Conjecture about the 2-Flavour QCD Phase Diagram

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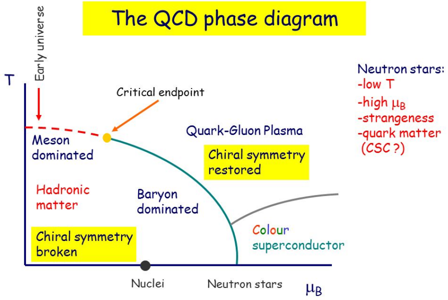
Motivation



- The QCD phase diagram is one of the most prominent outstanding mysteries within the Standard Model of particle physics.
- Quarks do have masses, but two flavours are very light compared to the intrinsic scale $\Lambda_{QCD} = 341(12)$MeV.

Global O(4) symmetry

It is of interest to describe the chiral symmetry breaking pattern

$$SU(N_f)_L \otimes SU(N_f)_R \rightarrow SU(N_f)_{L=R}, \quad (1)$$

for massles fermions by (locally) isomorphic orthogonal groups $O(N) \rightarrow O(n)$, $N > n$. This is possible solely for $N_f = 2$, where the transition 1 isomorphically corresponds to $O(4) \rightarrow O(3)$.

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The O(4) non-linear σ-model

Therefore the O(4) non-linear σ-model represents an effective theory for low-energy QCD with $N_f = 2$ massless quark flavours. Its Euclidean action

$$S[\vec{e}] = \frac{F_\pi^2}{2} \int d^d x \partial_\mu e(\vec{x}) \cdot \partial_\mu e(\vec{x}), \qquad e(\vec{x}) \in S^3 \tag{2}$$

corresponds to the QCD low-energy Lagrangian for $N_f = 2$ massless quark flavors. The global O(4) symmetry may break spontaneously down to O(3), which yields three NGBs, with fields in the coset space O(3), which is isomorphic to SU(2).

3d O(4) model

The goal of this project is a numerical study of the 3d O(4) model, considering in particular the role of the skyrmions as carriers of the baryon number. Since these are topological properties, such a study must be non-perturbative.
The standart lattice action is

$$S[\vec{e}] = \frac{F_\pi^2}{2} \sum_{<xy>} (1 - \vec{e_x} \cdot \vec{e_y}), \qquad \vec{e_x} \in S^3 \tag{3}$$

Lattice QCD

Once a lattice field theory has been formulated, the original field theory problem becomes one of statistical mechanics. The first step in understanding the theory is then to map out the phase diagram of the equivalent statistical mechanics system. The ground state of the theory changes qualitatively from one phase to another. In some of these theories there is a range of parameters for which the ground state cannot tolerate the presence of an isolated quark. This is the quark-confining phase of the cutoff theory.

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Phases

This phase may be separated by a critical surface from another phase in which quarks can be isolated. After the phases of the theory have been established, the behavior of the theory in the critical region must be determined. One must determine, for example, the order of the transition occurring between the two phases. Only if the transition is continuous can one obtain a continuum, relativistic field theory from the lattice system. If the transition is continuous, the system's mass gap vanishes as a critical point is approached.

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Critical point

At the critical point the theory loses memory of the lattice and the continuous space-time symmetries of the field theory are reestablished. In short, to use lattice formulations of field theory to construct continuum theories, one maps out the lattice theory's phase diagram, locates its critical points (lines or surfaces) of continuous phase transitions, and approaches the critical points in a well-defined, delicate fashion.

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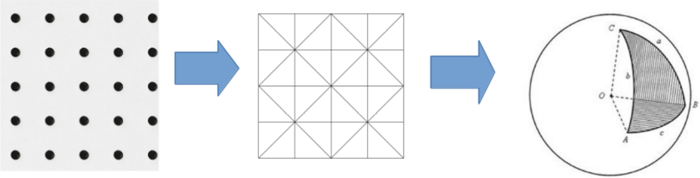
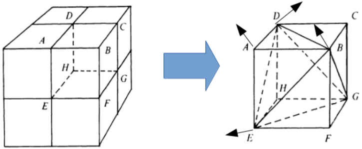
Topological charge

To define a topological charge on the lattice; in particular the geometric definition has the virtue of guaranteeing integer charges, $Q \in Z$, for all configurations (except for a subset of measure zero). It was suggested and successfully applied in the 2d O(3) model, where the lattice plaquettes are split into triangles, and the topological charge density is given by the (minimal) oriented area of the spherical triangle spanned by the spins at its vertices.

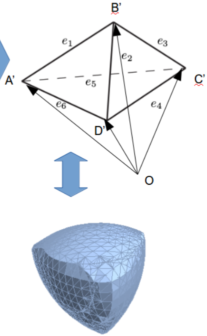


Topological charge

Cálculo de la carga topológica del modelo 3D O(4)



Espines de 4 dimensiones y norma 1



Todavía no existe el método analítico para este problema, por lo que se propone usar el método geométrico.

Única problema: Calculo del volumen del tetrahedro esférico

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To be continued...

- Find an orientation to asign a sign to the volumes of the spherical tetrahedron.
- Calculate the topological and magnetic suceptibilities.
- Find the divergence point (or points!) of the topological suceptibility and the point (or points!) where the magnetic susceptibility turns to zero.
- Get the order of the transition at the critical point (1st order, 2nd order)
- Write a thesis.

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