Electric dipole moment of the top quark within an effective theory

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Abstract. Using the effective Lagrangian approach, we develope the trilinear contributions originated in the dimension-six electroweak invariants $\tilde{O}_W = (1/3)\varepsilon_{ijk}W^{i\mu}{}_{\nu}W^{j\nu}{}_{\lambda}W^{k\lambda}{}_{\mu}$ and $\tilde{O}_{WB} = (1/2)\tilde{B}_{\alpha\beta}W^{c\alpha\beta}\Phi^{\dagger}\tau^{c}\Phi$, and then we insert the corresponding vertices in a one-loop ttV diagram, with V *off-shell*, generating the structure of the electric dipole moment. Using a nonlinear gauge, we prove that the results are gauge independent. Finally, we present the analytic expressions for the electric dipole form factors originated in each invariant introduced.

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1. INTRODUCTION

Very important information about the origin of CP violation may be extracted from the electric dipole moment (EDM) of elementary particles. This elusive electromagnetic property is very interesting, as it represents a net quantum effect in any renormalizable theory. In the standard model (SM), the only source of CP violation is the Cabbibo-Kobayashi-Maskawa phase, which however has a rather marginal impact on flavor-diagonal processes such as the EDM of elementary particles [1]. Although they are extremely suppressed in the SM, the EDMs can be very sensitive to new sources of CP violation, as it was shown recently for the case of the W boson in a model-independent manner using the effective Lagrangian technique [2][3]. In the present work, we are interested in studying the impact of a CP-violating WWV ($V=\gamma,Z$) vertex, with V *off-shell*, on the EDM of the top quark. As it was shown by Marciano and Queijeiro [4], the CP-odd electromagnetic properties of the W boson can induce large contributions on the EDM of fermions. In this work we will parametrize this class of effects in a model-independent manner via the effective Lagrangian approach [5], which is suited to describe those new physics effects that are quite suppressed or forbidden in the SM.

After spontaneous symmetry breaking, and assuming that the underlying physics respects the SM gauge symmetry, the WWV vertex can be parametrized by means of the following effective Lagrangian.

$$\mathscr{L}^{CP-odd}_{WWV} = -ig_V\left(\tilde{k}_V W^{+}_{\mu}W^{-}_{\nu}\tilde{V}^{\mu\nu} + \frac{\tilde{\lambda}_V}{m_W^2}W^{+}_{\mu\nu}W^{-\nu\rho}\tilde{V}_{\rho}{}^{\mu}\right), \tag{1}$$

where $W^{\pm}_{\mu\nu} = \partial_\mu W^{\pm}_{\nu} - \partial_\mu W^{\pm}_{\mu}$ and $\tilde{V}_{\mu\nu} = (1/2)\varepsilon_{\mu\nu\alpha\beta}V^{\alpha\beta}$. In this work we introduce two dimension-six operators that are electroweak invariants, generating, each one of them, a term of the above Lagrangian. These invariants are defined as follows.

$$\tilde{O}_W = \frac{1}{3!}\varepsilon_{ijk}W^{i\mu}{}_{\nu}W^{j\nu}{}_{\lambda}W^{k\lambda}{}_{\mu}, \tag{2}$$

$$\tilde{O}_{WB} = \frac{1}{2}\tilde{B}_{\alpha\beta}W^{c\alpha\beta}\Phi^{\dagger}\tau^{c}\Phi, \tag{3}$$

with τ^c $(c=1,2,3)$ the Pauli matrices, and Φ the Higgs doublet. Then we consider the following effective Lagrangian.

$$\mathscr{L}_{eff} = \mathscr{L}_{SM} + \frac{\tilde{\alpha}_W}{\Lambda^2}\tilde{O}_W + \frac{\tilde{\alpha}_{WB}}{\Lambda^2}\tilde{O}_{WB}, \tag{4}$$

where $\mathscr{L}_{SM}$ represents the SM Lagrangian. Once we have obtained the trilinear effective vertices generated by 4, we shall insert them into a one-loop ttV diagram.

2. THE INVARIANT $\tilde{O}_W$

The $\tilde{O}_W$ operator can be expressed as

$$\tilde{O}_W = -\sum_{V=\gamma,Z} \frac{ig_V}{g} W^{+\mu}{}_{\nu} W^{-\nu}{}_{\lambda} \tilde{V}^{\lambda}{}_{\mu} + \dots \tag{5}$$

with $g_V = s_W g$ if $V = \gamma$, and $g_V = c_W g$ if $V = Z$. Defining $\tilde{\varepsilon}_W = (v/\Lambda)^2 \tilde{\alpha}_W$, with v the Fermi scale, it can be obtained an effective Lagrangian that carries the three field contribution and which, after the application of the Feynman rules, leads to the effective vertex

$$\Gamma^{\tilde{\mathscr{O}}_W}_{\rho\lambda\mu}(k_1,k_2,k_3) = -\tilde{\varepsilon}_W \frac{ig_V g}{4m_W^2} \left(\varepsilon_{\rho\lambda\mu\sigma} k_2 \cdot k_3 + \varepsilon_{\rho\mu\theta\sigma} k_{3\lambda} k_2^{\theta} - \varepsilon_{\lambda\mu\theta\sigma} k_{2\rho} k_3^{\theta} \right) k_1^{\sigma}, \tag{6}$$

with all the momenta, k_1, k_2 and k_3, directed inwards in the WWV diagram. It is worth to emphasize that, for the case of this electroweak invariant, there is no more that one contribution to the process ttV at the one loop level. The only source of gauge dependence comes from the longitudinal part of the W boson propagator. The vertex 6 satisfies some Ward identities, by means of which this kind of gauge dependent terms vanishes, implying that the results are, in fact, gauge independent. Developing the vertex function associated with the diagram before referred and identifying the distinctive structure of the EDM, it can be proven that the electric dipole form factor (EDFF) originated in the $\tilde{O}_W$ contribution is

$$\begin{aligned} d_t^{\tilde{\mathscr{O}}_W} = -\tilde{\varepsilon}_W \frac{g^3 g_V}{256 m_W^2 \pi^2} \Bigg\{ & m_t + \frac{m_b^2 - m_W^2}{m_t} \left(B_0(0,m_b^2,m_W^2) - B_0(m_t^2,m_b^2,m_W^2)\right) \\ & + \frac{m_t(2m_t^2 - 2m_b^2 - 2m_W^2 + q^2)}{4m_t^2 - q^2} \left(B_0(q^2,m_W^2,m_W^2) - B_0(m_t^2,m_b^2,m_W^2)\right) \\ & - \frac{2m_t(m_t^4 - 2m_b^2 m_t^2 - q^2 m_t^2 + m_b^4 - m_W^4 + m_W^2 q^2)}{4m_t^2 - q^2} C_0(m_t^2,m_t^2,q^2,m_W^2,m_b^2,m_W^2) \Bigg\}. \end{aligned} \tag{7}$$

This result meets two remarkable features: it is gauge independent and free of divergences.

3. THE INVARIANT $\tilde{O}_{WB}$

This operator, when expanded, includes five couplings concerning three fields: W^+W^-V, $W^+G_W^-V$, $W^-G_W^+V$, $H\gamma V$ and HZV. We introduce a nonlinear gauge through the following gauge fixing functions.

$$f^a_{eff} = f^a_{SM} + \tilde{f}^a, \tag{8}$$

with $a = 1,2,3$, for the $SU_L(2)$ group, and [6]

$$f^B = \partial_\mu B^\mu + \xi \frac{ig'}{2}(\Phi^\dagger \Phi_0 - \Phi_0^\dagger \Phi), \tag{9}$$

for $U_Y(1)$. In the last definitions [6],

$$f^a_{SM} = (\delta^{ab}\partial_\mu - g'\varepsilon^{3ab}B_\mu)W^{b\mu} + \xi \frac{ig}{\sqrt{2}} \left(\Phi^\dagger(\sigma^a - i\varepsilon^{3ab}\sigma^b)\Phi_0 - \Phi_0^\dagger(\sigma^a + i\varepsilon^{3ab}\sigma^b)\Phi + i\varepsilon^{3ab}\Phi^\dagger\sigma^b\Phi \right), \tag{10}$$

$$\tilde{f}^a = -\frac{\tilde{\alpha}_{WB}}{g\Lambda^2} \varepsilon^{3ab} W^{b\mu\nu} \tilde{B_{\mu\nu}}, \tag{11}$$

and $\Phi_0^\dagger = (0, v/\sqrt{2})$. By means of this gauge fixing functions, all the non-physical vertices in the Lagrangians asociated to the dimension-six operators vanish. So, the only effective vertex functions obtained from $\tilde{O}_{WB}$ are those asociated to the couplings WWV, $H\gamma V$ and HZV:

$$\Gamma^{WWV}_{\rho\lambda\mu}(k_1,k_2,k_3;\xi) = -\tilde{\varepsilon}_{WB}\frac{ih(V)}{2}\left(\varepsilon_{\rho\lambda\mu\nu}k_1^\nu + \frac{1}{\xi m_W^2}(k_{3\rho}\varepsilon_{\lambda\mu\sigma\omega}k_1^\sigma k_2^\omega + k_{2\lambda}\varepsilon_{\mu\rho\sigma\omega}k_1^\sigma k_3^\omega)\right), \tag{12}$$

$$\Gamma^{H\gamma V}_{\rho\mu}(k_2,k_2) = -\tilde{\varepsilon}_{WB}\frac{ih(V)s_W}{2m_W}\varepsilon_{\mu\rho\eta\theta}k_1^\eta k_2^\theta, \tag{13}$$

$$\Gamma^{HZV}_{\rho\mu}(k_1,k_2) = -\tilde{\varepsilon}_{WB}\frac{ih(V)c_W}{2m_W}\varepsilon_{\mu\rho\eta\theta}k_1^\eta k_2^\theta, \tag{14}$$

where, as before, all the momenta are directed inwards. In the last expressions, $h(V) = (c_W/s_W)g_\gamma$ for $V = \gamma$, and $h(V) = -(s_W/c_W)g_Z$ if $V = Z$. The insertion of these effective vertices in the ttV diagram leads to two types of one-loop diagrams: self-energies and triangular. All the contributions coming from self-energy diagrams equal zero. The ttV triangular diagrams which include the $H\gamma V$ and HZV interactions are gauge independent, a feature that can be verified using some Ward identities satisfied by the corresponding vertex functions. Because of this, the only source of gauge dependence in the triangular diagrams comes from the longitudinal part of the W boson propagator and from the WWV effective vertex. This vertex function, which is the only one modified by the introduction of the non-linear gauge, satisfies some Ward identities which can be used to prove that the result is gauge independent. The electric dipole form factor, originated in the $\tilde{O}_{WB}$ invariant, is found to be

$$\begin{aligned}
d_t^{\tilde{O}_{WB}}(q^2) = {}& \varepsilon_{\tilde{WB}}\frac{h(V)g^2 m_t}{128\pi^2 m_W^2}\frac{1}{4m_t^2-q^2}\left[\frac{4m_t^2-q^2}{2} + 2(m_t^2+m_b^2+m_W^2)B_0(m_t^2;m_b^2,m_W^2)\right.\\
&+(2m_t^2-2m_b^2-2m_W^2-q^2)B_0(q^2;m_W^2,m_W^2) + \frac{8s_W^2 m_t^2(m_H^2-q^2)}{3q^2}B_0(m_t^2;0,m_t^2)\\
&+\frac{(8s_W^2-3)(m_H^2+m_Z^2-q^2)}{6}B_0(q^2;m_H^2,m_Z^2) - \frac{4s_W^2(m_H^2-q^2)}{3}B_0(q^2;0,m_H^2)\\
&-\frac{(8s_W^2-3)(2m_t^2(m_H^2-m_Z^2-q^2)+m_Z^2q^2)}{6q^2}B_0(m_t^2;m_t^2,m_Z^2)\\
&+\frac{3m_H^2q^2-2m_t^2(3m_H^2+3q^2+m_Z^2(8s_W^2-3))}{6q^2}B_0(m_t^2;m_t^2,m_H^2)\\
&-2((m_t^2-m_b^2)^2+m_b^2q^2-m_W^4)C_0(m_t^2,m_t^2,q^2;m_W^2,m_b^2,m_W^2)\\
&+\frac{8s_W^2 m_t^2(m_h^2-q^2)^2}{3q^2}C_0(m_t^2,m_t^2,q^2;m_H^2,m_t^2,0)\\
&\left.+\frac{(8s_W^2-3)(m_t^2(m_H^4-2m_H^2(m_Z^2+q^2)+(m_Z^2-q^2)^2)+m_H^2m_Z^2q^2)}{3q^2}C_0(m_t^2,m_t^2,q^2;m_H^2,m_t^2,m_Z^2)\right]
\end{aligned} \tag{15}$$

4. SUMMARY

We have presented the expressions for the electric dipole form factors generated by two dimension-six electroweak invariants that contribute to the one-loop ttV diagram, with V an *off-shell* photon or Z boson, through the trilinear vertices WWV and HVV'. We obtained these results using the effective Lagrangian technique and, introducing a nonlinear gauge, we found that the final expressions are gauge independent, which is a remarkable feature.

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