Gravitational Modification of the Coulomb-Breit Hamiltonian

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Abstract. In the poster session we presented a short review of our first results in the construction of the Coulomb-Breit Hamiltonian for a pair of fermions immersed in a background gravitational field which is described by General Relativity. Here we present a resume of that construction. We make a special stress on the objectives and the hypothesis used, but there is no special attention on the explicit form of the results because actually there is an updated and optimised version of our work in the edition process for publication; however we mention some special characteristics of the effect of the background gravitational field on the quantum nature of the system composed by fermions and its electromagnetic field, particularly the possibility of the observation of centre of mass effects in matter interferometry experiments.

Keywords: Feynman Green Function, Fermi Normal Coordinates, Effective Potential, Coulomb-Breit Hamiltonian.

1. MOTIVATIONS

Our main motivations to study this kind of problems were:

- The growing number of experiments establishing how quantum systems react on gravity and inertia in genuine quantum effects, this specially due to the fast development of matter (atomic and molecular) wave interferometry, which has proved to be very accurate in the detection of very small accelerations.
- The neccesity to look for new effects that could be detected in some experiments in order to enlarge the range of validity of GR to scales where the classical description of matter breaks down.
- The great advantage presented by the possibility of using the internal quantum numbers of atoms or molecules over the use of structureless particles such as photons and neutrons, for example.

2. OBJECTIVES AND HYPOTHESIS

Our main objectives on this problem were:

- Explore the consequences of a background classical gravitational field upon matter at the atomic level where a Quantum Mechanical (QM) description becomes mandatory for the description of the later.
- Consider the atom as a two-body system in a more canonical way, instead of a reduced mass evolving around one centre (One body approach of refs.[1, 2]) or the

non canonical two body approach of [3]

- Obtain an effective interaction Hamiltonian which includes the effect of the external curvature in a pseudo-relativistic regime up to order c^{-2}.

The main assumptions in our approach to the problem were:

- All components of the system are influenced by the background gravitational field on the quantum level. Particularly, the EM fields enters like a free Feynman propagator and not like a classical potential.
- There is one ideal observer in free fall in a region where there are no sources of the gravitational field, with his apparatus determines that the metric of the spacetime has the form
$$g_{\alpha\beta} = \eta_{\alpha\beta} + \mathscr{Q}_{\alpha b\beta c}x^b x^c \equiv \eta_{\alpha\beta} + h_{\alpha\beta}, \tag{1}$$
where
$$\mathscr{Q}_{0a0b} = -\breve{R}_{0a0b} \quad \mathscr{Q}_{0abc} = -\frac{2}{3}\breve{R}_{0abc} \quad \mathscr{Q}_{abcd} = -\frac{1}{3}\breve{R}_{abcd}. \tag{2}$$
The quantities $\breve{R}_{\alpha\beta\gamma\delta}$ are all very small, so in the final results we preserve only quantities up to first order in $h_{\alpha\beta}$.
- The spatial extension and the time duration of events related to observations in the quantum system are small compared with the characteristic lengths and times of appreciable change in the background curvature.

3. MAIN RESULTS

The Feynman Green function for the electromagnetic interaction in its Hadamard form [4], obtained in our first calculations and presented in the poster session, was

$$G_{\alpha\beta'}\left(x,x'\right) = \frac{1}{\sigma + i\varepsilon}\left\{\eta_{\alpha\beta} + \frac{1}{2}\left[h_{\alpha\beta}\left(x\right) + h_{\alpha\beta}\left(x'\right)\right]\right\} \tag{3}$$

being $\sigma\left(x,x'\right)$ the Synge world function for two arbitrary points in the coordinate patch of the observer. With this expression inserted in the S-matrix element corresponding to the exchange of one photon between the fermions we followed as close as possible to [5] and after the algebraic manipulations of the spinors and a reduction of terms up to the first order in the curvature, we got an effective potential which can be decomposed in tree terms, the first is *the modification of the Coulomb potential*, the second one *the modified Breit operator* and the last one was called *the CUrvature term*:

$$\mathscr{W}^{(\mathbf{1},\mathbf{2})} \equiv \frac{q_1 q_2}{\hbar c}\left[\mathscr{W}_C^{(\mathbf{1},\mathbf{2})} - \mathscr{W}_B^{(\mathbf{1},\mathbf{2})} + \mathscr{W}_{CU}^{(\mathbf{1},\mathbf{2})}\right]; \tag{4}$$

the $\mathscr{W}_{CU}^{(\mathbf{1},\mathbf{2})}$ operator is of order c^{-1} in the pseudo-relativistic approximation. Explicitly we showed for the *modification of the Coulomb potential*:

$$\mathscr{W}_C^{(\mathbf{1},\mathbf{2})} = \frac{1}{r} + \frac{1}{6r}\mathscr{Q}_{0a0b}\left[\mathscr{I}^{ab} + (\sigma_0)^a(\sigma_0)^b\right] - \frac{\mathscr{I}^{ab}}{2r^3}\mathscr{Q}_{iajb}(\sigma_0)^i(\sigma_0)^j, \tag{5}$$

with $\mathscr{I}^{ab} = (x+x')^a (x+x')^b$, $(\sigma_0)^a = (x-x')^a$ and $r^2 \equiv \delta_{ab} (\sigma_0)^a (\sigma_0)^b$. From the *modified Breit operator* we show only the last two terms:

$$\mathscr{W}_B^{(\mathbf{1},\mathbf{2})} = \cdots + \frac{(\sigma_0)_j}{4r} \left[\left(m_2 c^2\right) \underset{(2)}{\alpha^{\bar{j}} \beta} - \left(m_1 c^2\right) \underset{(1)}{\alpha^{\bar{j}} \beta} \right] + \frac{3(\sigma_0)_j}{2r} \mathscr{Q}_{0a0b} \left(\underset{(2)}{\alpha^{\bar{a}} x^b \alpha^{\bar{j}}} - \underset{(1)}{\alpha^{\bar{a}} x'^b \alpha^{\bar{j}}} \right) \quad (6)$$

because they are: *a mass coupling term* and *a spin-curvature-position coupling term*. And from the *the CUrvature term* here we show the last two terms:

$$\mathscr{W}_{CU}^{(\mathbf{1},\mathbf{2})} = \cdots - 2 \left[\mathscr{Q}_{0akj} (\sigma_0)^k \left(x'^a \alpha^{\bar{j}}_{(2)} - x^a \alpha^{\bar{j}}_{(1)} \right) + \left(\alpha^{\bar{j}}_{(2)} + \alpha^{\bar{j}}_{(1)} \right) \left(\mathscr{Q}_{0ajb} \mathscr{A}^{ab} - \frac{2}{3} \mathscr{Q}_{0ajk} (\sigma_0)^a (\sigma_0)^k - \frac{\mathscr{I}^{ab}}{2} \mathscr{Q}_{0akb} (\sigma_0)^k (\sigma_0)_j \right) \right] \quad (7)$$

In the last expression we found a *spin-curvature-position* term but now the *spin-position* part mixes particles. The last one, also shows a *spin-curvature-position* coupling but now the spin looks like the total spin of the system. In expressions (5)-(7), $\alpha^{\bar{j}}$ and β are the usual *alpha* and *beta* Dirac matrices corresponding to the signature $+2$ of the Minkowski metric.

4. FINAL COMMENTS

We want to underline that the poster session included only partial results of our work on quantum test of gravity, but we can mention two important characteristics of this kind of systems, the first one is the lack of reduction to relative quantities, which appears in (5) through the mid point coordinates and also in the first term of (7) through the mix between spin and position of the particles. Finally is remarkable the presence of coupling terms depending on the mass of the particles, which could give rise to effects related with the centre of mass. These centre of mass effects could be remarkable in atomic interferometry because of the importance of the classical trajectory in the analysis of the interferometer.

REFERENCES

1. L. Parker, Phys. Rev. D **22** No. 8, 1922 (1980).
   Phys. Rev. Lett. **44** No. 23, 1559 (1980).
2. J. Audrescht and K-P. Marzlin, Phys. Rev. A. **50** No. 3, 2080 (1994).
3. E. Fischbach B. S. Freeman and W-K Cheng, Phys. Rev. D **23** No. 10, 2157 (1981).
4. B.S. De Witt and R.W. Brehme, Ann. Phys. (N.Y) **9**, 220 (1960).
5. "Quantum Electrodynamics" 3d. Edition, pp. 370-374. Springer-Verlag Heidelberg, 2003.