# Unification and mass spectrum in a $B-L$ extended MSSM 

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#### Abstract

The simplest B-L extension of the minimum supersymmetric standard model (MSSM) may change some of the conceptions about the path for gauge unification as well as to affect the predicted spectrum of the supersymmetric particles at low energy. We present our results for the running of gauge coupling constants and mass parameter in this context.


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RGE. Including the gauge group $U(1)_{B-L}$ to the SM group, the general superpotential that can be written wich conserves the symmetry is,

$$
W=\bar{U} \mathbf{Y}_{u} Q H_{u}+\bar{D} \mathbf{Y}_{d} Q H_{d}+\bar{E} \mathbf{Y}_{e} L H_{d}+\bar{N} \mathbf{Y}_{N}^{D} L H_{u}+N \mathbf{Y}_{N}^{M} N \sigma_{1}+\mu H_{d} H_{u}+\mu^{\prime} \sigma_{1} \sigma_{2},
$$

where besides standard superfields we have additional fields, which under the symmetry group $S U(3)_{c} \times S U(2)_{L} \times U(1)_{Y} \times U(1)_{B-L}$, have the following representation, $\bar{N}=$ $(\mathbf{1}, 1,0,1), \sigma_{1}=(\mathbf{1}, \mathbf{1}, 0,-2), \sigma_{2}=(\mathbf{1}, 1,0,2)$.

The soft breaking terms that involve the new fields are

$$
\mathscr{L}_{S B}=1 / 2 M_{B-L} \tilde{Z}_{B-L} \tilde{Z}_{B-L}+\tilde{\bar{N}} \mathbf{h}_{N}^{D} \tilde{L} \tilde{H}_{u}+\tilde{N}^{\dagger} m_{N}^{2} \tilde{N}
$$

And finally, the scalar potential has as the only new terms,

$$
V\left(\sigma_{1}, \sigma_{2}\right)=m_{\sigma_{1}}^{\prime 2}\left|\sigma_{1}\right|^{2}+m_{\sigma_{2}}^{\prime 2}\left|\sigma_{2}\right|^{2}-\left(B^{\prime} \sigma_{1} \sigma_{2}+c . c .\right)+g^{\prime \prime}\left(\left|\sigma_{1}\right|^{2}-\left|\sigma_{2}\right|^{2}\right)^{2} / 8,
$$

where $m_{j}^{2}=m_{j}^{\prime 2}+\mu^{\prime 2}$ for $j=\sigma_{1}, \sigma_{2}$. For a general superpotential and soft breaking terms, the beta functions could be calculated, using the representations of the superfields[1]. The notation is $\Delta \beta_{f}=\beta_{f}-\beta_{f}^{M S S M}$, where $\beta_{f}=16 \pi^{2}(d f / d t)$, with $t=\ln \left(Q / Q_{0}\right)$. We present the RGE for the gauge couplings, Yukawa couplings, gaugino mass, $\mu$ and $\mu^{\prime}$ term,

$$
\begin{aligned}
& \beta_{g_{i}}=c_{i} g_{i}^{3}, \quad \beta_{M_{i}}=2 c_{i} g_{i}^{2} M_{i}, \quad \Delta \beta_{y_{t}}=y_{t}\left\{y_{D}^{2}-g_{B-L}^{2} / 6\right\}, \quad \Delta \beta_{y_{b}}=-y_{b} g_{B-L}^{2} / 6, \\
& \Delta \beta_{y_{\tau}}=y_{\tau}\left\{y_{D}^{2}-3 g_{B-L}^{2} / 2\right\}, \quad \beta_{y_{D}}=y_{D}\left\{4 y_{D}^{2}+3 y_{t}^{2}+y_{M}^{2}-3 g_{1}^{2} / 5-3 g_{2}^{2}-3 g_{B-L}^{2} / 2\right\}, \\
& \beta_{y_{M}}=y_{M}^{2}\left\{3 y_{M}^{2}+4 y_{D}^{2}-9 g_{B-L}^{2} / 2\right\}, \quad \Delta \beta_{\mu}=\mu y_{D}^{2}, \quad 2 \beta_{\mu^{\prime}}=\mu^{\prime}\left\{y_{M}^{2}-3 g_{B-L}^{2}\right\},
\end{aligned}
$$

where $i=1,2,3, B-L$, and we had made the following approximations, $\mathrm{Y}_{u}=$ $\operatorname{diag}\left(0,0, y_{t}\right), \mathrm{Y}_{d}=\operatorname{diag}\left(0,0, y_{b}\right), \mathrm{Y}_{e}=\operatorname{diag}\left(0,0, y_{\tau}\right), \mathrm{Y}_{N}^{D}=\operatorname{diag}\left(0,0, y_{D}\right), \mathrm{Y}_{N}^{M}=$ $\operatorname{diag}\left(0,0, y_{M}\right)$. For the corresponding anomalous dimensions to $\mathbf{h}_{i}$, it had been
used the approximations, $\mathrm{h}_{u}=\operatorname{diag}\left(0,0, a_{t}\right), \mathrm{h}_{d}=\operatorname{diag}\left(0,0, a_{b}\right), \mathrm{h}_{e}=\operatorname{diag}\left(0,0, a_{\tau}\right)$, $\mathrm{h}_{N}^{D}=\operatorname{diag}\left(0,0, a_{D}\right), \mathrm{h}_{N}^{M}=\operatorname{diag}\left(0,0, a_{M}\right)$. Then,

$$
\begin{aligned}
\Delta \beta_{a_{t}}= & a_{t}\left\{y_{D}^{2}-g_{B-L}^{2} / 6\right\}+y_{t}\left\{2 a_{D} y_{D}+g_{B-L}^{2} M_{B-L} / 3\right\}, \\
\Delta \beta_{a_{b}}= & -a_{b} g_{B-L}^{2} / 6+y_{b} g_{B-L}^{2} M_{B-L} / 3, \\
\Delta \beta_{a_{\tau}}= & a_{\tau}\left\{y_{D}^{2}-3 g_{B-L}^{2} / 2\right\}+y_{\tau}\left\{2 a_{D} y_{D}+3 g_{B-L}^{2} M_{B-L}\right\}, \\
\beta_{a_{D}}= & a_{D}\left\{12 y_{D}^{2}+3 y_{t}^{2}+2 y_{M}^{2}+y_{\tau}^{2}-3 g_{1}^{2} / 5-3 g_{2}^{2}-3 g_{B-L}^{2} / 2\right\} \\
& +y_{D}\left\{6 a_{t} y_{t}+2 a_{M} y_{M}+a_{\tau} y_{\tau}+6 g_{1}^{2} M_{1} / 5+6 g_{2}^{2} M_{2}+3 g_{B-L}^{2} M_{B-L}\right\}, \\
\beta_{a_{M}}= & a_{M}\left\{15 y_{M}^{2}+8 y_{D}^{2}-9 g_{B-L}^{2} / 2\right\}+\left\{8 a_{D} y_{D}+9 g_{B-L}^{2} M_{B-L}\right\} .
\end{aligned}
$$

For the mass anomalous dimensions the following approximations had been made, $m_{Q}^{2}=\operatorname{diag}\left(m_{Q_{1}}^{2}, m_{Q_{1}}^{2}, m_{Q_{3}}^{2}\right), m_{L}^{2}=\operatorname{diag}\left(m_{L_{1}}^{2}, m_{L_{1}}^{2}, m_{L_{3}}^{2}\right), m_{u}^{2}=\operatorname{diag}\left(m_{u_{1}}^{2}, m_{u_{1}}^{2}, m_{u_{3}}^{2}\right), m_{d}^{2}=$ $\operatorname{diag}\left(m_{d_{1}}^{2}, m_{d_{1}}^{2}, m_{d_{3}}^{2}\right), m_{e}^{2}=\operatorname{diag}\left(m_{e_{1}}^{2}, m_{e_{1}}^{2}, m_{e_{3}}^{2}\right), m_{N}^{2}=\operatorname{diag}\left(m_{N_{1}}^{2}, m_{N_{1}}^{2}, m_{N_{3}}^{2}\right)$. Then,

$$
\begin{aligned}
& \Delta \beta_{m_{H_{u}}^{2}}=2 y_{D}^{2}\left\{m_{H_{u}}^{2}+m_{L_{3}}^{2}+m_{N_{3}}^{2}\right\}+2 a_{D}^{2}, \quad \Delta \beta_{m_{H_{d}}^{2}}=0, \\
& \Delta \beta_{m_{Q_{3}}^{2}}=-g_{B-L}^{2} M_{B-L}^{2} / 3+g_{B-L}^{2} S^{\prime} / 4, \quad \Delta \beta_{m_{u_{3}}^{2}}=-g_{B-L}^{2} M_{B-L}^{2} / 3+g_{B-L}^{2} S^{\prime} / 4, \\
& \Delta \beta_{m_{d_{3}}^{2}}=-g_{B-L}^{2} M_{B-L}^{2} / 3+g_{B-L}^{2} S^{\prime} / 4, \quad \Delta \beta_{m_{e_{3}}^{2}}=-3 g_{B-L}^{2} M_{B-L}^{2}-3 g_{B-L}^{2} S^{\prime} / 4, \\
& \Delta \beta_{m_{L_{3}}^{2}}=2 y_{D}^{2}\left\{m_{H_{u}}^{2}+m_{L_{3}}^{2}+m_{N_{3}}^{2}\right\}+2 a_{D}^{2}-3 g_{B-L}^{2} M_{B-L}^{2}-3 g_{B-L}^{2} S^{\prime} / 4, \\
& \beta_{m_{\sigma_{1}}^{2}}=2 y_{M}^{2}\left\{m_{\sigma_{1}}^{2}+m_{N_{3}}^{2}\right\}+2 a_{M}^{2}-12 g_{B-L}^{2} M_{B-L}^{2}-3 g_{B-L}^{2} S^{\prime} / 2, \\
& \beta_{m_{\sigma_{2}}^{2}}=-12 g_{B-L}^{2} M_{B-L}^{2}+3 g_{B-L}^{2} S^{\prime} / 2, \\
& \beta_{m_{N_{3}}^{2}}=4 y_{D}^{2}\left\{m_{H_{u}}^{2}+m_{L_{3}}^{2}+m_{N_{3}}^{2}\right\}+4 y_{M}^{2}\left\{m_{\sigma_{1}}^{2}+m_{N_{3}}^{2}\right\}+4\left(a_{M}^{2}+a_{D}^{2}\right)-3 g_{B-L}^{2} M_{B-L}^{2} \\
& -3 g_{B-L}^{2} S^{\prime} / 4,
\end{aligned}
$$

where $S^{\prime}=2 m_{\sigma_{2}}^{2}-2 m_{\sigma_{1}}^{2}+\operatorname{Tr}\left[2 m_{Q}^{2}-2 m_{L}^{2}+m_{u}^{2}+m_{d}^{2}-m_{e}^{2}-m_{N}^{2}\right]$.
Unification. The RGE for $g_{i}$ can be rewritten in terms of $\alpha_{i}^{-1}$ and, at one loop order, the general solution is[2], $\alpha_{i}^{-1}(m)=c_{i}\left[\alpha_{i}^{-1}\left(m_{Z}\right)+(2 \pi)^{-1} b_{i} \ln \left(m / m_{Z}\right)\right]$. The Kac-Moody $\left(c_{1}, c_{2}, c_{3}, c_{B-L}\right)$ are $(3 / 5,1,1,3 / 8)$; and the $b_{i}$ are, $\left(b_{1}, b_{2}, b_{3}, b_{B-L}\right)=$ $(-11,-1,3,-24)$. It's know that $\alpha_{1}^{-1}\left(m_{Z}\right) \approx 98.33, \alpha_{2}^{-1}\left(m_{Z}\right) \approx 29.57$ and $\alpha_{3}^{-1}\left(m_{Z}\right) \approx$ 8.4[3]. Supposing unification for $\alpha_{B-L}^{-1}(m)$, the RGE have the solution shown in the figure below. Then $\alpha_{B-L}^{-1}\left(m_{Z}\right) \approx 191.1$, so, $g_{B-L}\left(m_{Z}\right) \approx 0.2565$.

Mass Spectrum. The RGE for the masses could be solved imposing initial conditions. As it is customized, those are fixed at the unification scale, $Q_{0} \approx 2.5 \times 10^{16} \mathrm{GeV}$. The already known solution for the MSSM[4] is recovered (for the $\beta_{f}^{M S S M}$ functions) as depicted in the plot below. When $B-L$ contributions are included, the initial conditions are forced to be different due to the phenomenology at low-energies, if one desires to have the SUSY breaking at the same scale, as it can be seen in the following figures. For the squarks and sleptons, the solid lines correspond to the third family, while the dashed lines do to the first and second families.


FIGURE 1. Running of gauge couplings.


FIGURE 2. Mass spectrum in MSSM and its $B-L$ extension.

Conclusions. It had been calculated the RGE for the $S U(3)_{c} \times S U(2)_{L} \times U(1)_{Y} \times$ $U(1)_{B-L}$ SUSY model. It had been shown that it is possible the unification of $\alpha_{B-L}^{-1}$ at $Q_{0} \approx 2.5 \times 10^{16} \mathrm{GeV}$. Slight modifications should be done to the initial conditions if the $B-L$ symmetry is included in the standard model, in order to preserve the phenomenology at low energies.

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