# PERTURBATIVE APPROACH FOR NON LOCAL AND HIGH ORDER DERIVATIVE THEORIES

Ana A. Avilez and J.David Vergara

*Instituto de Ciencias Nucleares,*
*Universidad Nacional Autonoma de Mexico*
*Apartado Postal 20-364, México 01000 D.F., Mexico.*

**Abstract.** We propose a reduction method of classical phase space of high order derivative theories in singular and non singular cases. The mechanism is to reduce the high order phase space by imposing suplementary constraints, such that the evolution takes place in a submanifold where high order degrees of freedom are absent. The reduced theory is ordinary and is cured of the usual high order theories diseases, it approaches well low energy dynamics.

**Keywords:** High Order Derivatives Theories, Perturbation Theory
**PACS:** 11.15.Kc Classical and semiclassical techniques

## INTRODUCTION

In quantum field theory the high order derivative theories (HODT) are usually considered in three common contexts: (i) they appear in effective low-energy Lagrangians; ii) they can be introduced to regularize a theory in the ultraviolet; iii) or for instance they are unavoidable because of the non locality for example in string theory field theory or noncommutative theories. However, high order derivative theories have no trivial technical problems, specially in the path of quantizing them. First, Noether energy associated to HODT by Ostrogradsky formalism is not bounded from below which implies that HODT are instable. Even at classical level this systems have badly behaved solutions and runaways, usually this fact implies at the quantum level that the S matrix is not unitary and in consequence physically unacceptable. However, in literature there are some attempts to deal with these difficulties, namely: 1.[Eliezer-Woodard] and 2.[Cheng-Ho] get a reduced theory which doesn't have high order derivatives. In both cases the procedure has been done in configuration space and for non singular cases. In this work on the one hand, we have generalized the method to phase space, in this hamiltonian scheme we consider singular and non singular theories, and the aim of the work is to consider gauge high order theories.

## HIGH ORDER DERIVATIVE THEORIES

### Local an Non Local Theories

A local theory action has as integrand a functional of the degrees of freedom evaluated at the same parameter point, hence it depends on finite number N of $q_i$ time derivatives

$$S[q_i] = \int dt L(q_i, \dot{q}_i, ..., q_i^{(N)})$$

N is usually called the degree of the theory. In turn, the most general non local theory is such that the functional $L$ depends on $q$'s evaluated at different values of $t$. We shall restrict ourselves on theories whose non local dependence appears in analytical expressions, hence we can take such theory as a limit of a local one when $N$ goes infinity.

In order to build the Hamiltonian the Ostrogradsky formalism has to be used. Under such formalism $q_i^{(N-1)}$ are proposed to be generalized coordinates each has its conjugated momentum defined by

$$p_n^i \equiv \sum_{k=n}^{N} (-\frac{d}{dt})^{k-n} \frac{\partial L}{\partial q^{(k)}} \tag{1}$$

$$n = 0, ..., N-1$$

In non singular case, when the relation between $p_N^i$ and $q^{(N)}$ is invertible the hamiltonian is defined by

$$H = \sum_{i,n}^{K,N-1} p_n^i q_i^{(n+1)} - L(q_i, \dot{q}_i, .., q_i^{(N-1)}, p_{N-1}^i) \tag{2}$$

The simplectic form which generates the hamiltonian flux is

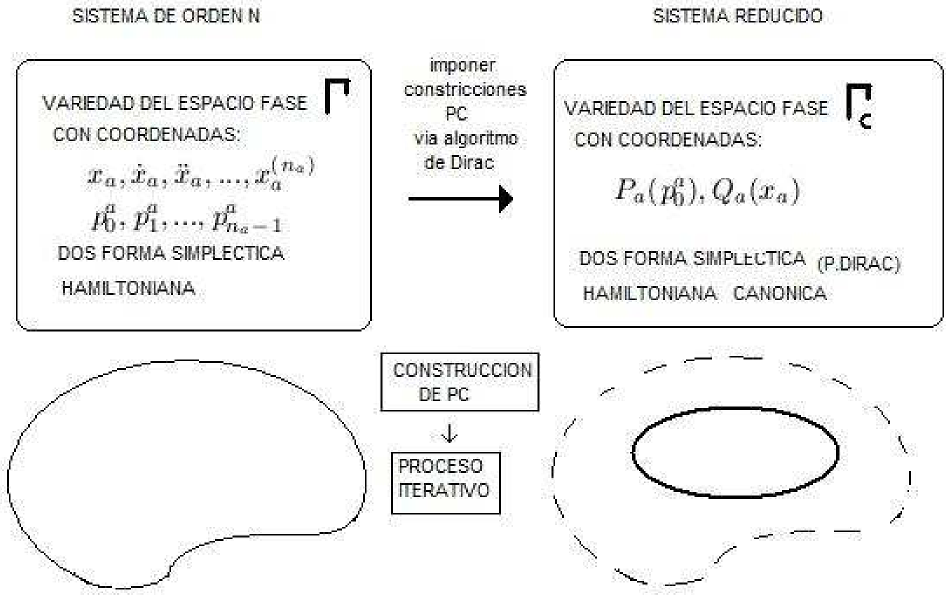
$$\Omega = \sum_{i,n}^{K,N-1} dp_n^i \wedge dq_i^{(n)} \tag{3}$$

associated with the usual Poisson brackets.

## PERTURBATIVE APPROACH IN HAMILTONIAN SCHEME

The aim of this approach is to get a reduced theory with no high order degrees of freedom as dynamical variables. In order of such aim, the classily way is to restrict the underlying phase space to a subspace where high order degrees of freedom are absent, this can be done through in the Dirac way. The method sight is to build up such "perturbative constraints" ($C_p$) order to order from Hamilton-Ostrogradski equations.

We begin from a given subyacent theory ($Gamma(\xi_o, \xi_h), H, \Omega$) where




## Non Singular Case

- $\Gamma \rightarrow T^*Q$ , whose independent coordinates are classified as following: $\xi_o$ lowest order in derivatives canonical coordinates and momenta,$\xi_h$ high order derivative variables.
- $\Omega$ Poisson simplectic form
- $H$ evolution generator

$$H = \sum_{a,s_a=0}^{N,n_a-1} p_{m_a} q_a^{m_a+1} + \tilde{H}(q_a^{r_a}, p_{s_a}^a).$$

$$\tilde{H} = -\frac{1}{2}(\dot{q}_a^0)^2 + \frac{\omega^2}{2}(q_a^0)^2 + V(q_a^{s_a}, p_{n_a-1}^a).$$

where high order derivative terms are perturbations while they are heightened by $\alpha \ll 1$.

### *Construction of Supplementary Constraints*

From Hamilton fundamental equations the following recurrence relation is derived

$$[q_a^2]_{n+1} + \omega^2 q_a^0 = \sum_{k_a=0}^{n_a-1} (-1)^{k_a+1} \left[ \frac{d^{k_a}}{dt^{k_a}} \left( \frac{\partial V}{\partial q_a^{k_a}} \right) \right]_n \tag{4}$$

$$[\dot{p}_1^a]_{n+1} = -p_0^a + \dot{q}_a^0 - \left[\frac{\partial V}{\partial q_a^1}\right]_n \tag{5}$$

Notation $[X]_n \to X$ order $n$ in $\alpha$.
From which one gets the following constraints

$$\Phi_a^n = q_a^2 - \Lambda_1(p_0^a, q_a^0) \approx 0 \tag{6}$$
$$\phi_a^n = -p_0^a + \dot{q}_a^0 + \Lambda_2(p_0^a, q_a^0) \approx 0. \tag{7}$$

Together with the other Hamilton equations

$$\psi_{s_a}^n = p_{s_a}^a - \Sigma_{s_a}(p_0^a, q_a^0) \tag{8}$$
$$\tilde{\psi}_{s_a} = q_a^{s_a} - \dot{q}_a^{s_a-1} \tag{9}$$

- The mechanism is rewriting the underlying equations of motion iteratively, and set them in some order in $\alpha$, which are precisely (6) y (8). They can be imposed as second class Dirac-like constraints. $C_p = \{\Phi, \phi, \psi, \tilde{\psi}\}$ in the underlying phase space.

### *Reduction of the Underlying Theory*

- After make $C_p$ strongly null, the appropiate Dirac reduced theory correspond to the effective theory we looked for, wich describes the dynamics in the low energy limit.

$$\Omega_r = (1+C)dp_0^a \wedge dq_a^0$$

$C \to$ are corrections dependent of $\Lambda_{1,2}$ and $\Sigma_{s_a}$

## Singular Case

### *Second Class Constraints*

Underlying Theory: $(\Gamma_c(\xi_o, \xi_h), H_c, \Omega_D)$

- $\Gamma_c$ sub-manifold of $T^*Q$ defined by second class constraints $(\xi)$, whose non independent frame of coordinates are classified as following: $\xi_o$ lowest order in derivatives canonical coordinates and momenta,$\xi_h$ high order derivative variables.
- $\Omega_D$ Dirac brackets simplectic form
- $H_c$ evolution generator on $\Gamma_c$.

From the fundamental evolution equations

$$\dot{A} = \{A, H_c\}_D \tag{10}$$

From these evolution scheme $C_p$ are obtained.

## CONCLUSIONS

- Through perturbative approach one gets a reduced theory without usual high order (and non local) diseases.
- Perturbative approach performed in phase space shows clearly physical nature of the reduced theory from a given underlying theory. In such scheme, generalization for gauge theories is straightforward.
- Taking out high order degrees of freedom through perturbative method restrict the predictive power of the reduced theory to describe just low energy phenomena.
- A significative advance is that unlike usual perturbative theory , through this method structure from underlying simplectic form is recovered into reduced theory.