Electromagnetic multipole moments of spin 3/2 particles in NKR formalism

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Abstract. In this work we present results for Compton scattering off spin 3/2 particles in NKR formalism and relate the behavior of the cross section to the electromagnetic multipole moments of the particle included by this formalism. We obtain expressions for such multipole moments using model independent definitions.

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INTRODUCTION

Recently, a new formalism for the description of particles with spin was proposed in [1]. The special case of spin 3/2 shows advantages with respect to the commonly used Rarita-Schwinger (RS) formalism. Unlike the RS method, in the formalism of Ref. [1] (NKR formalism here on) one gets a causal theory in the presence of an electromagnetic field.

In a previous work we calculated Compton Scattering off spin 3/2 particles in the NKR framework, we obtain the correct classical limit and the calculated cross section satisfy the unitarity constraints for center of mass energies up to the mass of the spin 3/2 particle. However, in the high energy limit, we obtain a cross section that violates unitarity. In this work we calculate the electromagnetic moments of a NKR particle.

SPIN 3/2 LAGRANGIAN

The e.o.m. in NKR formalism for spin 3/2 particles is based on the $\left[\left(\frac{1}{2},\frac{1}{2}\right)\otimes\left[\left(\frac{1}{2},0\right)\oplus\left(0,\frac{1}{2}\right)\right]\right.$ representation, a general construction results in a two parameter expression [1]:

$$[\Gamma_{\alpha\beta\mu\nu}p^{\mu}p^{\nu}-m^{2}g_{\alpha\beta}]\psi^{\beta}=0,$$

where $\Gamma_{\alpha\beta\mu\nu}=\frac{1}{3}\left[(i\sigma_{\alpha\beta}+2g_{\alpha\beta})g_{\mu\nu}+(i\sigma_{\beta\nu}-2g_{\beta\nu})g_{\alpha\nu}-i\sigma_{\alpha\mu}g_{\beta\mu}\right]-ig[M_{\mu\nu}]_{\alpha\beta}-if\gamma^{5}\varepsilon_{\alpha\beta\mu\nu}$, with $M_{\mu\nu}$ being the generators of the representation. This equation implies the constrictions: $(p^{2}-m^{2})\psi_{\alpha}=0, \quad \gamma^{\alpha}\psi_{\alpha}=0, \quad p^{\alpha}\psi_{\alpha}=0$. The propagator can be found as the inverse of the $\Gamma_{\alpha\beta\mu\nu}p^{\mu}p^{\nu}-m^{2}g_{\alpha\beta}$ operator. Studying the propagation of the spin 3/2 waves in an electromagnetic background , it is shown in [1] that the only values of g leading to causal propagation are $g=0,2$. The latter value is chosen as appropriate for the description of charged particles and a striking connection between causality and the gyromagnetic ratio $g = 2$ is established this way. The Lagrangian density associated with the equation of motion is

$$L = (\partial^\mu \overline{\psi}^\alpha)\Gamma_{\alpha\beta\mu\nu}\partial^\nu \psi^\beta - m^2 \overline{\psi}^\alpha \psi_\alpha,$$

gauging this Lagrangian we get the interacting part as

$$L_{\text{int}} = j_\mu A^\mu + e^2 \overline{\psi}^\alpha \Gamma_{\alpha\beta\mu\nu} \psi^\beta A^\mu A^\nu,$$

and using $\psi_\alpha = u_\alpha(p)e^{-ip\cdot x}$, we find the transition current in momentum space as

$$j_\mu = e\bar{u}^\alpha(p')(\Gamma_{\alpha\beta\nu\mu} p'^\nu + \Gamma_{\alpha\beta\mu\nu} p^\nu)u^\beta(p).$$

COMPTON SCATTERING

One can use the Compton Scattering process to get more information about the undetermined parameters appearing in the interaction current obtained for spin 3/2. In the classical limit, the cross section in the lab frame for this process is independent of undetermined parameters:

$$\sigma_{s=3/2}(f,g)|_{\eta\to 0} = \frac{8\pi}{3} r_0^2 \equiv \sigma_T,$$

where $\eta = \omega/m$, ω being the energy of the incident photon, $r_0 = \alpha/m$ is the so called classical radius of the particle and σ_T the Thompson cross section. This result is what one expects since at low energies the particle interacts only with its charge. For Higher energies, the cross section diverges for arbitrary values of the undetermined parameters. In a previous work we have shown that in this cases it is useful to integrate the differential cross section from $x = -1 + \varepsilon$ to $x = 1 - \varepsilon$, where $x = \cos\theta_{\text{lab}}$, in order to detect the specific values that preserve unitarity when ε goes to zero [2]. Following this procedure we find that the partially integrated cross section is

$$\sigma_{s=3/2}(f,g)|_{\eta\to\infty} = \frac{\text{sign}(f^2 + fg + g^2)^2 \text{sign}(\varepsilon - 1)}{\text{sign}(\varepsilon - 2)} \times \infty,$$

(result given by the Feyncalc package) this expression means that the cross section diverges independently of the choice of parameters. In the region of high energy, the cross section must be dominated by the higher multipole moments. The main objective of this work is to calculate these electromagnetic moments, aiming to understand the high energy behavior of the Compton cross section.

ELECTROMAGNETIC MOMENTS OF SPIN 3/2 PARTICLES IN NKR FORMALISM.

The Gordon decomposition of the transition current (with even parity terms only) is:

$$j_\mu(p',p) = e\bar{u}^\alpha(p')\left\{g_{\alpha\beta}(p'+p)_\mu - ig[M_{\mu\nu}]_{\alpha\beta}(p'-p)^\nu + if\gamma^5\varepsilon_{\alpha\beta\mu\nu}(p'-p)^\nu\right\}u^\beta(p),$$

all nonzero multipole moments of this current can be extracted by its classical definition, for example, the charge is $E_1 = \lim_{\mathbf{q}\to 0}\rho(\mathbf{q}) = -e$, and for higher moments we use the definitions [3]

$$
\begin{aligned}
M_2 &= \lim_{\mathbf{q}\to 0}\left[\frac{i}{2}\frac{\partial}{\partial q_3}\frac{\partial}{\partial \mathbf{q}}\cdot(\mathbf{j}\times\mathbf{q})\right], \\
E_4 &= \lim_{\mathbf{q}\to 0}\left[\left(\frac{\partial^2}{\partial q_1^2}+\frac{\partial^2}{\partial q_2^2}-2\frac{\partial^2}{\partial q_3^2}\right)\rho(\mathbf{q})\right], \\
M_8 &= \lim_{\mathbf{q}\to 0}\left[-\frac{3i}{4}\frac{\partial}{\partial q_3}\left(3\frac{\partial^2}{\partial q_1^2}+3\frac{\partial^2}{\partial q_2^2}-2\frac{\partial^2}{\partial q_3^2}\right)\frac{\partial}{\partial \mathbf{q}}\cdot(\mathbf{j}\times\mathbf{q})\right],
\end{aligned}
$$

where M_2 is the magnetic dipole moment, E_4 the electric quadrupole moment and M_8 the magnetic octupole moment. The charge and current densities are defined in terms of the interaction current between states in the Breit frame:

$$
j^\mu(\mathbf{q}) = j^\mu(p',p)/\omega, \quad p' = (\omega/2, \mathbf{q}/2) \quad p = (\omega/2, -\mathbf{q}/2),
$$

where $j^\mu(\mathbf{q}) = (\rho(\mathbf{q}), \mathbf{j}(\mathbf{q}))$. With a magnetic field aligned in the z direction, and considering even parity contributions only, the results obtained for the multipole moments of a spin 3/2 particle between states of polarization $\lambda = 3/2, 1/2, -1/2, -3/2$ are

$$
M_2 = \lambda\frac{eg}{2m}, \quad E_4 = \frac{1}{3}\left(3\lambda^2 - \frac{15}{4}\right)\frac{e(-f+g-1)}{m^2}, \quad M_8 = \lambda\left(\frac{15}{2}\lambda^2 - \frac{123}{8}\right)\frac{e(2f+g)}{m^3},
$$

reproducing the expected result for the dipole magnetic moment. All other moments for example E_2 or M_4 are zero. One can demonstrate also that parity odd terms for example $\bar{u}^\alpha(p')\gamma^5[M_{\mu\nu}]_{\alpha\beta}(p'-p)^\nu u^\beta(p)$ do not contribute to the physical moments above.

CONCLUSIONS

We have found that this formulation produces an electromagnetic interaction that includes the expected number of nonzero multipole moments of a spin 3/2 particle. In this formalism, apart from the charge, particles interact with the electromagnetic field via its magnetic dipole moment, electric quadrupole moment, and, in addition with respect to the vector case, via its octupole magnetic moment. We found that the g-term in the current contain contributions to all the relevant multipole moments. We have also found that the new f-term coming from a general formulation can contribute to the electric quadrupole moment and the magnetic octupole moment of the particle. The next step in the Compton scattering problem is to study the origin of violation of unitarity of the scattering of a spin 3/2 particle on the light of the present results.

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