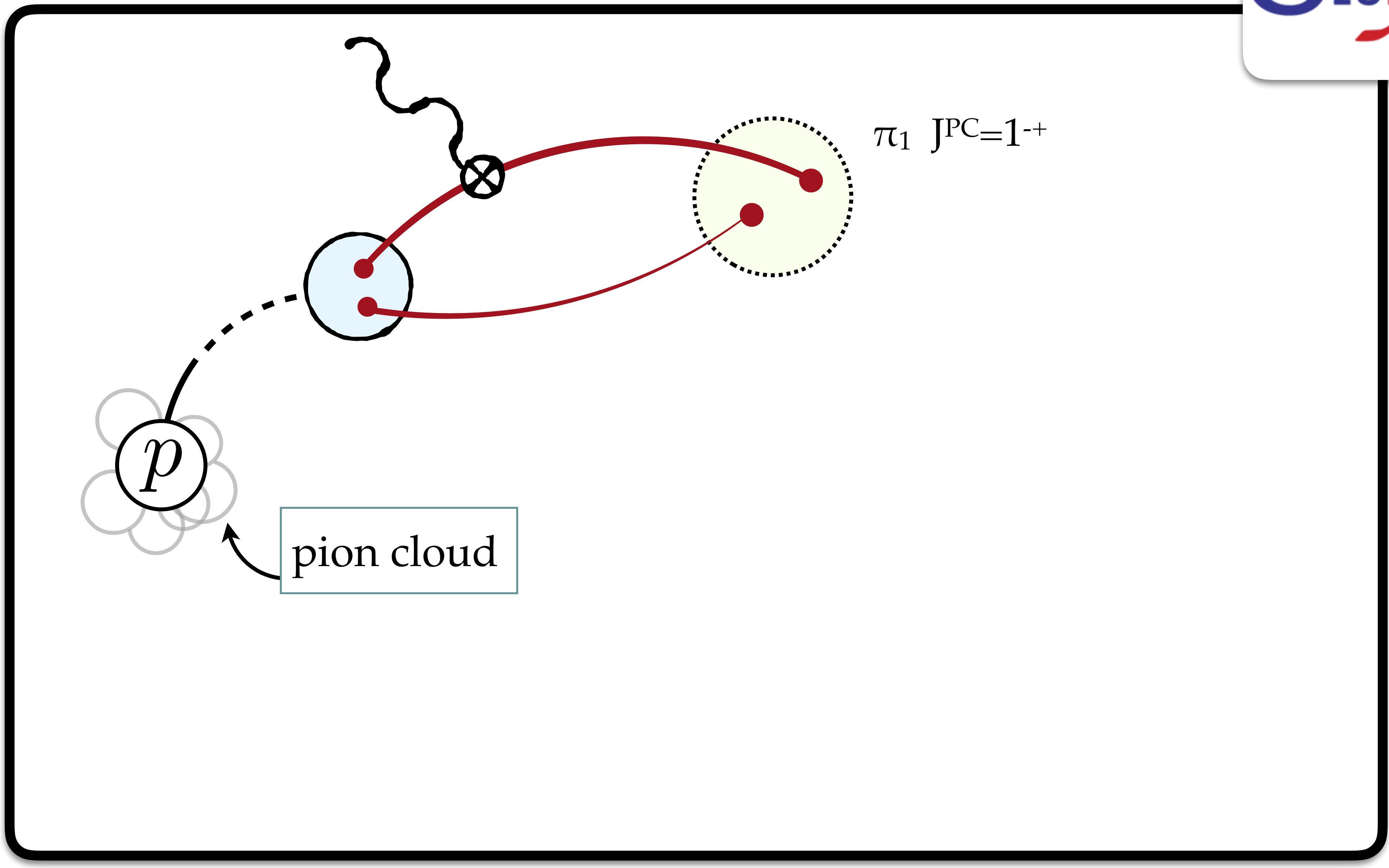
Few-hadron systems in QCD

- The vast majority of QCD states are **composite states**, which are either:
  - *accidentally* stable (bound states, e.g. nuclei)
  - *accidentally* unstable under QCD (resonances)
  - depending on the QCD parameters, a state can *transition*
- multi-hadron forces

I'll touch on this at the end…



Resonances in experiment



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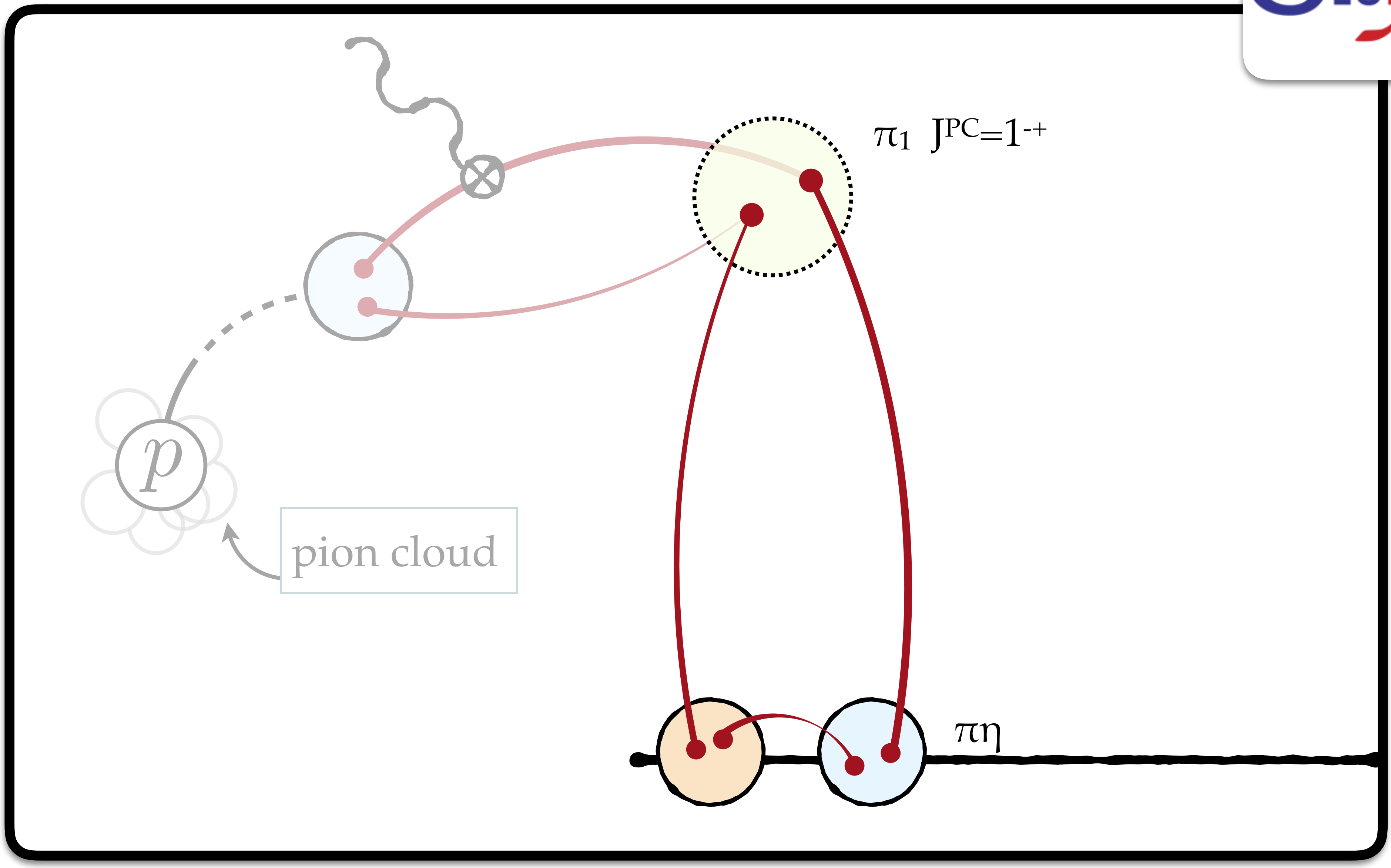

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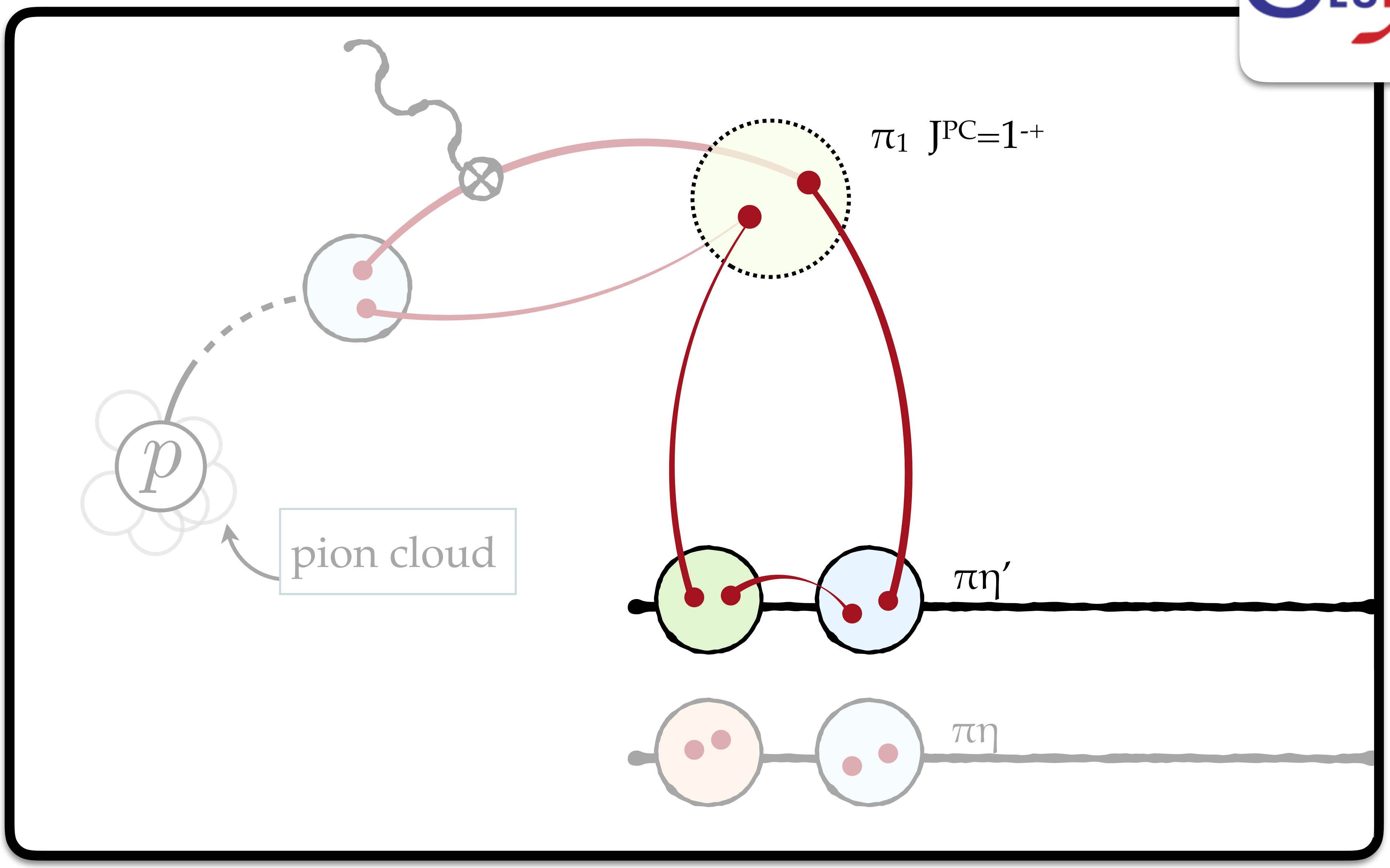
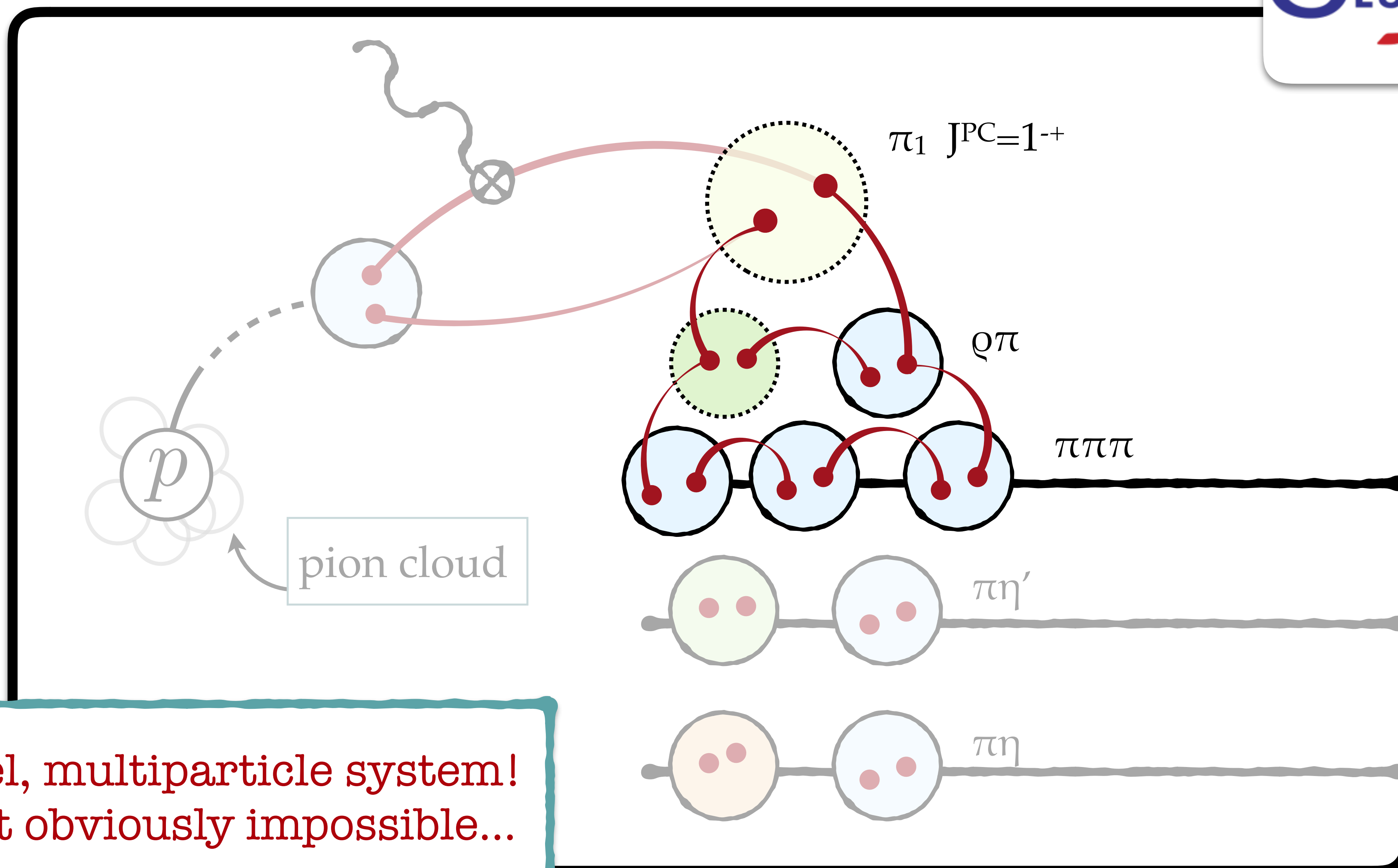


Resonances in experiment



multichannel, multiparticle system!
hard, but not obviously impossible...

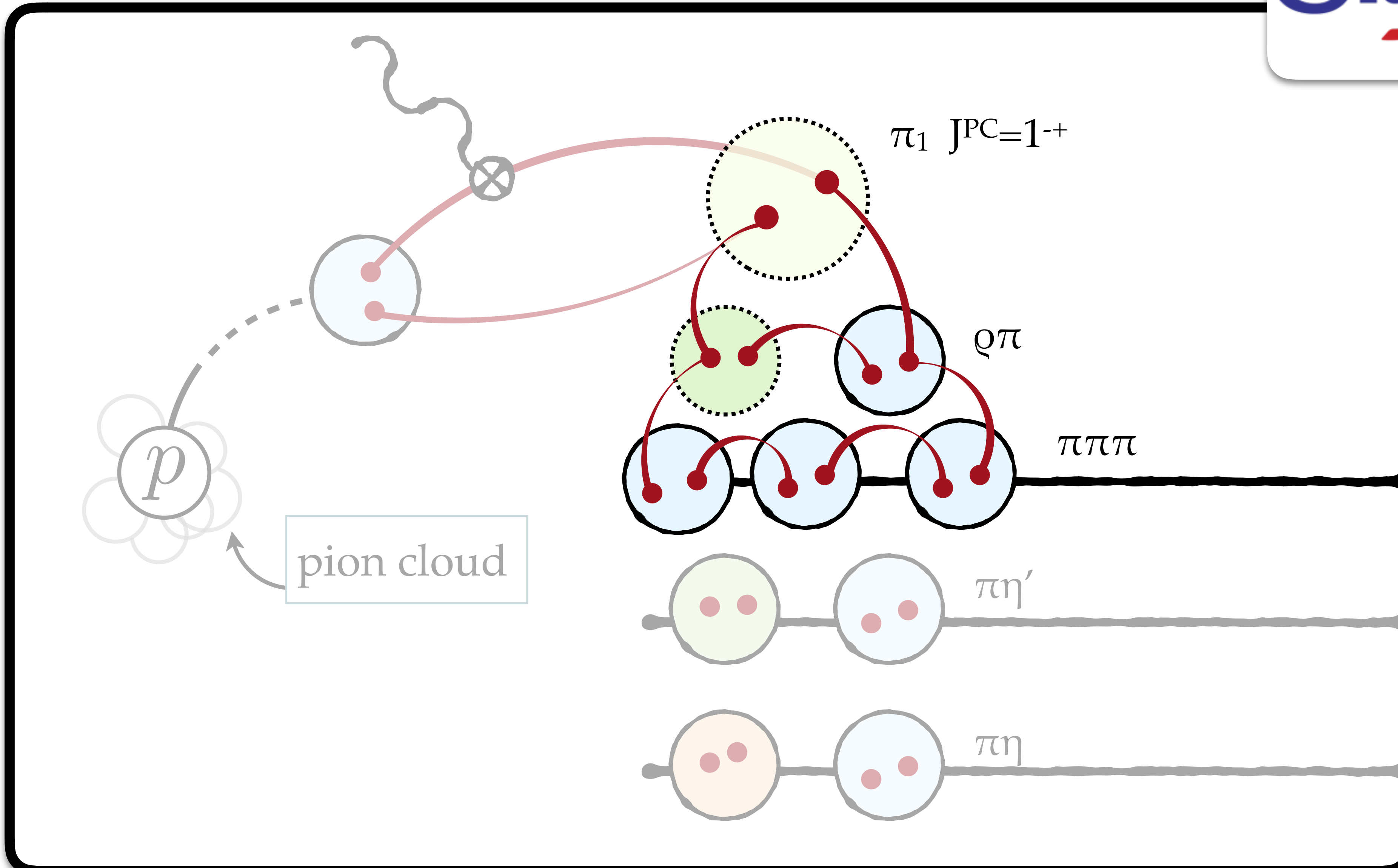
experimental needs

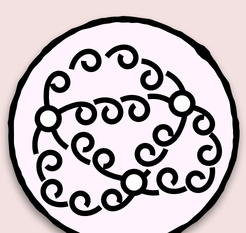
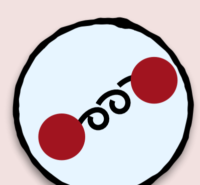
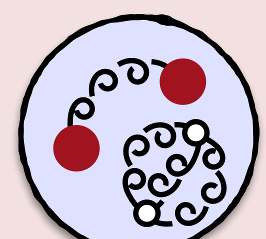
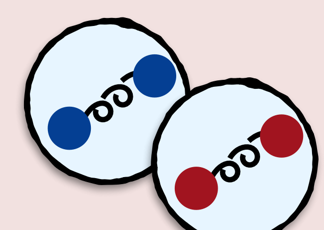
- confirmation
- production mechanism [couplings]
- identification of prominent decay channels
- couplings to decay channels

theoretical needs

- structural understanding

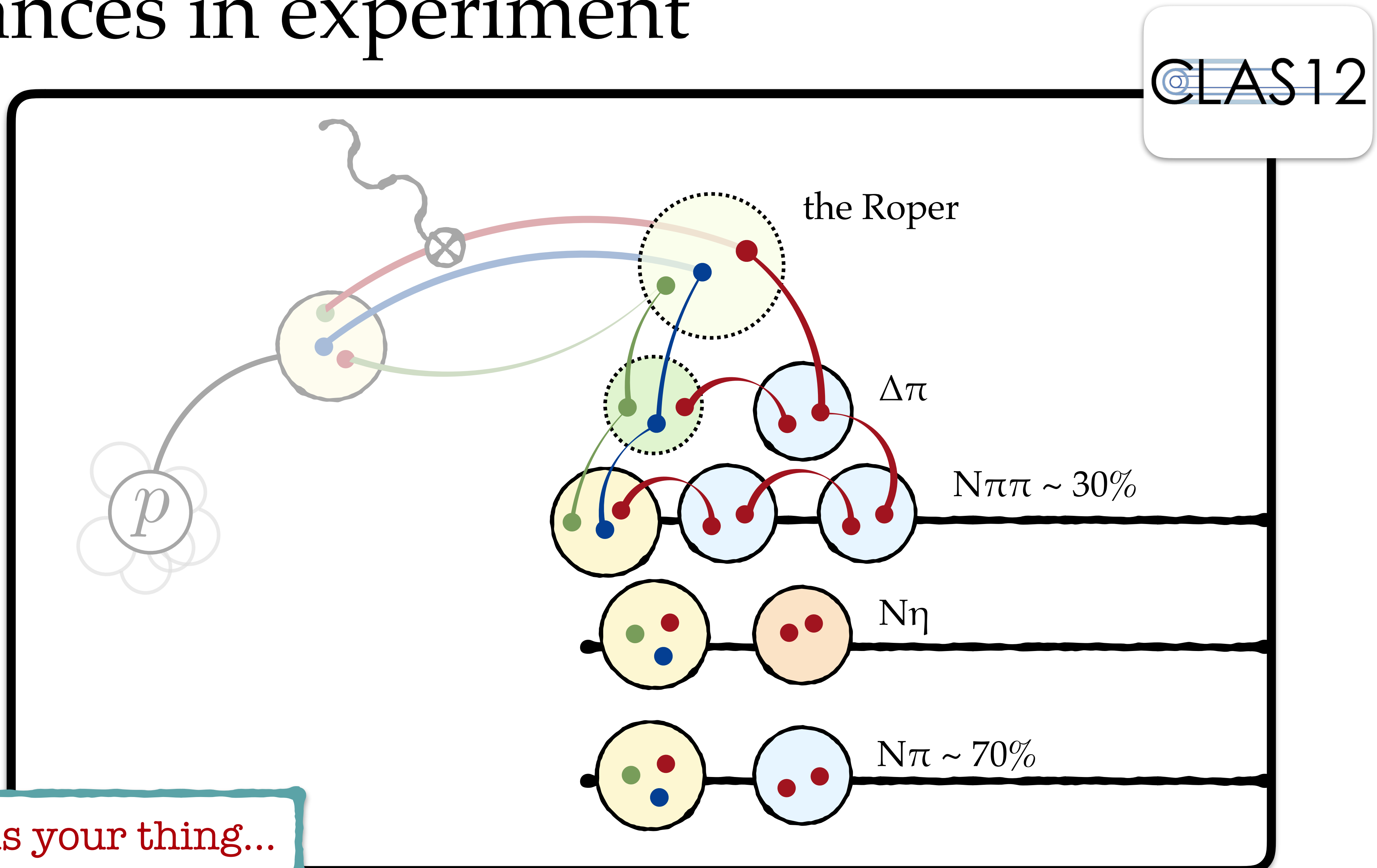
Resonances in experiment



$|n\rangle_{\rm QCD} = c_0$  $+\ c_1$  $+\ c_2$  $+\ c_3$  $+\ \cdots$

theoretical needs — structural understanding

Resonances in experiment



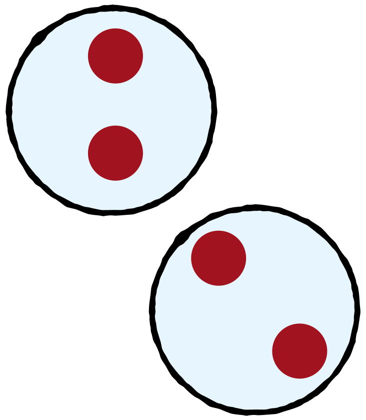
if baryons is your thing...

demand for lattice:

- Stable states generated *"exactly"*
- Resonant/non-resonant amplitudes are generated *"exactly"*
- QED/weak can be introduced perturb. or non-perturb.

Lattice QCD

- Wick rotation [Euclidean spacetime]: $t_M \to -it_E$
- Monter Carlo sampling



Lattice QCD

- Wick rotation [Euclidean spacetime]: $t_M \to -it_E$
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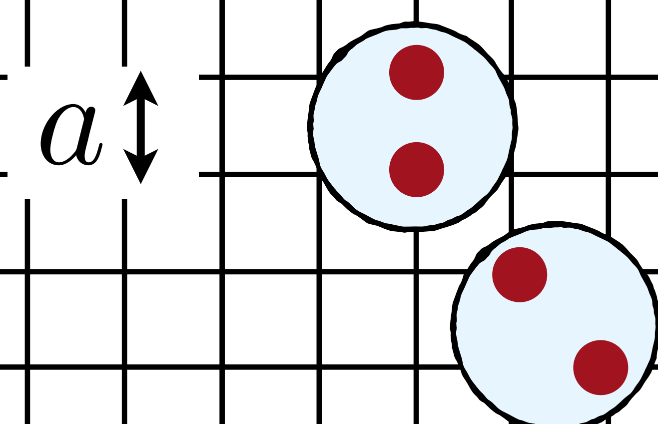
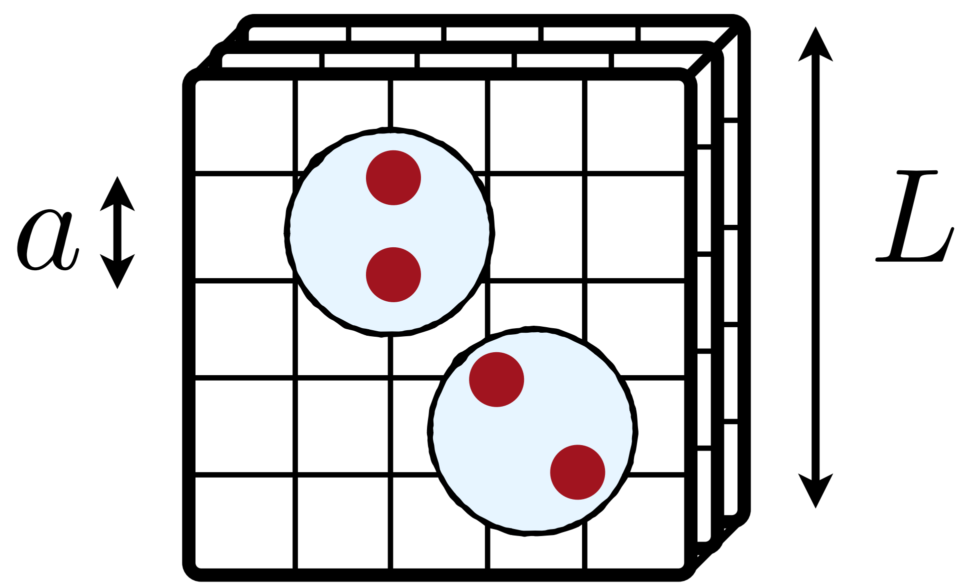
- lattice spacing: $a \sim 0.03 - 0.15$ fm



Lattice QCD

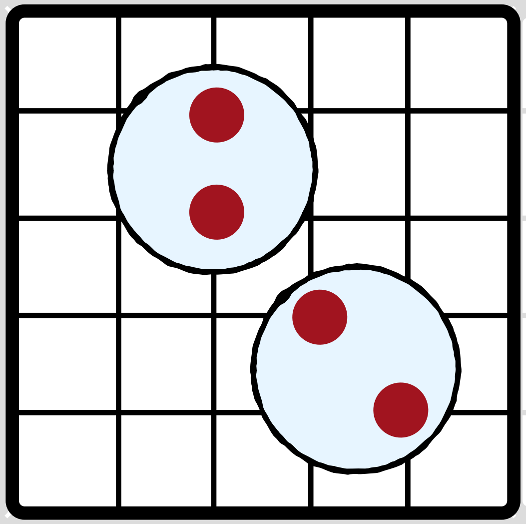
- Wick rotation [Euclidean spacetime]: $t_M \to -it_E$
- Monter Carlo sampling
- lattice spacing: $a \sim 0.03 - 0.15$ fm
- finite volume

$$D_\mu = \Big(\qquad \Big) \updownarrow (L/a)^3 \times (T/a)$$



Lattice QCD

- Wick rotation [Euclidean spacetime]: $t_M \to -it_E$
- Monter Carlo sampling
- lattice spacing: $a \sim 0.03 - 0.15$ fm
- finite volume [periodic…]



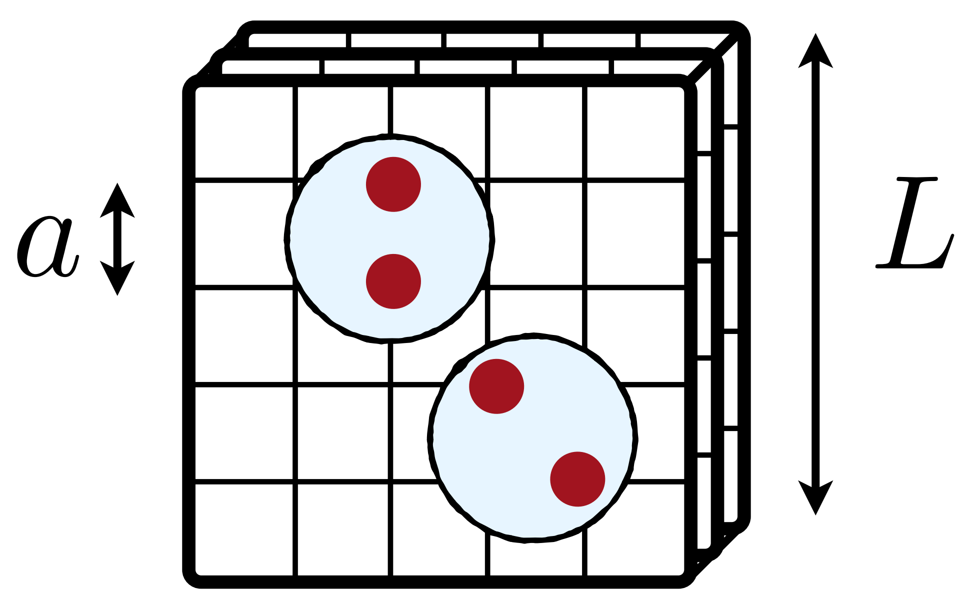
Never free!

No asymptotic states!

No scattering!

Lattice QCD

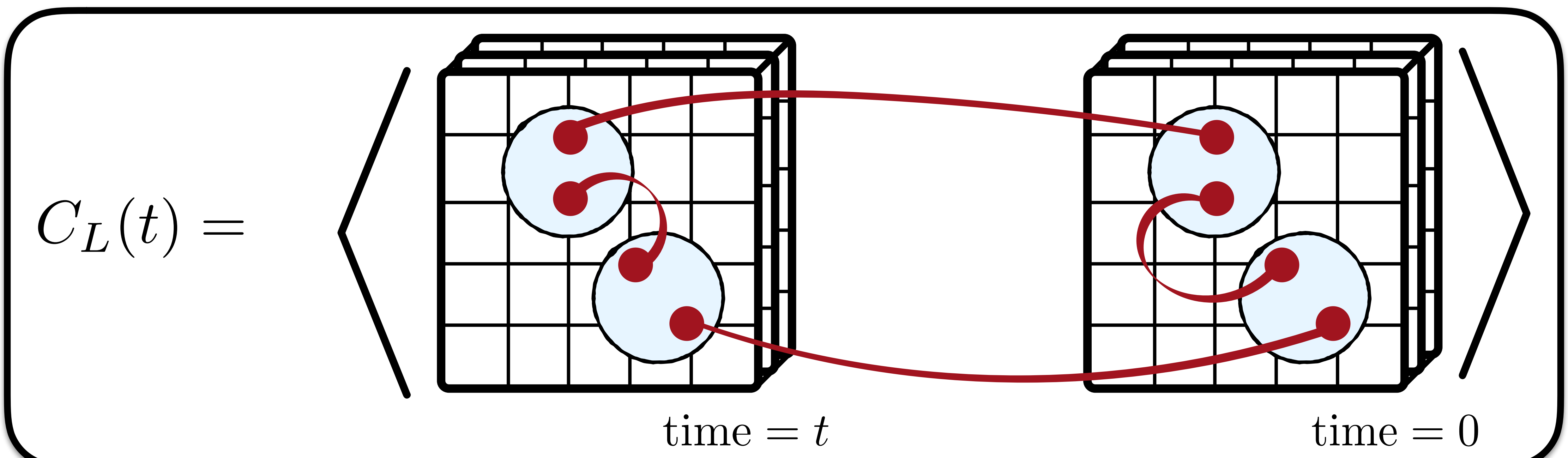
- Wick rotation [Euclidean spacetime]: $t_M \to -it_E$
- Monter Carlo sampling
- lattice spacing: $a \sim 0.03 - 0.15$ fm
- finite volume
- quark masses: $m_q \to m_q^{\text{phys.}}$



Advantage over experiment!

Lattice QCD

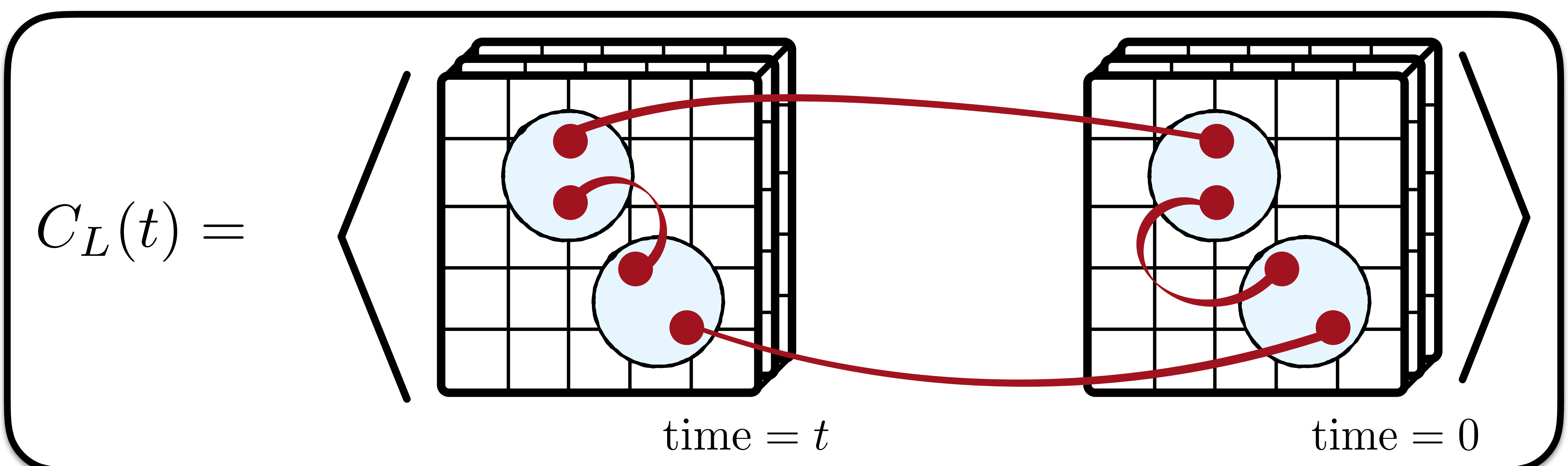
- Wick rotation [Euclidean spacetime]: $t_M \to -it_E$
- Monter Carlo sampling
- lattice spacing: $a \sim 0.03 - 0.15 \text{ fm}$
- finite volume
- quark masses: $m_q \to m_q^{\text{phys.}}$
- Correlation functions: spectrum, matrix elements



On correlation functions

$$\langle \hat{\mathcal{O}}(t)\hat{\mathcal{O}}^\dagger(0)\rangle = \sum_n \langle 0|\hat{\mathcal{O}}(t)|n\rangle\langle n|\hat{\mathcal{O}}^\dagger(0)\rangle$$

inserting complete set of states

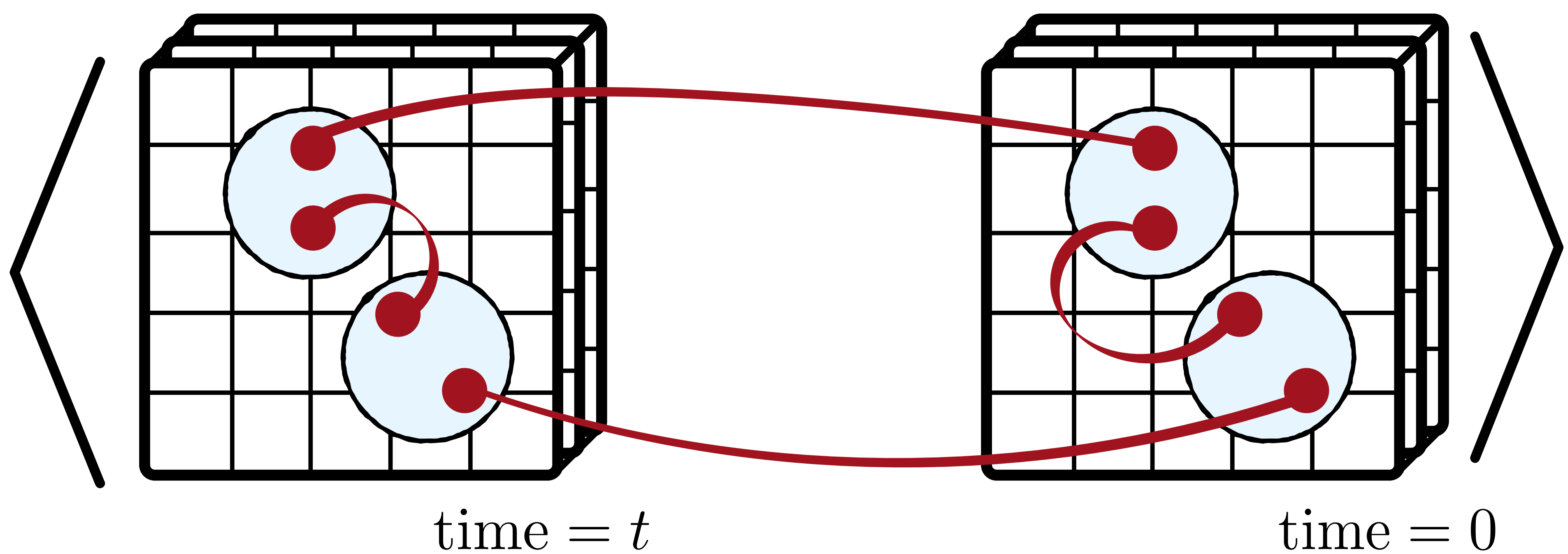


On correlation functions

$$\langle \hat{\mathcal{O}}(t)\hat{\mathcal{O}}^\dagger(0)\rangle = \sum_n \langle 0|\hat{\mathcal{O}}(t)|n\rangle\langle n|\hat{\mathcal{O}}^\dagger(0)\rangle$$
$$= \sum \langle 0|e^{tH_{QCD}}\hat{\mathcal{O}}(0)e^{-tH_{QCD}}|n\rangle\langle n|\hat{\mathcal{O}}^\dagger(0)\rangle$$

time-evolution in Euclidean space

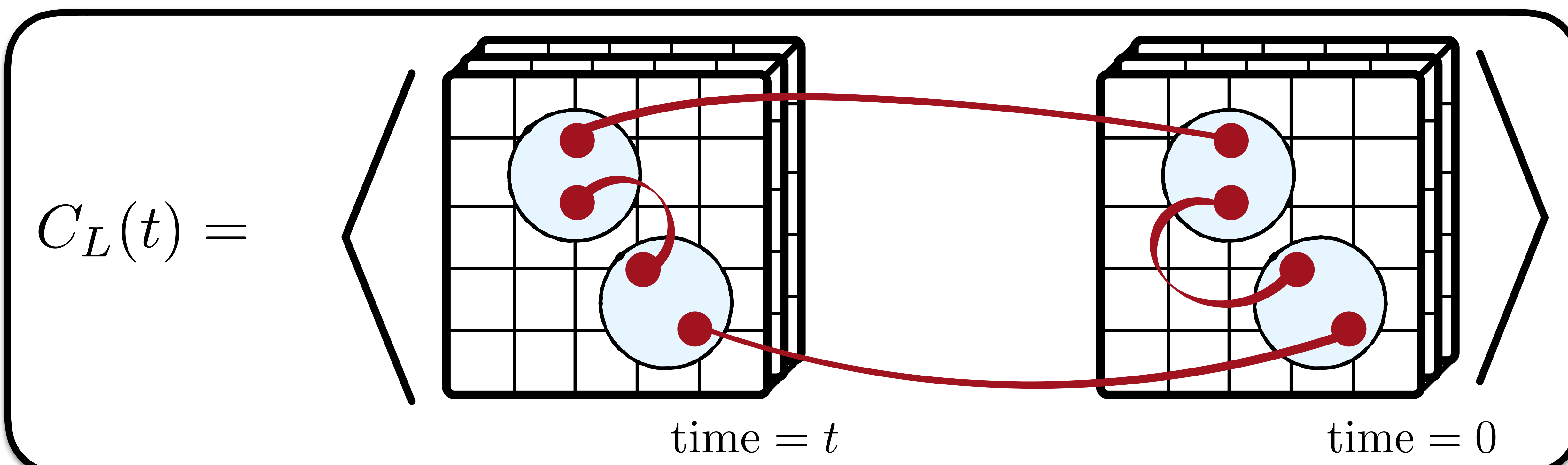
$C_L(t) =$



On correlation functions

$$\begin{aligned}
\langle \hat{\mathcal{O}}(t)\hat{\mathcal{O}}^\dagger(0)\rangle &= \sum_n \langle 0|\hat{\mathcal{O}}(t)|n\rangle\langle n|\hat{\mathcal{O}}^\dagger(0)\rangle \\
&= \sum_n \langle 0|e^{tH_{QCD}}\hat{\mathcal{O}}(0)e^{-tH_{QCD}}|n\rangle\langle n|\hat{\mathcal{O}}^\dagger(0)\rangle \\
&= \sum_n e^{-tE_n}|\langle 0|\hat{\mathcal{O}}(0)|n\rangle|^2 \\
&= \mathcal{Z}_{QCD}^{-1}\int \mathcal{D}U\,\mathcal{D}q\,\mathcal{D}\overline{q}\;\; e^{-S_E} f[U, q\overline{q}, t]
\end{aligned}$$

← **path-integral representation**



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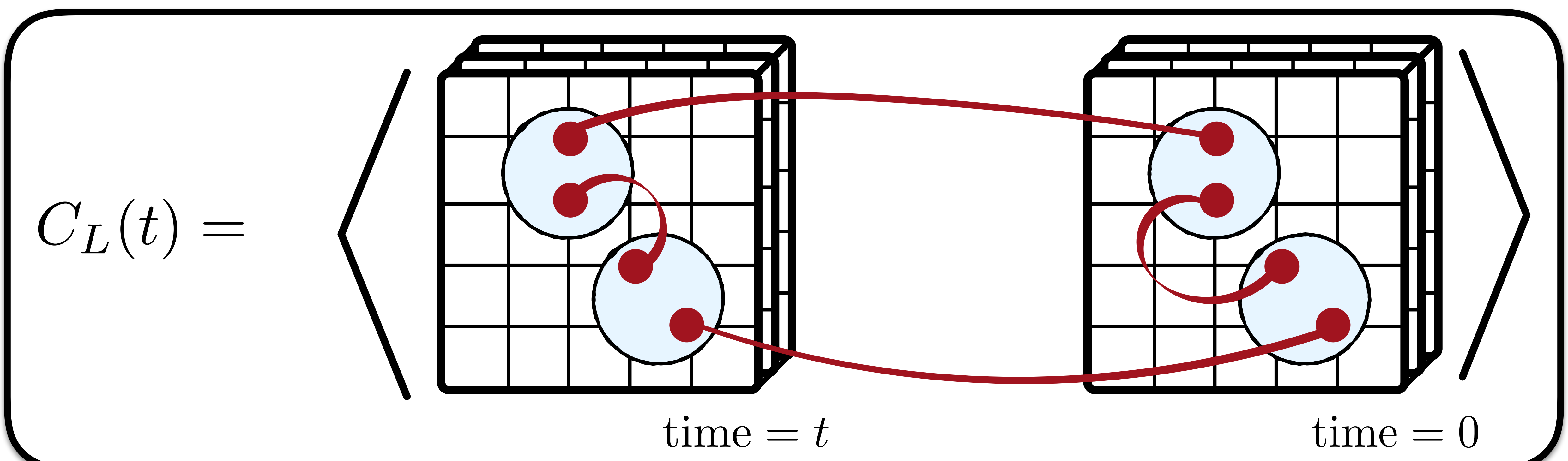
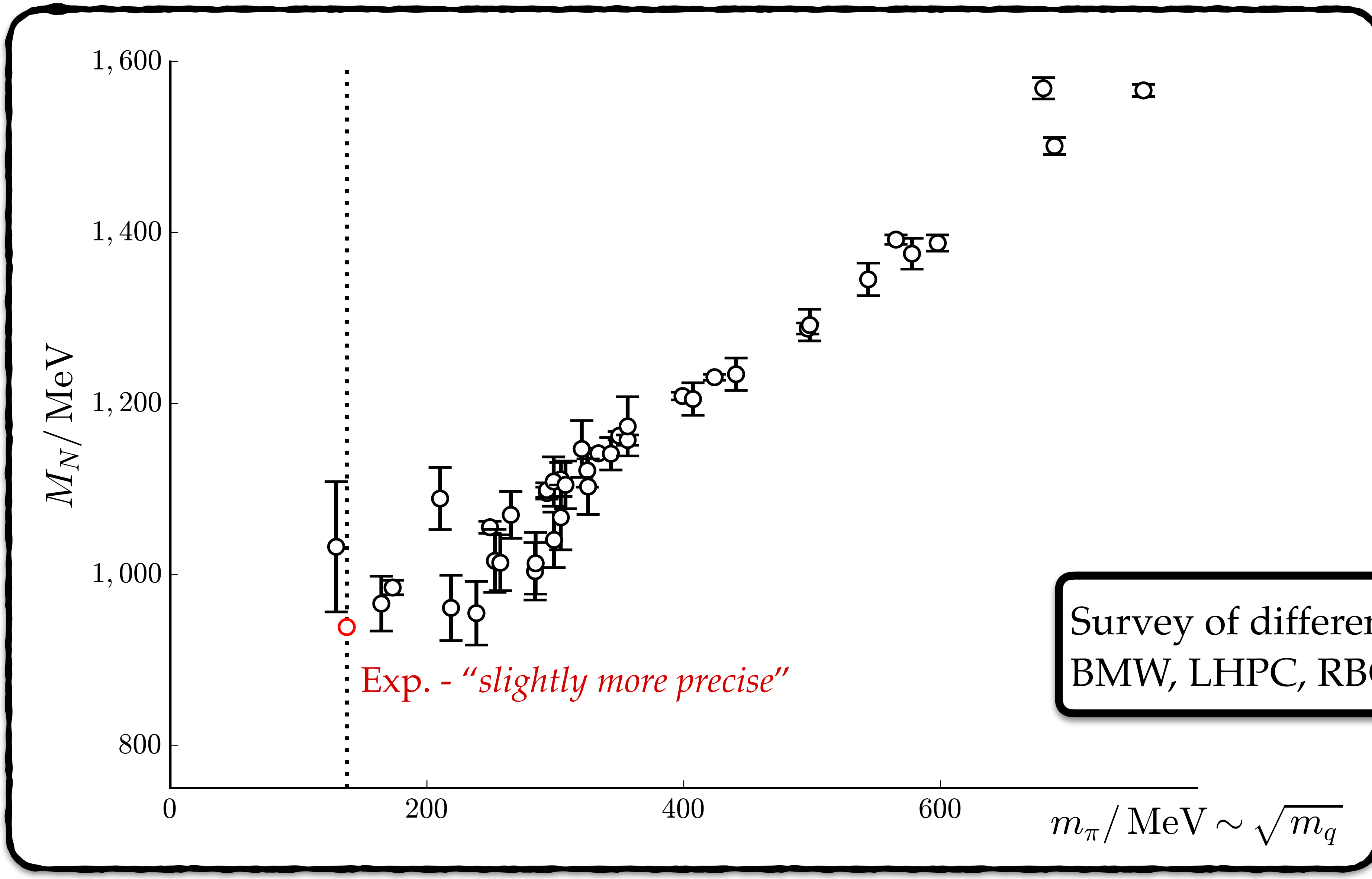
&= \mathcal{Z}_{QCD}^{-1}\int \mathcal{D}U\,\mathcal{D}q\,\mathcal{D}\overline{q}\;\; e^{-S_E} f[U, q\overline{q}, t] \\
&\approx \frac{1}{N}\sum_{n_U} f[n_U, t]
\end{aligned}$$

averaging over Monte Carlo samples



Status of the field

- *Simple* properties of QCD stable states [non-composite states]
  - physical or lighter quark masses [down to m_π~120 MeV] ✓
  - non-degenerate light-quark masses: N_f=1+1+1+1 ✓
  - dynamical QED ✓



Survey of different groups:
BMW, LHPC, RBC, & χQCD

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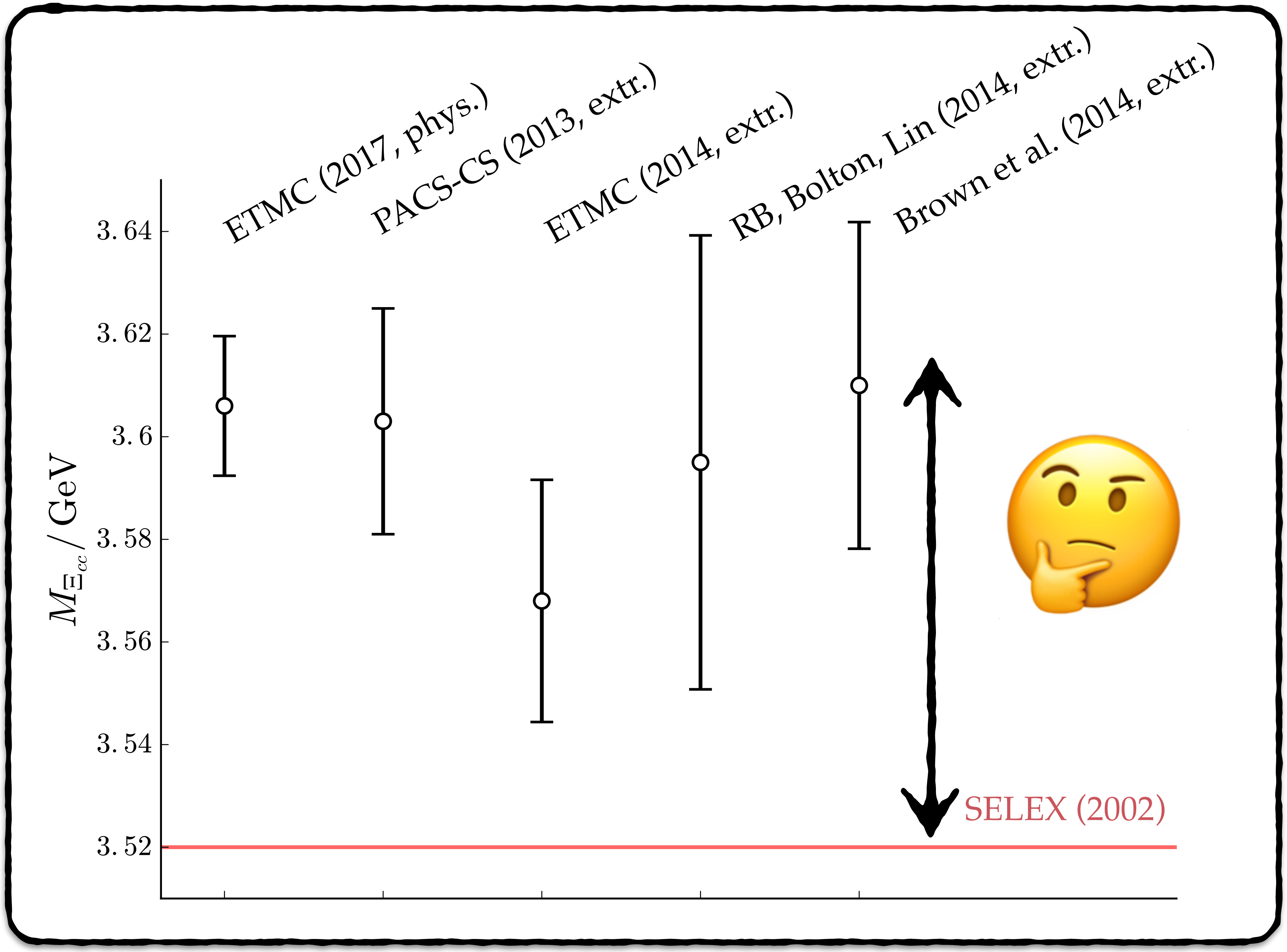

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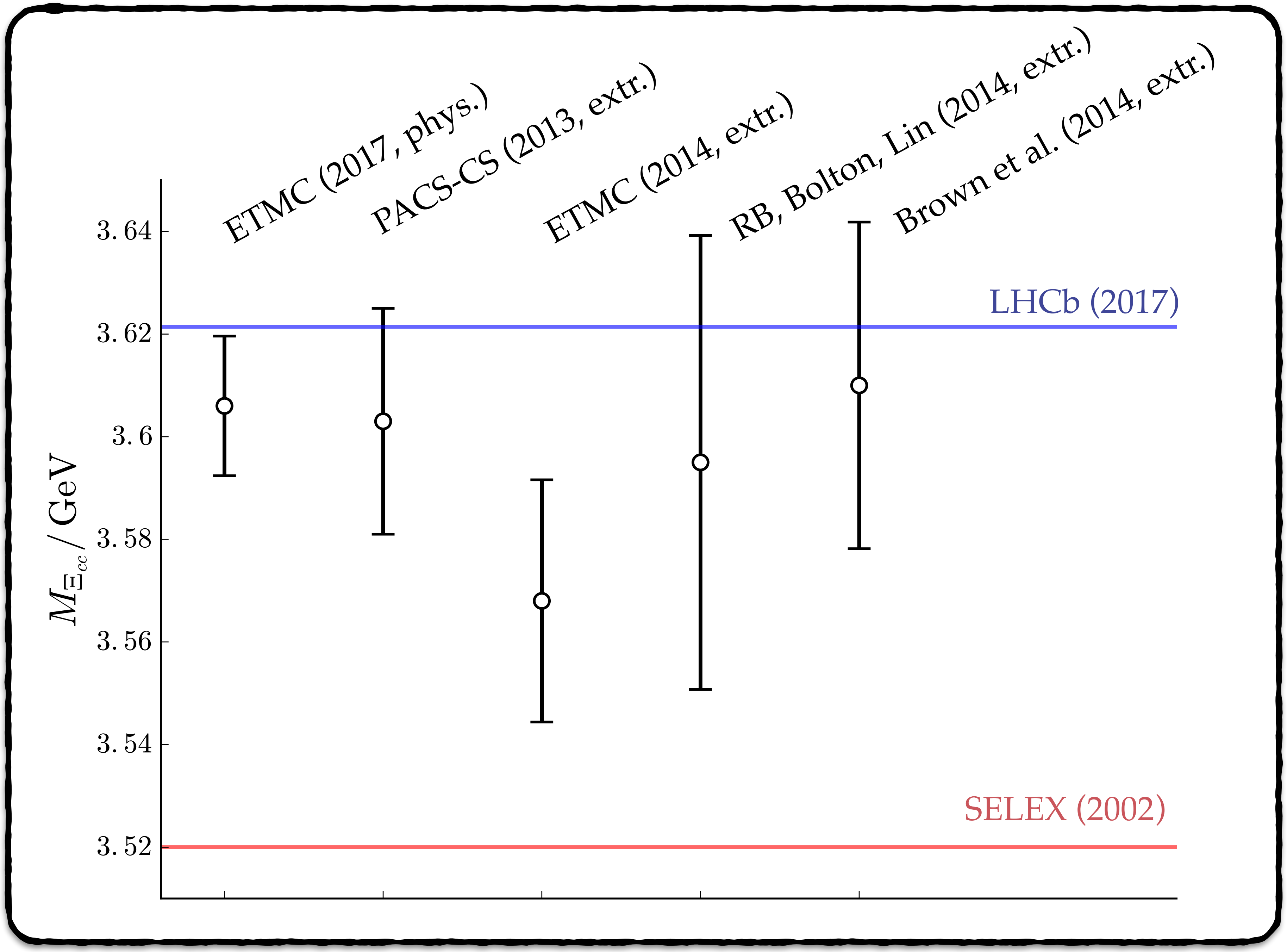

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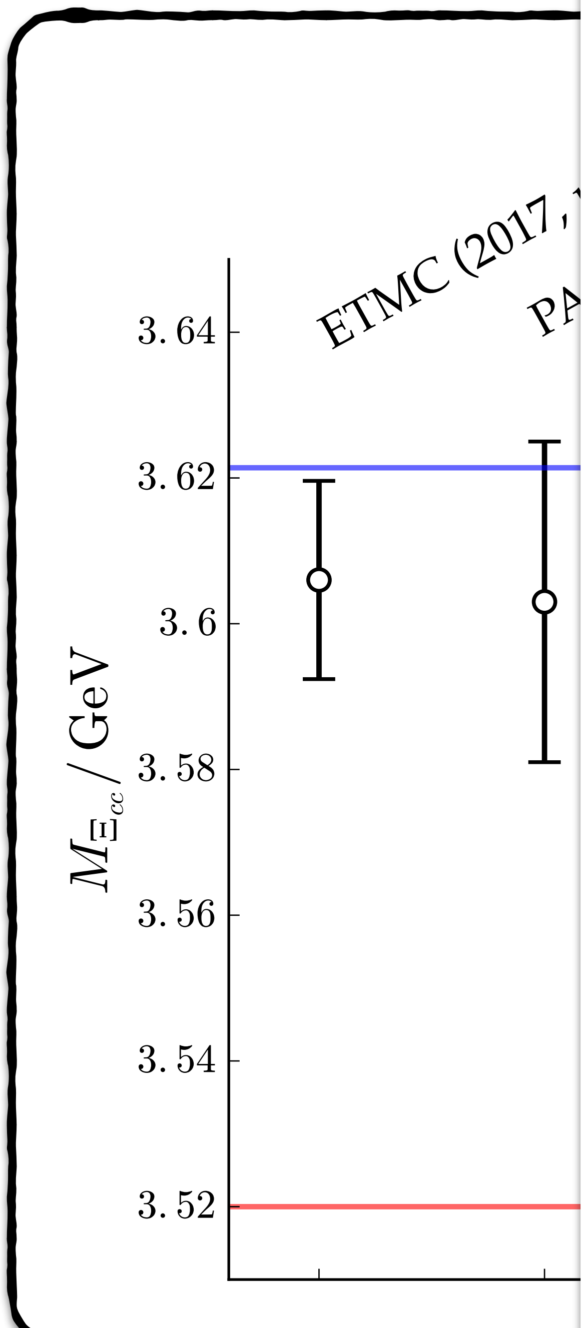


Selected for a Viewpoint in *Physics*
PRL **119,** 112001 (2017) PHYSICAL REVIEW LETTERS week ending 15 SEPTEMBER 2017

Observation of the Doubly Charmed Baryon Ξ_{cc}^{++}

R. Aaij *et al.*[*]
(LHCb Collaboration)

(Received 6 July 2017; revised manuscript received 2 August 2017; published 11 September 2017)

A highly significant structure is observed in the $\Lambda_c^+ K^- \pi^+ \pi^+$ mass spectrum, where the Λ_c^+ baryon is reconstructed in the decay mode $pK^-\pi^+$. The structure is consistent with originating from a weakly decaying particle, identified as the doubly charmed baryon Ξ_{cc}^{++}. The difference between the masses of the Ξ_{cc}^{++} and Λ_c^+ states is measured to be $1334.94 \pm 0.72(\text{stat.}) \pm 0.27(\text{syst.})\ \text{MeV}/c^2$, and the Ξ_{cc}^{++} mass is then determined to be $3621.40 \pm 0.72(\text{stat.}) \pm 0.27(\text{syst.}) \pm 0.14(\Lambda_c^+)\ \text{MeV}/c^2$, where the last uncertainty is due to the limited knowledge of the Λ_c^+ mass. The state is observed in a sample of proton-proton collision data collected by the LHCb experiment at a center-of-mass energy of 13 TeV, corresponding to an integrated luminosity of $1.7\ \text{fb}^{-1}$, and confirmed in an additional sample of data collected at 8 TeV.

DOI: 10.1103/PhysRevLett.119.112001

The quark model [1–3] pr
lets of baryon and meson sta
the lightest four quarks (u,
[4]. Numerous states with c
or $C = 1$ have been disc
expected $q\bar{q}$ and qqq grou
decaying qqq states with C
doublet ($\Xi_{cc}^{++} = ccu$ and
singlet ($\Omega_{cc}^{+} = ccs$), each
The properties of these bary
a variety of theoretical mode
the Ξ_{cc} states are predicted
3700 MeV/c^2 [6–33]. The
states are expected to differ
approximate isospin symme
for the lifetime of the Ξ_{cc}^{+}
250 fs, and the lifetime of th
three to four times longer a
37–40]. While both are expe
colliders [41–43], the longe
should make it significantly
baryon in such experiments
(online) event-selection req
backgrounds originating
tion point.
Experimentally, there is a
system. Observations of the
2 MeV/c^2 with signal yield
background in the final state

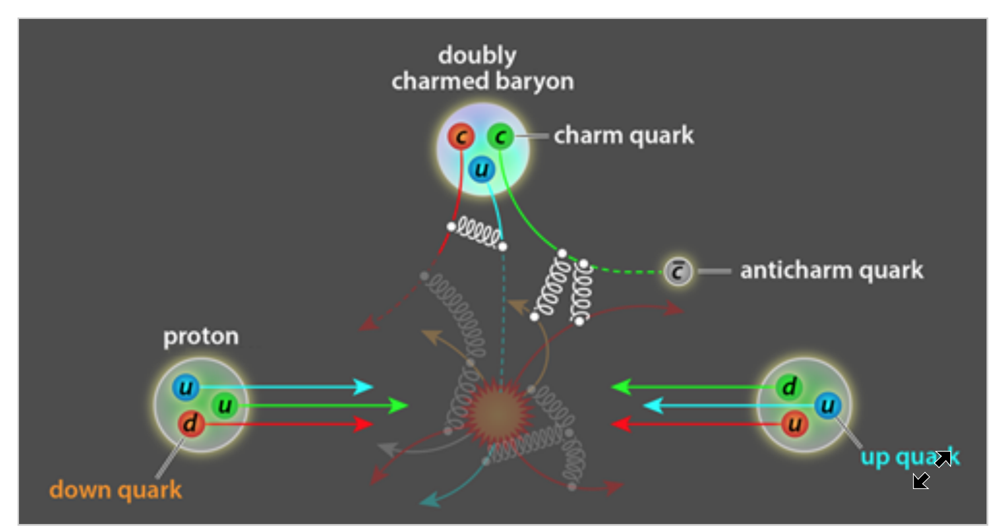
APS physics | Physics

Viewpoint: A Doubly Charming Particle

Raúl A. Briceño, Department of Physics, Old Dominion University, Norfolk, VA 23529, USA and Thomas Jefferson National Accelerator Facility, Newport News, VA 23606, USA

September 11, 2017 • *Physics* 10, 100

High-precision experiments at CERN find a new baryon containing two charm quarks.



APS/Alan Stonebraker

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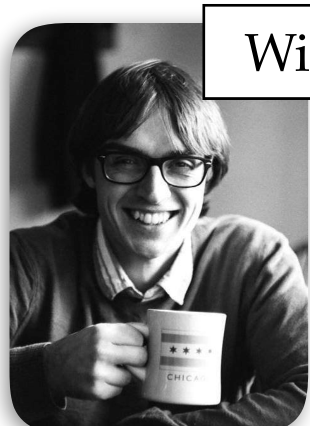
  - physical or lighter quark masses [down to m_π~120 MeV] ✓
  - non-degenerate light-quark masses: N_f=1+1+1+1 ✓
  - dynamical QED ✓
- One of the frontiers of lattice QCD: multi-particle physics
  - scattering / reactions
  - composite states
    - bound states
    - hadronic resonances
    - form factors

Formal development:
- under way
- more needed

Benchmark calculations:
- unphysical quark masses [m_π=236, 391 MeV]
- exploratory
- proof of principle
- …

Lattice QCD calculations with multi-hadron states
in the mesonic isoscalar sector



Wilson (Marie Curie / Royal fellow / Trinity)

← *a familiar and friendly face!*

Dudek (W&M / JLab)

Edwards (JLab)

hadspec

PRL **118,** 022002 (2017) PHYSICAL REVIEW LETTERS week ending 13 JANUARY 2017

Isoscalar $\pi\pi$ Scattering and the σ Meson Resonance from QCD

Raul A. Briceño,[1,*] Jozef J. Dudek,[1,2,†] Robert G. Edwards,[1,‡] and David J. Wilson[3,§]

(for the Hadron Spectrum Collaboration)

[1] *Thomas Jefferson National Accelerator Facility, 12000 Jefferson Avenue, Newport News, Virginia 23606, USA*
[2] *Department of Physics, College of William and Mary, Williamsburg, Virginia 23187-8795, USA*
[3] *Department of Applied Mathematics and Theoretical Physics, Centre for Mathematical Sciences,*

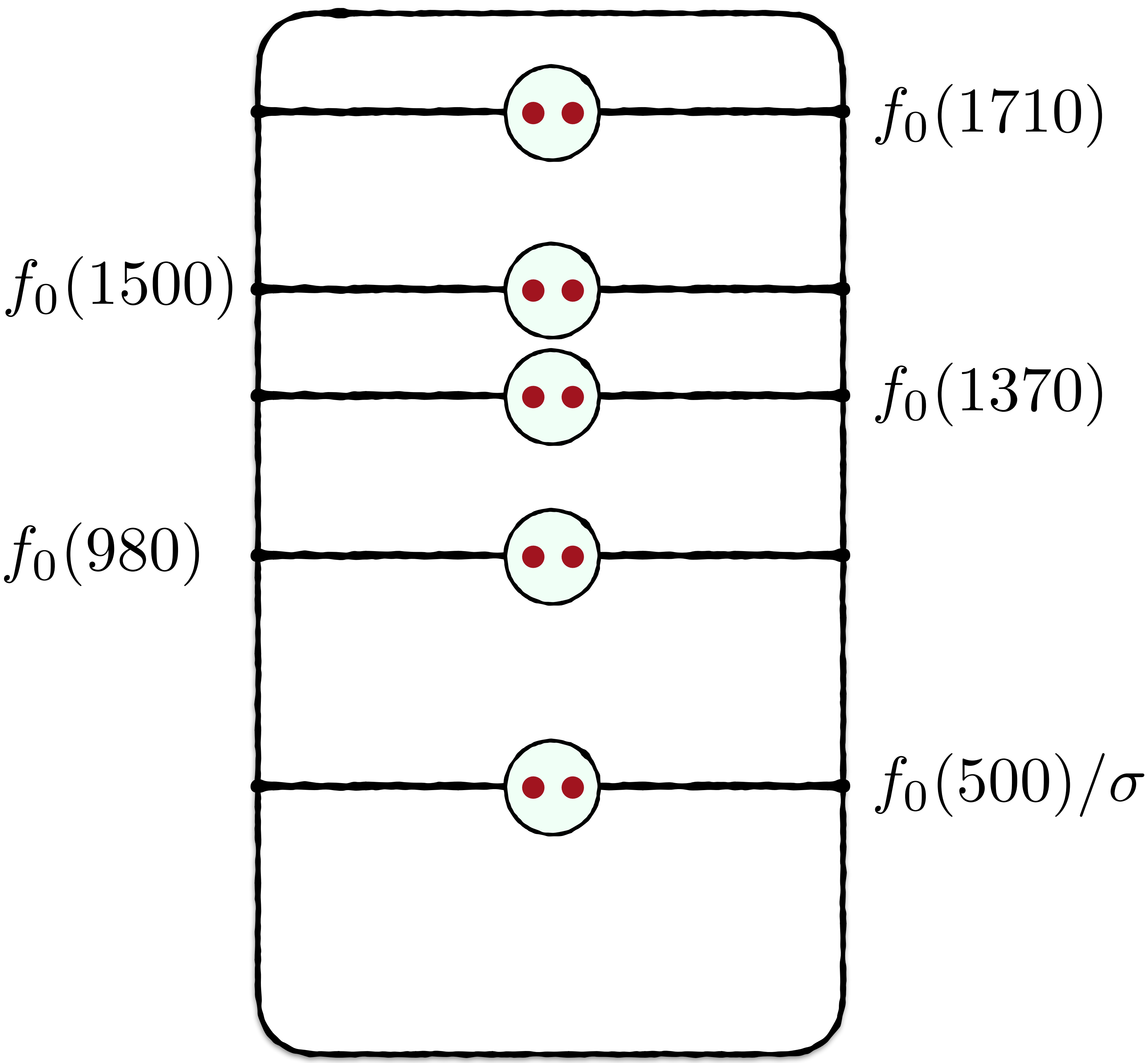
JLAB-THY-17-2534

Isoscalar $\pi\pi, K\overline{K}, \eta\eta$ scattering and the σ, f_0, f_2 mesons from QCD

Raul A. Briceño,[1,2,*] Jozef J. Dudek,[1,3,†] Robert G. Edwards,[1,‡] and David J. Wilson[4,§]
(for the Hadron Spectrum Collaboration)
[1] *Thomas Jefferson National Accelerator Facility, 12000 Jefferson Avenue, Newport News, VA 23606, USA*
[2] *Department of Physics, Old Dominion University, Norfolk, VA 23529, USA*
[3] *Department of Physics, College of William and Mary, Williamsburg, VA 23187, USA*
[4] *School of Mathematics, Trinity College, Dublin 2, Ireland*
(Dated: August 23, 2017)

We present the first lattice QCD study of coupled isoscalar $\pi\pi, K\overline{K}, \eta\eta$ S- and D-wave scattering extracted from discrete finite-volume spectra computed on lattices which have a value of the quark mass corresponding to $m_\pi \sim 391$ MeV. In the $J^P = 0^+$ sector we find analogues of the experimental σ and $f_0(980)$ states, where the σ appears as a stable bound-state below $\pi\pi$ threshold, and, similar to what is seen in experiment, the $f_0(980)$ manifests itself as a dip in the $\pi\pi$ cross section in the vicinity of the $K\overline{K}$ threshold. For $J^P = 2^+$ we find two states resembling the $f_2(1270)$ and $f_2'(1525)$, observed as narrow peaks, with the lighter state dominantly decaying to $\pi\pi$ and the heavier state to $K\overline{K}$. The presence of all these states is determined rigorously by finding the pole singularity content of scattering amplitudes, and their couplings to decay channels are established using the residues of

The isoscalar, scalar sector



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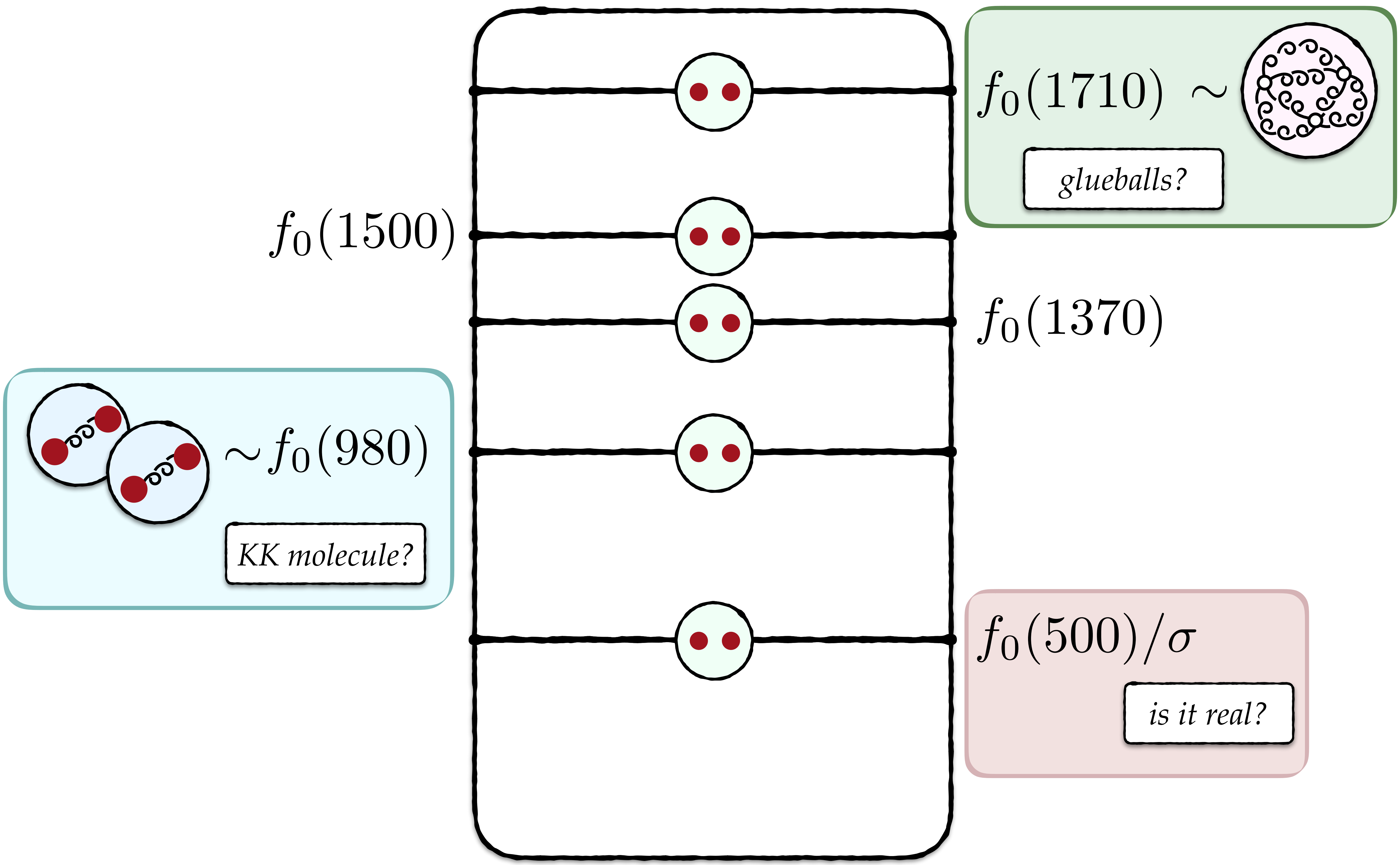

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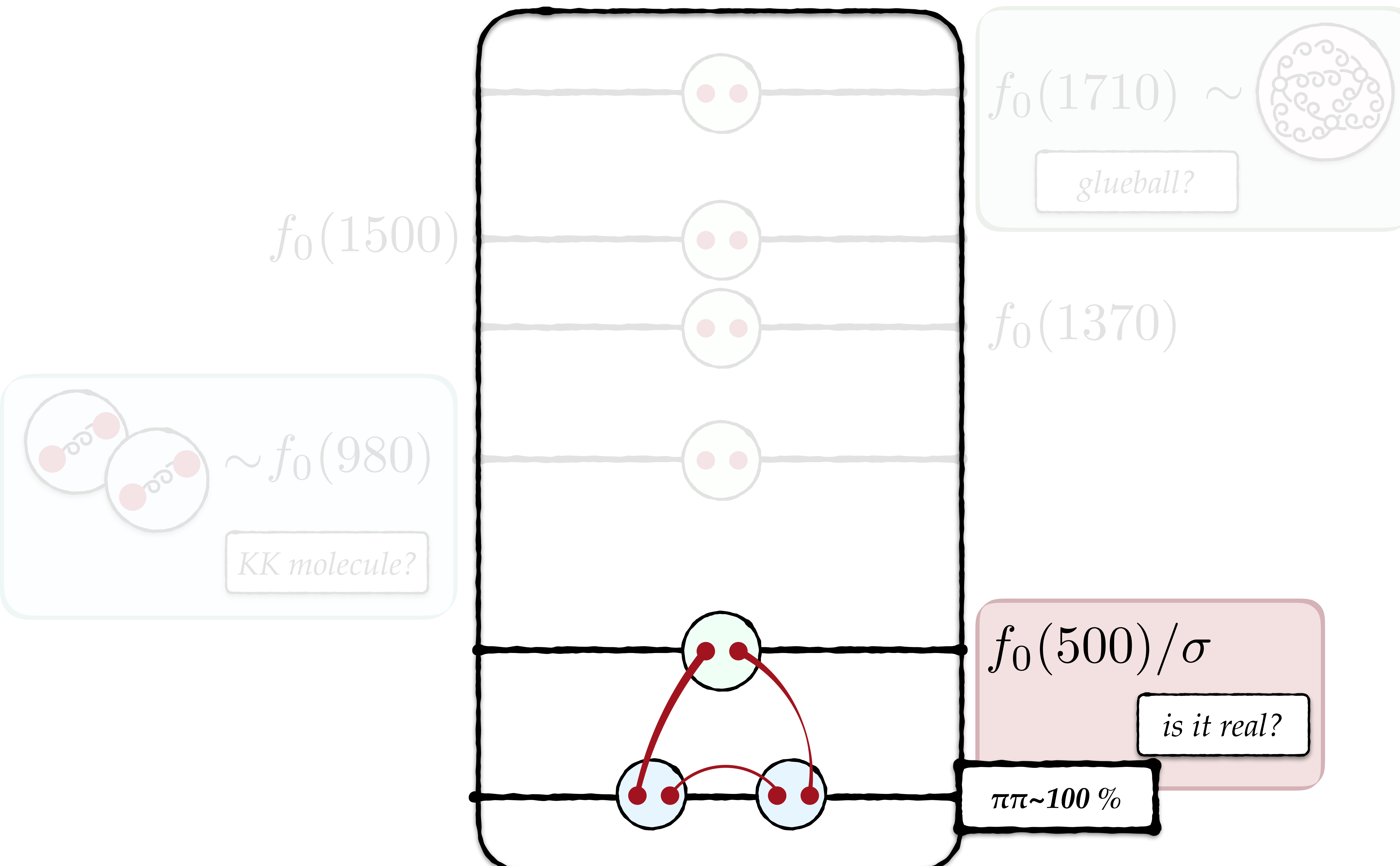

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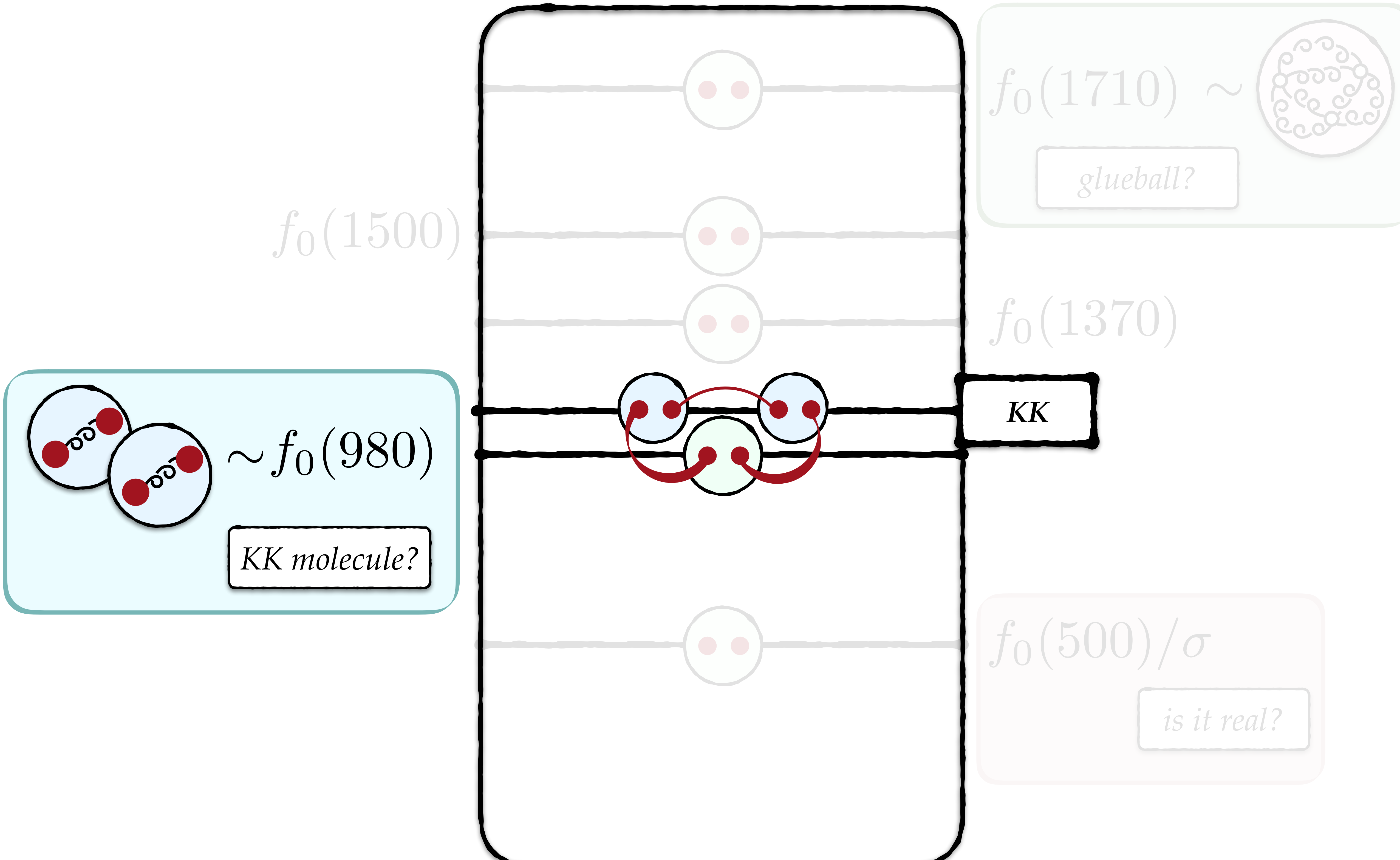

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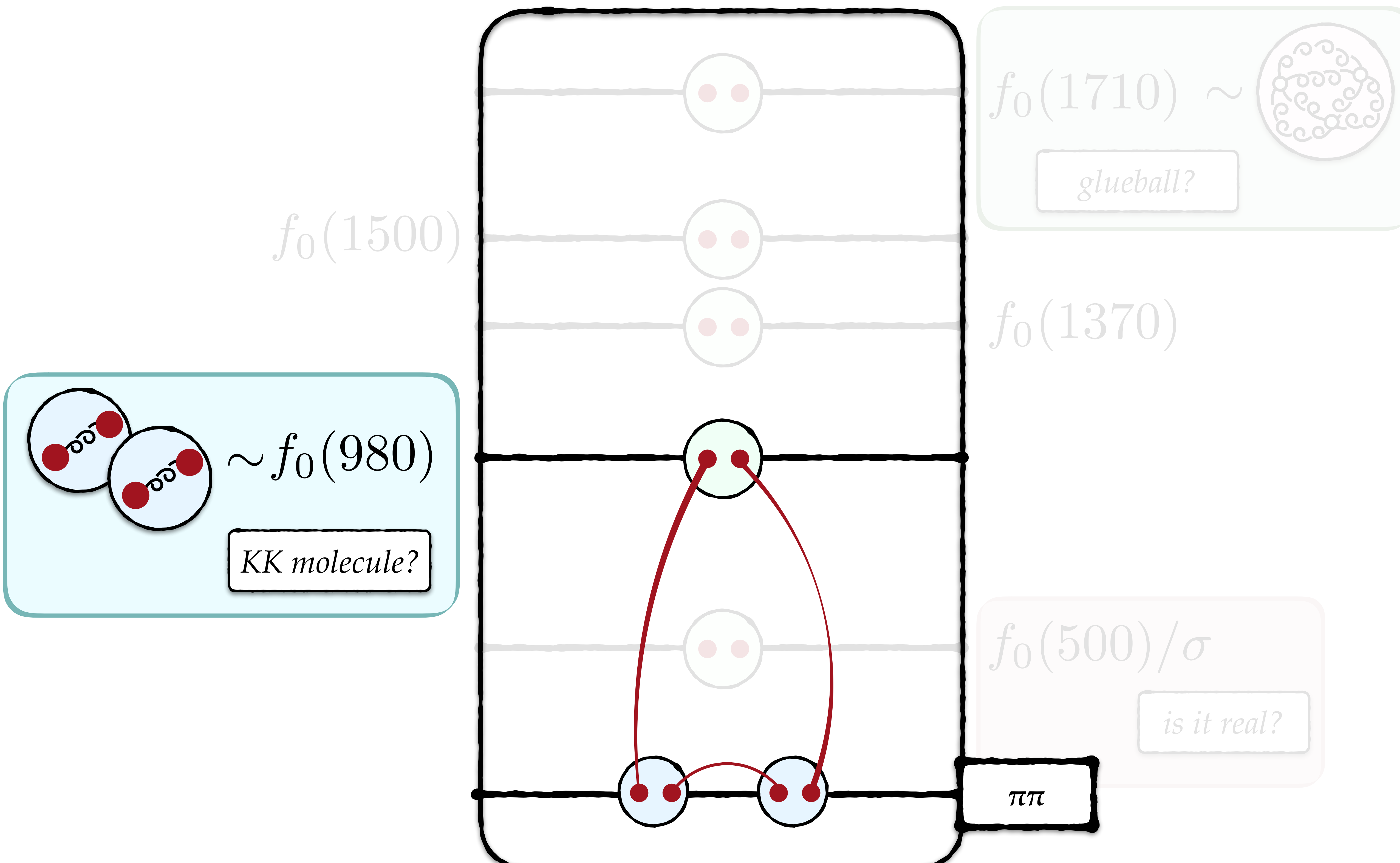
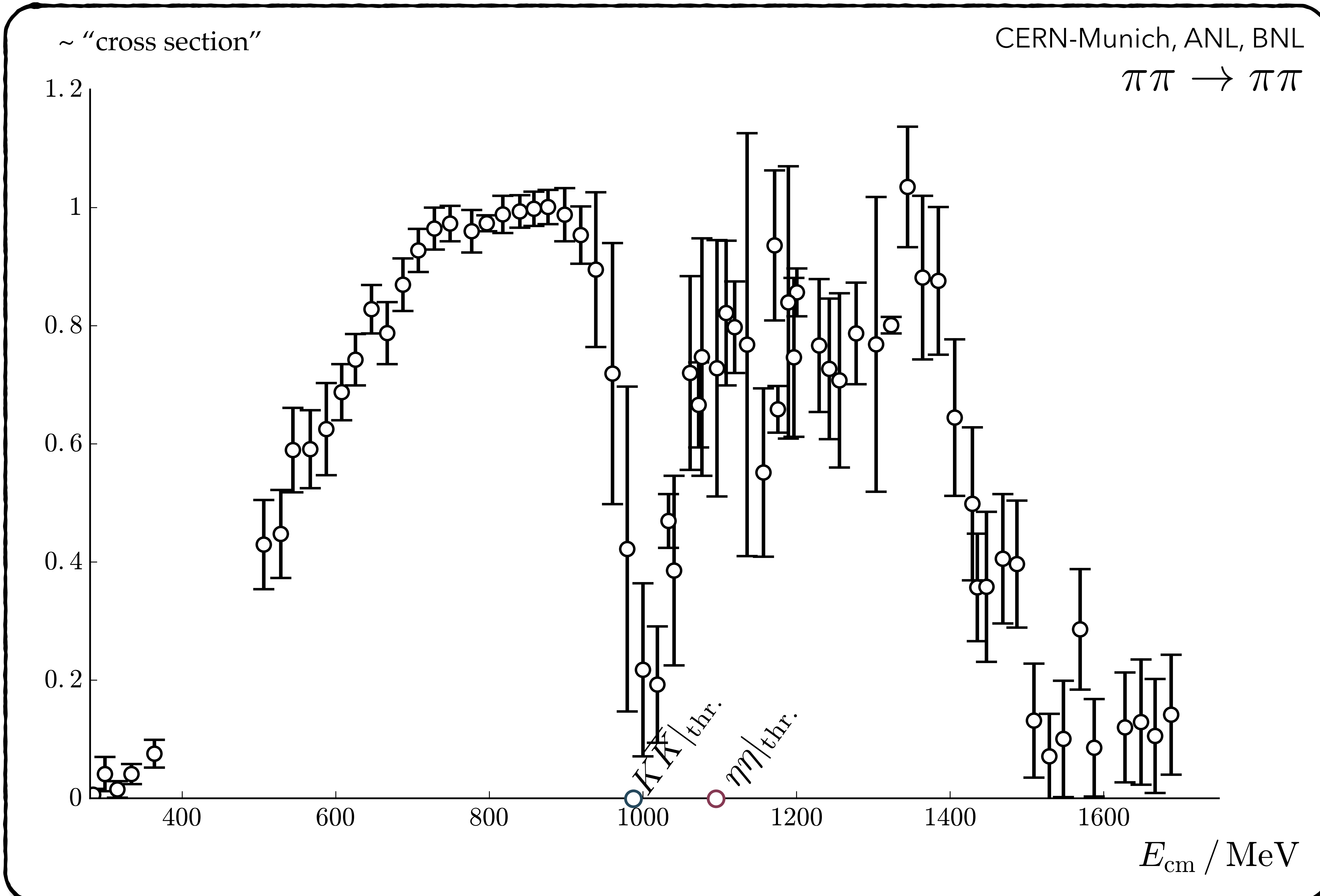
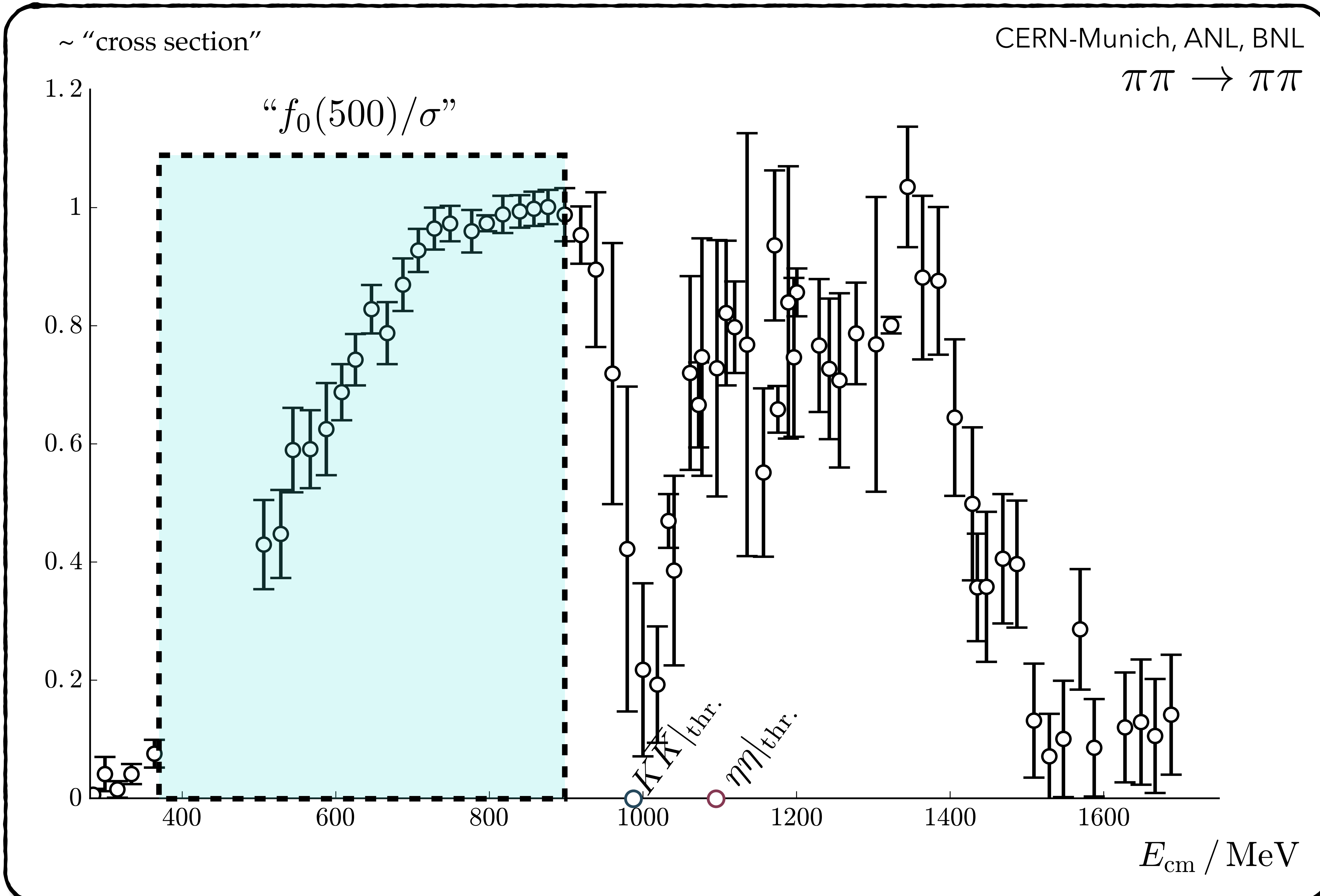


Experimental manifestation



Experimental manifestation



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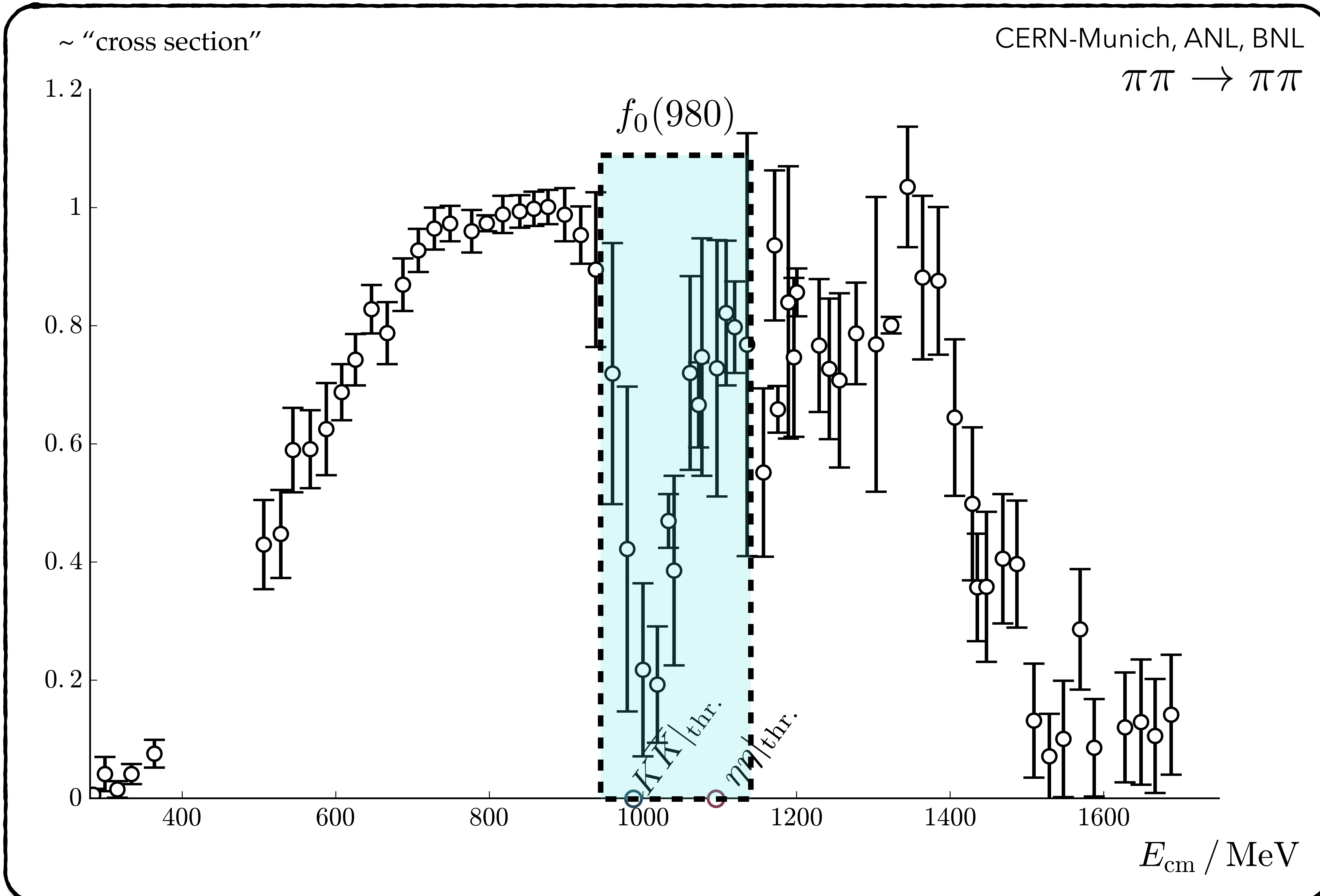
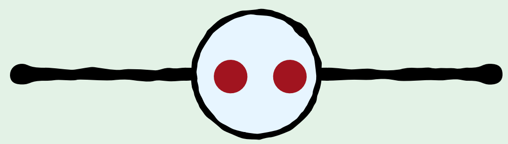


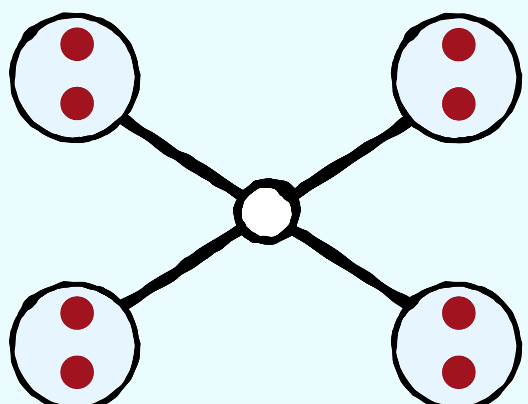
Quantitative definition

propagator:



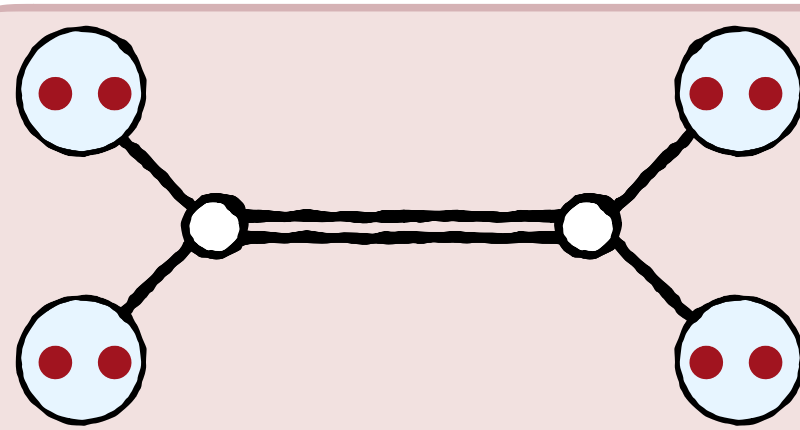
$$\sim \frac{iZ}{p^2 - m^2} = \frac{iZ}{s - m^2}$$

scattering amplitude:



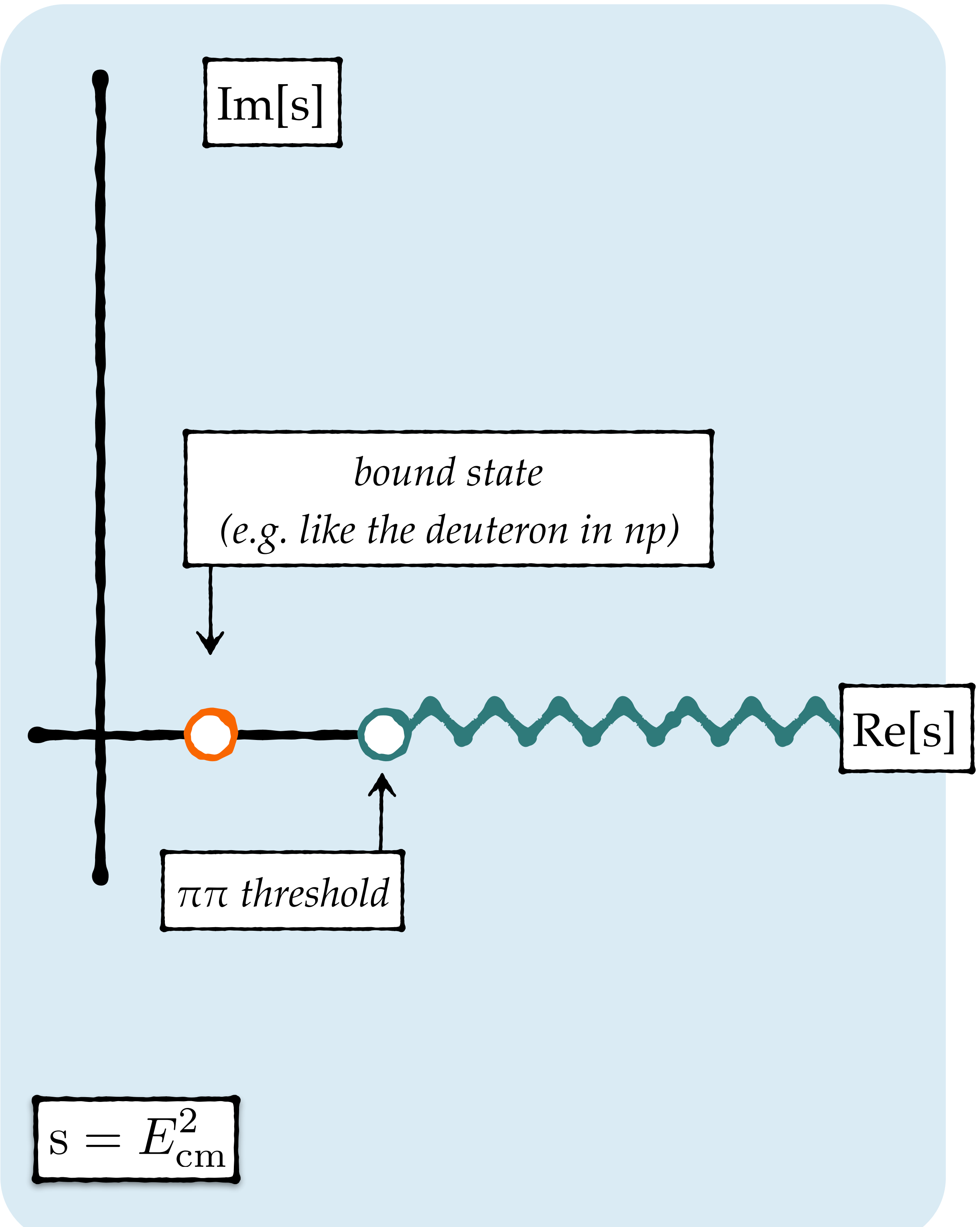
$$= i\mathcal{M}$$

…near bound state or resonance:



$$\sim \frac{ig^2}{s_0 - s}, \quad s_0 = (E_0 - \tfrac{i}{2}\Gamma)^2$$

Sheets and composite states



unitarity: $\mathcal{M}_{\pi\pi}^{-1} \propto p_{\pi\pi} \cot\delta - i p_{\pi\pi}$

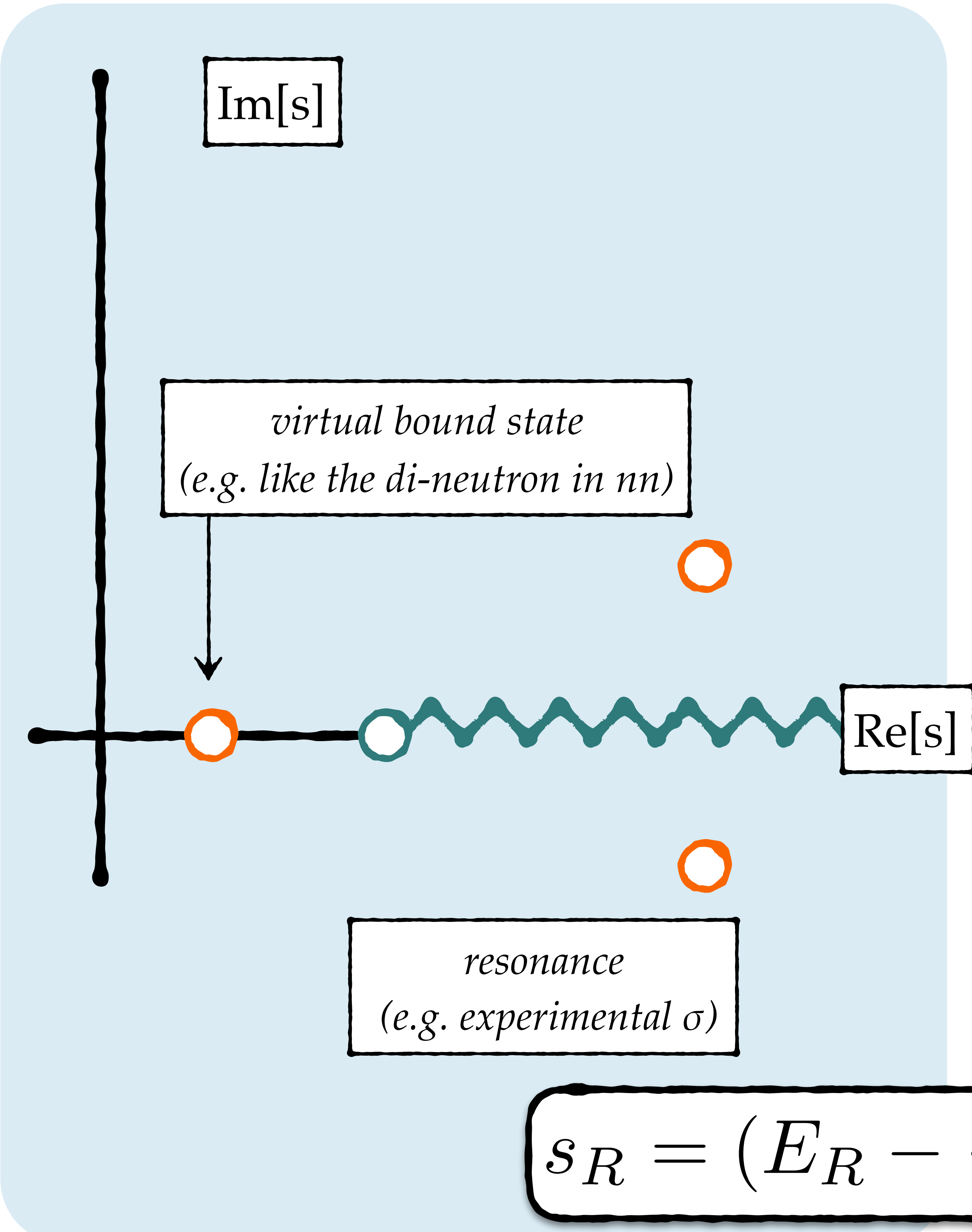
square-root singularity at each threshold:

$$p_{\pi\pi} = \frac{1}{2}\sqrt{s - s_{\pi\pi,th}}$$

unphysical sheet:

sheet	Im $p_{\pi\pi}$
I	+

Sheets and composite states



$$s_R = (E_R - \tfrac{i}{2}\Gamma_R)^2$$

unitarity: $\mathcal{M}^{-1}_{\pi\pi} \propto p_{\pi\pi} \cot\delta - i p_{\pi\pi}$

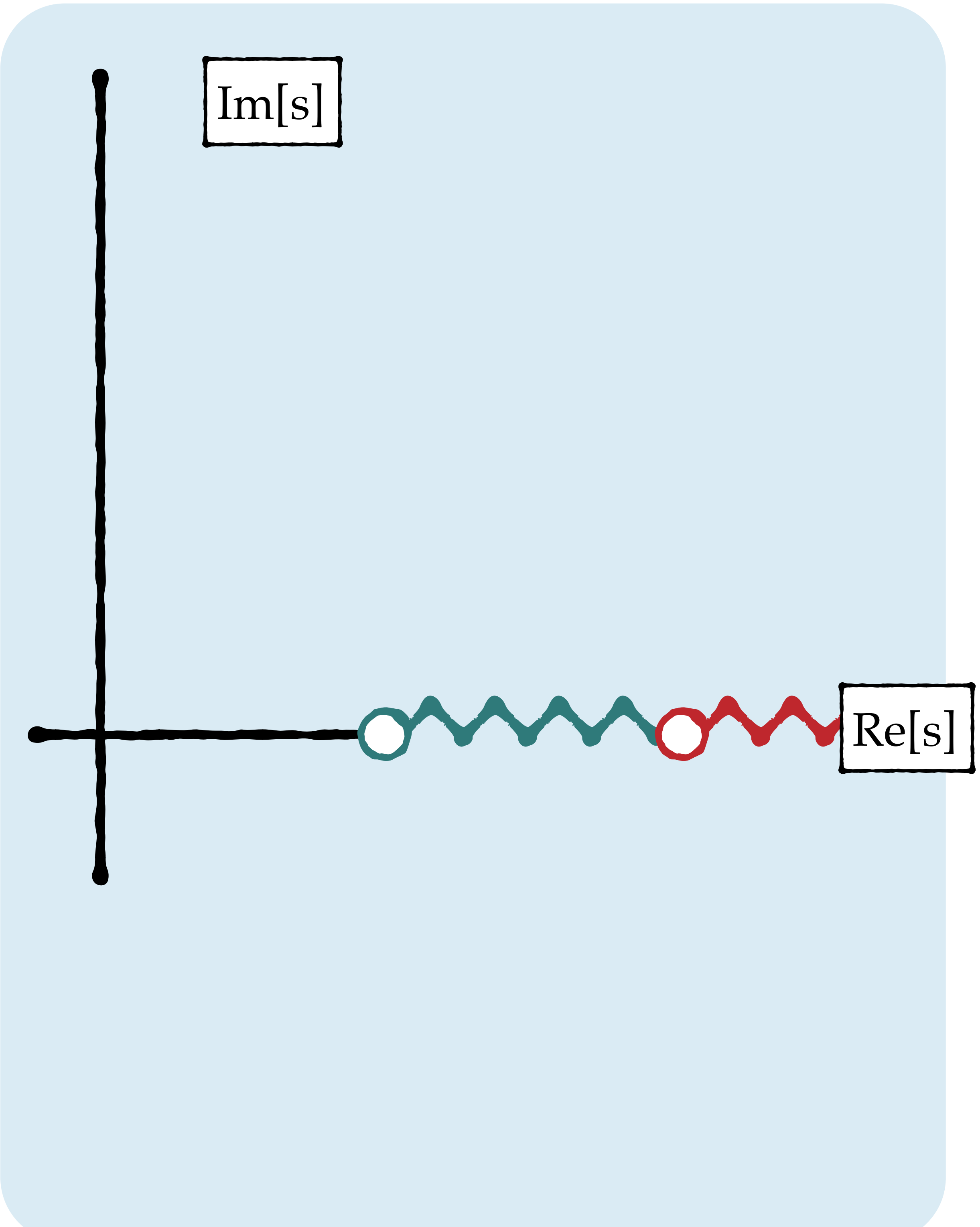
square-root singularity at each threshold:

$$p_{\pi\pi} = \frac{1}{2}\sqrt{s - s_{\pi\pi,th}}$$

unphysical sheet:

sheet	Im $p_{\pi\pi}$
I	$+$
II	$-$

Sheets and composite states



unitarity: $\mathcal{M}_{ab}^{-1} \propto \mathcal{K}_{ab}^{-1} - i p_a \, \delta_{ab}$

$a\,b = \pi\pi \text{ or } K\overline{K}$

square-root singularity at each threshold:

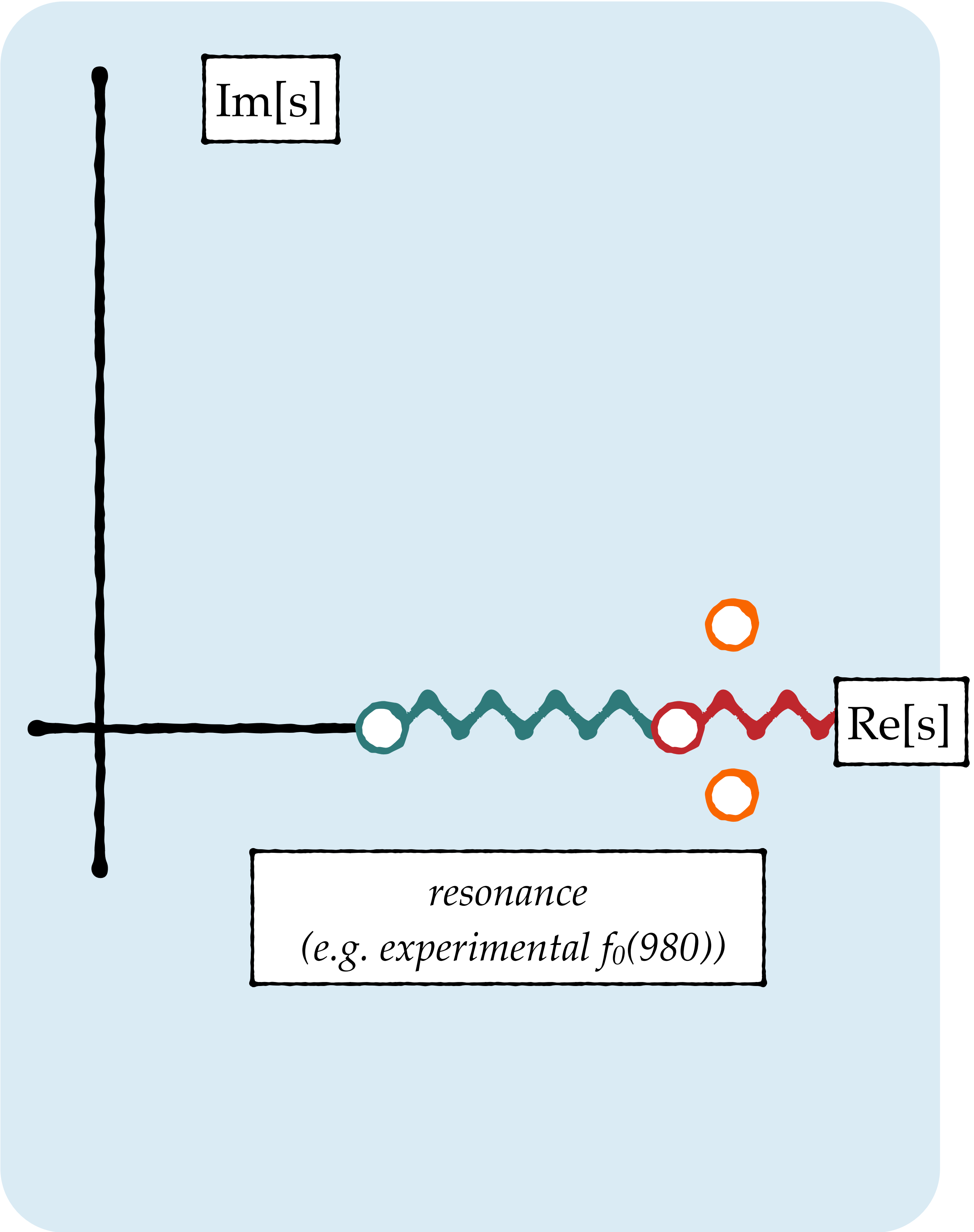
$$p_{\pi\pi} = \frac{1}{2}\sqrt{s - s_{\pi\pi,th}}$$

$$p_{K\overline{K}} = \frac{1}{2}\sqrt{s - s_{K\overline{K},th}}$$

physical sheet:

sheet	Im $p_{\pi\pi}$	Im $p_{K\overline{K}}$
I	$+$	$+$

Sheets and composite states



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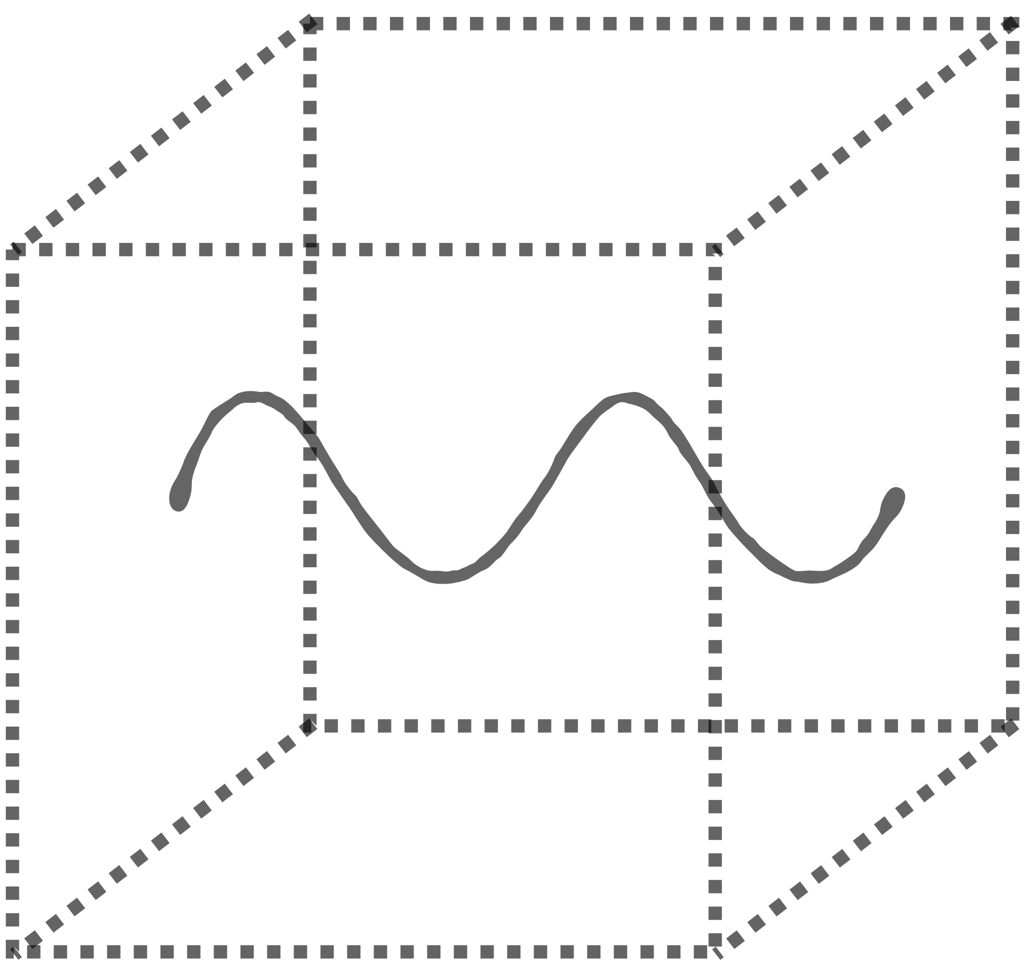
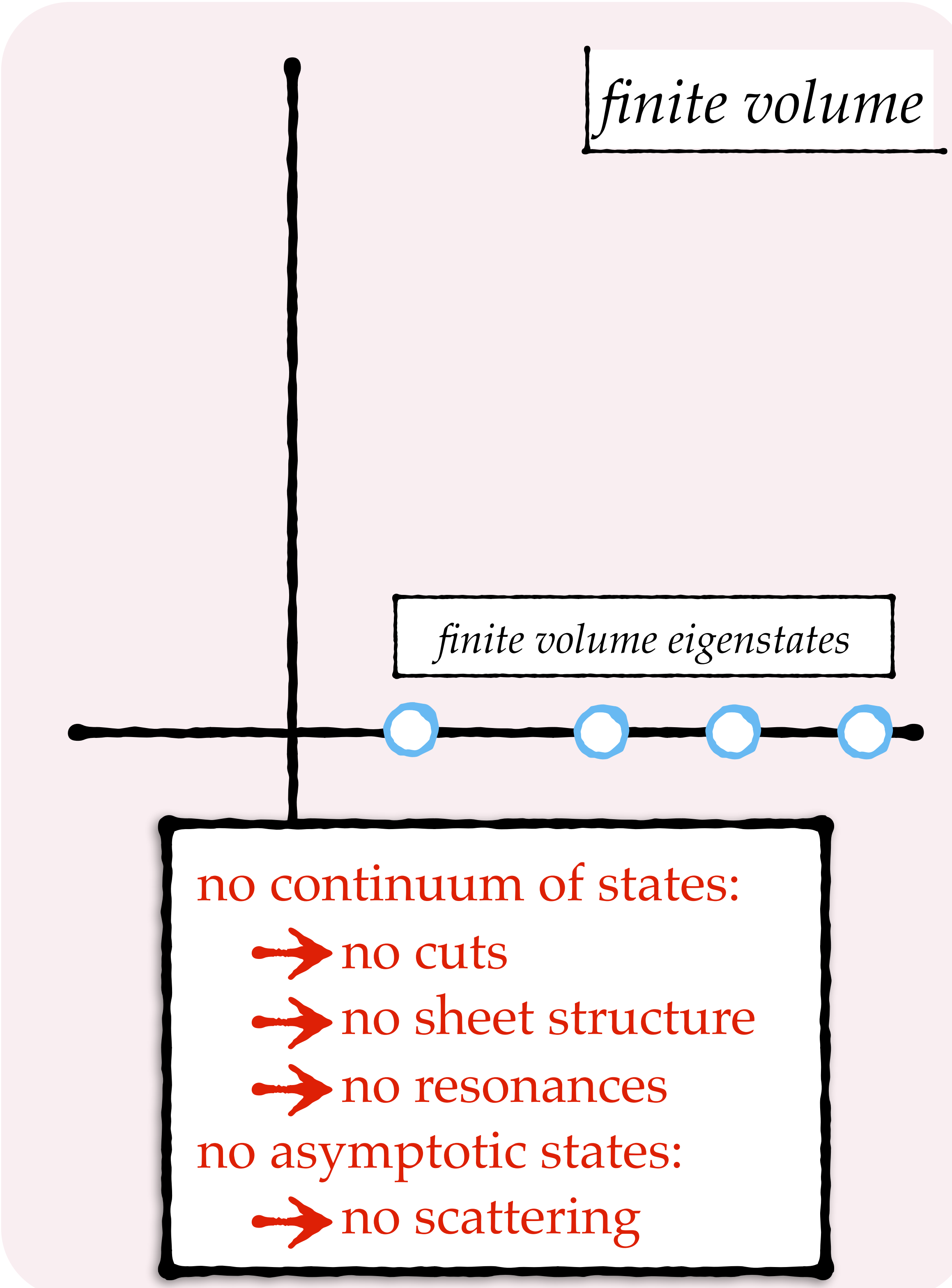
unphysical sheets:

sheet	Im $p_{\pi\pi}$	Im $p_{K\overline{K}}$
I	+	+
II	−	+
III	−	−
IV	+	−

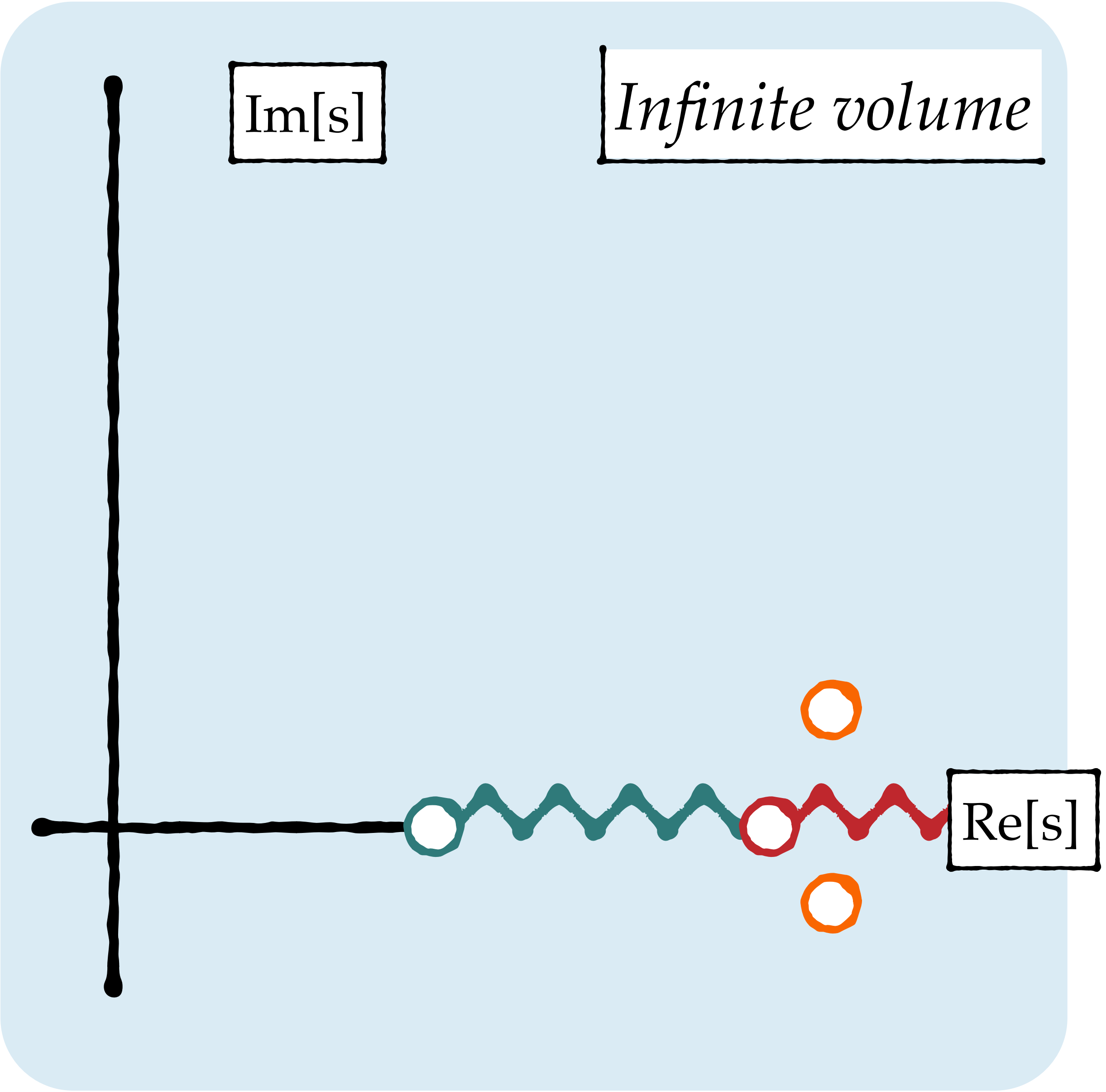
Lattice QCD is QCD in a finite volume

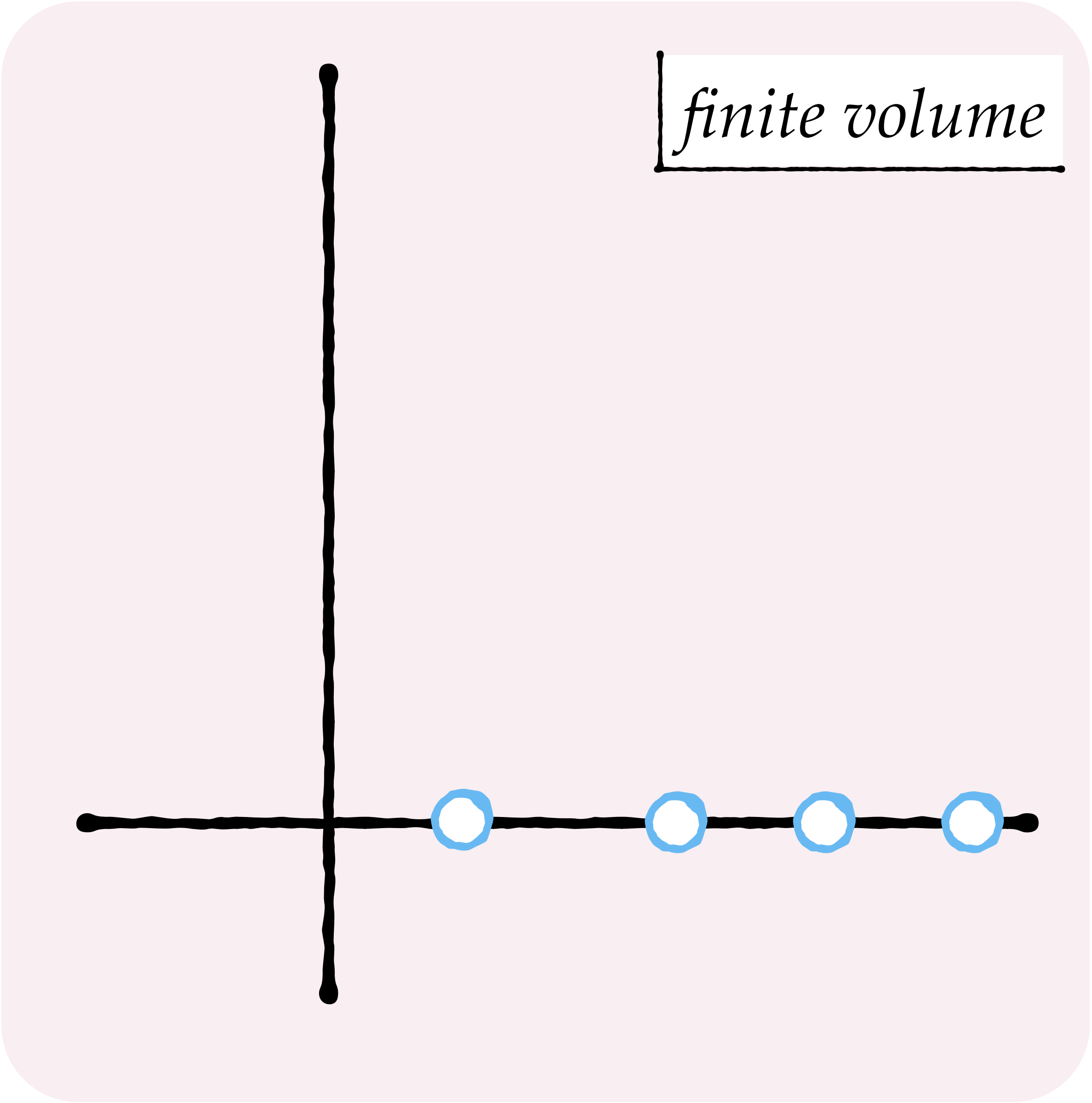


"only a discrete number of modes can exist in a finite volume"



Finite vs. infinite volume spectrum

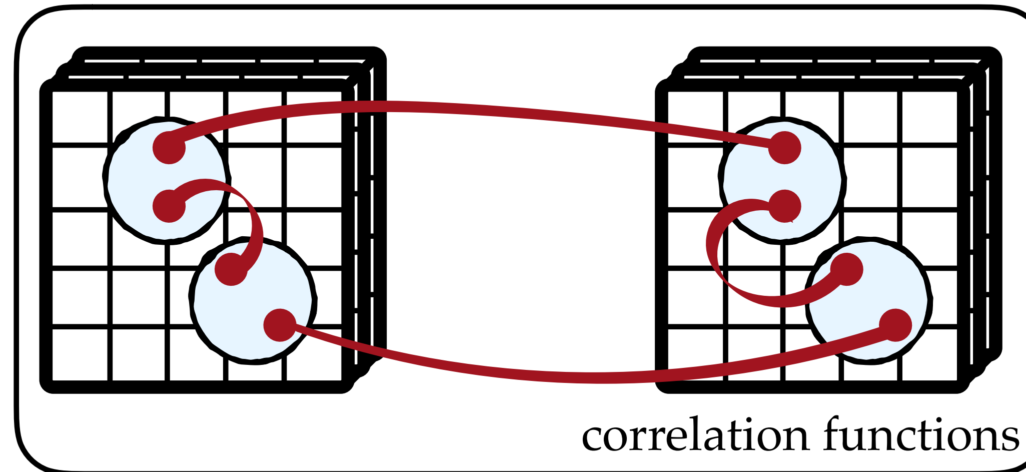


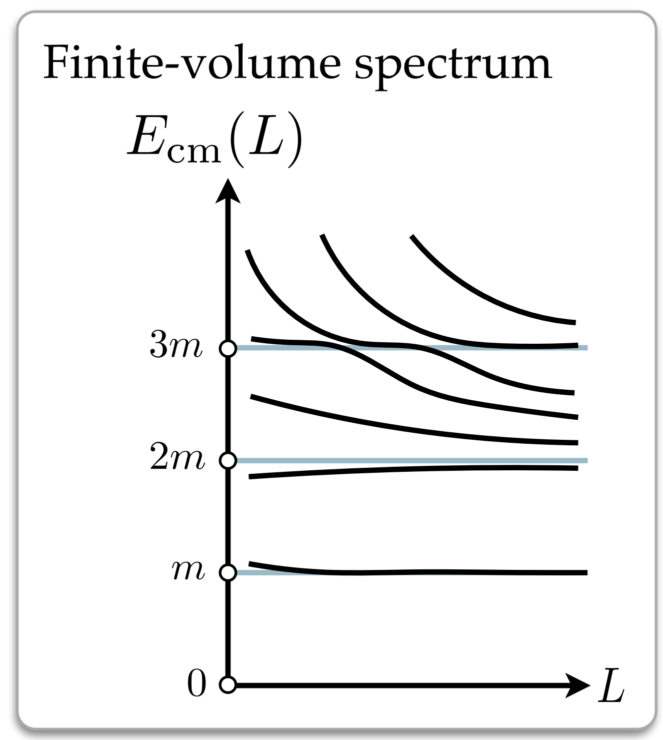


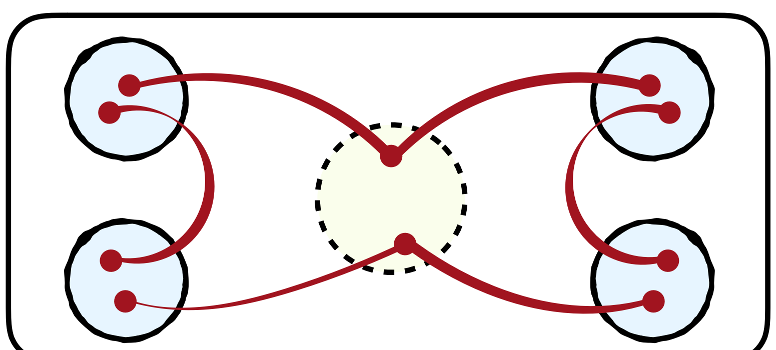
both pictures *are* QCD, and they must be related!

Obtaining the QCD spectrum

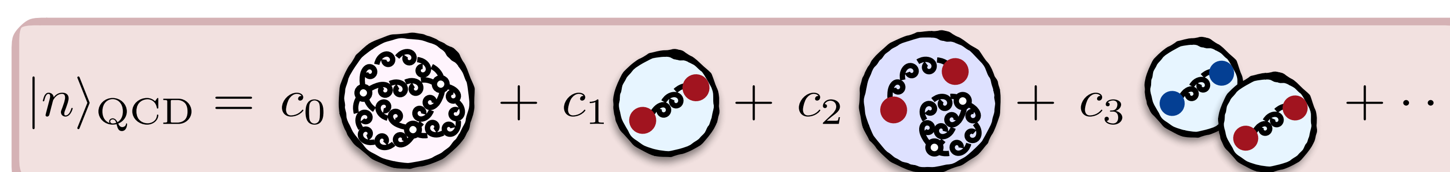
QCD → Lattice QCD →







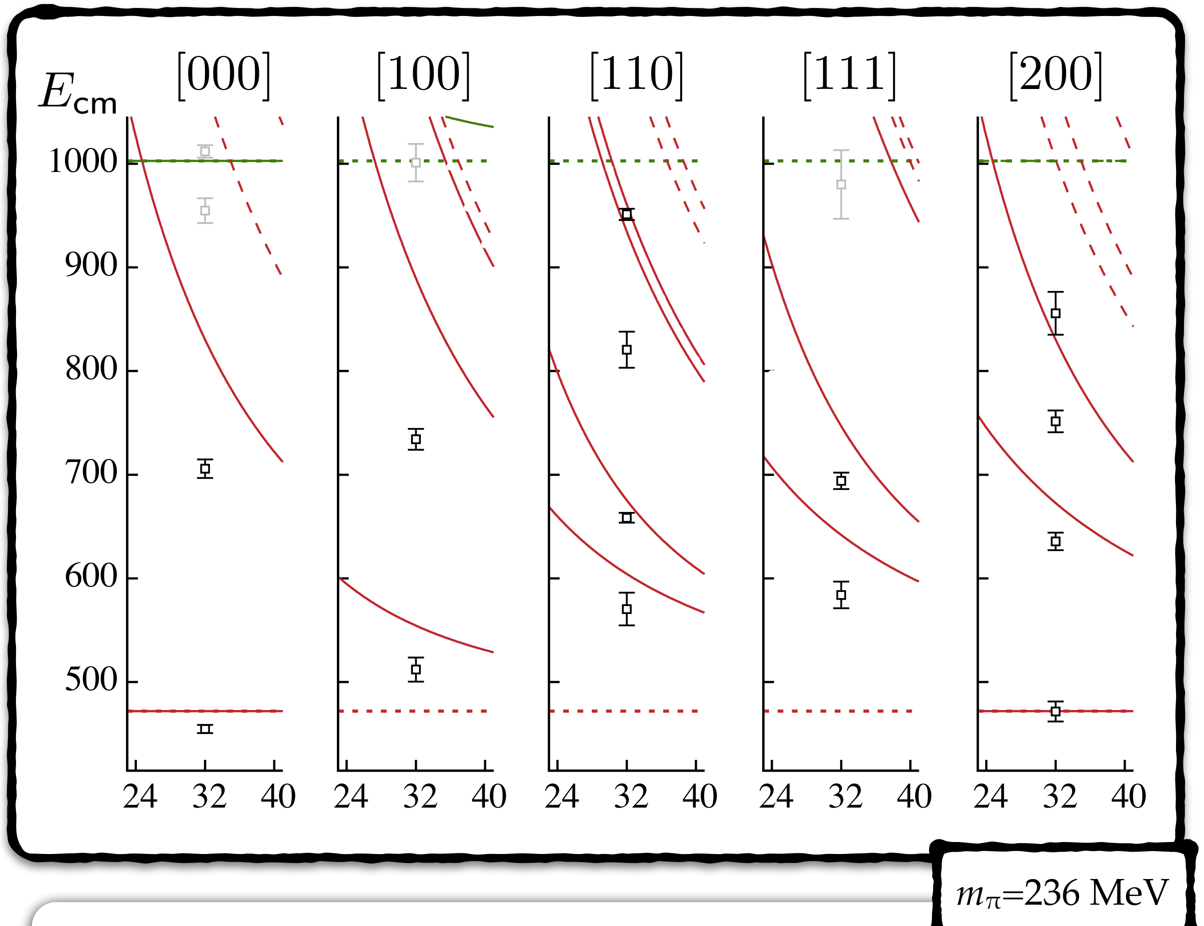
resonance poles →

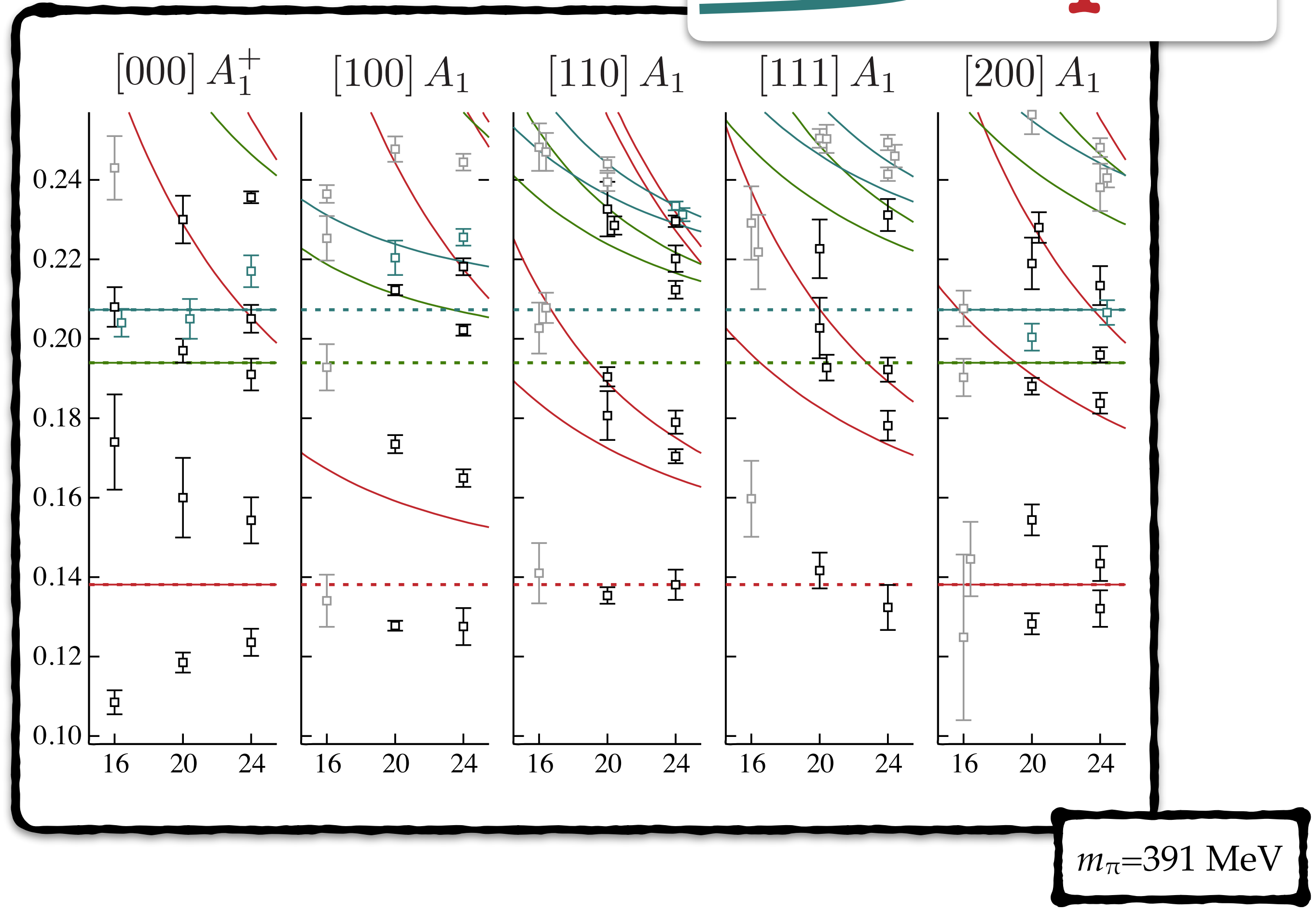


Extracting the spectrum

$$C_{ab}^{2pt.}(t,\mathbf{P}) \equiv \langle 0|\mathcal{O}_b(t,\mathbf{P})\mathcal{O}_a^\dagger(0,\mathbf{P})|0\rangle = \sum_n Z_{b,n} Z_{a,n}^\dagger e^{-E_n t}$$

- *Use local and multi-hadron ops ~ 20-30 ops*
- Evaluate all Wick contraction: **distillation** [Peardon, *et al.* (2009)]
- *Variationally* optimize operators [Michael (1985), Lüscher & Wolff (1990)]
- extract ~ 30 - 100 energy levels



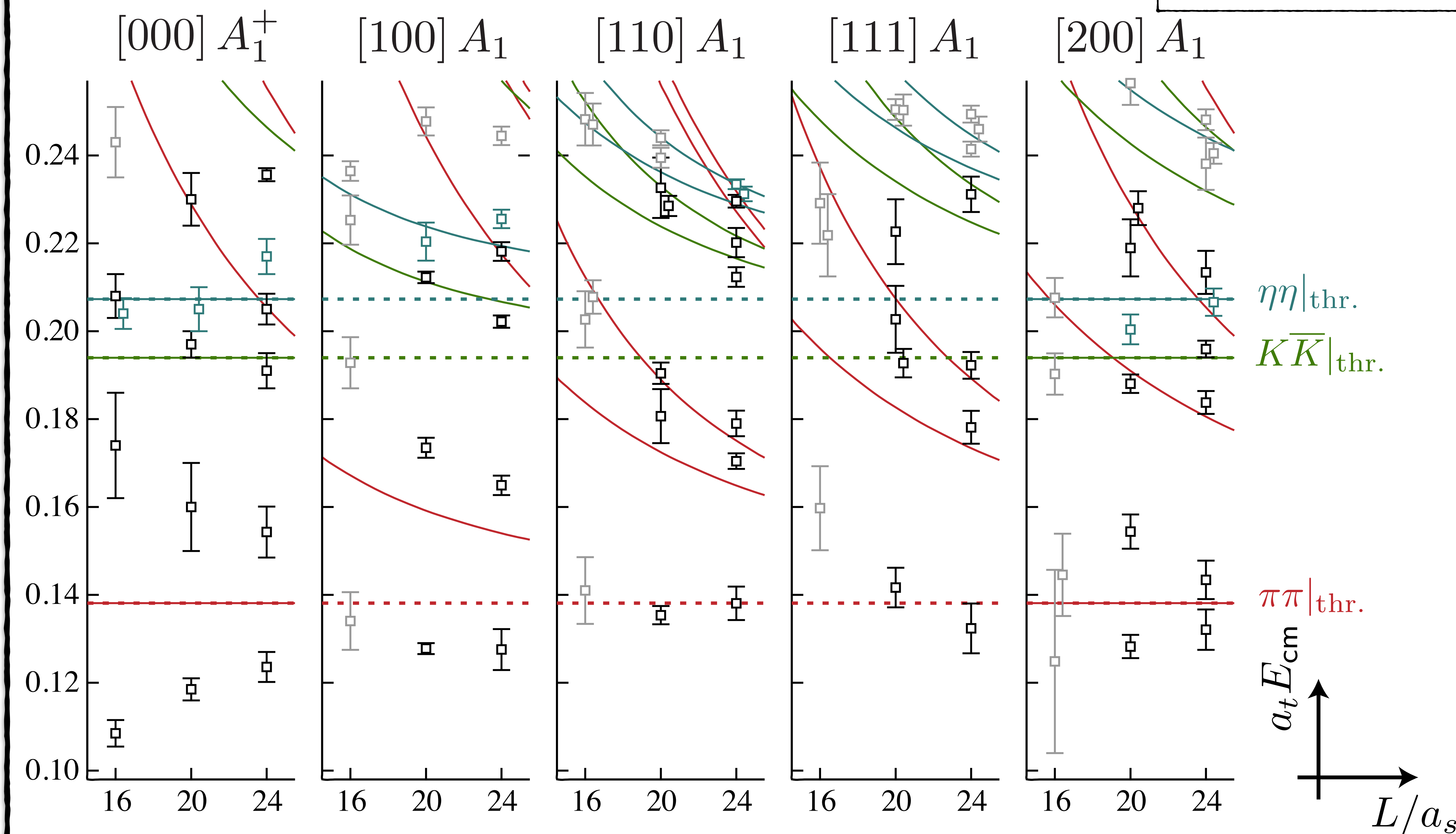


RB, Dudek, Edwards, Wilson - PRL (2017)
RB, Dudek, Edwards, Wilson - arXiv (2017)

Isoscalar spectra: S-wave dominant

- Multi-meson ops. are crucial
- Spectrum including a larger basis: $\{\pi\pi, K\overline{K}, \eta\eta, \ell\bar{\ell}, s\bar{s}\}$

Record number of states worldwide!



m_π=391 MeV

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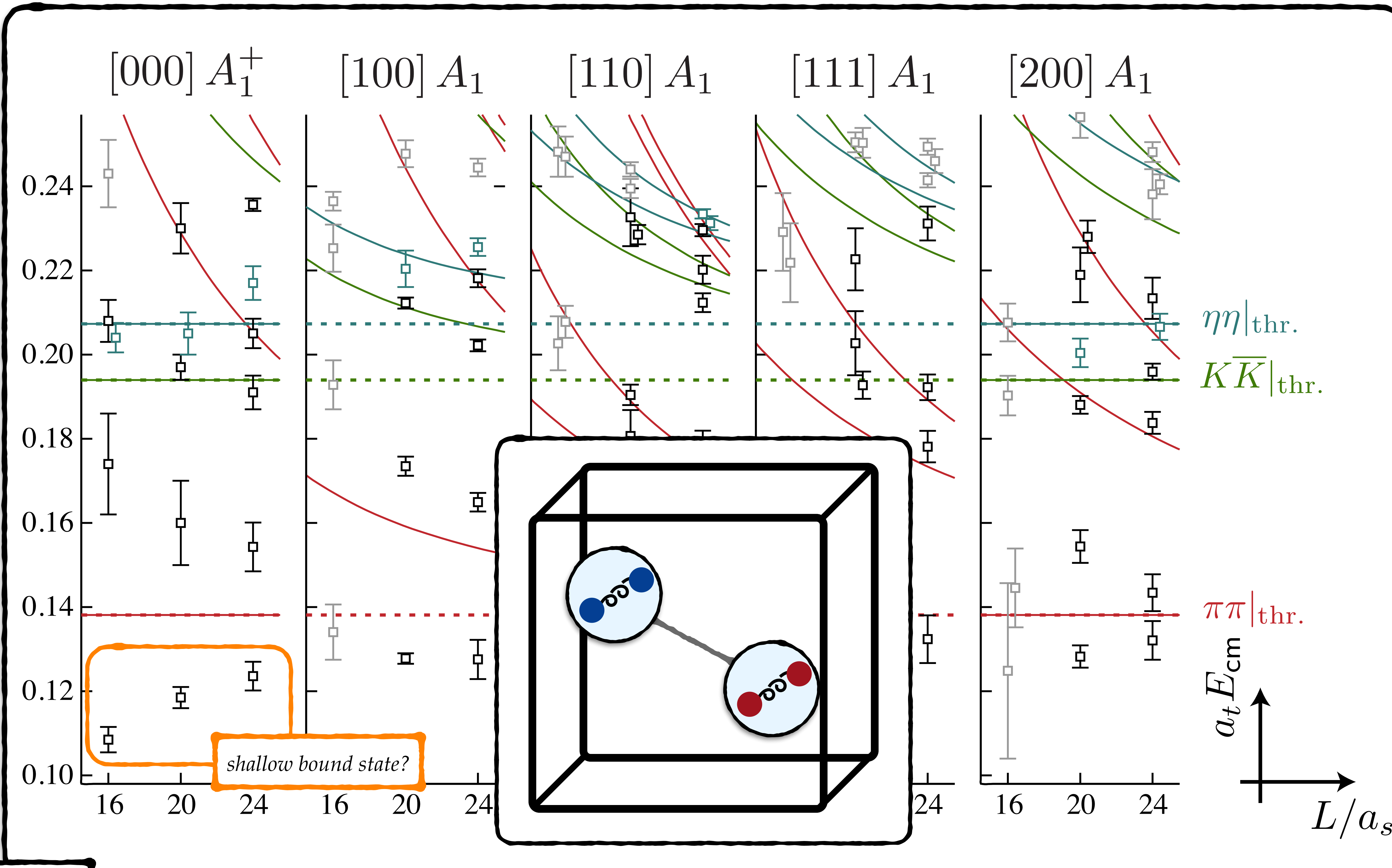

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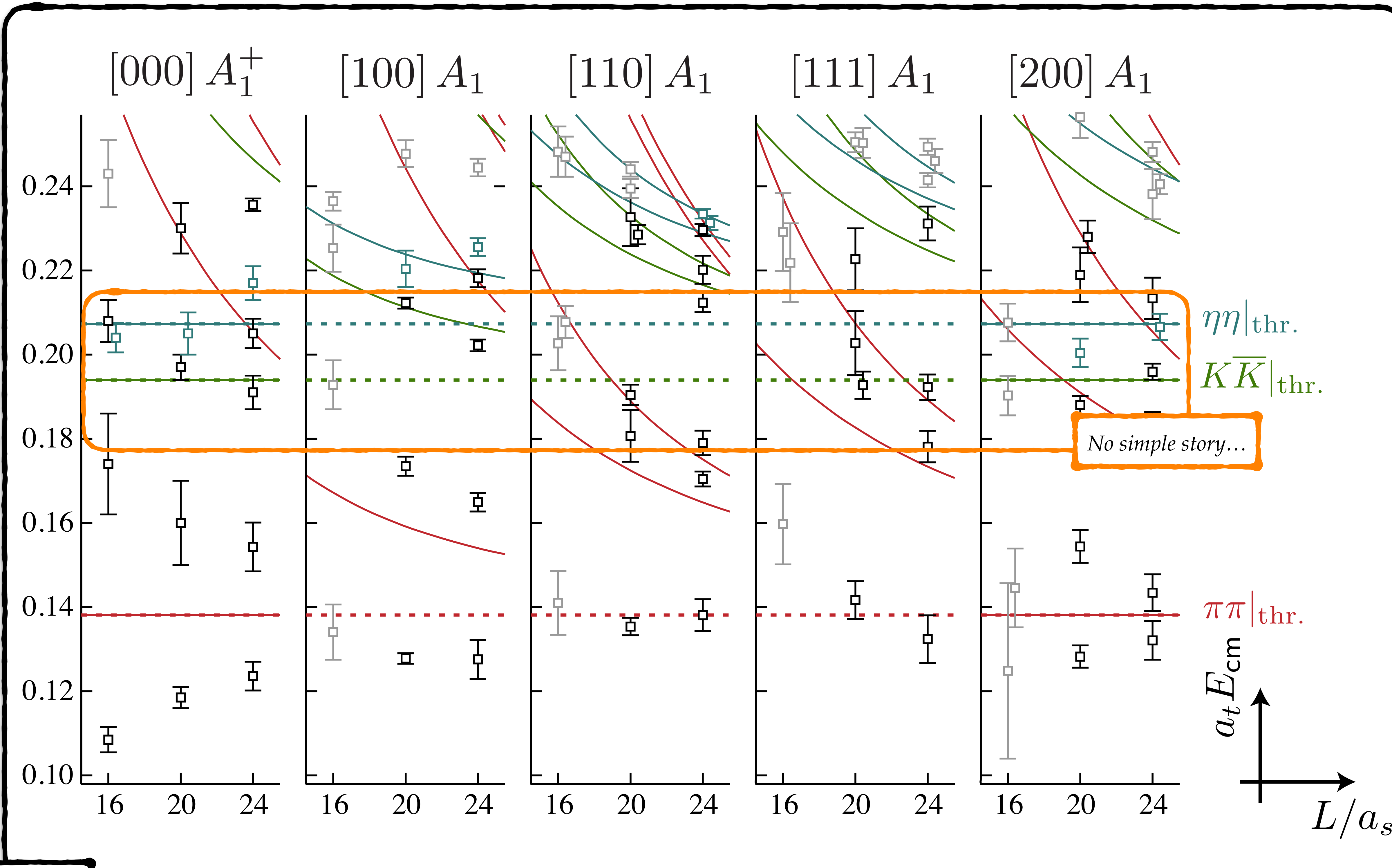

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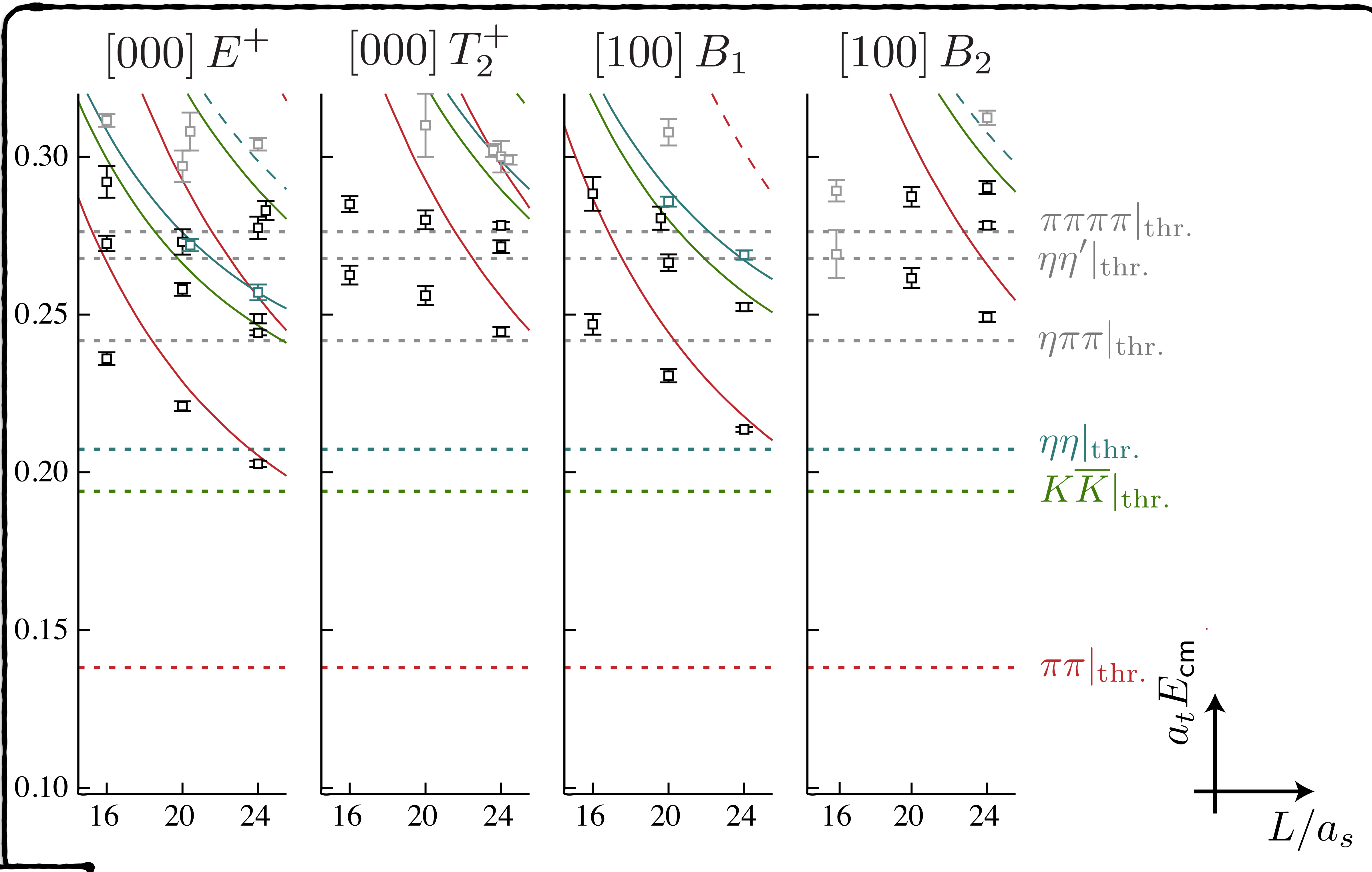

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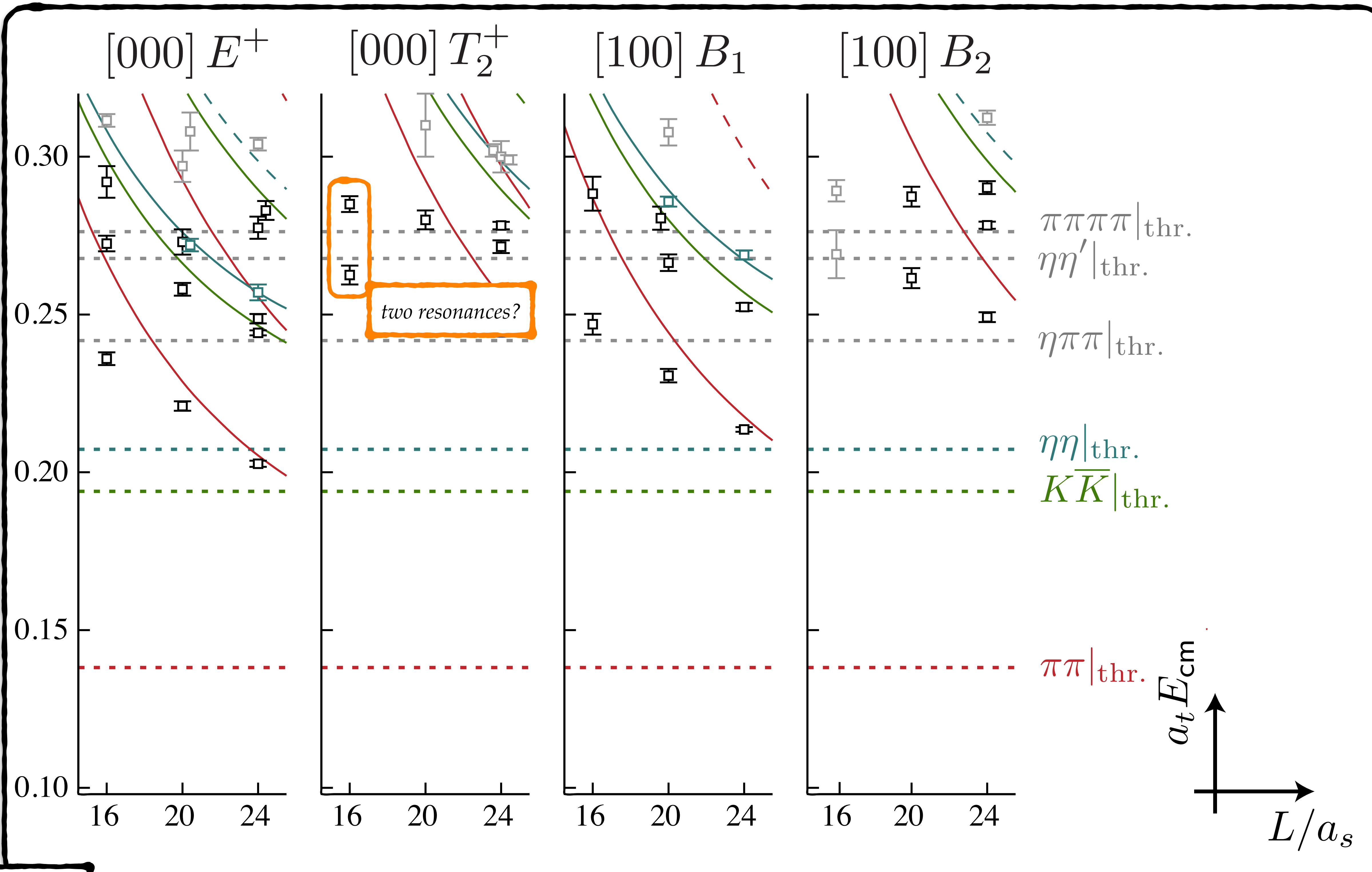

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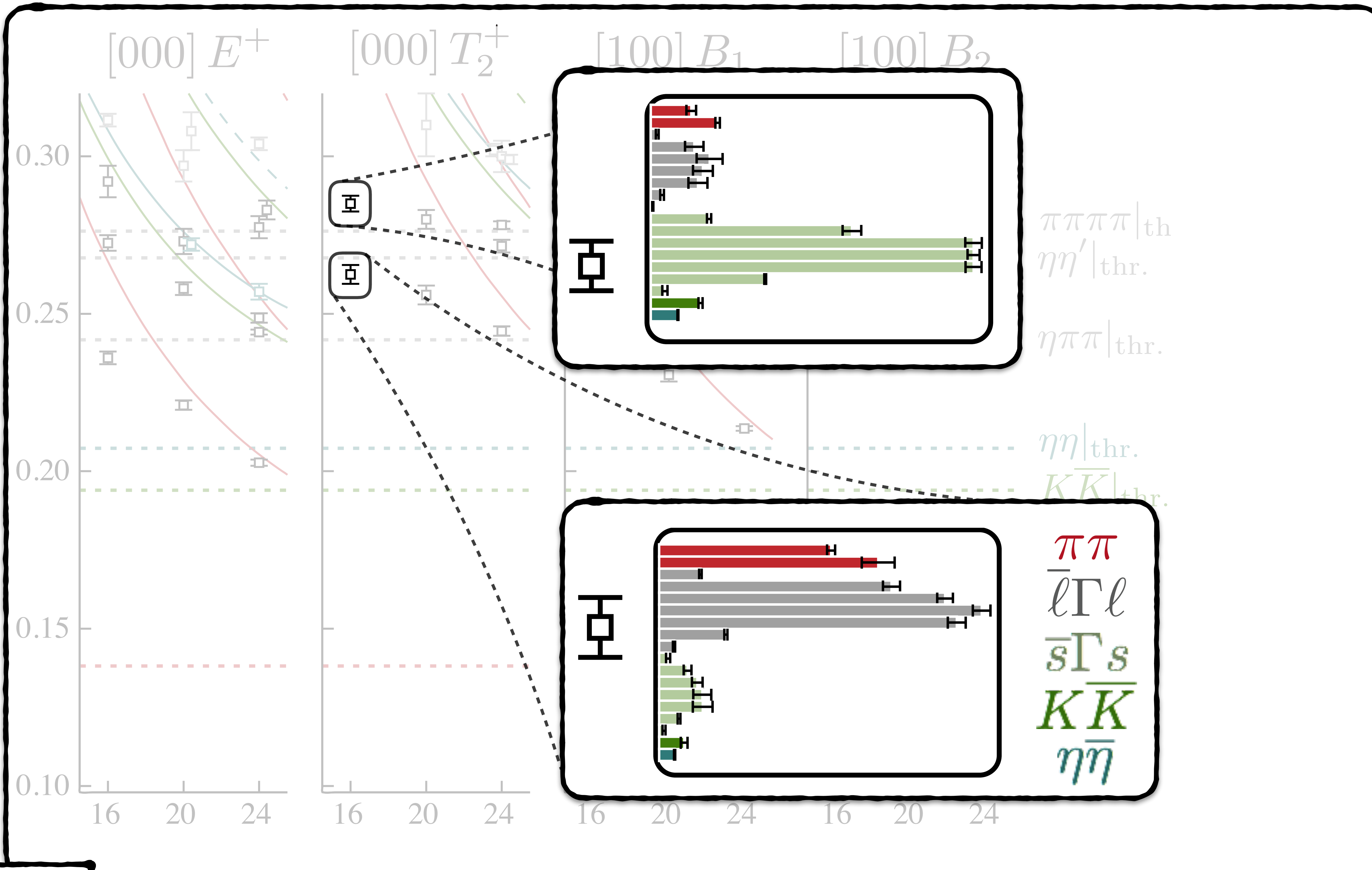

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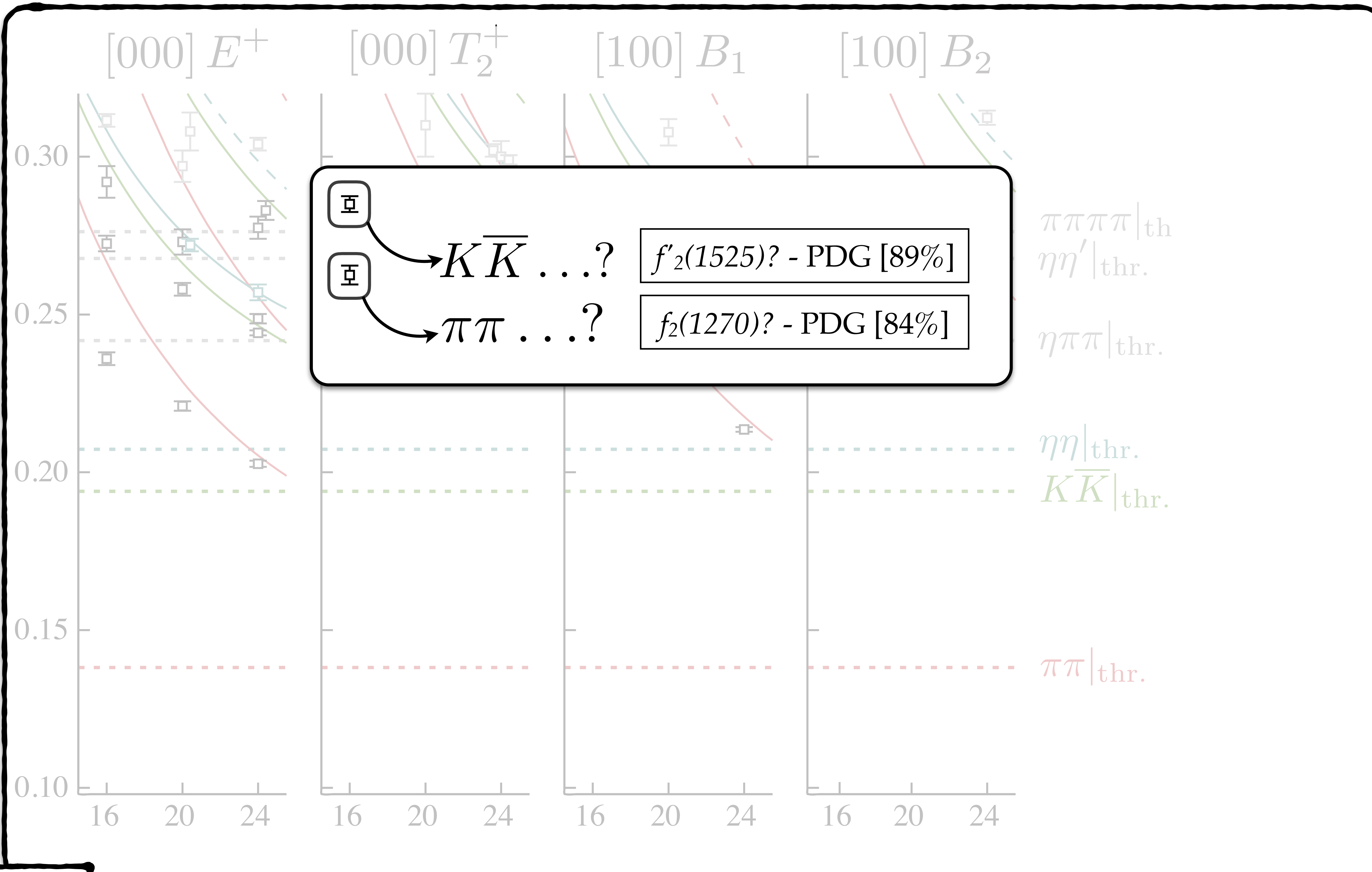
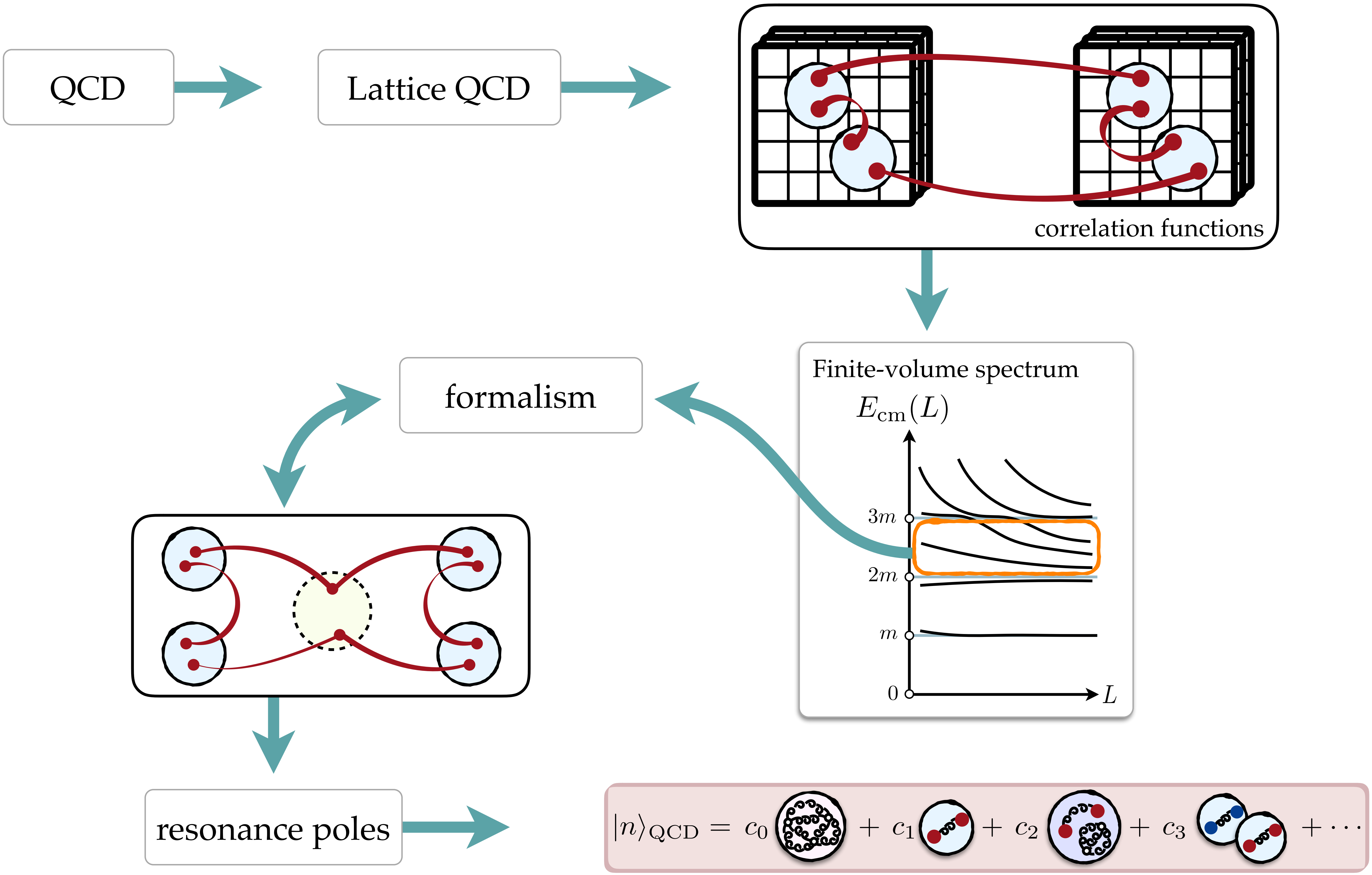
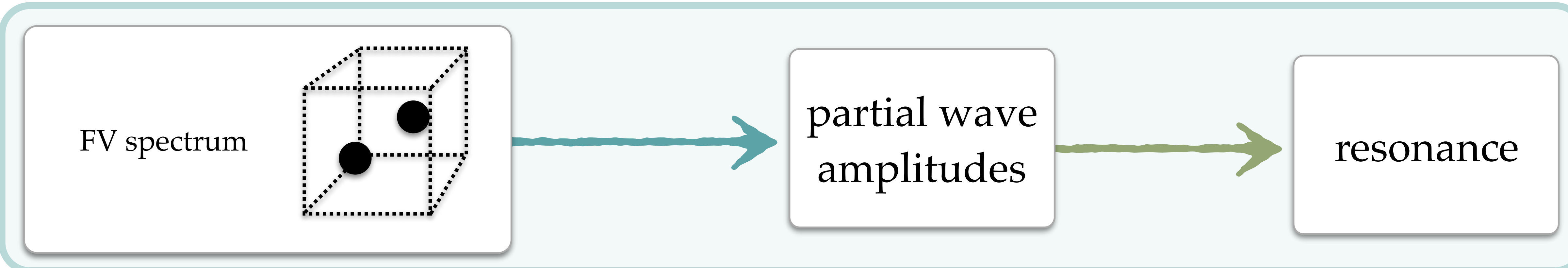


m_π=391 MeV

Obtaining the QCD spectrum



Scattering amplitudes

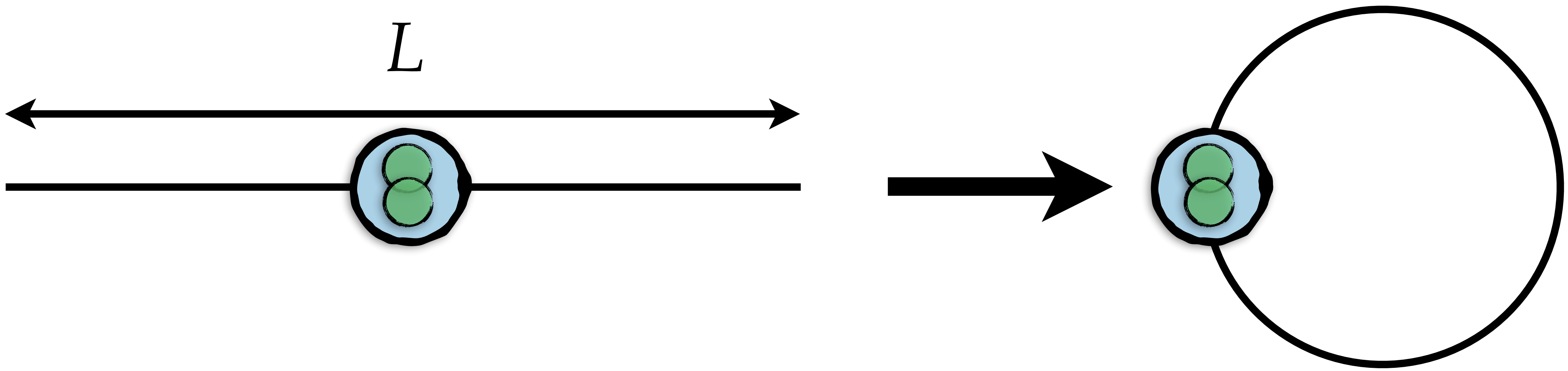


$$\det[F^{-1}(E_L, L) + \mathcal{M}(E_L)] = 0$$

E_L = finite volume spec.
L = finite volume
F = known function
$\mathcal{M}$ = scattering amp.

- Lüscher (1986, 1991) [elastic scalar bosons]
- Rummukainen & Gottlieb (1995) [moving elastic scalar bosons]
- Kim, Sachrajda, & Sharpe/Christ, Kim & Yamazaki (2005) [QFT derivation]
- Feng, Li, & Liu (2004) [inelastic scalar bosons]
- Hansen & Sharpe / RB & Davoudi (2012) [moving inelastic scalar bosons]
- RB (2014) [general 2-body result]

Physics in a 1D-box

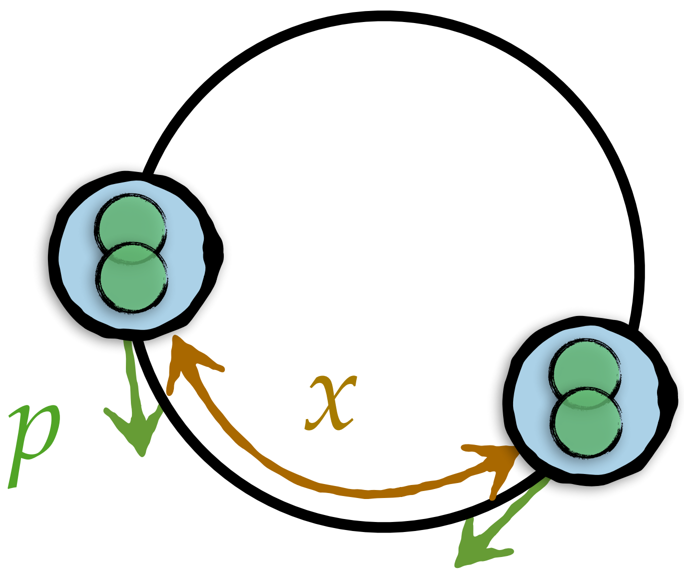


$$\phi(x) \sim e^{ipx}$$

Periodicity: $$L\, p_n = 2\pi n$$

Physics in a 1D-box

Two identical particles:



infinite volume scattering phase shift

$$\psi(x) \sim \cos(p|x| + \delta(p))$$

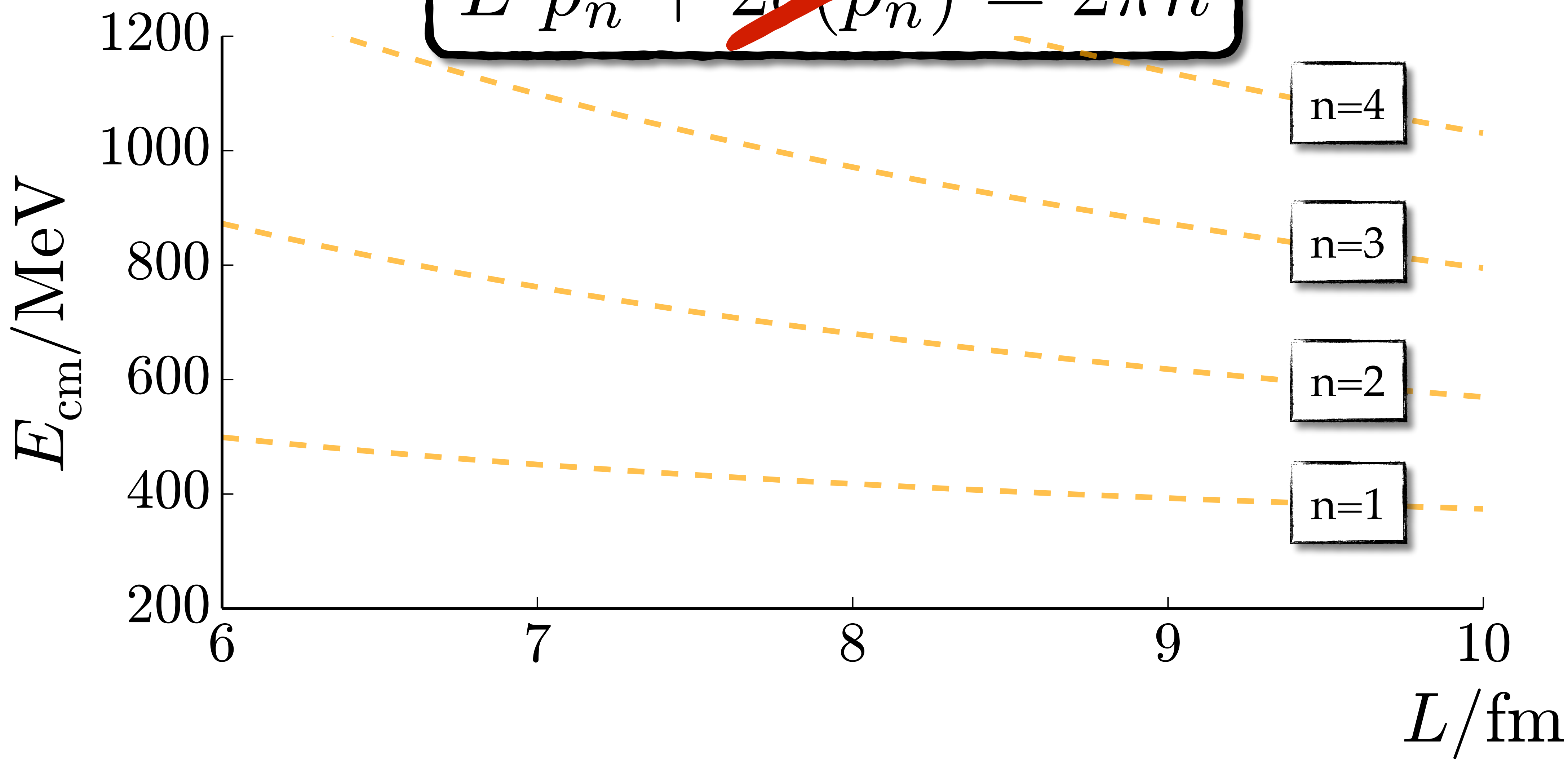
asymptotic wavefunction

Periodicity:

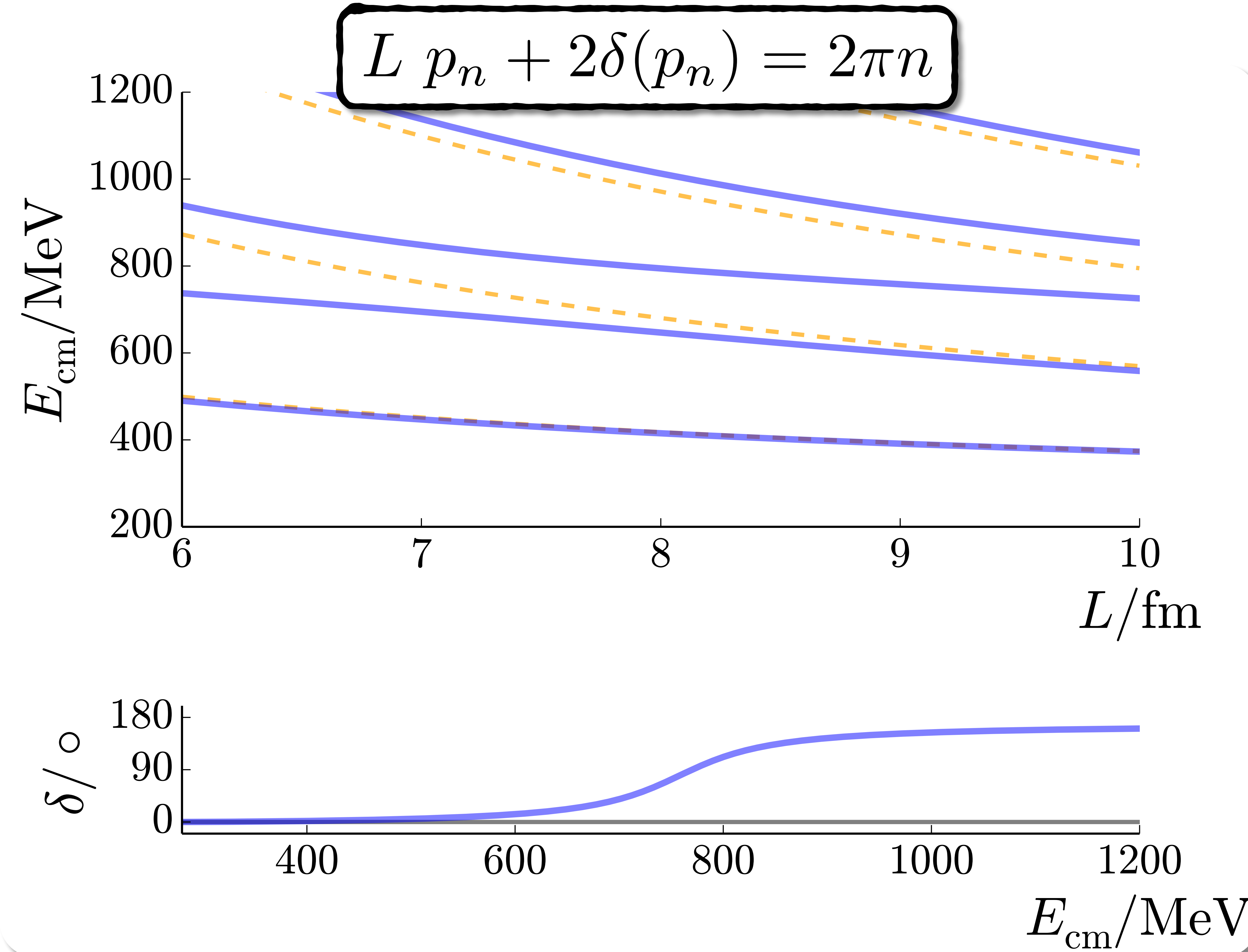
$$L\, p_n + 2\delta(p_n) = 2\pi n$$

Spectrum in a 1+1D box

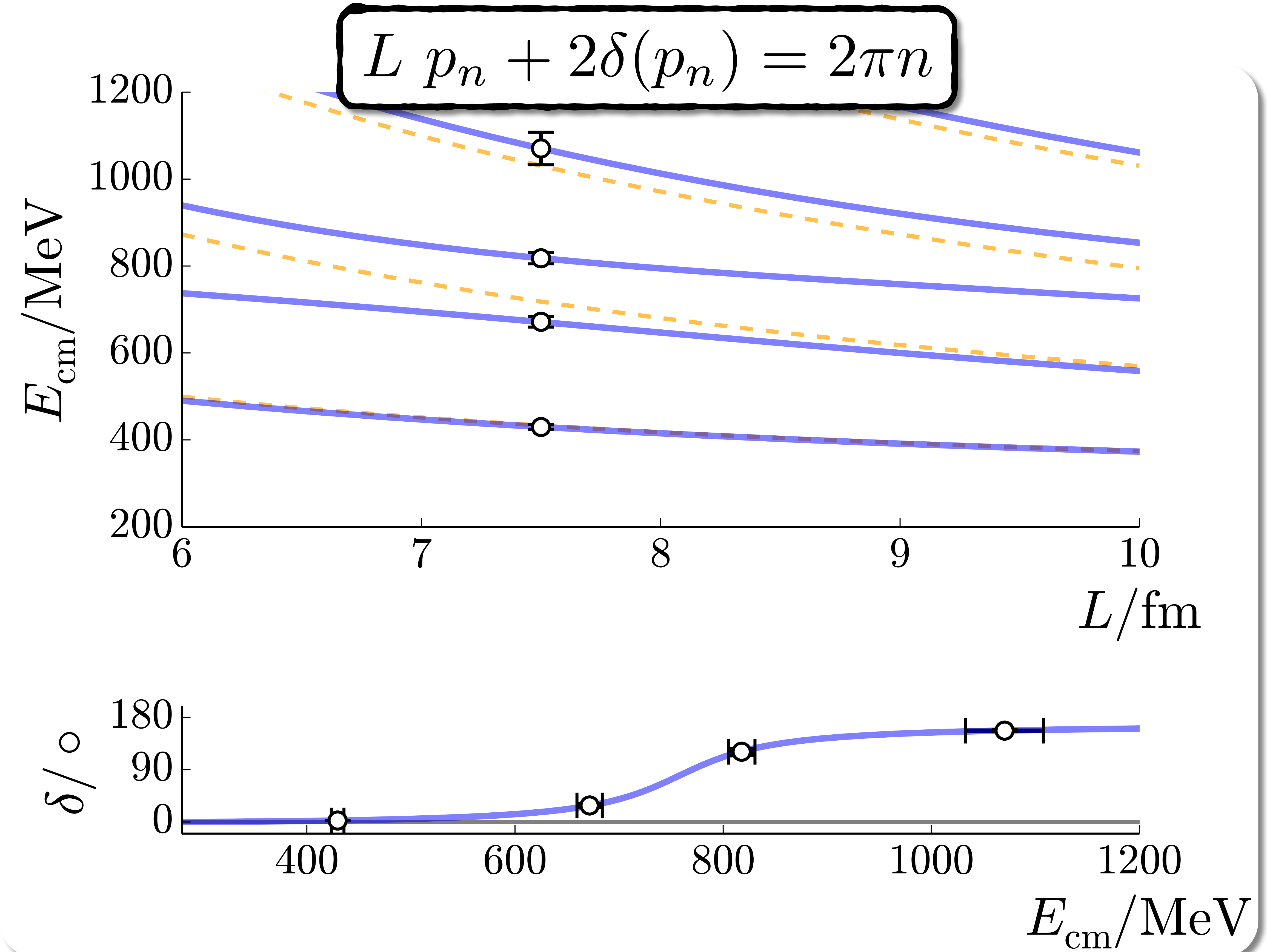
$$L\ p_n + 2\delta(p_n) = 2\pi n$$



Spectrum in a 1+1D box

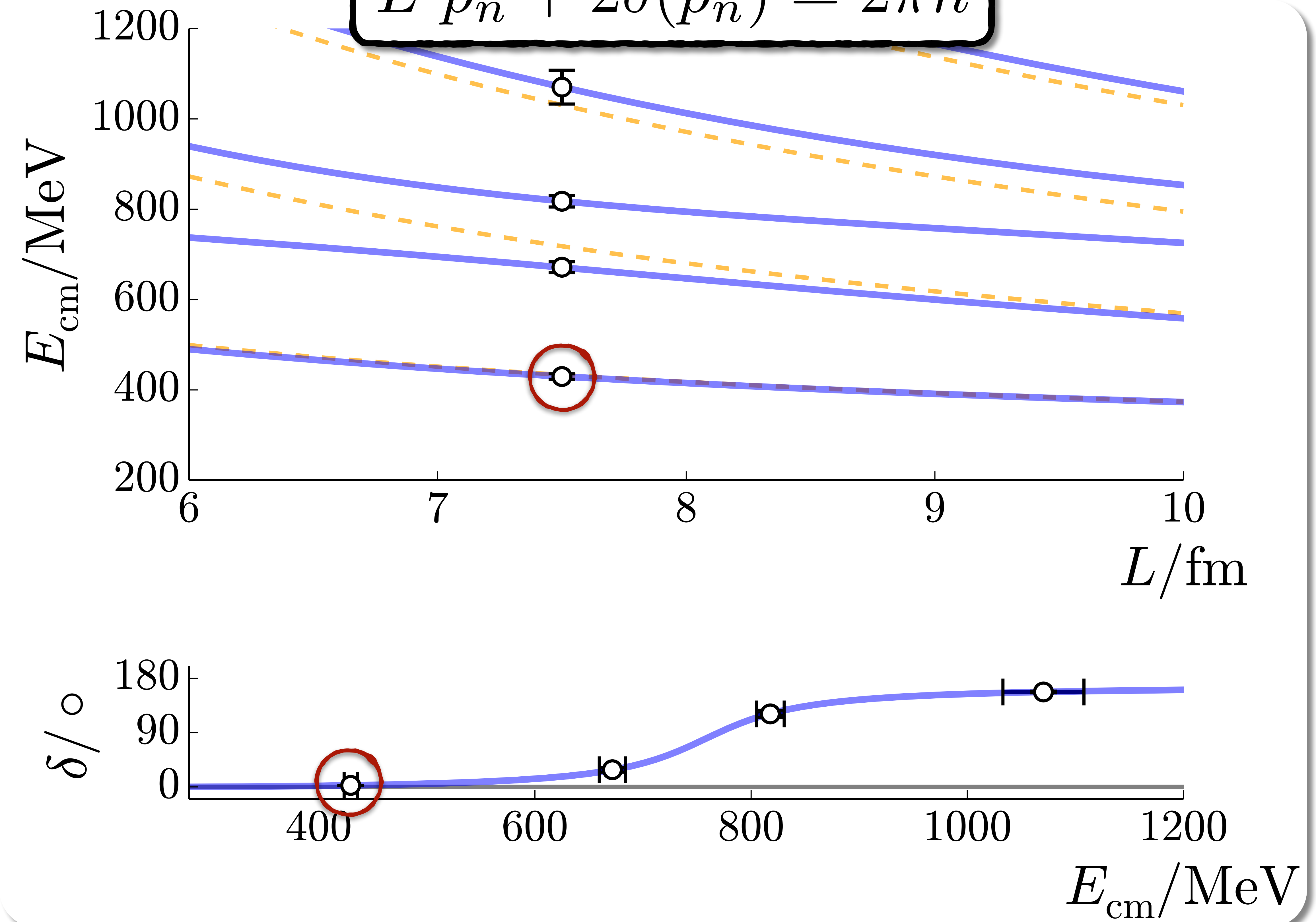


Spectrum in a 1+1D box

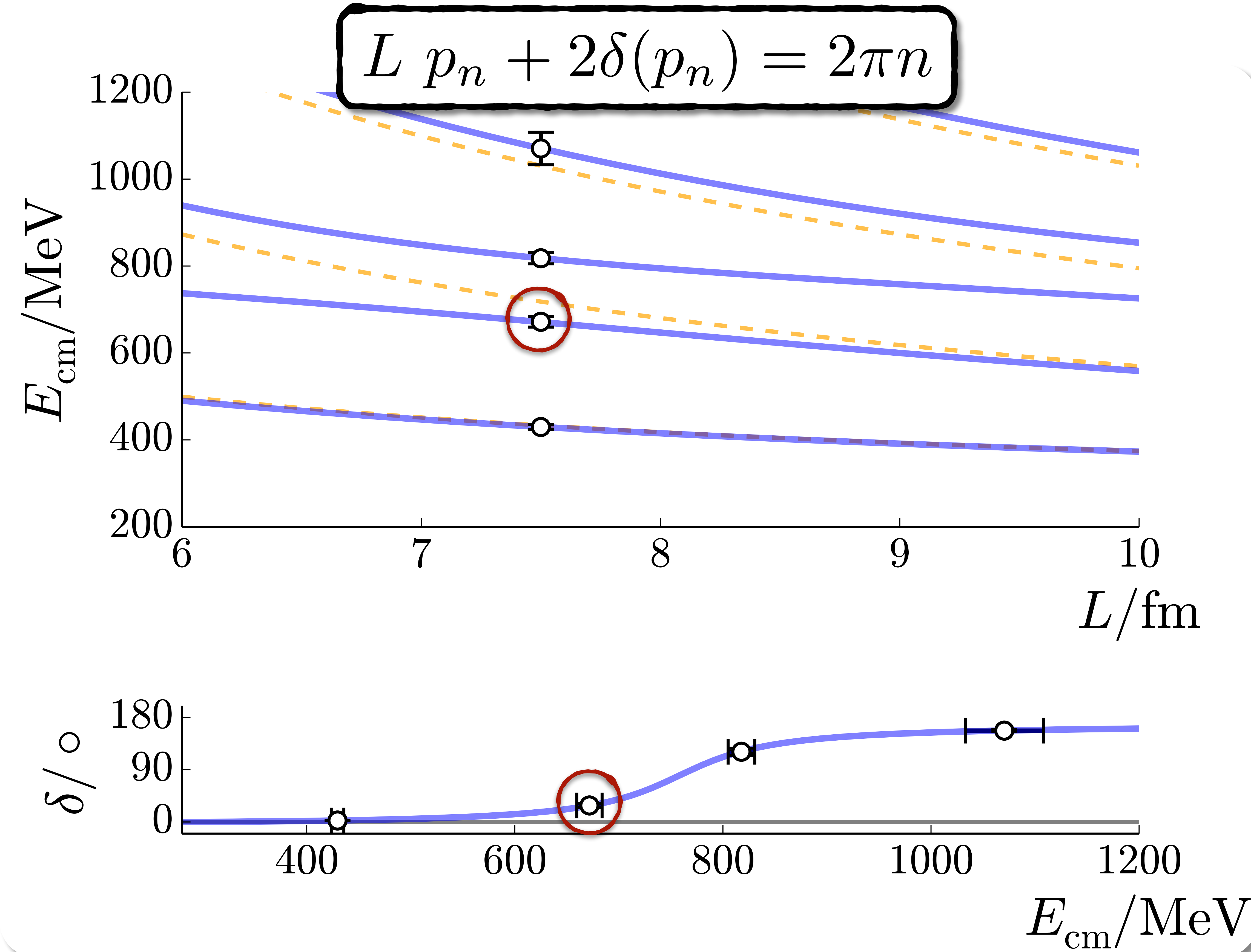


Spectrum in a 1+1D box

$$L\ p_n + 2\delta(p_n) = 2\pi n$$



Spectrum in a 1+1D box



Spectrum in a 1+1D box

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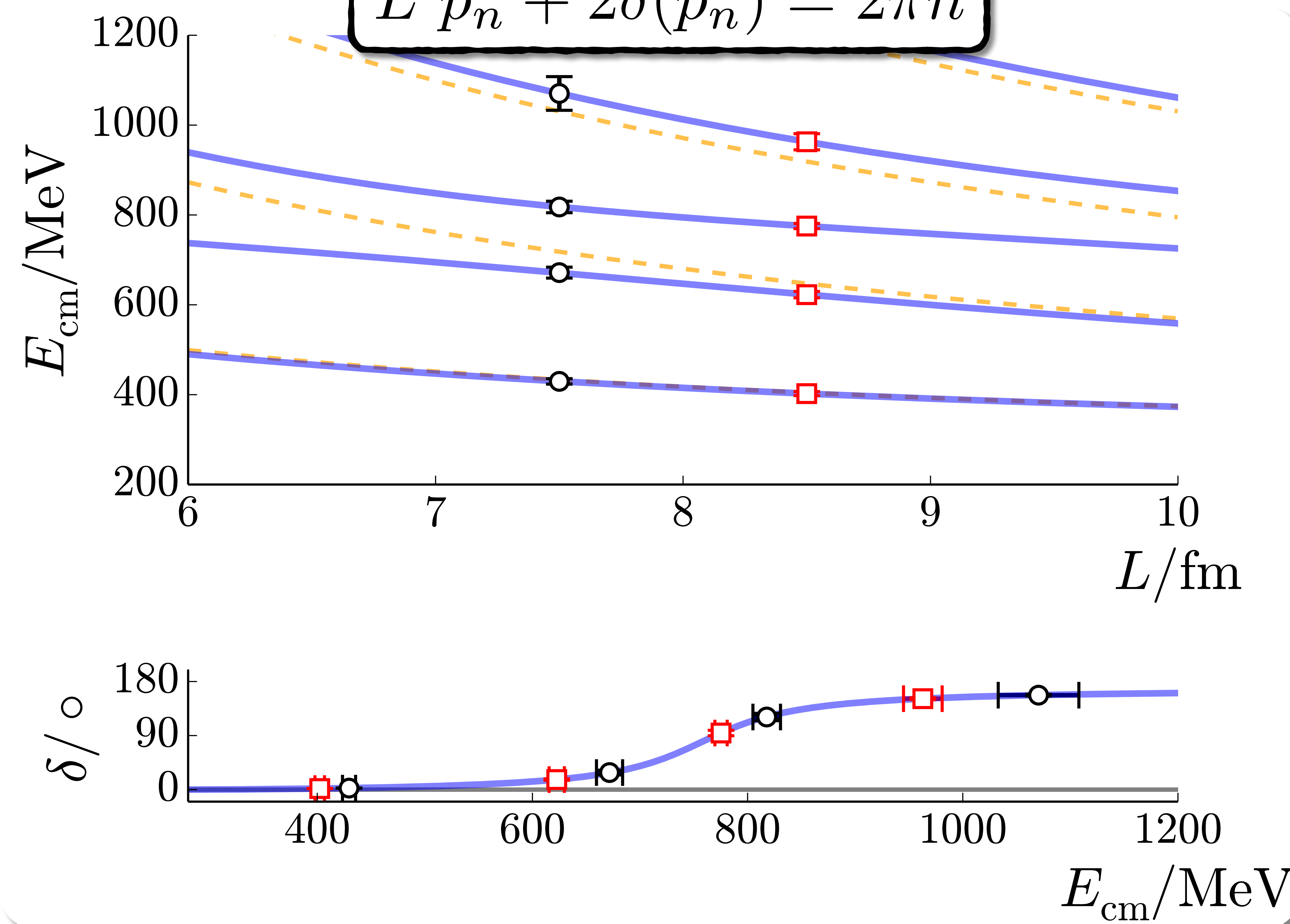
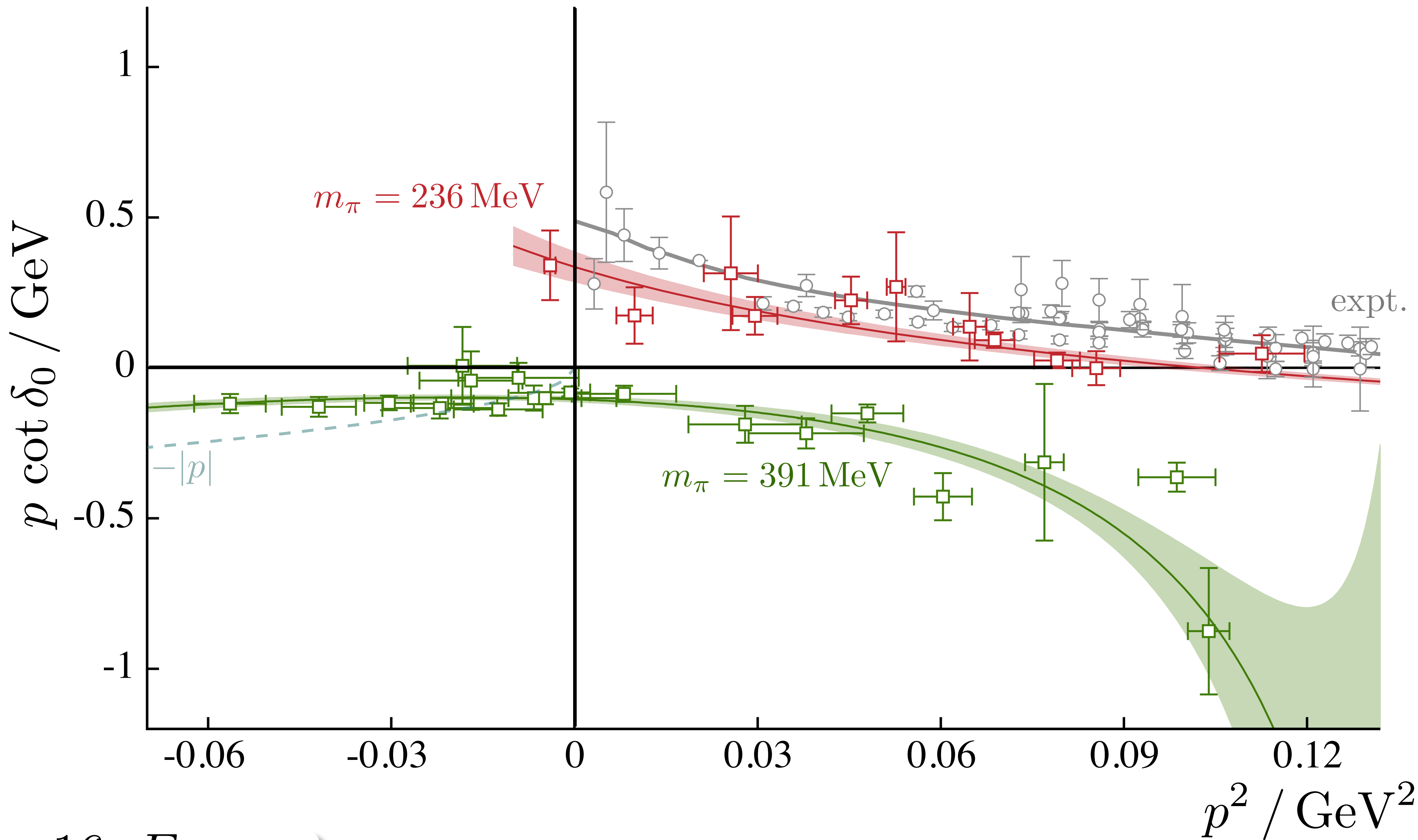


Isoscalar $\pi\pi$ scattering: elastic region



$$\mathcal{M}_0 = \frac{16\pi E_{\rm cm}}{p \cot \delta_0 - ip}$$

RB, Dudek, Edwards, Wilson - PRL (2017)

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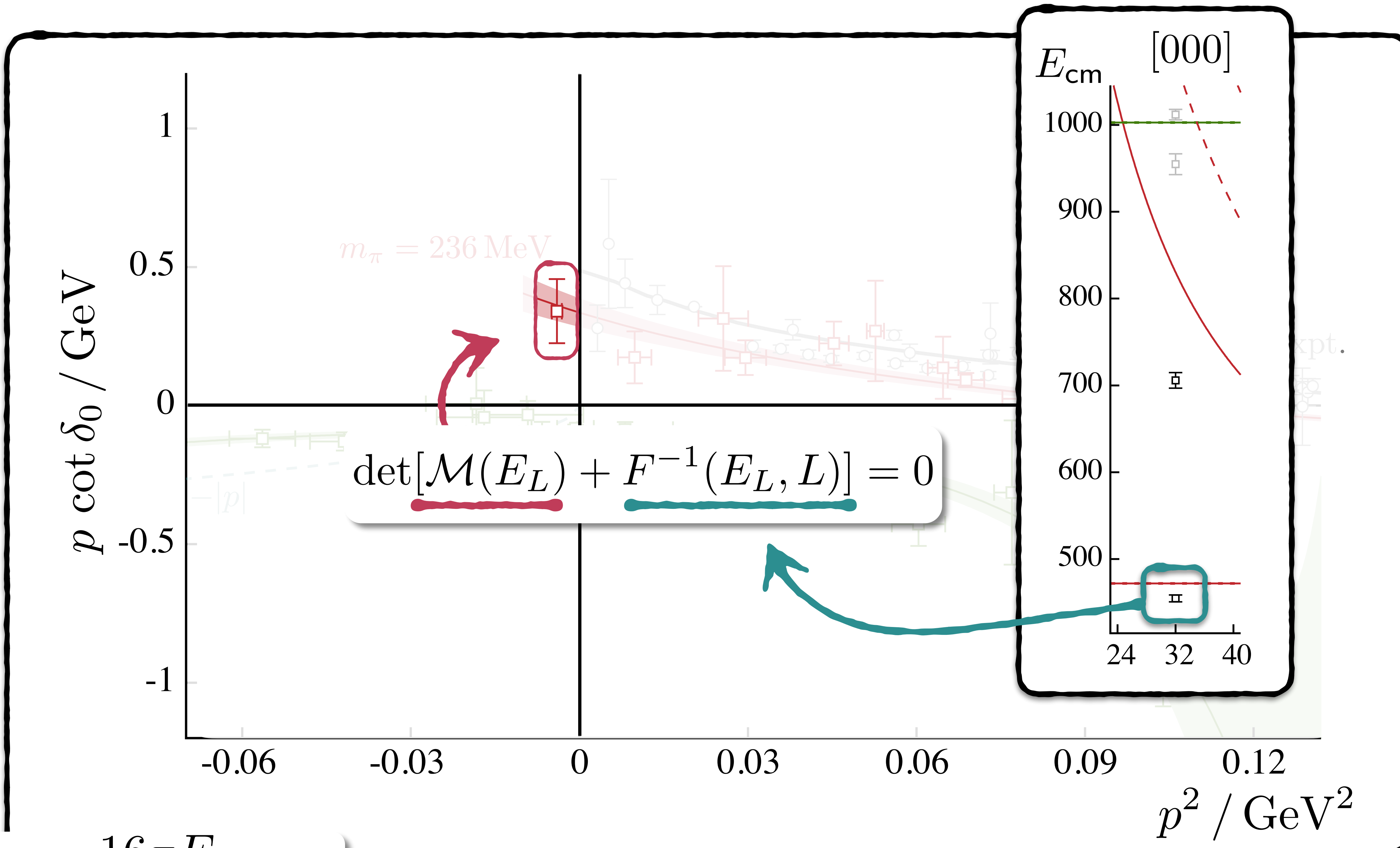
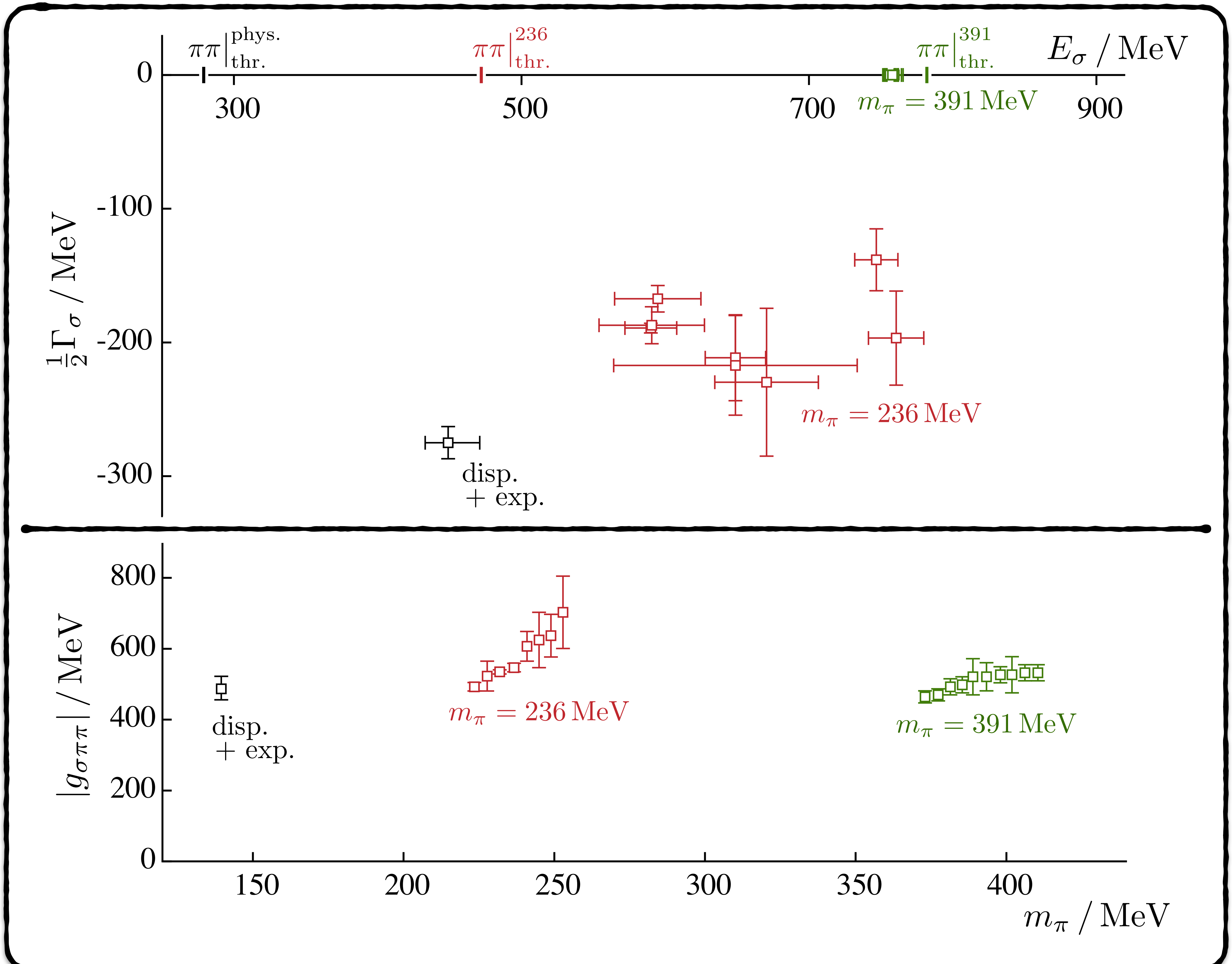


$$\mathcal{M}_0 = \frac{16\pi E_{\rm cm}}{p \cot \delta_0 - ip}$$

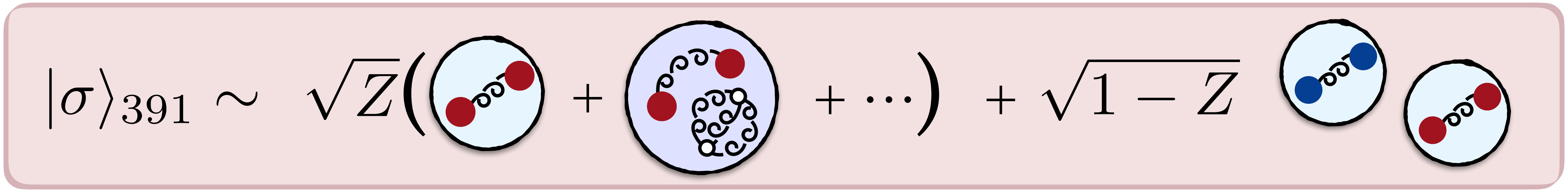
RB, Dudek, Edwards, Wilson - PRL (2017)

The σ / $f_0(500)$ vs m_π



Weinberg compositeness criterion for the σ

- For the heavier ensemble, the σ is a bound state, so we can apply Weinberg's criterion

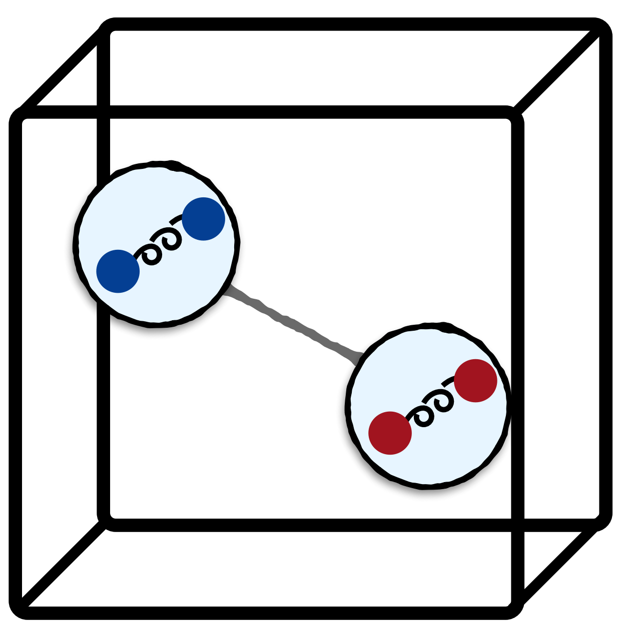


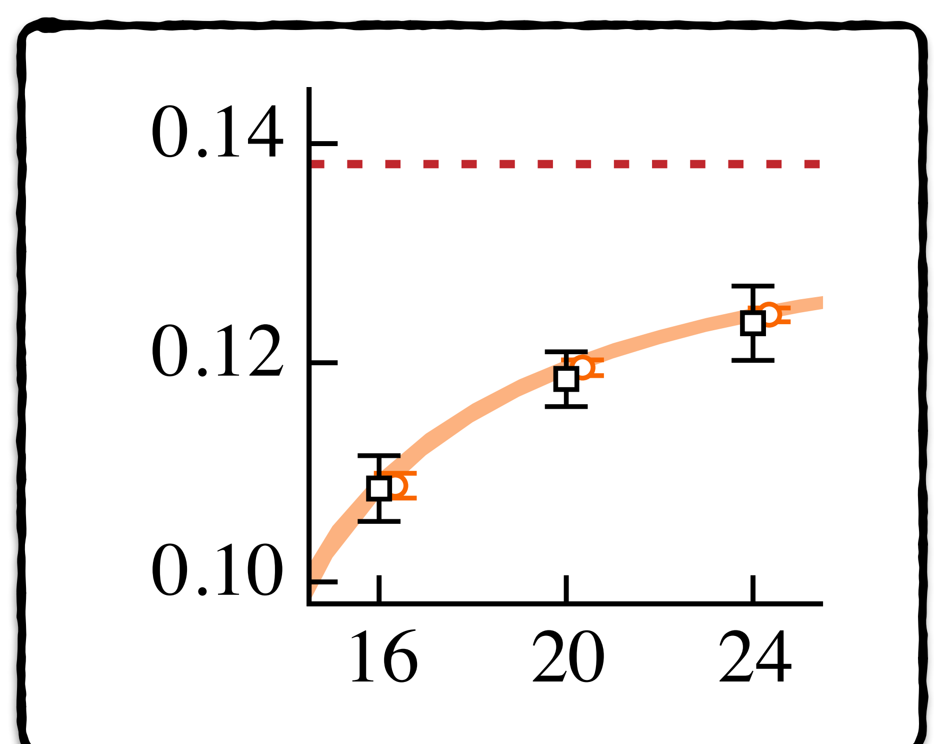
- Can relate Z to scattering information

$$a = -2\frac{1-Z}{2-Z}\frac{1}{\sqrt{m_\pi B_\sigma}}, \qquad r = -\frac{Z}{1-Z}\frac{1}{\sqrt{m_\pi B_\sigma}}$$

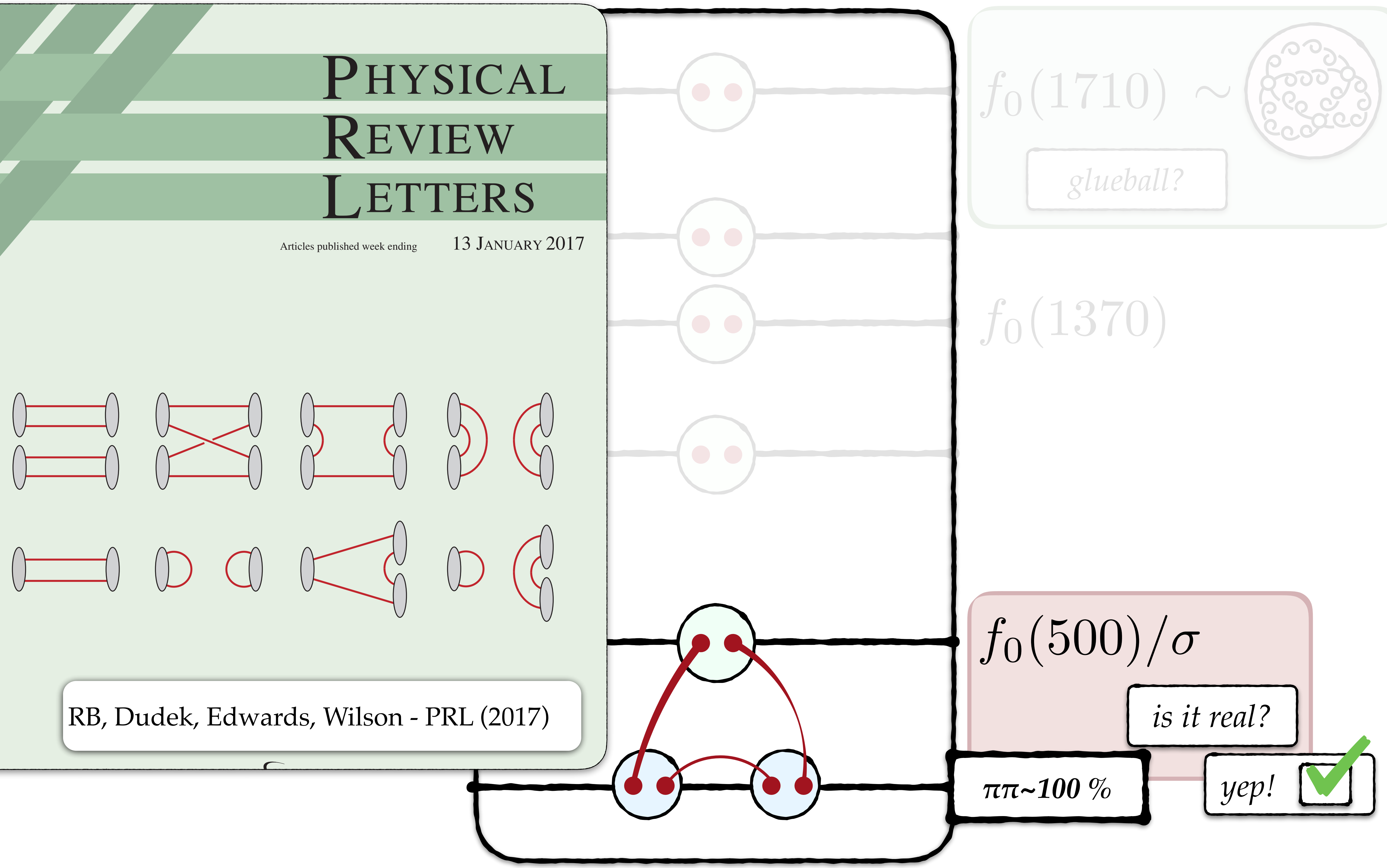
- To obtain: $Z \sim 0.3(1)$

- Consistent with the large FV effects

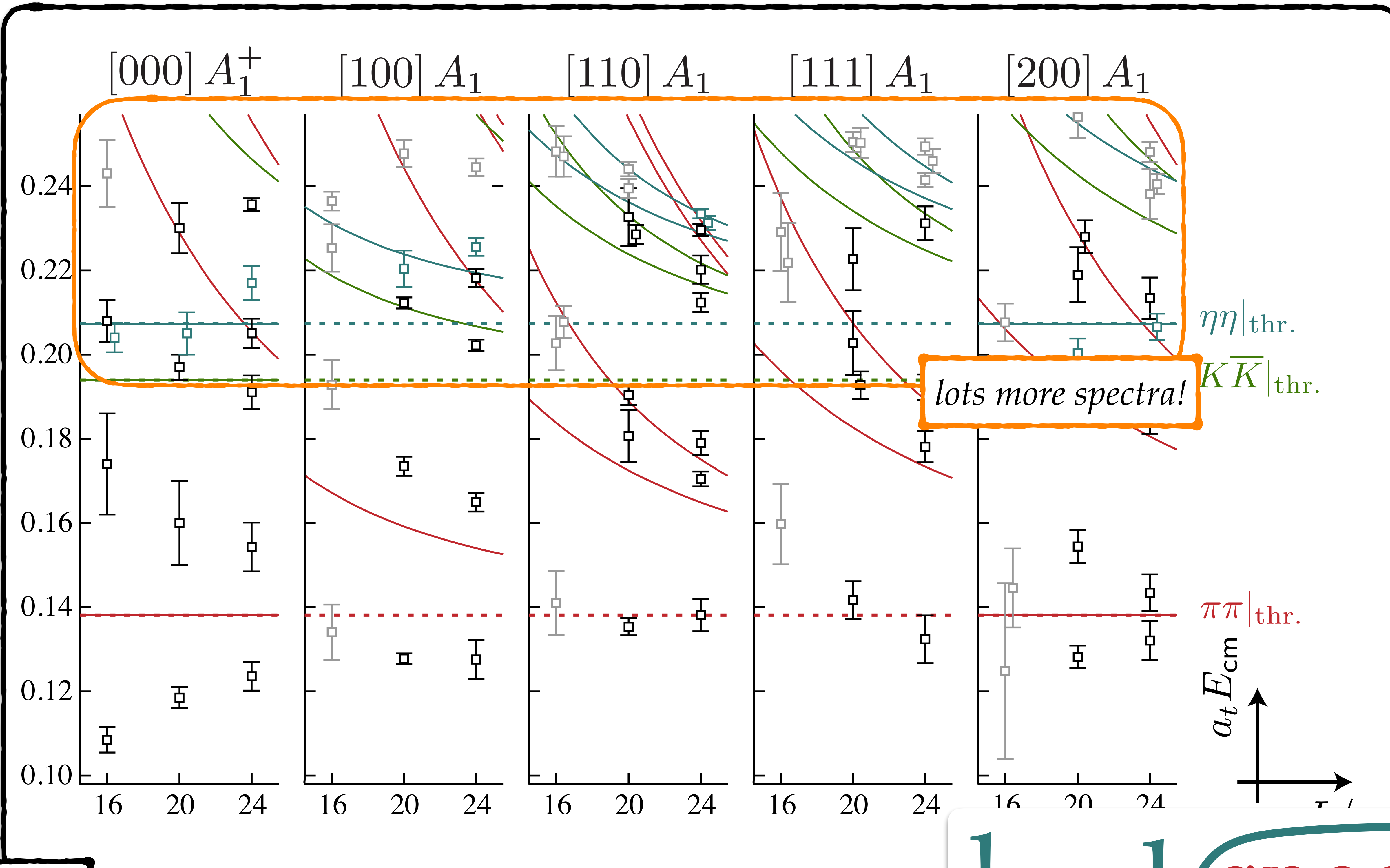




The isoscalar, scalar sector



Multi-channel systems - the cutting edge!



m_π=391 MeV

hadspec

Multi-channel systems - the cutting edge!

- the *necessary* formalism for doing coupled-channel scattering of

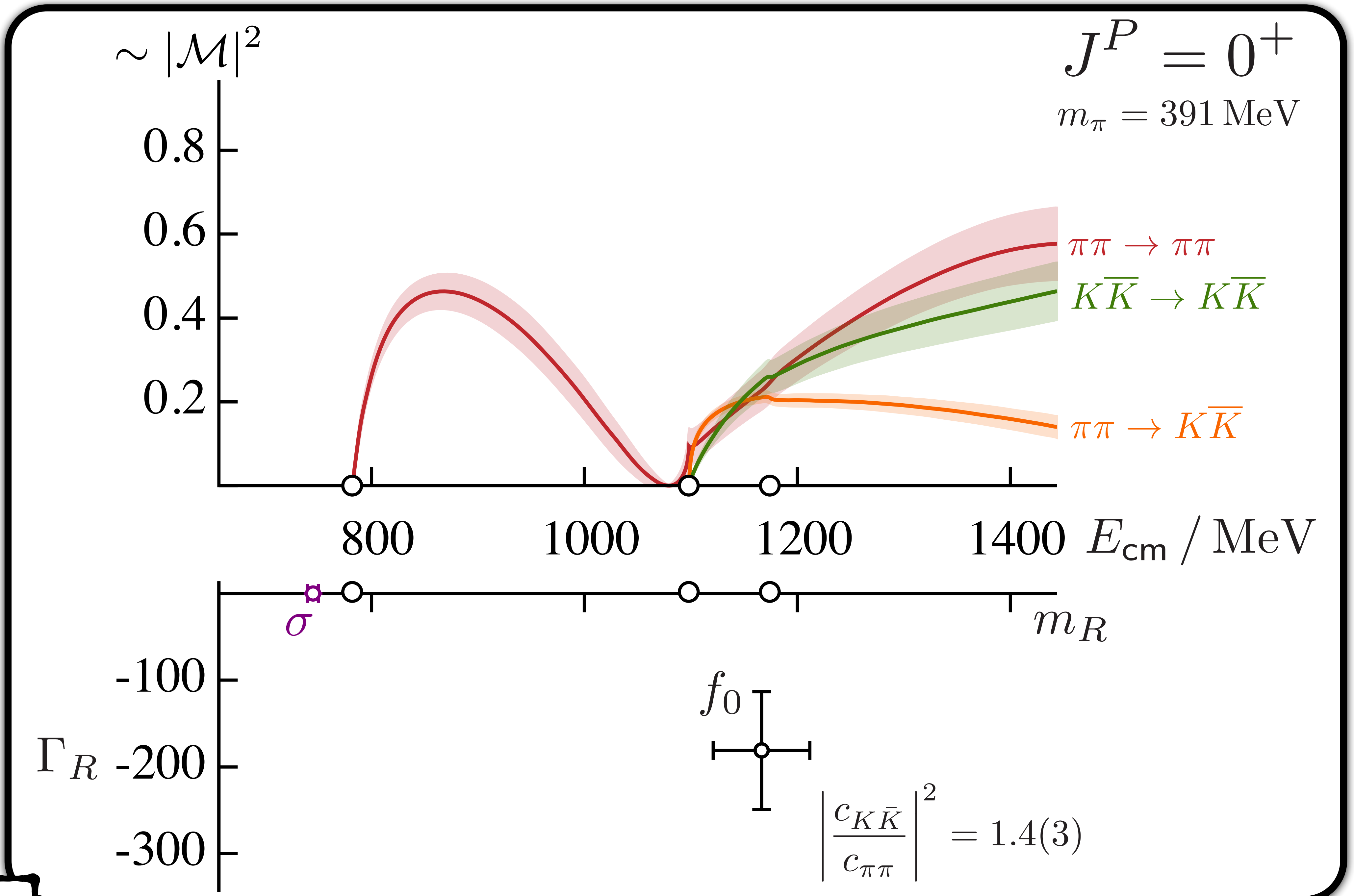
Feng, Li, & Liu (2004) [inelastic scalar bosons]
Hansen & Sharpe / RB & Davoudi (2012) [moving inelastic scalar bosons]
RB (2014) [general 2-body result]

- to date, the ***Hadron Spectrum* collaboration** is the only one to have extracted coupled-channel scattering amplitude information from QCD

$\pi\pi$, KK, $\eta\eta$ [isoscalar]:	RB, Dudek, Edwards, Wilson - PRL (2017)
	RB, Dudek, Edwards, Wilson - PRD (2018)
$K\pi$, $K\eta$:	Dudek, Edwards, Thomas, Wilson - PRL (2015)
	Wilson, Dudek, Edwards, Thomas - PRD (2015)
$\pi\eta$, KK:	Dudek, Edwards, Wilson - PRD (2016)
$D\pi$, $D\eta$, D_sK:	Moir, Peardon, Ryan, Thomas, Wilson - JHEP (2016)
$\pi\pi$, KK [isovector]:	Wilson, RB, Dudek, Edwards, Thomas - PRD (2015)

Multi-channel systems - the cutting edge!

- *Coupled channels:* e.g., S-wave $\pi\pi$, $K\overline{K}$



m_π=391 MeV

RB, Dudek, Edwards & Wilson (2017)

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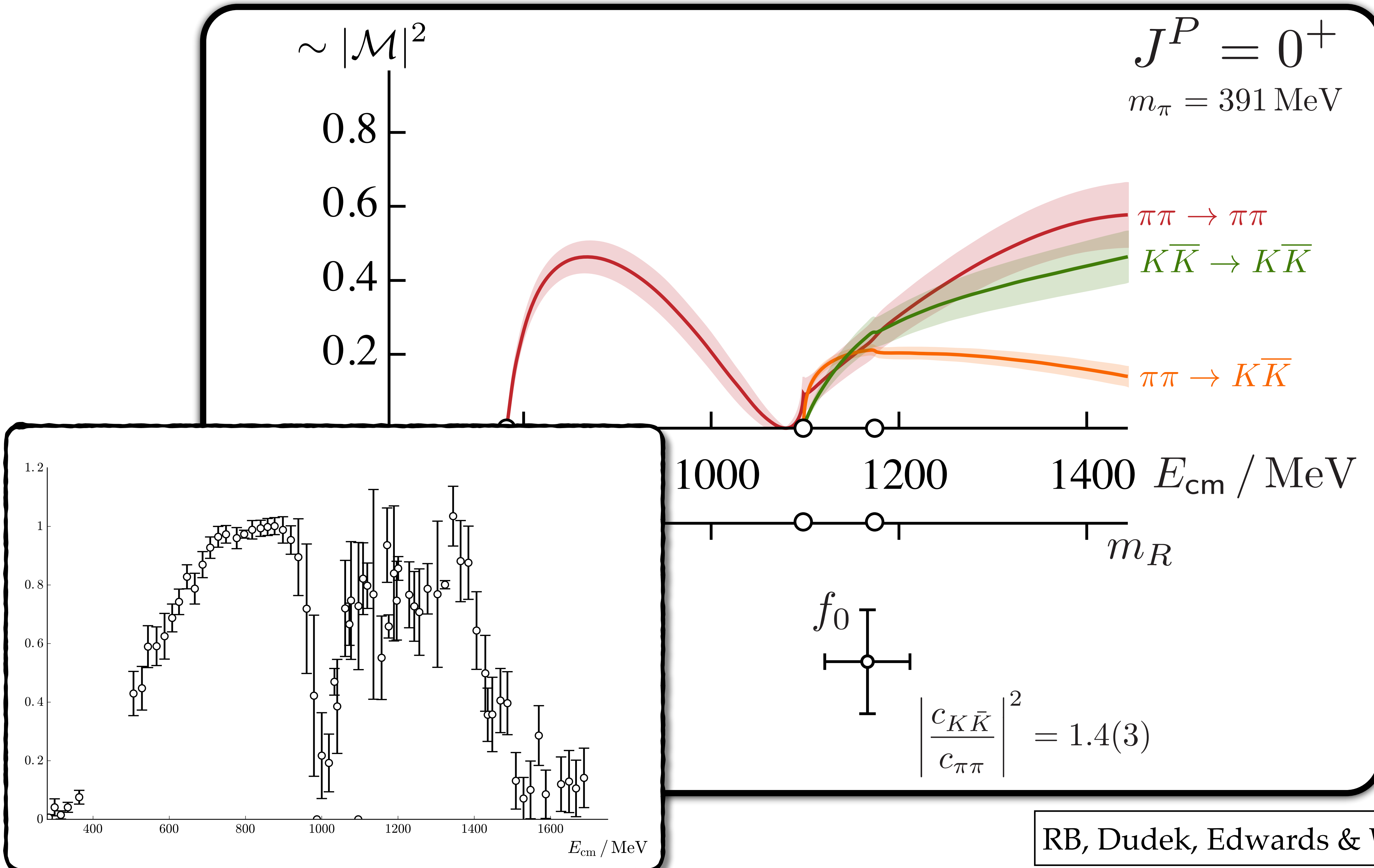

RB, Dudek, Edwards & Wilson (2017)

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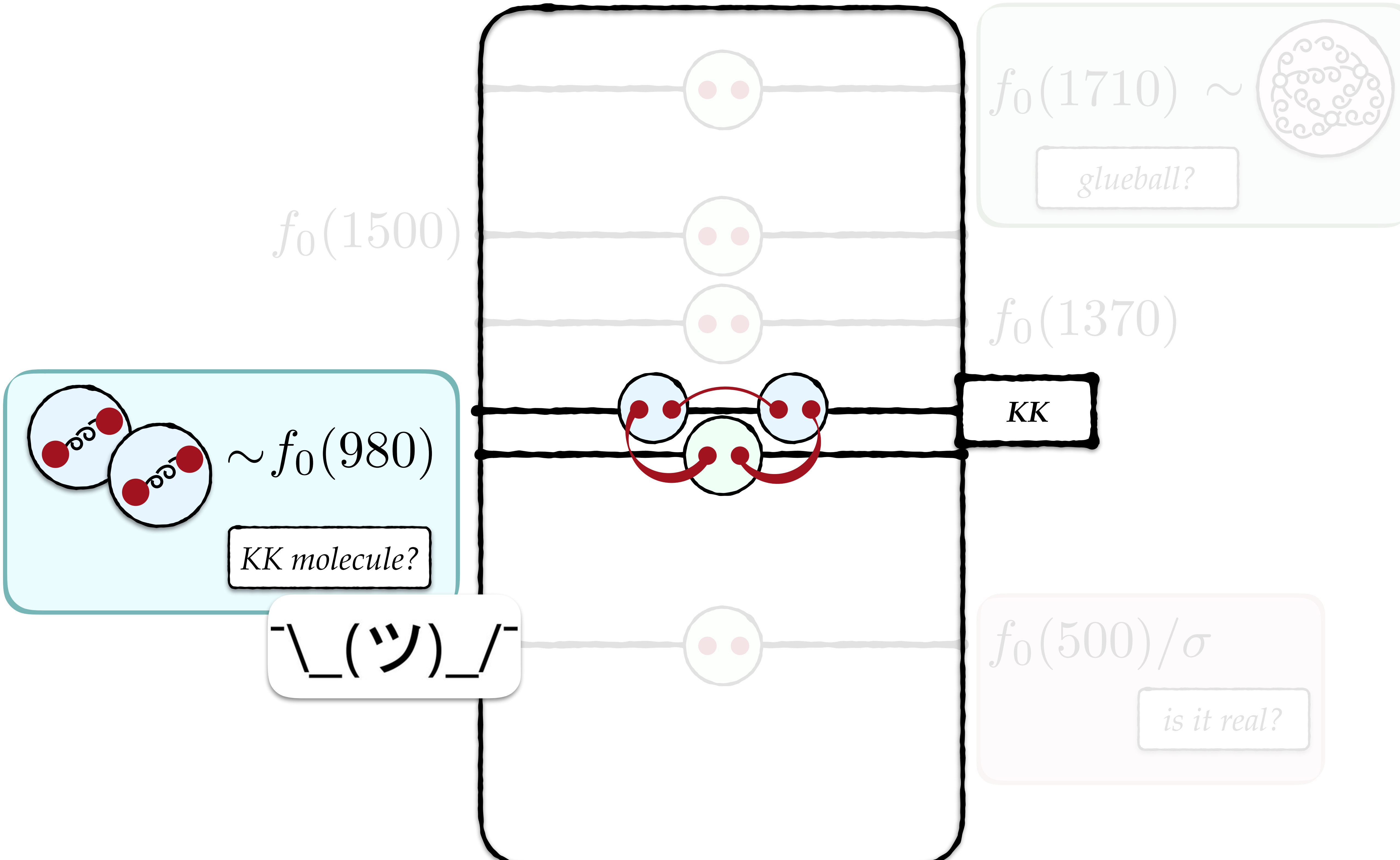

Multi-channel systems - the cutting edge!

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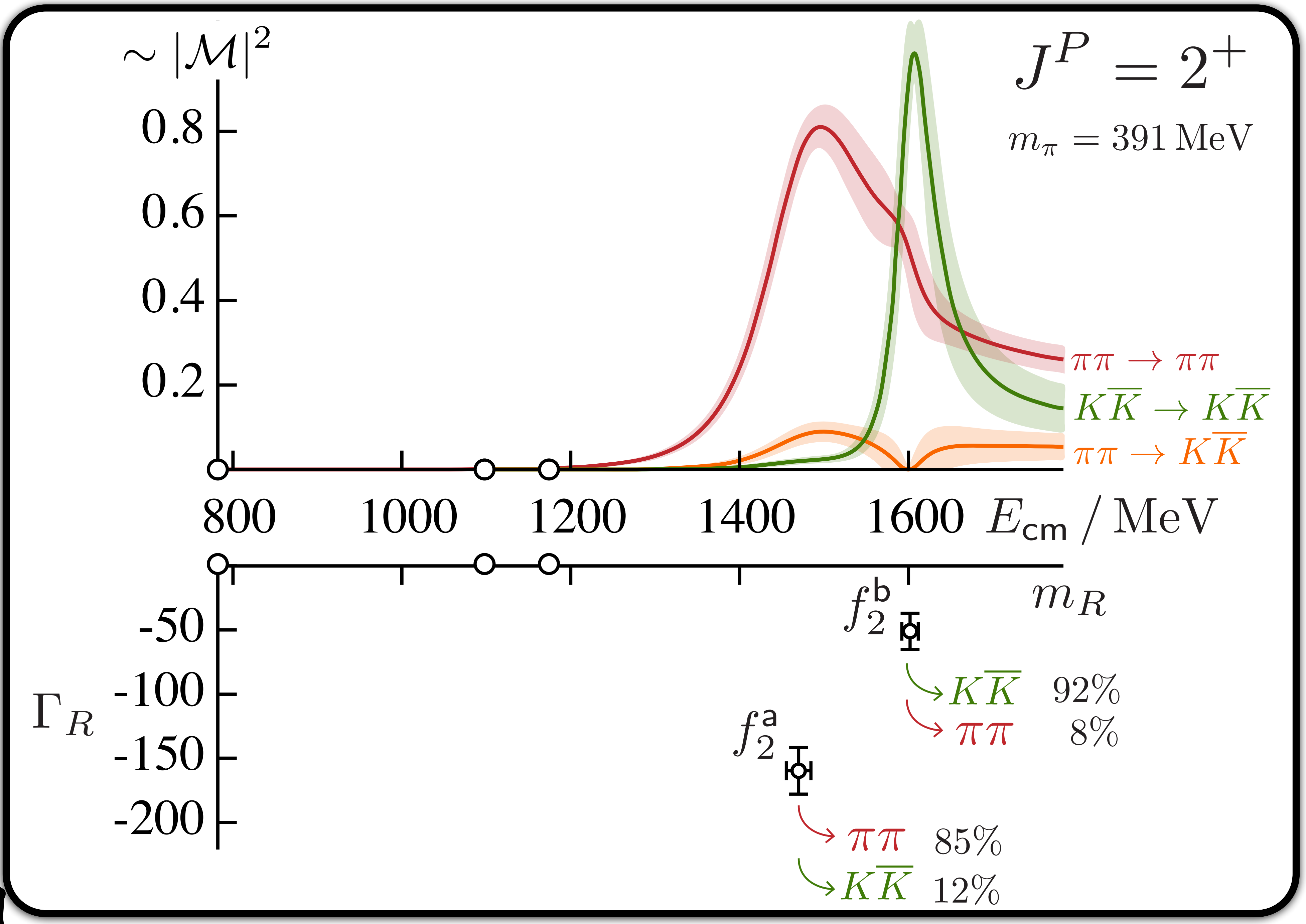

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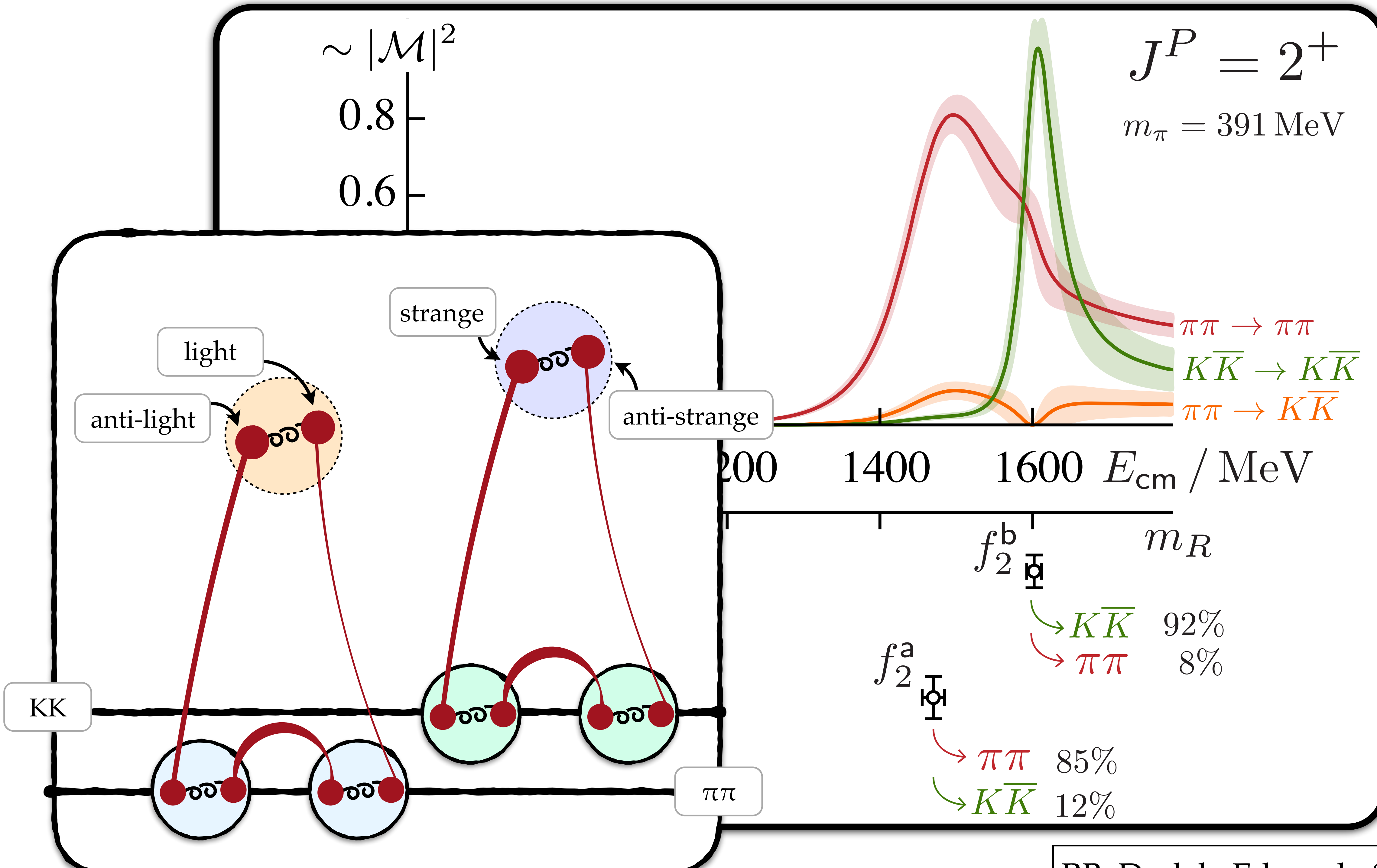
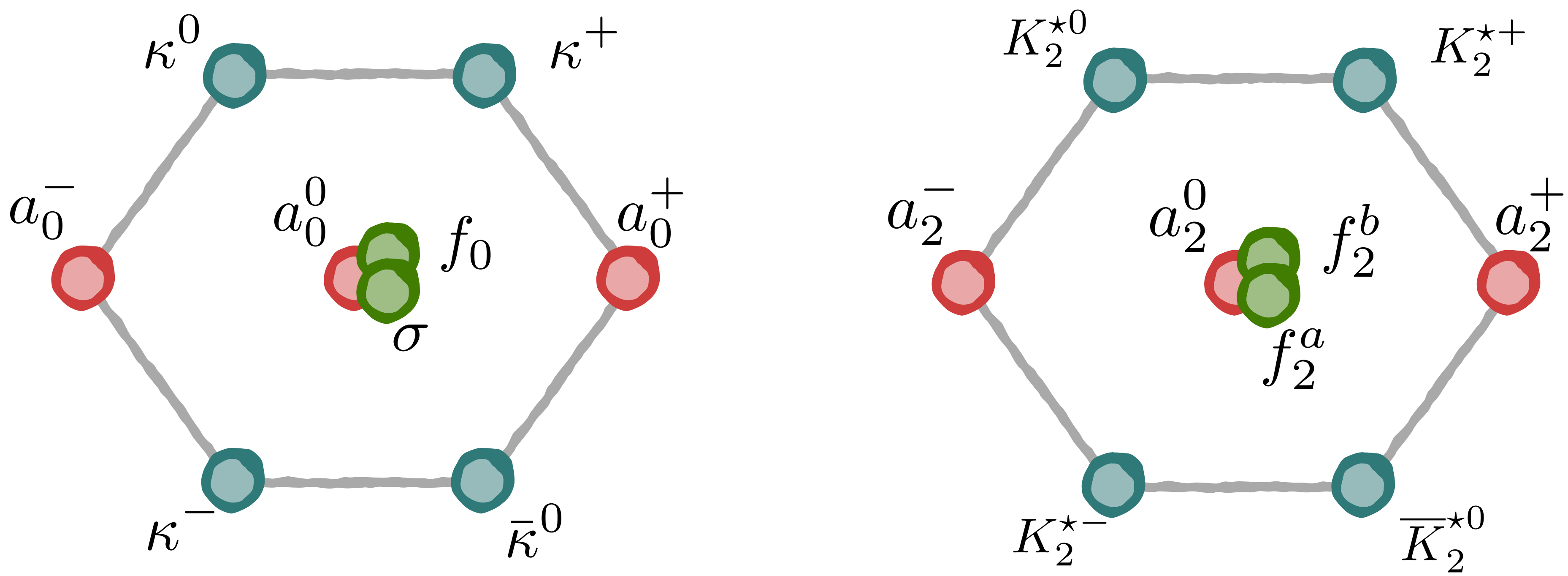


RB, Dudek, Edwards & Wilson (2017)

Tensor and scalar nonets

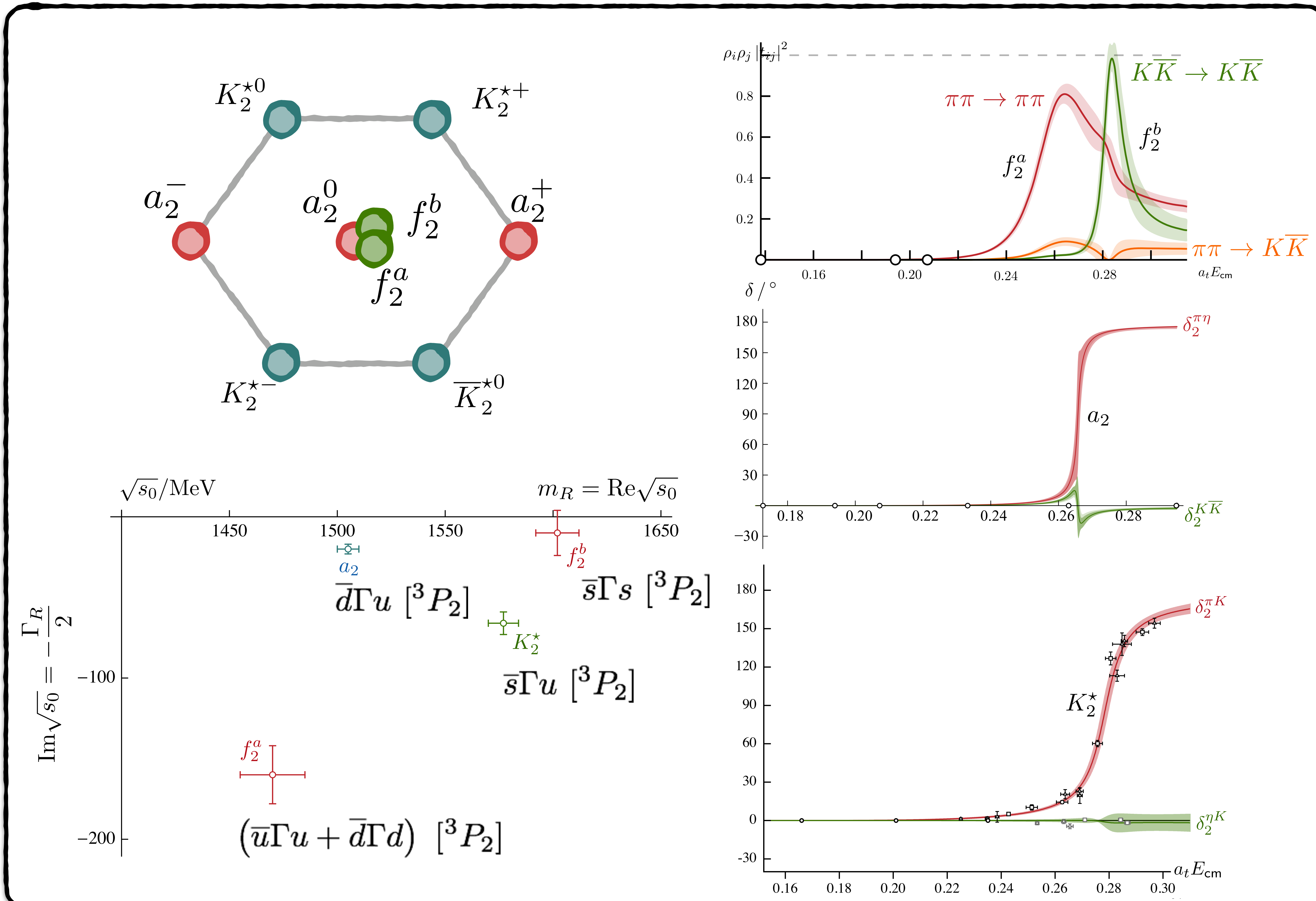
- First complete determination of the scalar and tensor nonets from LQCD :



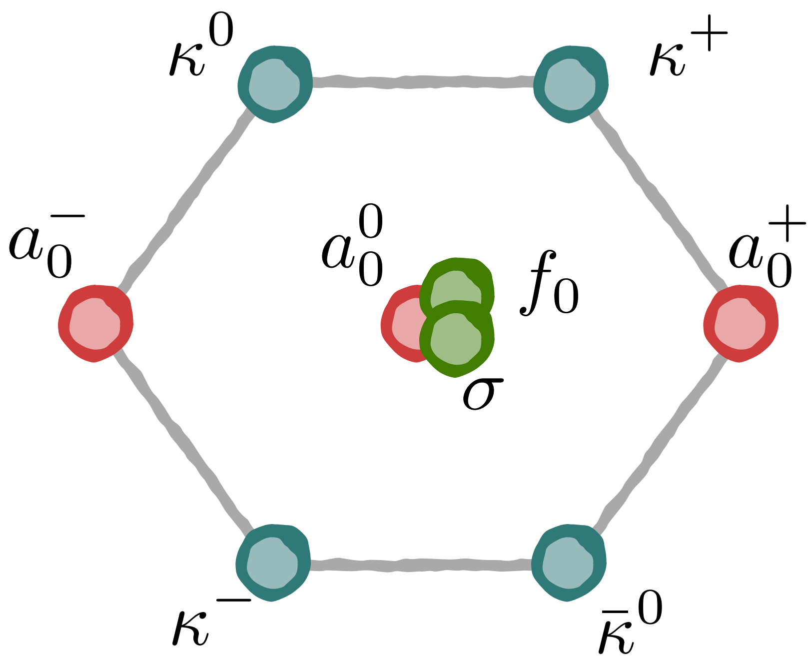
ππ, KK, ηη:	RB, Dudek, Edwards - PRL (2017)
	RB, Dudek, Edwards - arXiv (2017)
Kπ, Kη:	Dudek, Edwards, Thomas, Wilson - PRL (2015)
	Wilson, Dudek, Edwards, Thomas - PRD (2015)
πη, KK:	Dudek, Edwards, Wilson - PRD (2016)

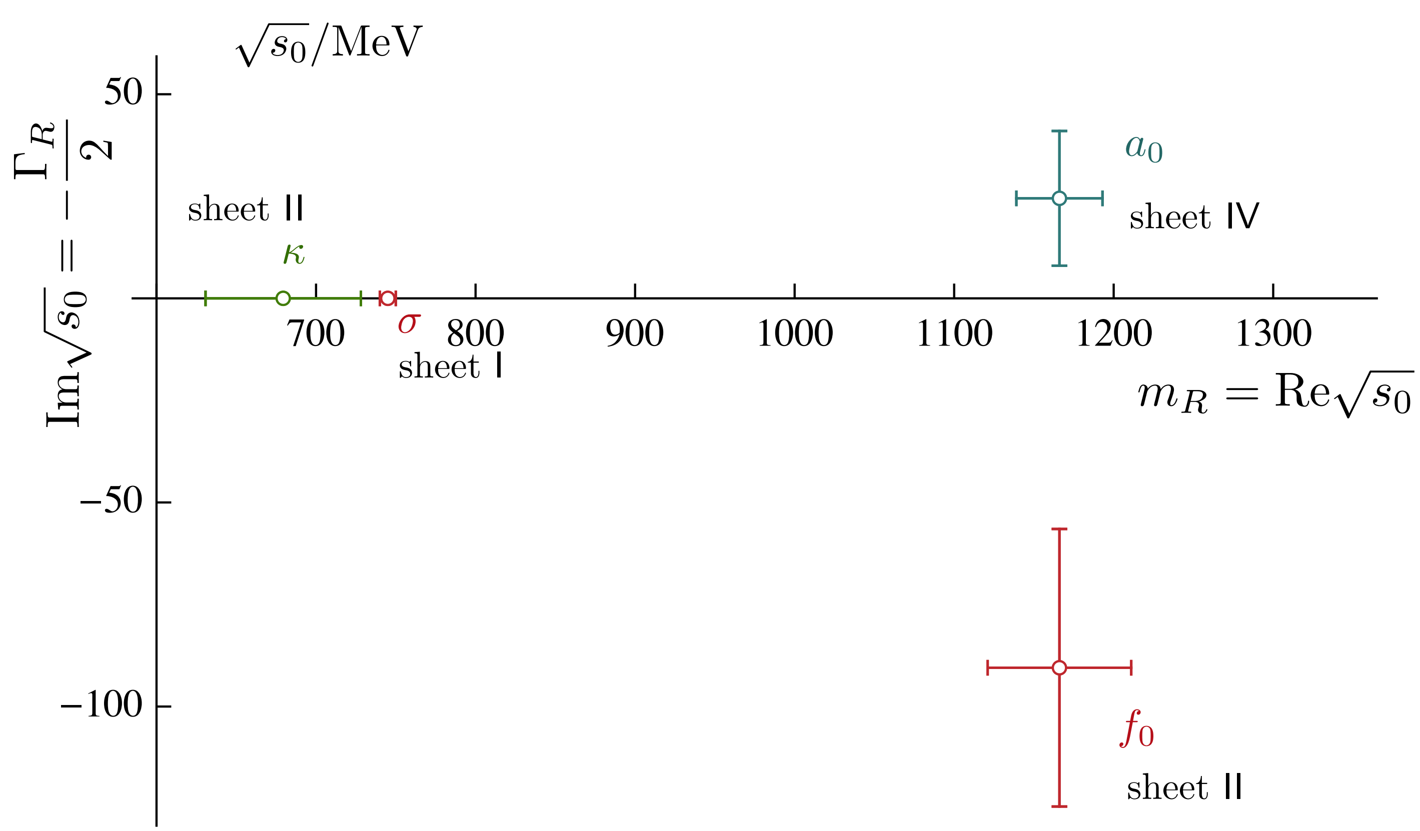
hadspec

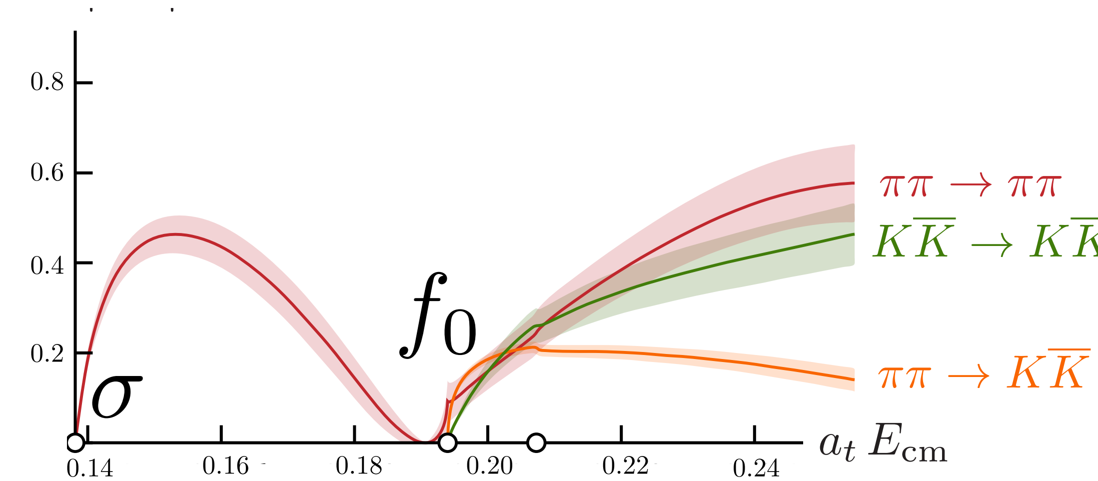
Tensor nonet

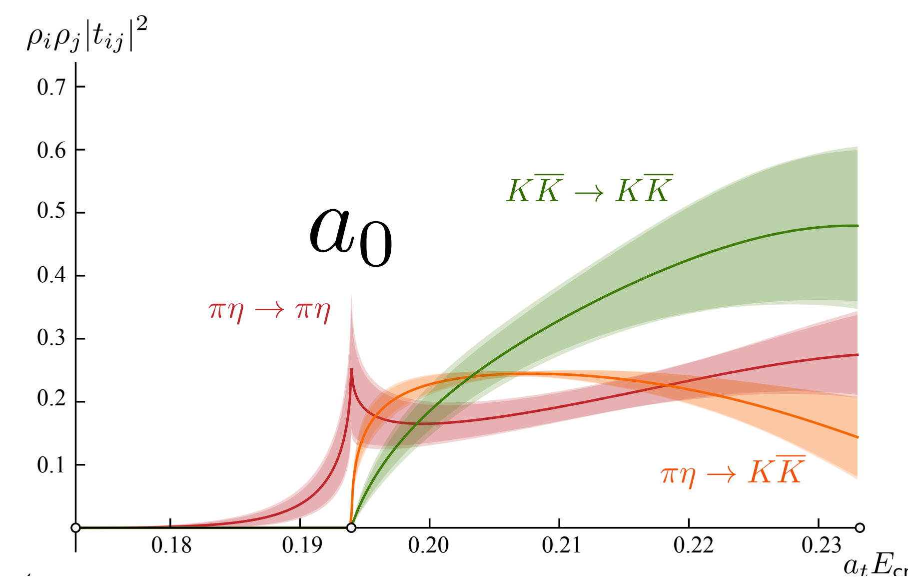


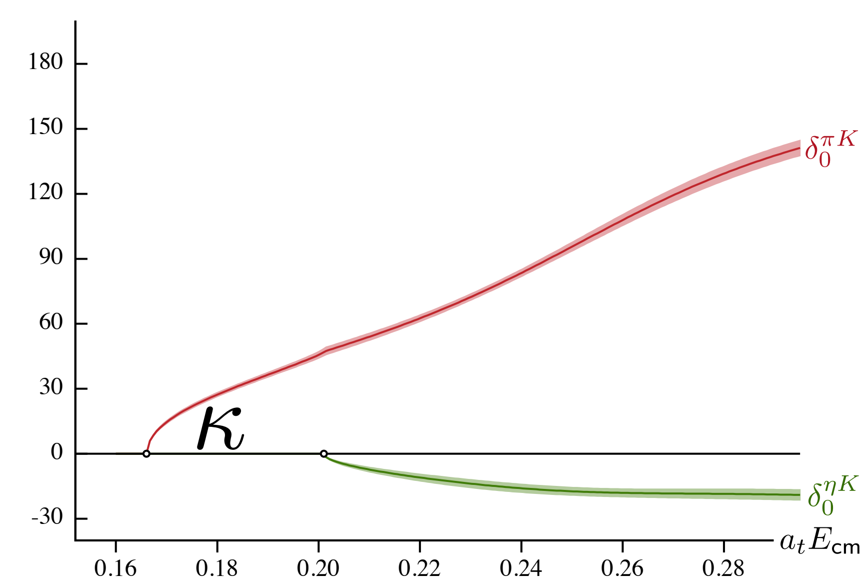
Scalar nonet





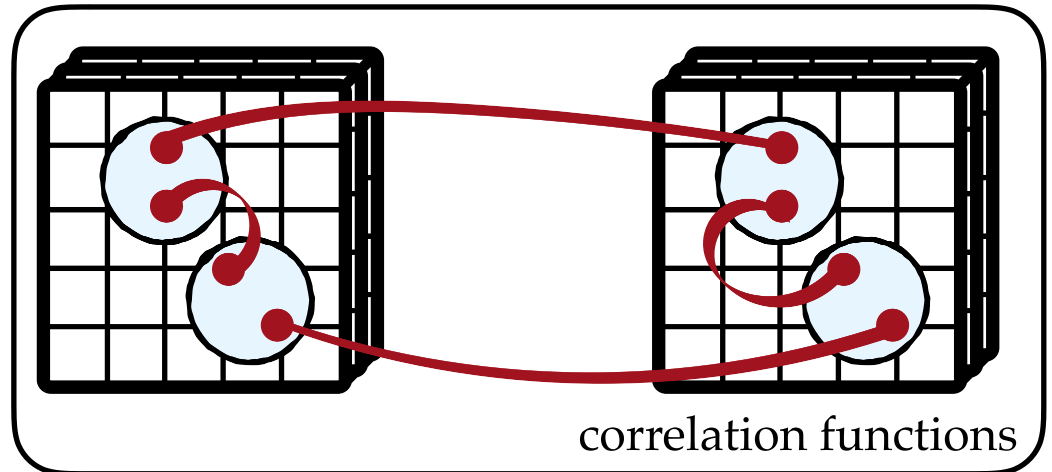


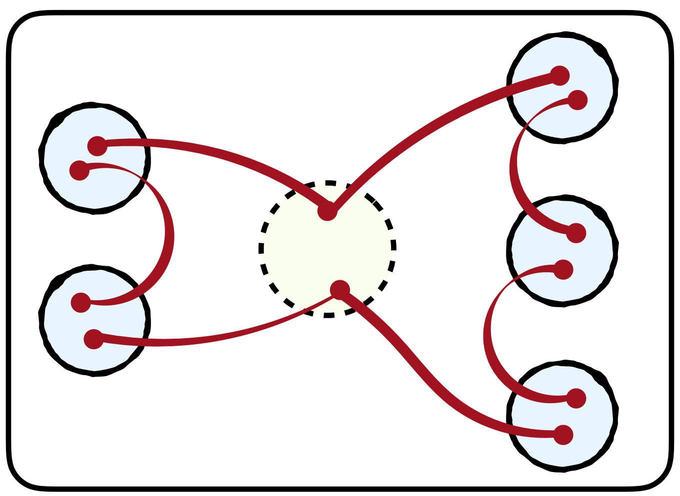


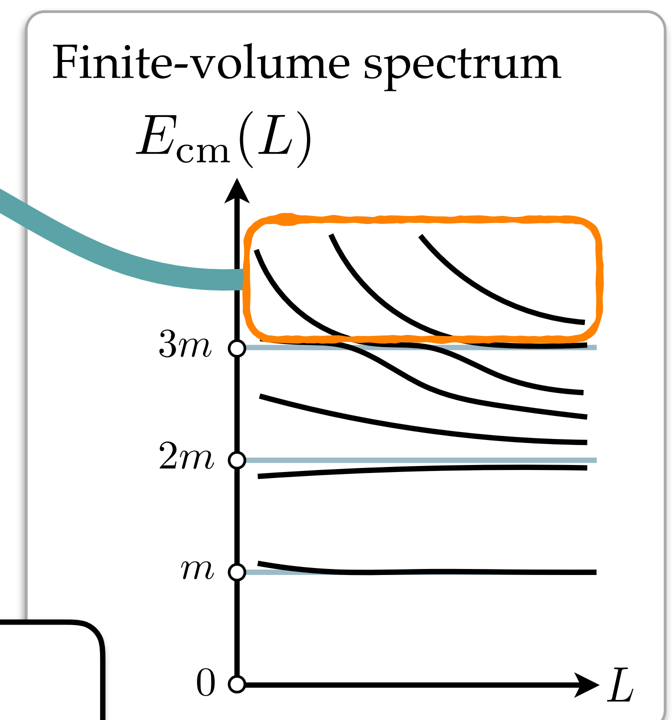


Obtaining the QCD spectrum

QCD → Lattice QCD →



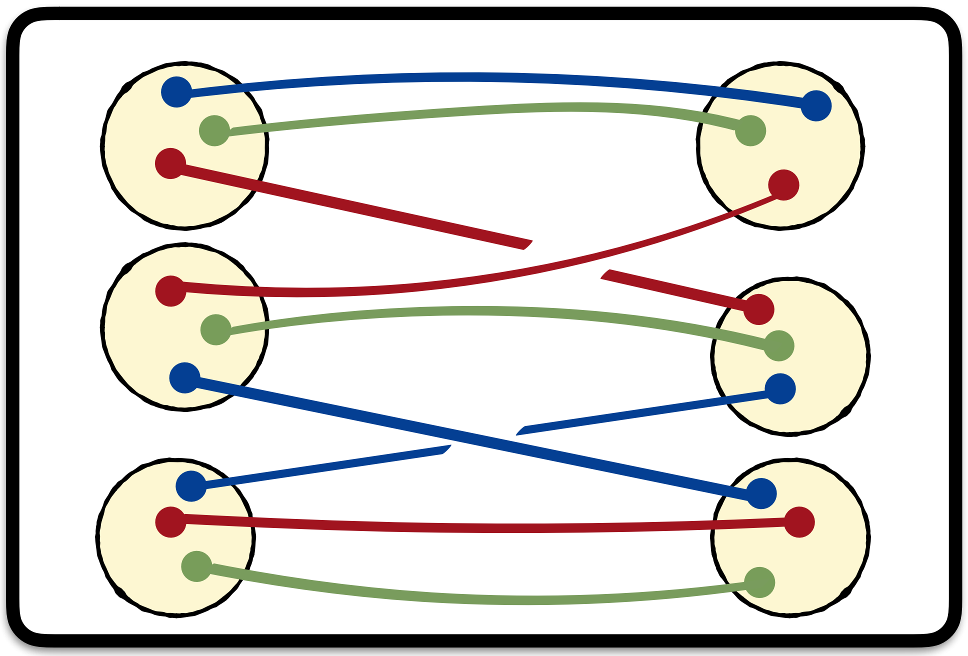


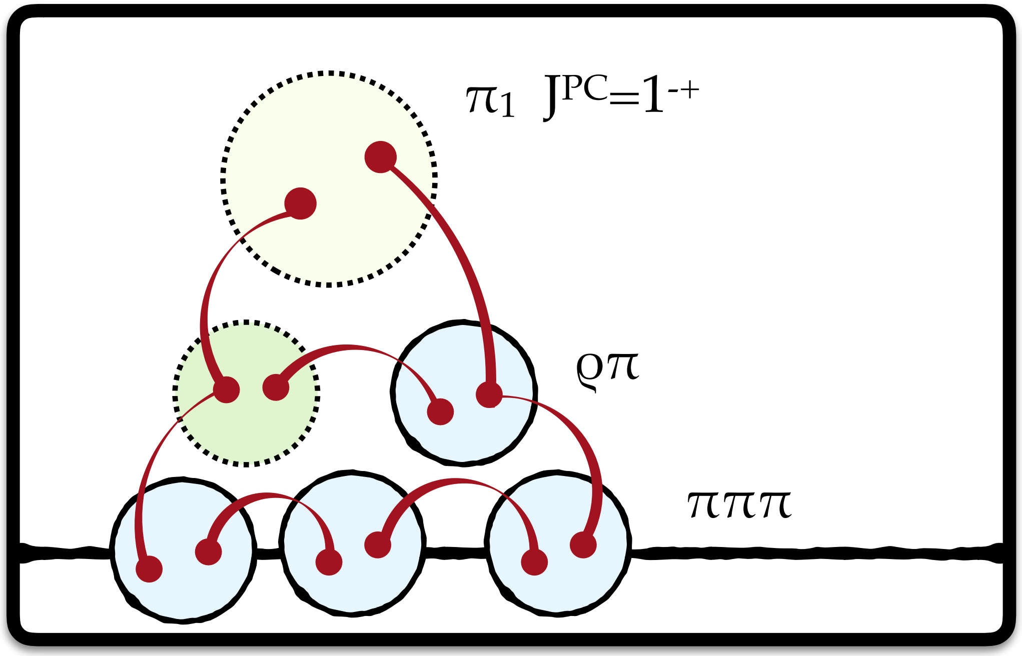


- RB & Davoudi (2013) [1+shallow bound states]
- Hansen & Sharpe (2014-15) [relativistic $\pi\pi\pi$]
- Hammer, Pang, Rusetsky (2017) [NR EFT parameterization]
- RB, Hansen & Sharpe (2016) [relativistic coupled, 2-,and 3-mesons]

Need for three-body formalism

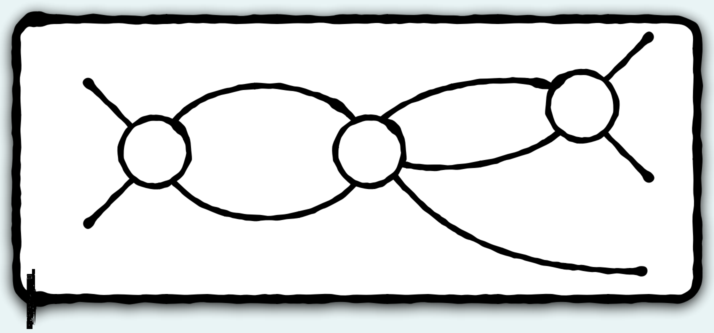
- Needed for:
  - resonances [e.g., the Roper]
  - 3N-force



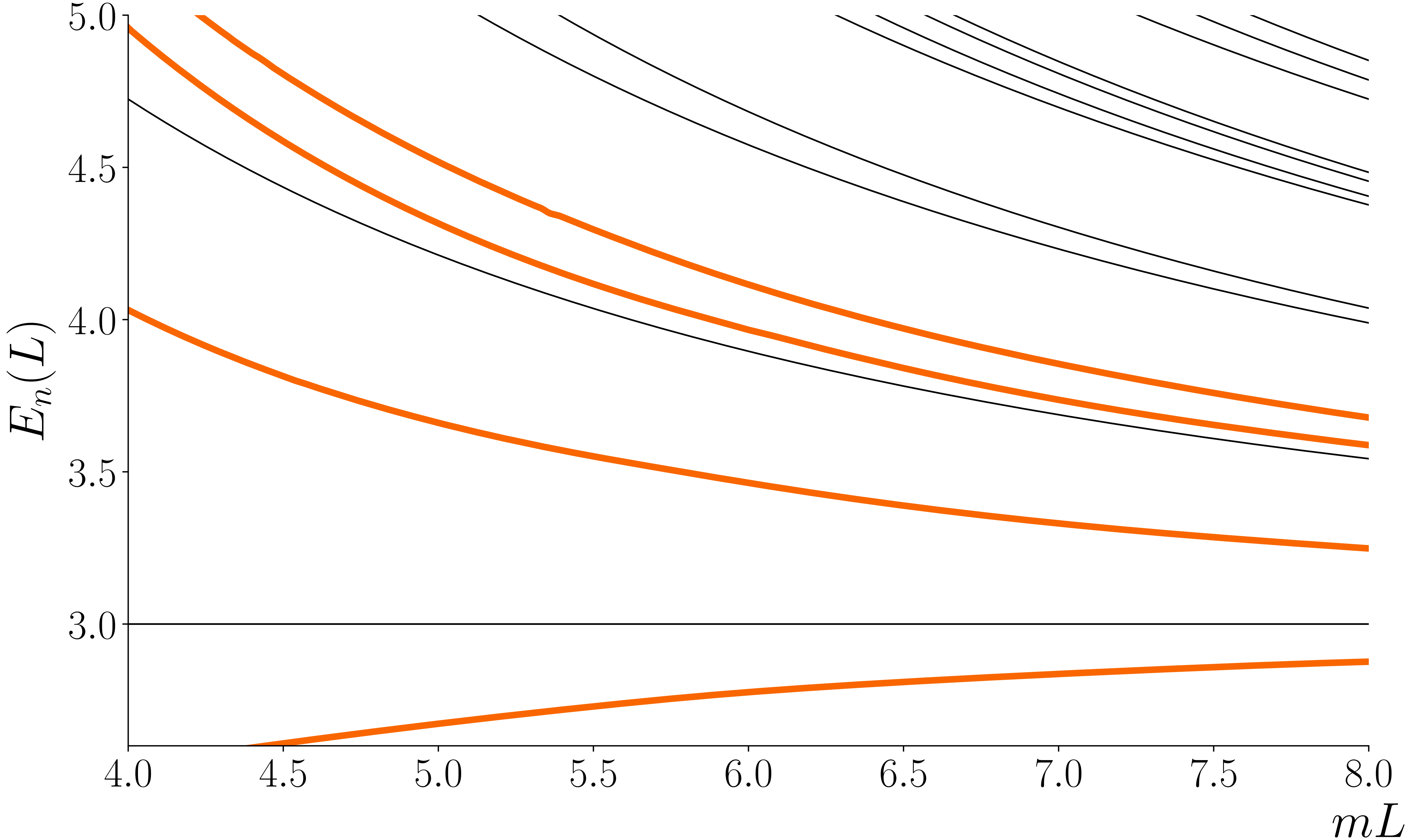


$$\det\left[1+\begin{pmatrix} F_2 & 0 \\ 0 & F_3 \end{pmatrix}\begin{pmatrix} \mathcal{K}_2 & \mathcal{K}_{23} \\ \mathcal{K}_{32} & \mathcal{K}_{\mathrm{df},3} \end{pmatrix}\right]=0$$

RB, Hansen & Sharpe (2017)



Need for three-body formalism



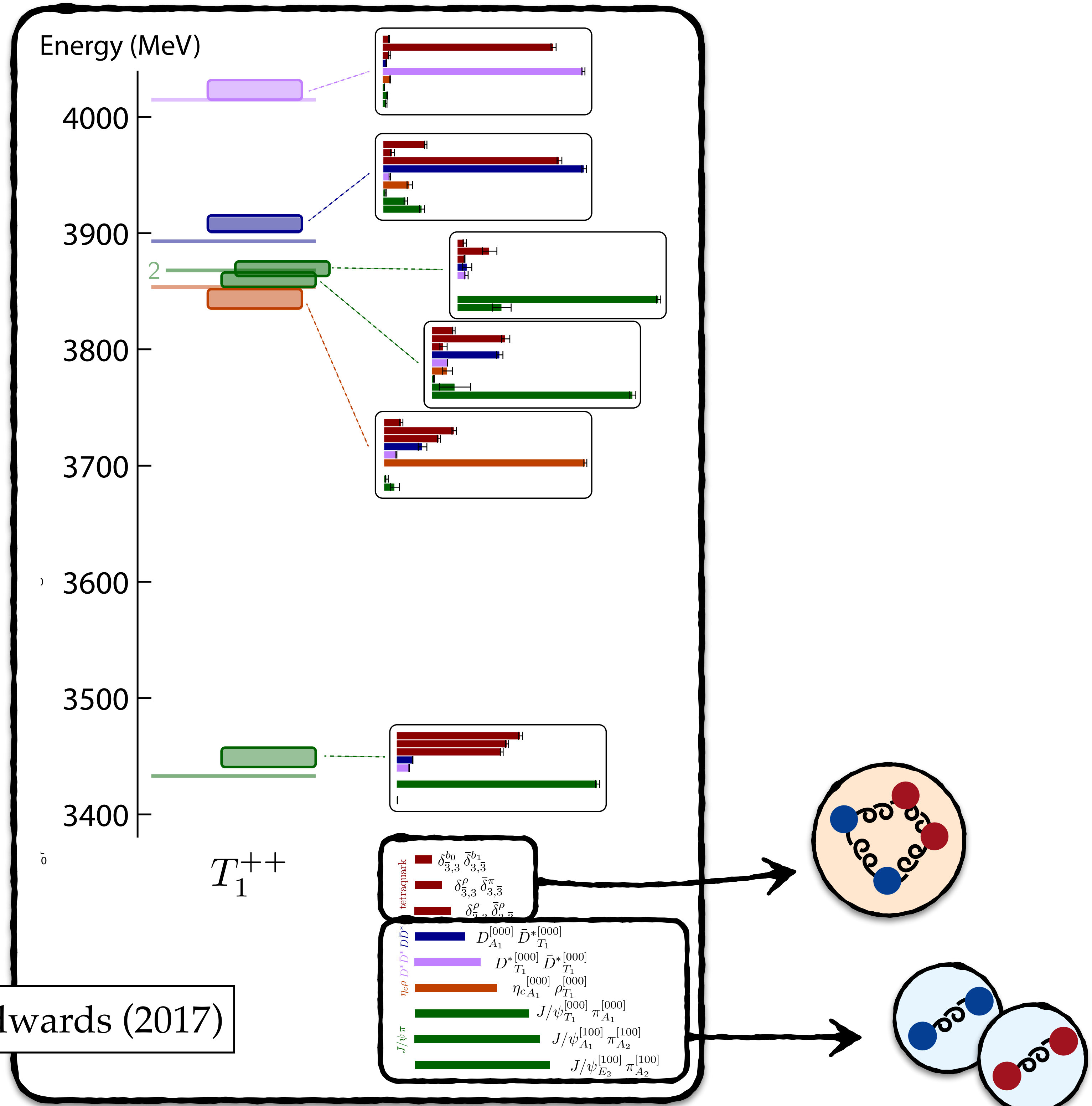
analysis code under construction!

RB, Hansen, Sharpe (to appear)

Remaining questions:

Operator basis:

- tetraquarks, pentaquarks…
- 3 particles or more
- glueballs,
- …



Cheung , Thomas, Dudek & Edwards (2017)

Remaining questions:

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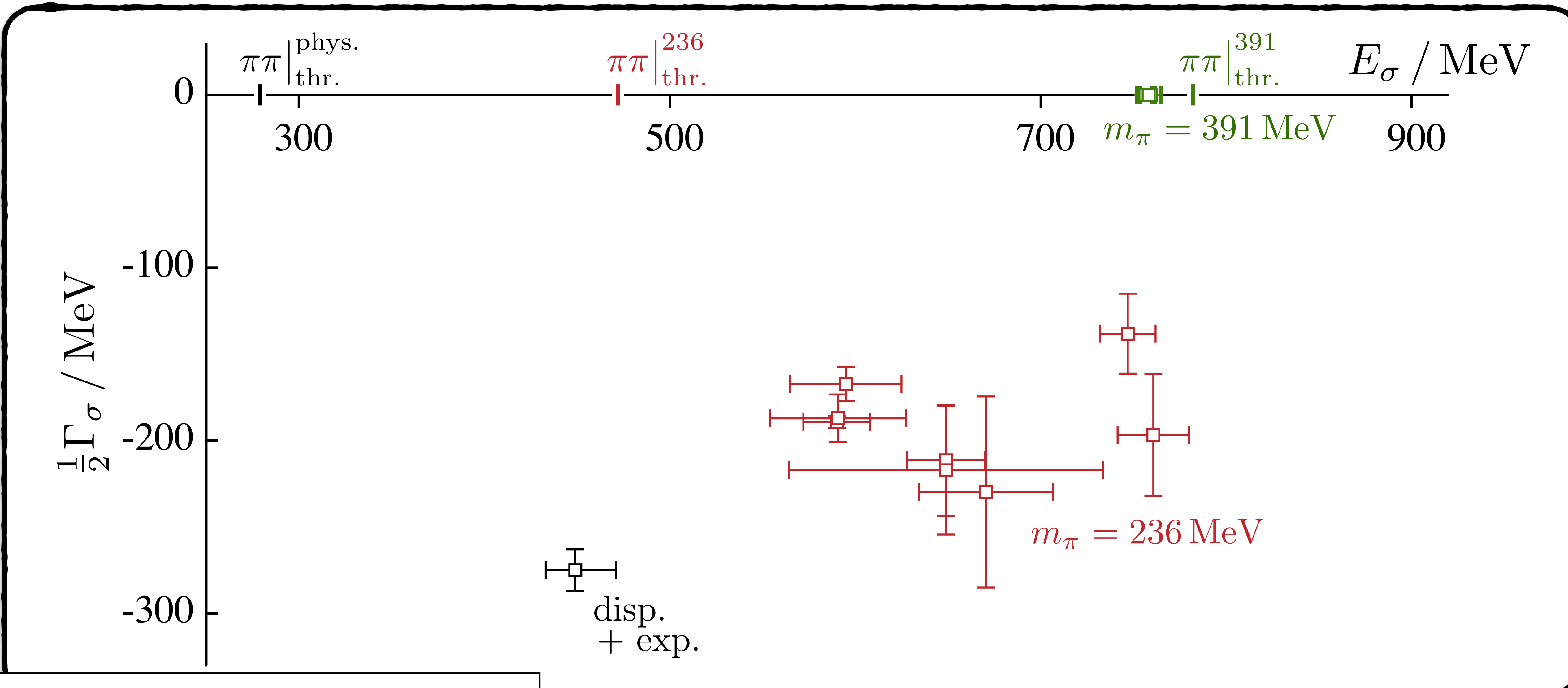
- …

Amplitude analysis:

- 3 particles or more
- dispersive techniques



RB, Dudek, Edwards & Wilson (2016)

Remaining questions:

Operator basis:

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- 3 particles or more
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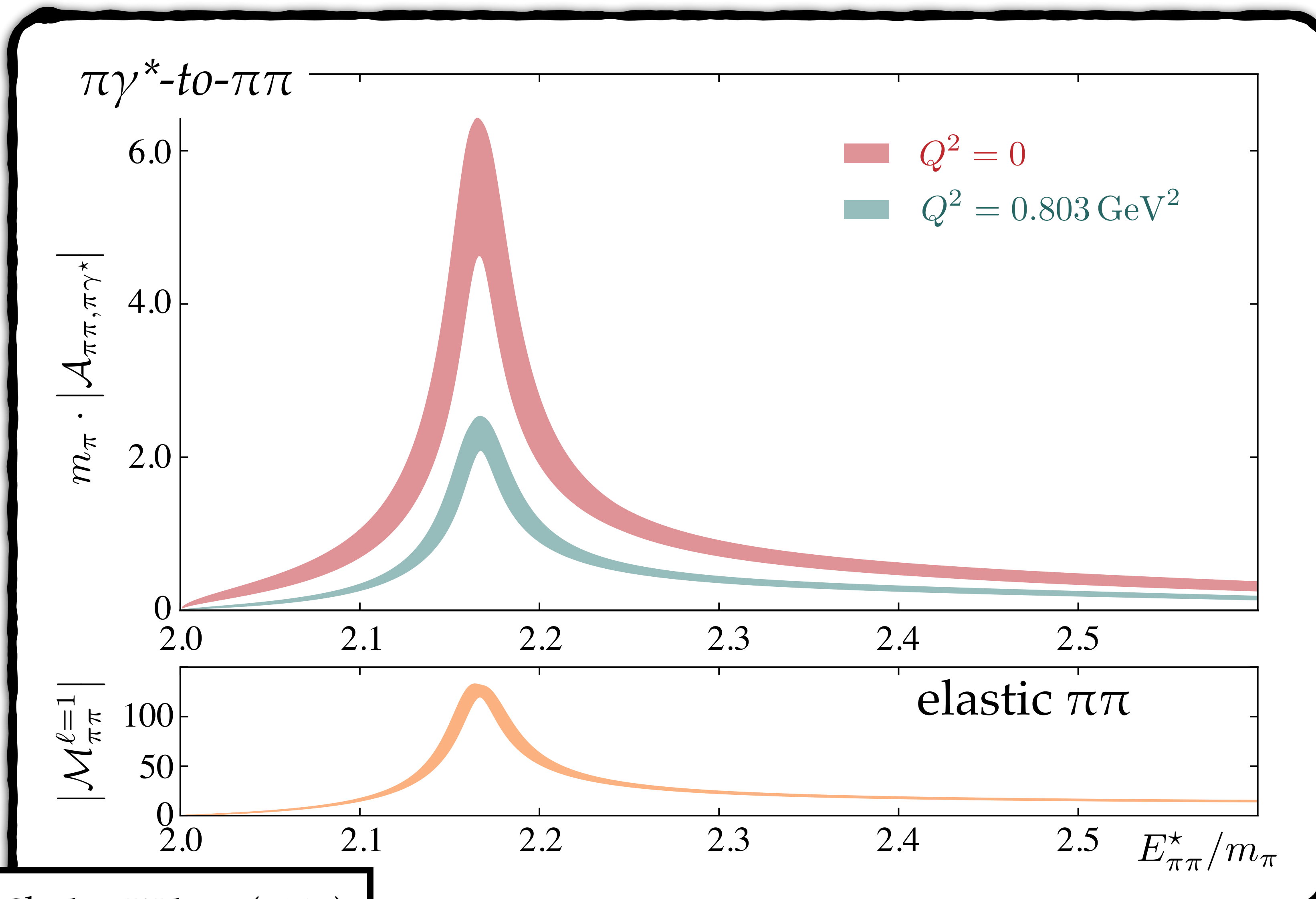
Coupling to QED currents:

- transition processes



RB, Dudek, Edwards, Thomas, Shultz, Wilson (2015)

Remaining questions:

Operator basis:

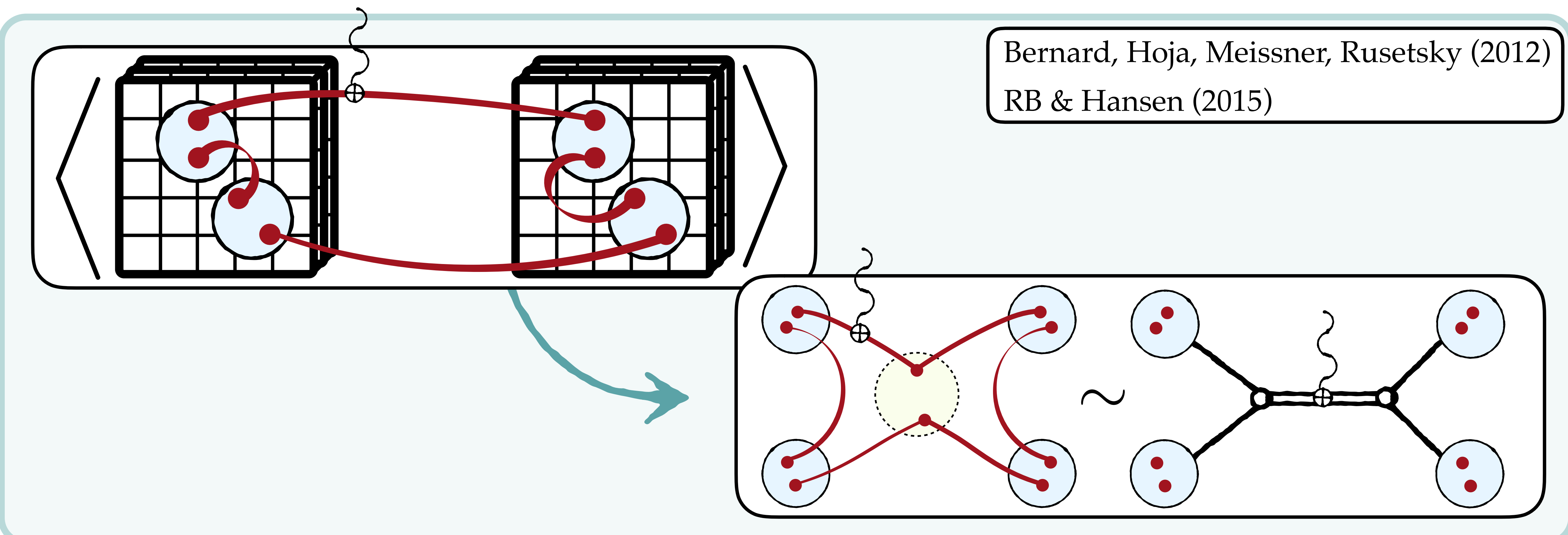
- tetraquarks, pentaquarks…
- 3 particles or more
- glueballs,
- …

Amplitude analysis:

- 3 particles or more
- dispersive techniques

Coupling to QED currents:

- transition processes
- elastic processes - form factors of resonances (the future)



A review / introduction

Scattering processes and resonances from lattice QCD

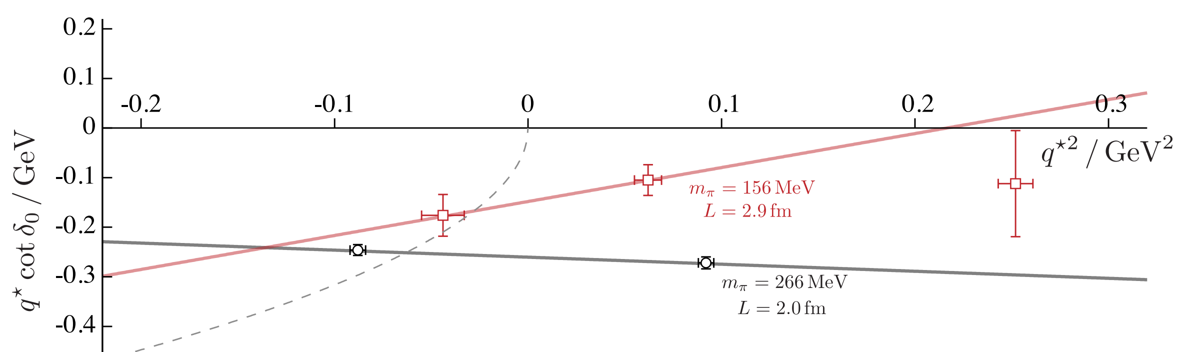
Raúl A. Briceño,[1,*] Jozef J. Dudek,[1,2,†] and Ross D. Young[3,‡]

[1] *Thomas Jefferson National Accelerator Facility, 12000 Jefferson Avenue, Newport News, Virginia 23606, USA*

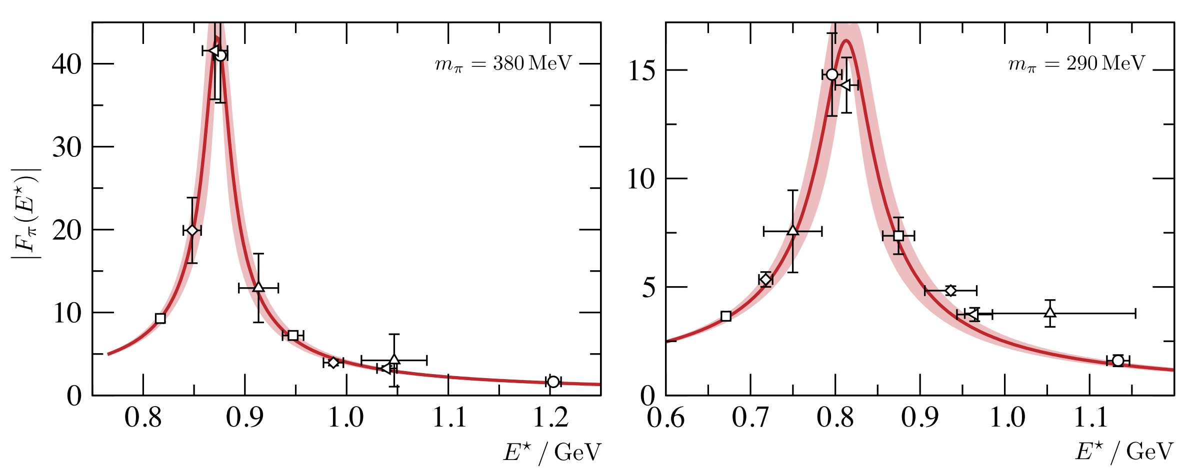
[2] *Department of Physics, College of William and Mary, Williamsburg, Virginia 23187, USA*

[3] *Special Research Center for the Subatomic Structure of Matter (CSSM), Department of Physics, University of Adelaide, Adelaide 5005, Australia*

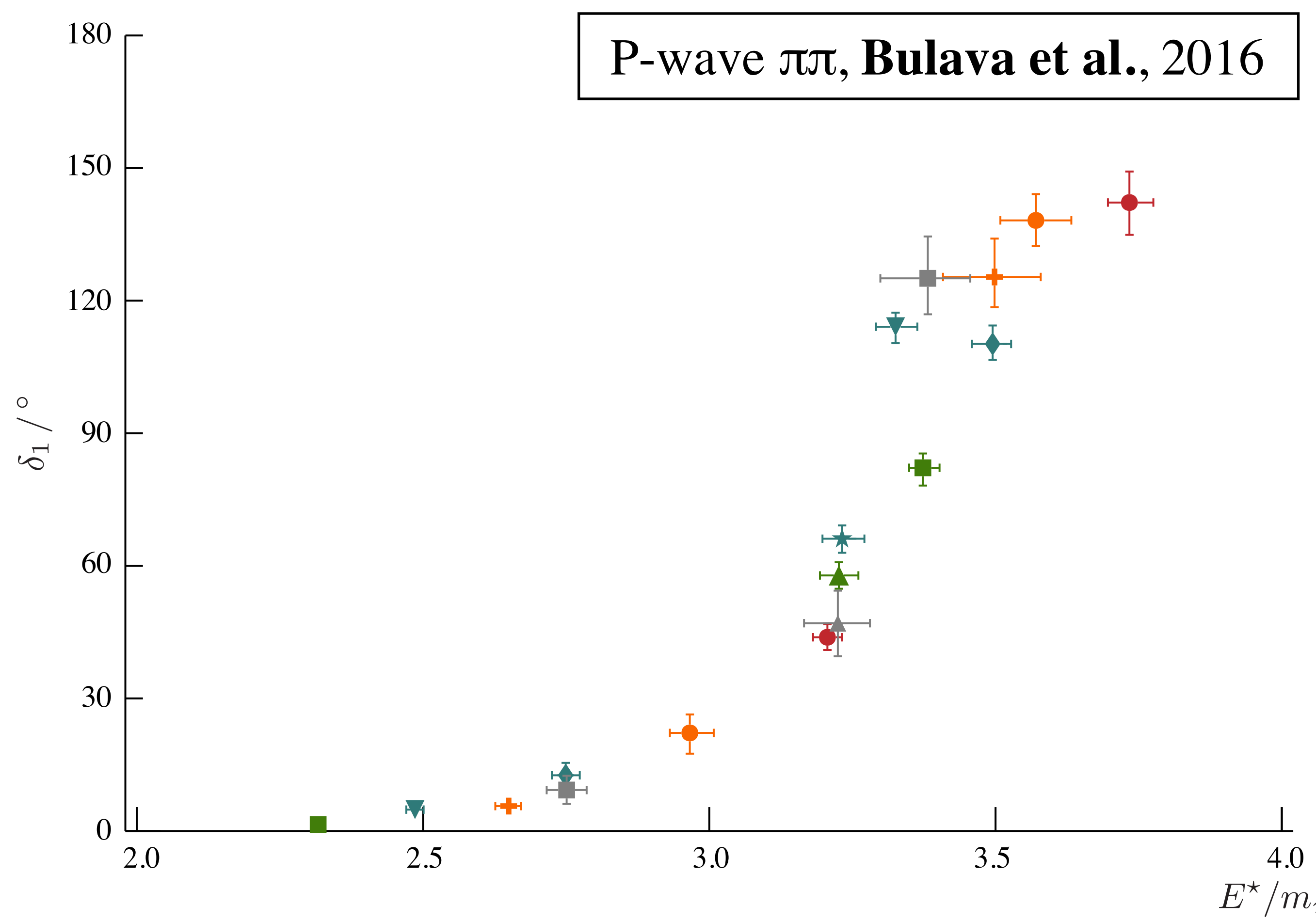
Dudek Young



S-wave DK and the $D^\star_{s0}$, **Lang et al.**, 2014



γ*-to-ππ, **Feng et al.**, 2015



P-wave ππ, **Bulava et al.**, 2016

The team and some references

more numerical - JLab

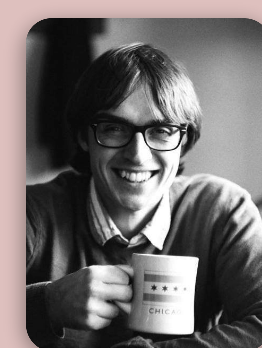

Dudek


Edwards


Winter


Joó

more numerical - Europe


Wilson


Peardon


Ryan


Thomas

more formal


Hansen


Sharpe

Meson Spectrum

JHEP05 021 (2013)
PRD88 094505 (2013)
JHEP07 126 (2011)
PRD83 111502 (2011)
PRD82 034508 (2010)
PRL103 262001 (2009)

Baryon Spectrum

PRD91 094502 (2015)
PRD90 074504 (2014)
PRD87 054506 (2013)
PRD85 054016 (2012)
PRD84 074508 (2011)

Scattering

arXiv:1708.06667
PRL118 022002 (2017)
JHEP011 1610 (2016)
PRD93 094506 (2016)
PRD92 094502 (2015)
PRD91 054008 (2015)
PRL113 182001 (2014)
PRD87 034505 (2013)
PRD86 034031 (2012)
PRD83 071504 (2011)

Electroweak

PRD93 114508 (2016)
PRL115 242001 (2015)
PRD91 114501 (2015)
PRD90 014511 (2014)

Techniques

arXiv:1709.01417
PRD85 014507 (2012)
PRD80 054506 (2009)
PRD79 034502 (2009)

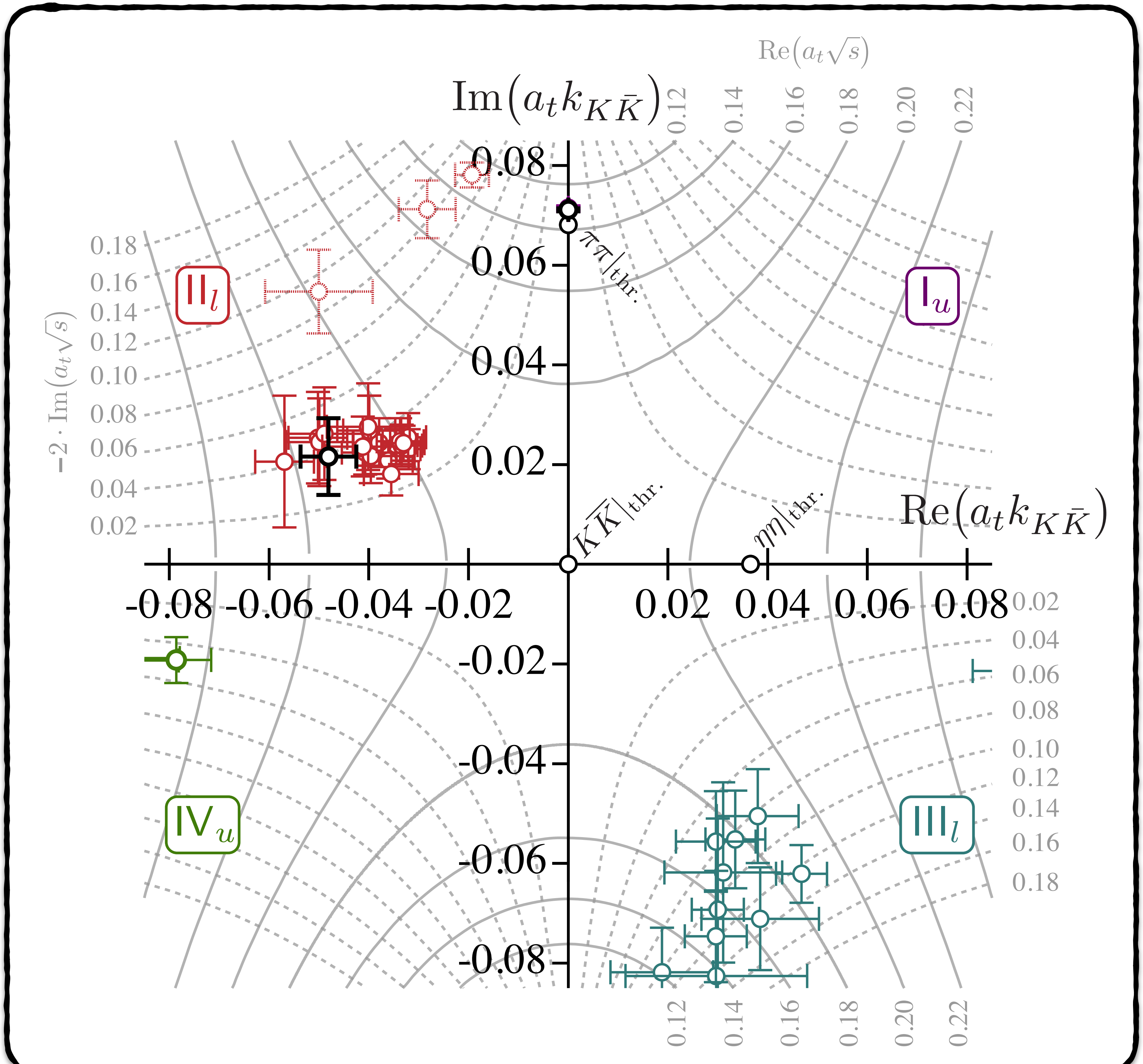
Students:
Johnson, Radhakrishnan, Cheung, Moss, O Hara, Tims

Formalism

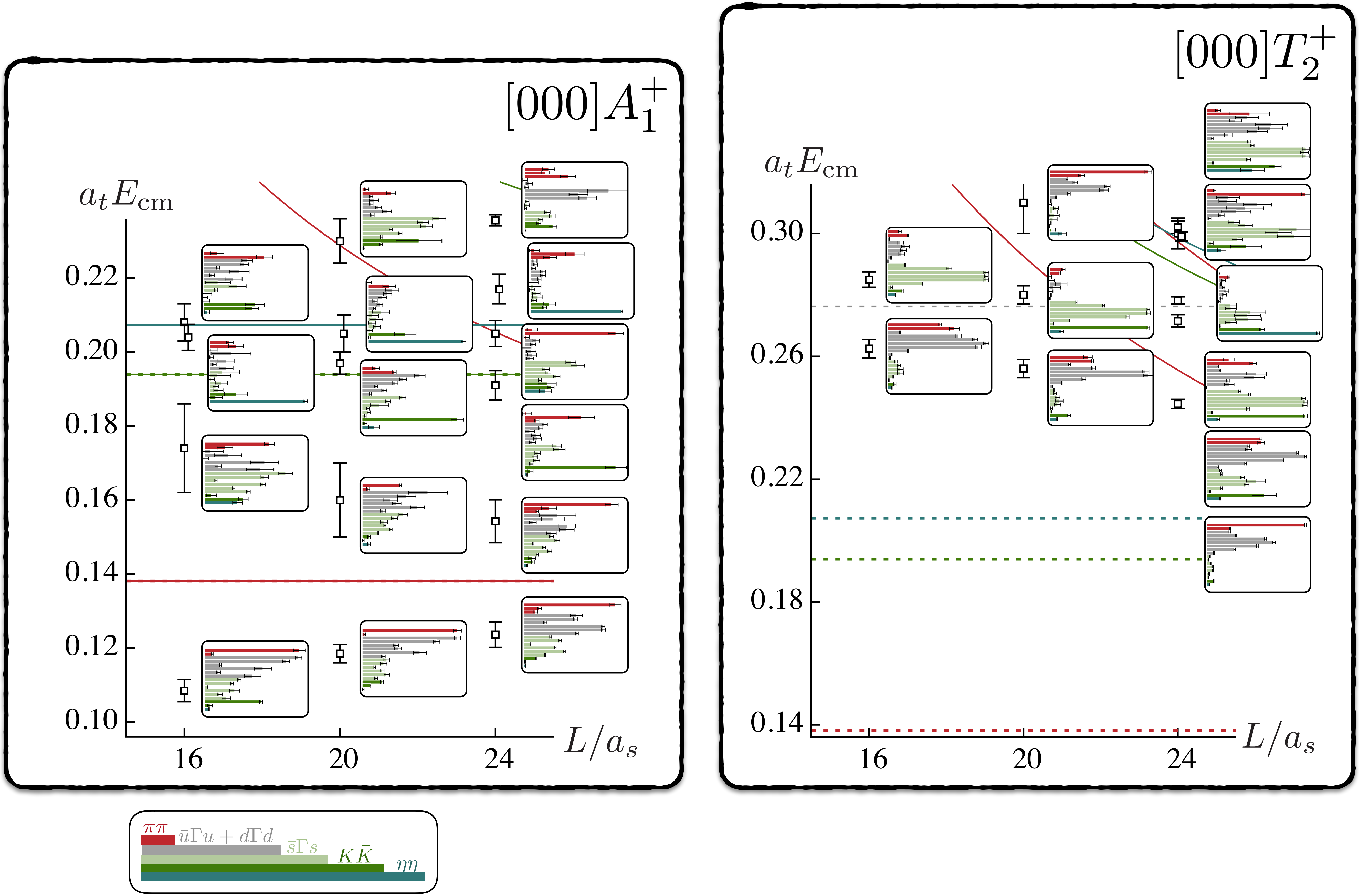
PRD95 074510 (2017)
PRD94 013008 (2016)
PRD92 074509 (2015)
PRD91 034501 (2015)
PRD89 074507 (2014)

back-up slides

Complex momentum plane



Overlaps

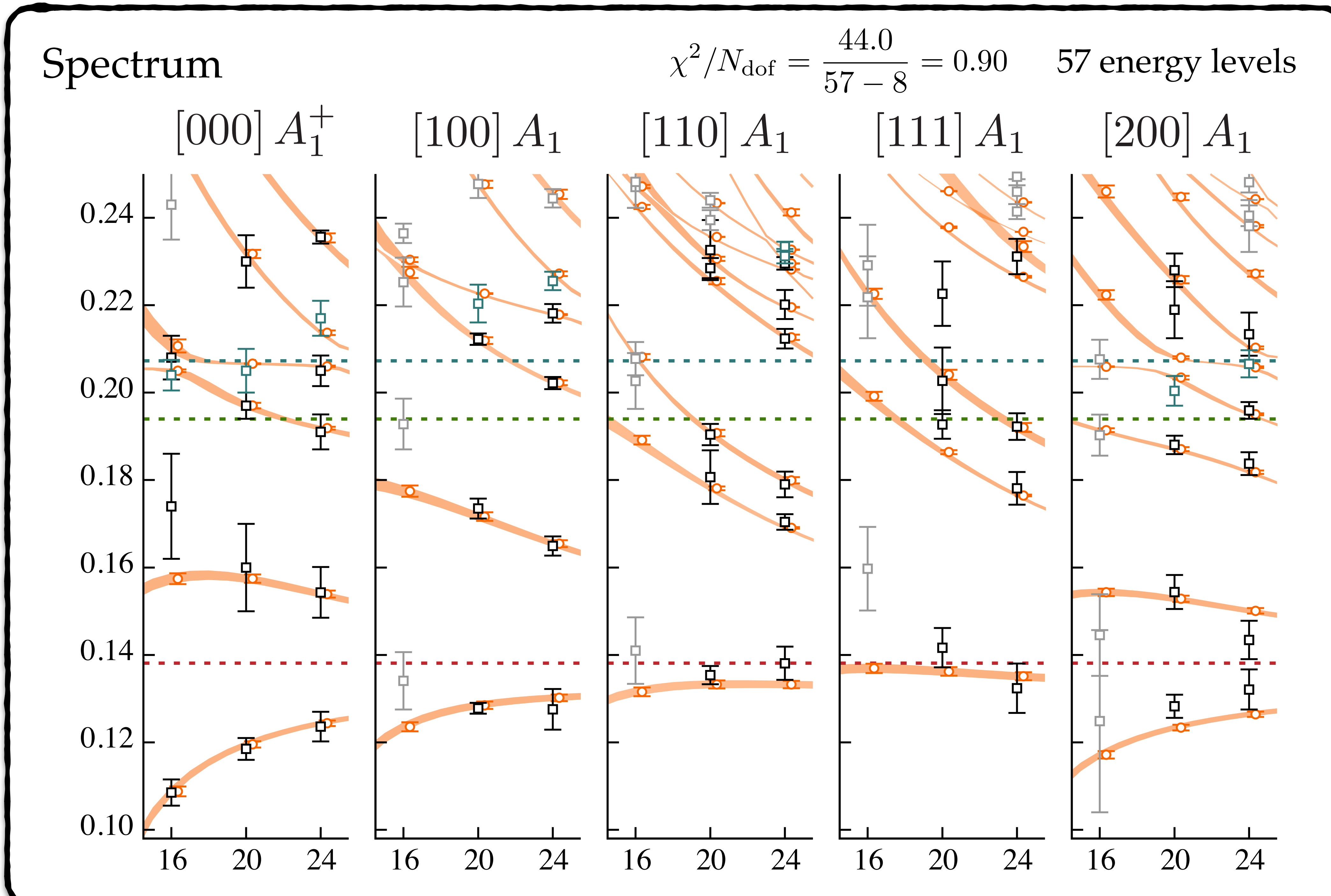


[000]A_1^+
$a_t E_{\mathrm{cm}}$
0.22
0.20
0.18
0.16
0.14
0.12
0.10
16
20
24
L/a_s
[000]T_2^+
$a_t E_{\mathrm{cm}}$
0.30
0.26
0.22
0.18
0.14
16
20
24
L/a_s
$\pi\pi$
$\bar{u}\Gamma u + \bar{d}\Gamma d$
$\bar{s}\Gamma s$
$K\bar{K}$
$\eta\eta$

Coupled-channels analysis

- S-wave above $2m_\pi$, $2m_K$, and $2m_\eta$

- Ansatz $\mathbf{K}^{-1}(s) = \begin{pmatrix} a+bs & c+ds & e \\ c+ds & f & g \\ e & g & h \end{pmatrix}$



Coupled-channels analysis

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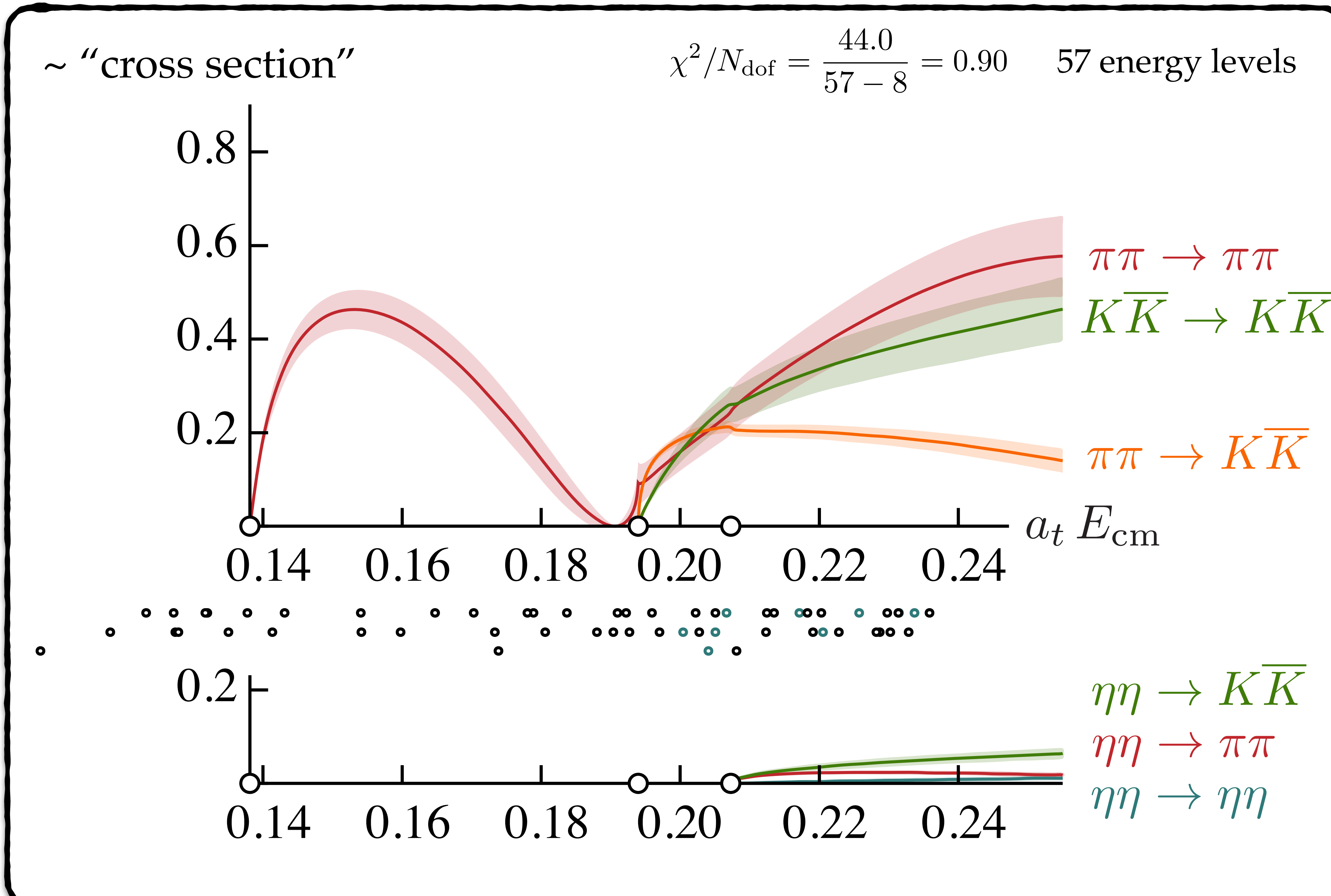
- Ansatz $\mathbf{K}^{-1}(s) = \begin{pmatrix} a+bs & c+ds & e \\ c+ds & f & g \\ e & g & h \end{pmatrix}$

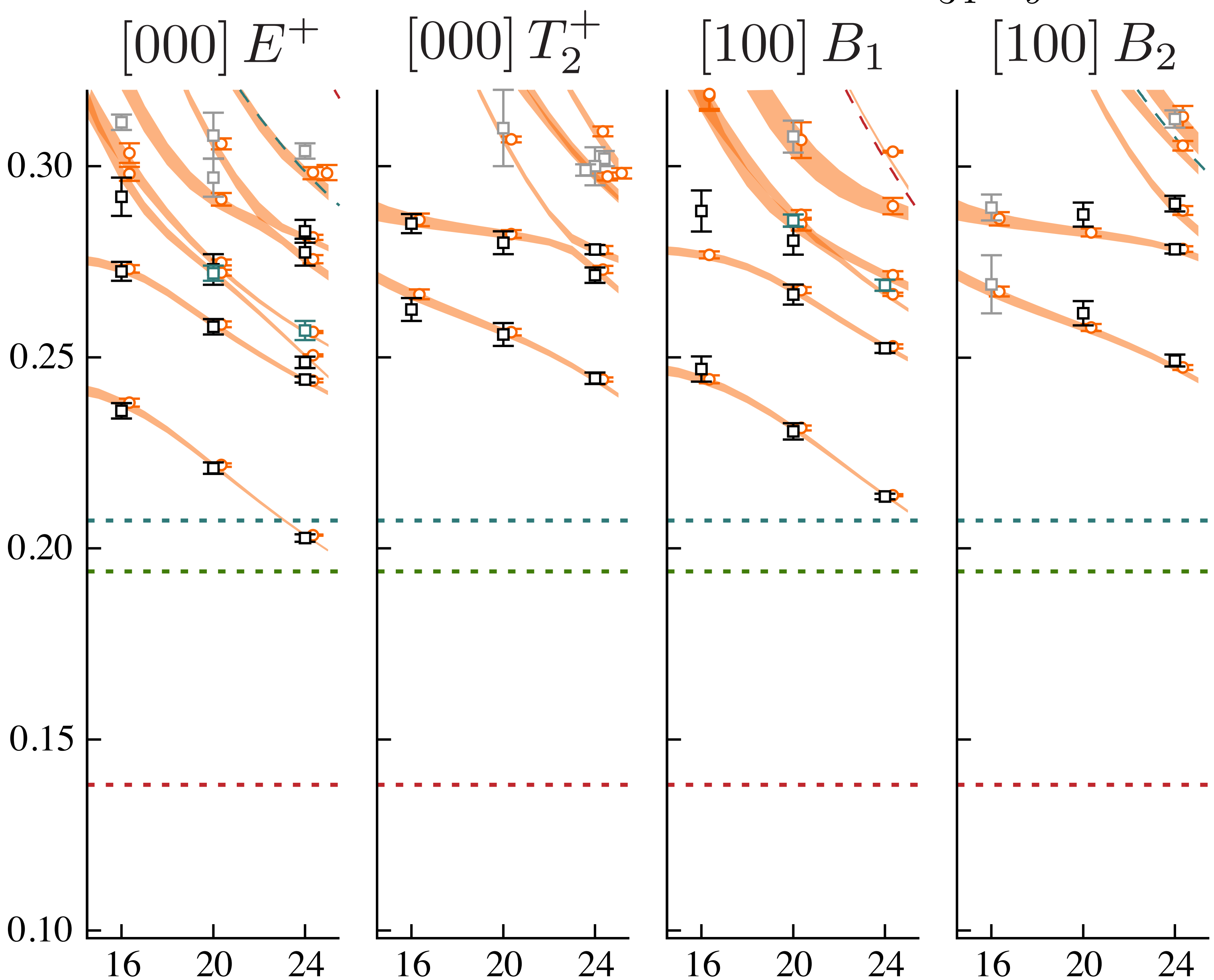


Coupled-channels analysis

- D-wave above $2m_\pi$, $2m_K$, and $2m_\eta$

- Ansatz $$K_{ij}(s) = \frac{g_i^{(1)} g_j^{(1)}}{m_1^2 - s} + \frac{g_i^{(2)} g_j^{(2)}}{m_2^2 - s} + \gamma_{ij}$$ $\gamma_{\eta\eta} \neq 0$, $\gamma_{ij} = 0$ otherwise

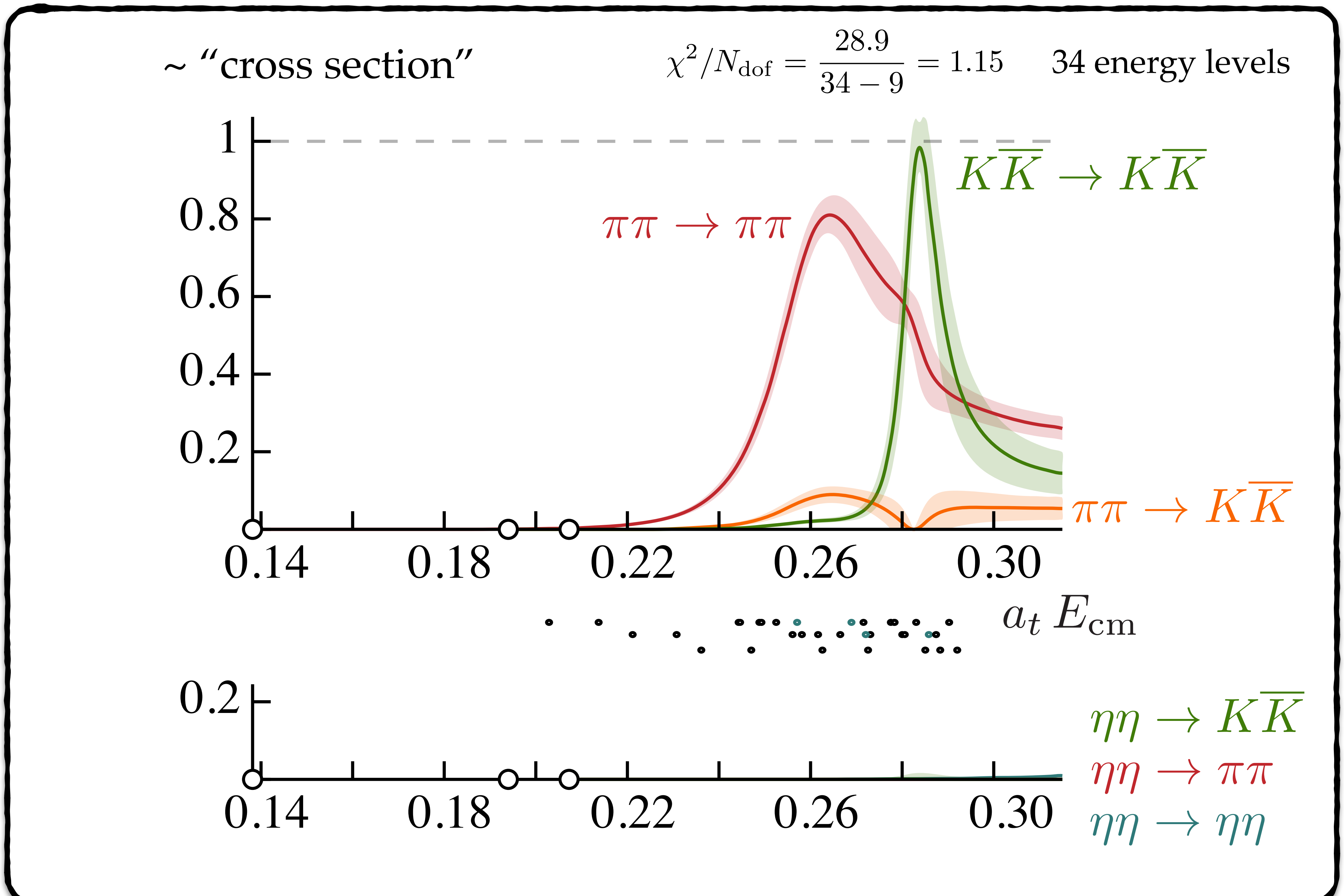
Spectrum $\chi^2/N_{\rm dof} = \frac{28.9}{34-9} = 1.15$ 34 energy levels



Coupled-channels analysis

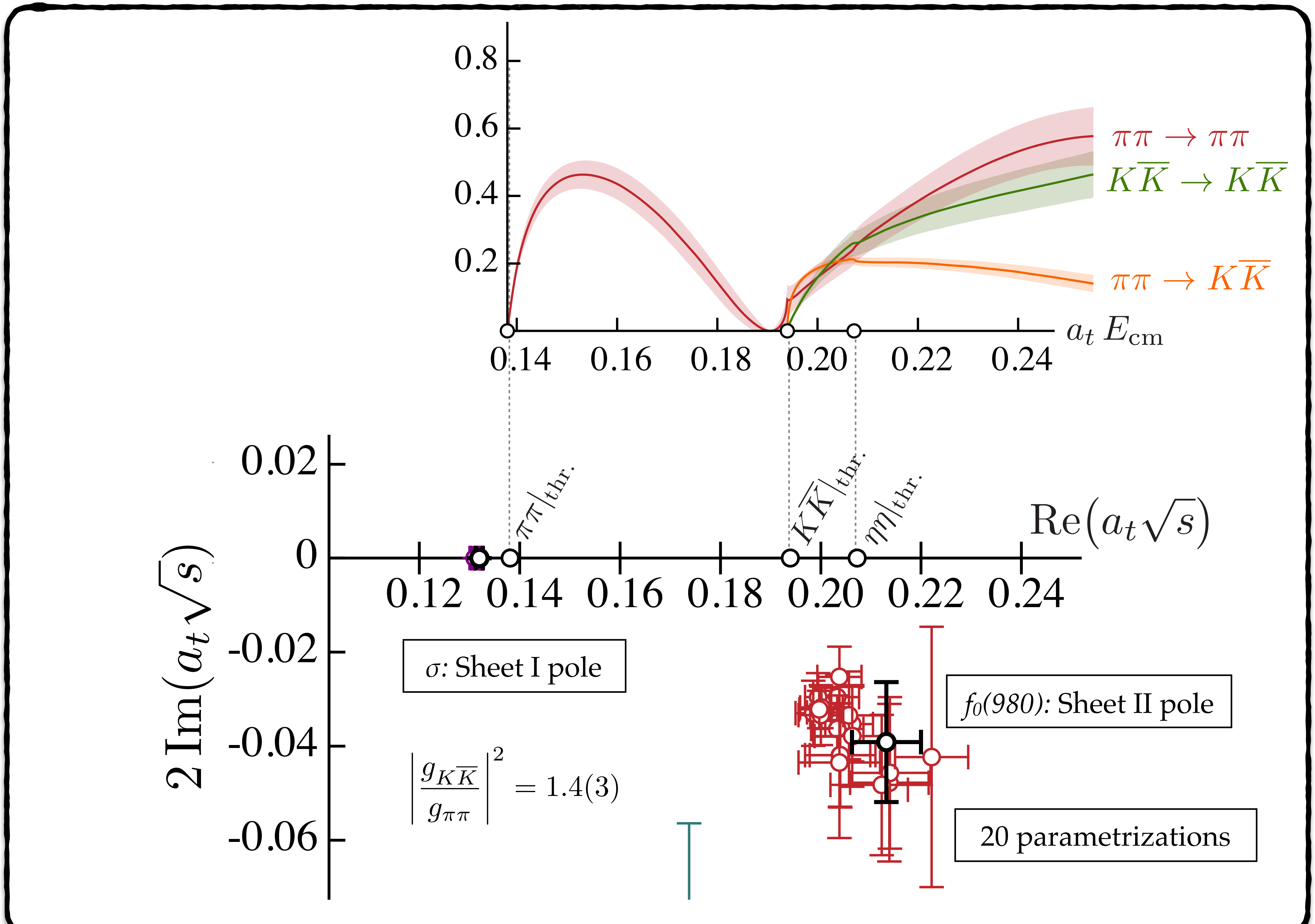
- D-wave above *$2m_\pi$, $2m_K$, and $2m_\eta$*

- Ansatz $$K_{ij}(s) = \frac{g_i^{(1)} g_j^{(1)}}{m_1^2 - s} + \frac{g_i^{(2)} g_j^{(2)}}{m_2^2 - s} + \gamma_{ij}$$ $\gamma_{\eta\eta} \neq 0$, $\gamma_{ij} = 0$ otherwise



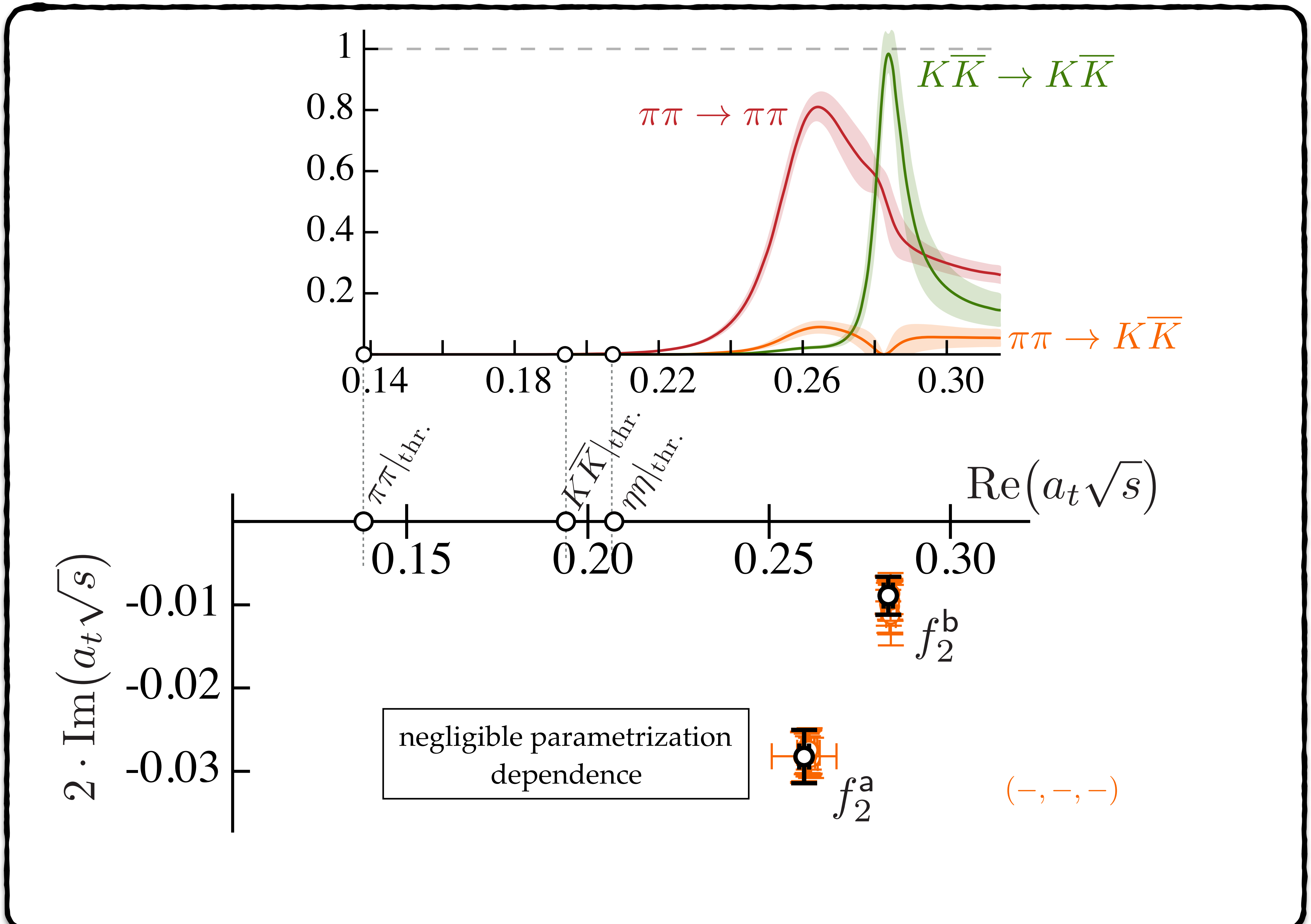
Scalar poles: σ and *f₀(980)*

- Near poles: $\mathcal{M} \sim \frac{g^2}{s_0 - s}$



Tensor poles: the f_2's

- Near poles: $\mathcal{M} \sim \dfrac{g^2}{s_0 - s}$



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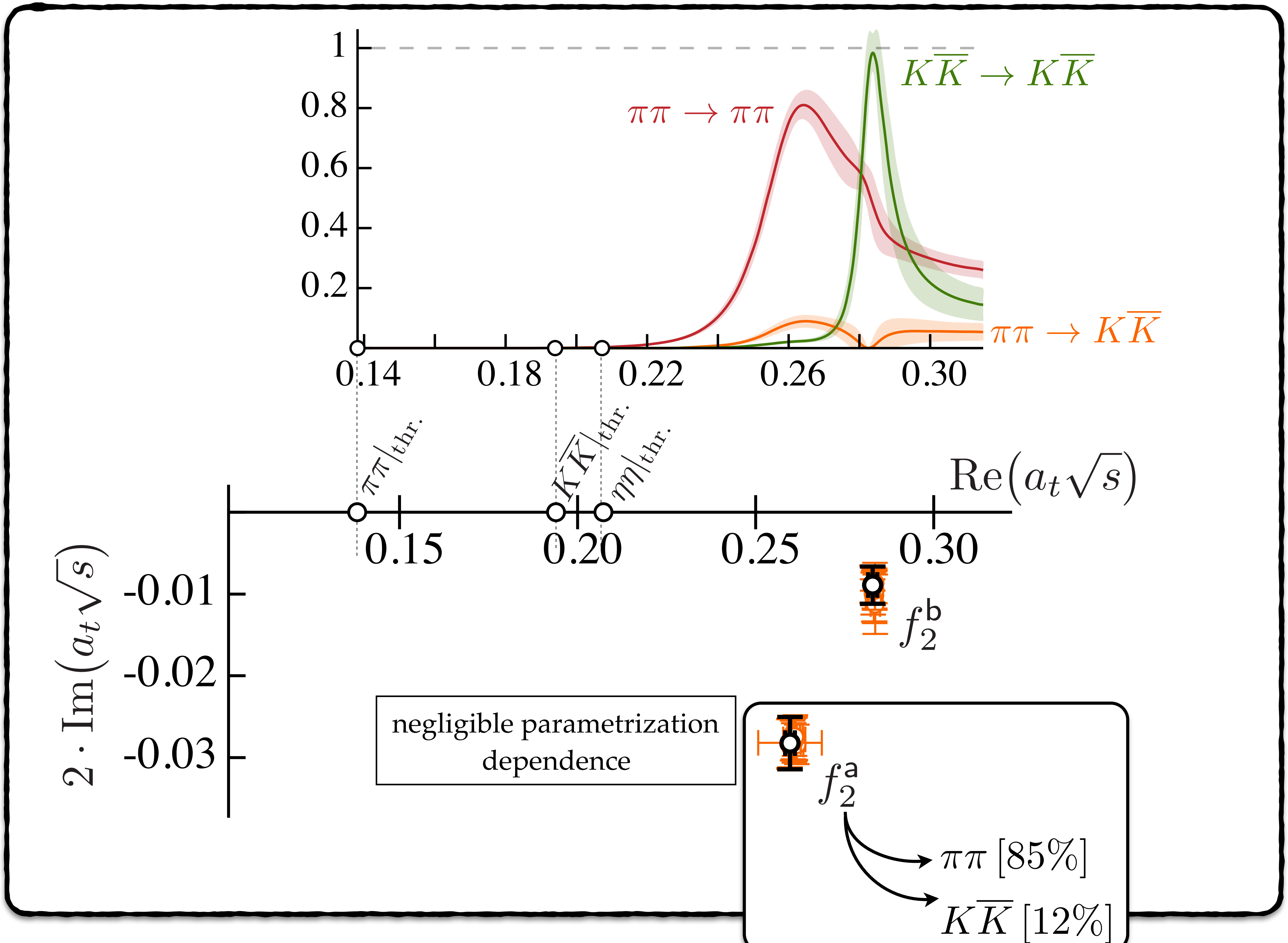

Tensor poles: the f_2's

- Near poles: $\mathcal{M} \sim \dfrac{g^2}{s_0 - s}$