# Thermodynamics of the Schwarzschild Black Hole in Noncommutative Space

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**Abstract.** In this paper we study noncommutative black holes. In particular, we use a deform Schwarzschild solution in noncommutative gauge theory of gravity. By means of euclidean quantum gravity we obtain the entropy, temperatute and the time of evaporation of the noncommutative black hole.

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## INTRODUCTION

Recently there has been interest in the properties of black holes in noncommutative space-time(see for example [1], and references there in), where several approaches have been proposed, in particular in [2] the authors obtain a deformed Schwarzschild solution for a noncommutative gauge theory of gravity. The solution is constructed from a deformation of the gravitational fields by gauging the noncommutative de Sitter SO(4,1) group and using the Seibeg-Witten map [3]. The new gauge fields are obtain up to second order in the noncommutative parameters $\Theta^{\mu\nu}$. Finally they calculte the new gravitational gauge potentials by contracting the noncommutative gauge group SO(4,1) to the Poincaré group ISO(3,1).

In this paper we will follow the approach as in [2] to calculate thermodynamical properties. As we will show the temperature, entropy and time of evaporation get corrected by the nature of the noncommutative solution.

**Noncommutative Solutions:** To calculate the thermodynamical properties of the Schwarzschild black hole in the classical picture, one can use the euclidean quantum gravity approach. In this formalism the path integral procedure is employed to obtain the partition function of the system and then the thermodynamical properties of interest (for details see for example [4]). In this work we proceed in the same manner to calculate the temperature, entropy and time of evaporation for the noncommutative black hole. As mentioned on the introduction, we start with a deformed Schwarzschild solution in noncommutative gauge theory of gravity [2], where the non-zero components of the metric have the following form

$$\hat{g}_{00} = -\left(1 - \frac{\alpha}{r}\right) - \frac{\alpha(8\alpha - 11\alpha)}{16r^4}\Theta^2, \tag{1}$$

$$\hat{g}_{11} = \left(1 - \frac{\alpha}{r}\right)^{-1} + \frac{\alpha(4r - 3\alpha)}{16r^2(r-\alpha)^2}\Theta^2, \tag{2}$$

$$\hat{g}_{22} = r^2 + \frac{(r^2 - 17\alpha r + 17\alpha)}{32(r-\alpha)}\Theta^2, \tag{3}$$

$$\hat{g}_{33} = r^2 \sin^2\vartheta + \frac{(r^2 + \alpha r - \alpha^2)\cos^2\vartheta - \alpha(2r-\alpha)}{16r(r-\alpha)}\Theta^2, \tag{4}$$

where $\Theta$ is the noncommutative parameter and $\alpha = 2M$ (in units of $c = G = 1$). The noncommutative action will be

$$I^E_{NC} = \frac{1}{16\pi}\int_{\mathscr{M}} \sqrt{-\hat{g}}\hat{R}d^4x + \frac{1}{8\pi}\int_{\partial\mathscr{M}} \sqrt{-\hat{h}}\hat{K}d^3x, \tag{5}$$

where $\hat{g}_{\mu\nu}$ is the noncommutative metric, $\hat{R}$ is the curvature scalar corresponding to the noncommutative metric, $\hat{h}_{\mu\nu}$ is the noncommutative metric induced in the boundary and $\hat{K}$ the noncommutative extrinsic curvature. In the limit $\Theta \to 0$ we recover the commutative action. We know that the Schwarzschild space-time is asymptotically flat, and so we can compute the action by using flat space-time as a reference background. For both space-times, the Ricci scalar $R = 0$ hence $\hat{R} = 0$, so the first part of (5) vanish, so we have

$$I^E_{NC} = \frac{1}{8\pi}\int_{\partial\mathscr{M}} \left[\sqrt{-\hat{h}}\hat{K}_{00}\hat{g}^{00}d^3x + \sqrt{-\hat{h}}\hat{K}_{22}\hat{g}^{22}d^3x + \sqrt{-\hat{h}}\hat{K}_{33}\hat{g}^{33}d^3x\right], \tag{6}$$

where

$$\sqrt{-\hat{h}} = r^2\sin\vartheta\left(1 - \frac{\alpha}{r}\right)^{1/2}\left[1 + \left(\frac{q_0(r)}{2} + \frac{q_2(r)}{2} + \frac{q_3(r)}{2}\right)\Theta^2\right] \tag{7}$$

and

$$q_0(r) = \frac{\alpha(8r - 11\alpha)}{32r^4(r-\alpha)}\left(1 - \frac{\alpha}{r}\right)^{-1}, \qquad q_2(r) = \frac{2r^2 - 17r\alpha + 17\alpha^2}{32r^2(r-\alpha)},$$

$$q_3(r) = \frac{(r^2 - r\alpha - \alpha^2)\cos\vartheta - \alpha(2r-\alpha)}{16r(r-\alpha)r^2\sin^2\vartheta}. \tag{8}$$

Finally the action takes the form

$$I^E_{NC} = \frac{\beta M}{2} + \frac{\beta M}{2}R(r,\alpha)\Theta^2, \tag{9}$$

where $\beta = 1/T$ and $R(r,\alpha)$ is

$$\begin{aligned} R(r,\alpha) \quad &= \Big\{-\tfrac{3}{r^2} + \tfrac{11\alpha}{4r^3} - q_0(r) - \tfrac{3}{2}q_1(r) + \tfrac{1}{\alpha}\left(1 - \tfrac{\alpha}{r}\right) \\ &\times \Big[\tfrac{r^2}{16r(r-\alpha)} + \tfrac{1}{16(r-\alpha)} + \tfrac{17\alpha}{32(r-\alpha)^2} - \tfrac{17\alpha^2}{32r(r-\alpha)^2} \\ &- \tfrac{17\alpha^2}{32r^2(r-\alpha)} - 2q_2(r) - \tfrac{4}{3\beta M}\left(1 - \tfrac{\alpha}{r}\right)\tfrac{\alpha(2r-\alpha)}{16r(r-\alpha)}\Big]\Big\}. \end{aligned} \tag{10}$$

Having calculated the noncommutative action, lets proceed to calculate the temperature. The problem of determening the temperature is reduced to removing the singularity in the metric [4]. One way to achieve this is by calculating the poles of the metric (the spatial part). So we have

$$\beta = \frac{8\alpha^2 - \Theta^2}{8\alpha}, \tag{11}$$

in the limit $\Theta \to 0$ we obtain the commutative result.

Gathering the previous results, the energy is

$$\hat{E} = \frac{\partial I^E_{NC}}{\partial \beta} = M + \frac{\partial}{\partial \beta}\left(\frac{\beta M}{2} R(r,\alpha)\Theta^2\right), \tag{12}$$

and the entropy

$$S_{NC} = \beta \hat{E} - I^E_{NC} = \frac{\mathscr{A}}{4} + \left[\beta \frac{\partial}{\partial \beta}\left(\frac{\beta M}{2} R(r,\alpha)\right) + \frac{\beta M}{2} R(r,\alpha)\right]\Theta^2. \tag{13}$$

The time of evaporation for the noncommutative black hole is

$$\tau_{nc} = \frac{c^2}{3}\left[\frac{M_i^3}{\kappa} + \frac{M_i \Theta^2}{8} + \mathscr{O}(\Theta^3)\right], \tag{14}$$

where $\kappa = \sigma \hbar^4 c^8 / 256\pi^3 G^2 k^4$ and $M_i$ is the initial mass of the black hole.

## CONCLUSIONS

We have show that thermodynamical properties such as the temperature, entropy and time of evaporation using the deform Schwarzschild solution in noncommutative theory of gravitation, leads to corrections in such quantities. This corrections are proportional to the square of the noncommutative prameter $\Theta$. In the limit when $\Theta \to 0$ we recover the classical results.

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## REFERENCES

1. J. C. Lopez-Dominguez, O. Obregon, M. Sabido and C. Ramirez, Phys. Rev. D **74**, 084024 (2006) [arXiv:hep-th/0607002].
2. M. Chaichian, A. Tureanu and G. Zet, Phys. Lett. B **660**, 573 (2008) [arXiv:0710.2075 [hep-th]].
3. N. Seiberg and E. Witten, JHEP **9909**, 032 (1999) [arXiv:hep-th/9908142].
4. C.V. Johnson,“ *D-Branes*”, Cambridge University Press, 2003.