Exceptional point and degeneracy of the neutral Higgs boson system H-A

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Abstract. We analyze the masses and mixings of the isolated neutral and heavy Higgs fields H and A of the Minimal Supersymmetric Standard Model (MSSM) with CP violation, which have opposite CP parities and nearly degenerate masses. At the degeneracy point, the hypersurfaces that represent the physical masses as functions of the system parameters have a rank one algebraic branch point, and the real and imaginary parts have branch cuts, both starting at the same exceptional point but extending in opposite directions in parameter space. Associated with this singularity, the propagator for the mixed neutral Higgs system $H-A$ has a double pole in the non-physical sheet of the squared energy complex plane s. The continuity of the transition amplitude matrix at the exact degeneracy of the masses is examined.

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NEUTRAL AND HEAVY HIGGS BOSON SYSTEM $H-A$

The minimal supersymmetric standard model [1–3] has two complex Higgs doublets. The most general two Higgs doublet $SU(2)$ potential with CP violation is

$$\begin{aligned} L_V &= \mu_1^2(\Phi_1^\dagger\Phi_1)+\mu_2^2(\Phi_2^\dagger\Phi_2)+m_{12}^2(\Phi_1^\dagger\Phi_2)+\lambda_1(\Phi_1^\dagger\Phi_1)^2+\lambda_2(\Phi_2^\dagger\Phi_2)^2 \\ &\quad +\lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)+\lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)+\lambda_5(\Phi_1^\dagger\Phi_2)^2+[\lambda_6(\Phi_1^\dagger\Phi_1) \\ &\quad +\lambda_7(\Phi_2^\dagger\Phi_2)](\Phi_1^\dagger\Phi_2)+h.c.. \end{aligned} \tag{1}$$

After soft supersymmetry and electroweak gauge symmetry breaking, and considering $\lambda_5=\lambda_6=\lambda_7=0$ and all other coupligs real, the potential will lead to five physical Higgs bosons: the neutral Higgs bosons, two of which are CP-even, h and H, and one CP-odd, A, as well as a pair of charged Higgs bosons $H^\pm$, which corresponds to a CP-invariant model.

At one-loop level it is possible to have complex couplings in L_V of the form [4]

$$m_{12}^2=m_{12}^{2R}+im_{12}^{2I}, \qquad \lambda_{5,6,7}=\lambda_{5,6,7}^R+i\lambda_{5,6,7}^I. \tag{2}$$

Hence, at an order higher than tree level, the mixing $H-A$ of neutral Higgses generates new sources of CP violation (CPV) [5].

The spontaneous gauge symmetry breaking and the soft supersymmetry breaking generate a first order 3×3 neutral Higgs bosons mass matrix $\mathscr{M}^2(s)$. In the decoupling limit, $M_A^2 >> |\lambda_i|\mathrm{v}^2$, all couplings are at the electroweak scale [6]. Then, the $\mathscr{M}^2(s)$ matrix is reduced to two block matrices, one for the light Higgs boson, and the other one for the masses of the two states H and A. The mass matrix of the coupled system $H-A$ is a symmetric non-Hermitian 2×2 matrix

$$\mathscr{M}_{HA}^2 = \begin{pmatrix} M_H^2 - iM_H\Gamma_H & \Delta_{HA}^2 \\ \Delta_{HA}^2 & M_A^2 - iM_A\Gamma_A \end{pmatrix}. \tag{3}$$

The eigenvalues of this matrix are given by the zeroes of the $\det[\mathscr{M}_{HA}^2 - m^2\mathbf{1}]$, as follows

$$m_{2,3}^2 = \frac{1}{2}\left[(M_H^2 - iM_H\Gamma_H) + (M_A^2 - iM_A\Gamma_A)\right] \pm \sqrt{\left(\vec{R} - i\frac{1}{2}\vec{\Gamma}\right)^2}, \tag{4}$$

where $\vec{R} = (\Re e\ \Delta_{HA}^2, 0, 1/2(M_H^2 - M_A^2))$ and $\vec{\Gamma} = (-2\ \Im m\ \Delta_{HA}^2, 0, (M_H\Gamma_H - M_A\Gamma_A))$. From (4) the degeneracy conditions can be calculated, and are $R^2 - \frac{1}{4}\Gamma^2 = 0$, and $\vec{R}\cdot\vec{\Gamma} = 0$.

The matrix elements are expressed as functions of the model parameters. In the decoupling limit [4]

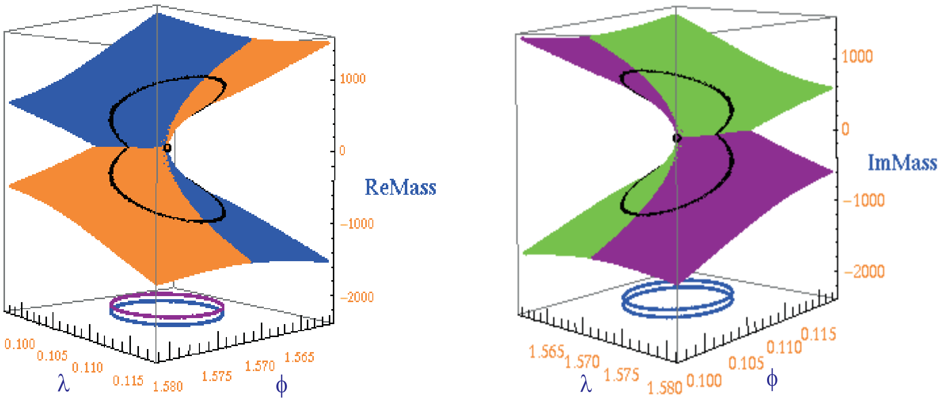
$$\begin{aligned} M_H^2 - M_A^2 &\approx \lambda v^2 \cos\phi, & (5)\\ 32\pi[M_H\Gamma_H - M_A\Gamma_A] &\approx [\Delta_t + 9\lambda^2 v^2 \cos 2\phi], & (6)\\ \Re e\ \Delta_{HA}^2 &\approx -\frac{1}{2}\lambda v^2 \sin\phi, & (7)\\ 32\pi\ \Im m\ \Delta_{HA}^2 &\approx -\frac{9}{2}\lambda^2 v^2 \sin 2\phi, & (8) \end{aligned}$$

where we have taken the magnitudes of all λ_i equal, $|\lambda_i| = \lambda$, ϕ is the CP violating common phase of $\lambda_{5,6,7}$, $\Delta_t = -12M_{H/A}^2(m_t/v)^2(1-\beta_t^2)\beta_t$ is the one loop contribution of the top quark, $v = (v_1^2 + v_2^2)^{1/2} = [\sqrt{2}G_F]^{-1/2}$ and $\tan\beta = v_2/v_1$.

UNFOLDING OF THE EXCEPTIONAL POINT

Equation (4) defines the masses of the heavy neutral Higgs bosons as functions of the parameters λ and ϕ. If we neglect the weak s dependence of the elements of $\mathscr{M}_{HA}^2$, this function, eq. (4), gives the position of the pole of the Higgs $H-A$ propagator in the complex $s-$plane. The term under the square root in the right hand side of eq. (4) is a regular function of its arguments and may be expanded in a Taylor series around the exceptional point. Keeping only the first order terms we get

$$\mu_{2,3}^2(\lambda,\phi) = \frac{1}{2}\sqrt{c_1^{(1)}(\lambda - \lambda^\star) + c_2^{(1)}(\phi - \phi^\star)}, \tag{9}$$



(a) (b)

FIGURE 1. The figure shows the mass hypersurfaces representing (a) real and (b) imaginary parts of $\mu^2_{2,3}$ as function of the parameters λ and ϕ in the neighbourhood of the exceptional point λ^*, ϕ^*.

where the $c_k^{(1)}$'s are the derivatives of the term in the square root in eq. (4). The real and imaginary parts of $\mu^2_{2,3}$ are

$$\Re e\,\mu^2_{2,3} = \pm\frac{1}{2\sqrt{2}}|\vec{\zeta}|^{1/2}\left[\sqrt{(\vec{\mathscr{R}}\cdot\hat{\zeta})^2+(\vec{\mathscr{I}}\cdot\hat{\zeta})^2}+(\vec{\mathscr{R}}\cdot\hat{\zeta})\right]^{1/2}, \qquad (10)$$

$$\Im m\,\mu^2_{2,3} = \pm\frac{1}{2\sqrt{2}}|\vec{\zeta}|^{1/2}\left[\sqrt{(\vec{\mathscr{R}}\cdot\hat{\zeta})^2+(\vec{\mathscr{I}}\cdot\hat{\zeta})^2}-(\vec{\mathscr{R}}\cdot\hat{\zeta})\right]^{1/2} \qquad (11)$$

with

$$\vec{\mathscr{R}} = \left(\Re e\,c_1^{(1)}, \Re e\,c_2^{(1)}\right), \quad \vec{\mathscr{I}} = \left(\Im m\,c_1^{(1)}, \Im m\,c_2^{(1)}\right), \quad \vec{\zeta} = \begin{pmatrix} \lambda-\lambda^\star \\ \phi-\phi^\star \end{pmatrix}. \qquad (12)$$

Figure 1(a) shows the mass hypersurface representing the real part of $\mu^2_{2,3}$ and figure 1(b) shows the imaginary part of $\mu^2_{2,3}$, both as function of the parameters λ and ϕ in the neighbourhood of the exceptional point. Its topological structure presents a rank one branch point and branch cuts that start at the exceptional point but extend in opposite directions. The surfaces are in orthogonal spaces [7, 8]. By performing a complete excursion around the exceptional point on the hypersurfaces a double rotation in the parameter space, 4π, is required to get to the original point.

Transition matrix and propagator

In the electroweak basis, the transition matrix between states with CP violation via a resonant Higgs exchange contains a scalar field propagator of the form [9–12] $\Delta(s) = (s-\mathscr{M}^2_{HM})^{-1}$. We transform $\mathscr{M}^2_{HA}(s)$ to the mass basis by means of a complex ortogonal matrix written in terms of a rotation by a complex mixing angle

$$\cos\theta = (m_2^2 - m_3^2)^{-1/2}\{1/2(m_2^2 - m_3^2) + [1/4(m_2^2 - m_3^2)^2 - \Delta_{HA}^2]^{1/2}\}.$$

In this basis the resonant transition amplitude takes the form [13]

$$\mathscr{T}^{res} = \tilde{V}_2^P \frac{1}{s - m_2^2} \tilde{V}_2^D + \tilde{V}_3^P \frac{1}{s - m_3^2} \tilde{V}_3^D, \tag{13}$$

where $\tilde{V}_i^P = (V^P O)_i$ and $\tilde{V}_j^D = (O V^D)_j$. A straightforward computation shows that, at the degeneracy limit, i.e. $m_2^{\star 2} = m_3^{\star 2} = m_d^2$

$$\lim_{m_2^2 \to m_3^2} \mathscr{T}^{res} = \frac{d}{dm^2}\left(\frac{F(m^2)}{s - m^2}\right)\Big|_{m^2 = m_d^2} = \frac{1}{s - m_d^2}\frac{dF(m_d^2)}{dm_d^2} - \frac{F(m_d^2)}{(s - m_d^2)^2}. \tag{14}$$

Thus, it is evident that, at the degeneracy $\mathscr{T}^{res}$ is continuous and has two terms, one corresponding to a single pole and the other to a double pole in the complex variable s.

SUMMARY AND CONCLUSIONS

We analized and displayed the behaviour of a degeneracy of the mass matrix of the heavy neutral Higgs boson system of the MSSM in parameter space. We showed that the transition matrix is continuous at the exact coalescence, allowing thus to calculate the CP violation in a specific cross section at the exact degeneracy.

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REFERENCES

1. H. P. Nilles, "Supersymmetry, Supergravity And Particle Physics," Phys. Rept. **110**, 1 (1984).
2. H. E. Haber and G. L. Kane, "The Search For Supersymmetry: Probing Physics Beyond The Standard Model," Phys. Rept. **117**, 75 (1985).
3. S. P. Martin, "A Supersymmetry Primer," arXiv:hep-ph/9709356.
4. S. Y. Choi, J. Kalinowski, Y. Liao and P. M. Zerwas, Eur. Phys. J. C **40**, 555 (2005) [arXiv:hep-ph/0407347].
5. A. Pilaftsis, Phys. Lett. B **435**, 88 (1998) [arXiv:hep-ph/9805373].
6. J. F. Gunion and H. E. Haber, Phys. Rev. D **67** (2003) 075019 [arXiv:hep-ph/0207010].
7. E. Hernández, A. Jáuregui, A. Mondragón and L. Nellen, Int. Jour. Theor. Phys.**46** No.6, 1666 (2007).
8. E. Hernández, A. Jáuregui and A. Mondragón, J. Phys. A **39**, 10087 (2006).
9. A. Pilaftsis, Nucl. Phys. B **504**, 61 (1997) [arXiv:hep-ph/9702393].
10. A. Pilaftsis, arXiv:hep-ph/9908373.
11. A. Pilaftsis and C. E. M. Wagner, Nucl. Phys. B **553** (1999) 3 [arXiv:hep-ph/9902371].
12. J. Bernabeu, D. Binosi and J. Papavassiliou, JHEP **0609**, 023 (2006) [arXiv:hep-ph/0604046].
13. M. S. Carena, J. R. Ellis, A. Pilaftsis and C. E. M. Wagner, Nucl. Phys. B **625** (2002) 345 [arXiv:hep-ph/0111245].