Anomalies, Beta Functions, and GUT's

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Abstract. In the framework of supersymmetric Grand Unified theories it is possible to extend the minimal Higgs sectors of the models by introducing high dimension (anomaly free) representations. For example, in the minimal SU(5) supersymmetric Grand Unified Model, this is done to obtain phenomenological viable fermion mass relations and/or to solve the doublet-triplet splitting problem. In this work we explore models with different anomaly free combinations of SU(5) representations motivated by the flavour problem as well as their effect on perturbative validity of the gauge coupling evolution.

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1. INTRODUCTION

Supersymmetric Grand Unified Theories (SGUT) [1, 2] have achieved some degree of success: unification of gauge couplings, charge quantization, prediction of the weak mixing angle, mass-scale of neutrinos. Although this degree of success is already present in the minimal models (SO(10) or some variant of SU(5)), there are open problems that suggest the need to incorporate more elaborate constructions [3, 4, 5], specifically the use of high dimension representations in the Higgs sector. For instance, a **45** representation is needed to obtain correct mass relations for the first and second families of d-type quarks and leptons [6].When one adds these higher-dimensional Higgs representation within the context of $\mathcal{N} = 1$ SUSY GUTs, one must verify the cancellation of anomalies associated to their fermionic partners, i.e., the Higgsinos. The most straightforward solution to anomaly-cancellation is obtained by including vector-like representations i.e., including both ψ and $\bar{\psi}$ chiral supermultiplets. It is one of the purposes of this work to find alternatives to this option, namely to create an anomaly free Higgs sector including some representation for ψ and a set of other representations of lower-dimension for $\{\phi_1, \phi_2, ...\}$. It turns out that both choices are not equivalent in terms of the beta functions corresponding to each model.

It is also known that the unification condition imposes some restrictions on the GUT-scale masses of the gauge bosons, gauginos, Higgses, and Higgsinos [11]. Although the addition of complete multiplets does not change the unification of gauge couplings, and neither modifies the unification scale. In turn, the evolution of the gauge couplings above the GUT scale, up to the Planck scale, depends on the matter and Higgs content of the model, thus the perturbative validity of the model affected by the inclusion of additional multiplets.

We study the effect on the evolution of the gauge coupling up to the Planck scale due to the different sets of fields and representations that can render an anomaly-free model[7, 8, 9].

2. ANOMALIES IN GAUGE THEORIES

Anomalies can be associated with both global and local symmetries, the latter being most dangerous for the consistency of the theory. The need to require anomaly cancellation in any theory stems from the fact that their presence destroys the unitarity of the theory.For a given representation of a gauge group G, the anomaly can be written as $A(D)d^{abc} \equiv Tr\left[\left\{T_a^{D_i}, T_b^{D_i}\right\} T_c^{D_i}\right]$, where $T_a^{D_i}$ denotes the generators of the gauge group G in the representation D_i, and d^{abc} denotes the anomaly associated to the fundamental representation [10, 11]. The anomaly coefficients $A(D)$ for most common representations are shown in [10, 11] for SU(N) groups known in the literature. We have extended these results to include higher-dimensional representation [9]. Results for SU(5) most common representations are shown in Table 1.

3. ANOMALY CANCELLATION IN SU(5)

In the minimal SU(5) SUSY GUT model the mass relations $m_{d_i} = m_{e_i}$, are predicted by the **5**-$\bar{\mathbf{5}}$ Higgs sector, this relation works well for the third family, but not for the first and second family. One way to solve this problem is to add a **45** representation, which couples to the d-type quarks, but not to the up-type, then one obtains the Georgi-Jarlskog factor [6] and the correct mass relations. Most models that attempt to obtain this relations involve an extended Higgs sector; one possible choice, free of anomalies, include both representations $\mathbf{45} + \mathbf{45}$ to cancel anomalies [12]. This is however not the only possibility, and this is one of the main results of our work.

TABLE 1. Dimension and anomaly coefficients for different representations of SU(5).

Irrep	Multiplet	dim(R)	A(r)	2T(R)
$[5]$	(0,0,0,0)	1	0	0
$[1]$	(1,0,0,0)	5	1	1
$[2]$	(0,1,0,0)	10	1	3
$[1,1]$	(2,0,0,0)	15	9	7
Ad		24	0	10
$[4,1]$	(1,0,0,1)	24	0	10
$[1,1,1]$	(3,0,0,0)	35	44	28
$[2,1]$	(1,1,0,0)	40	16	22
$[3,1]$	(1,0,1,0)	45	6	24
$[2,2]$	(0,2,0,0)	50	15	35

Consider the representations of SU(5) (and their conjugates) listed in Table 1; we can see that the **45** anomaly coefficient is 6, then taking into consideration that the **5** and the **10** have the same anomaly, $A = 1$, we can obtain anomaly-free models through the combinations: $A(\mathbf{45}) + A(\mathbf{45}) = 0$, $A(\mathbf{45}) + 6A(\mathbf{5}) = 0$, $A(\mathbf{45}) + 6A(\mathbf{10}) = 0$, $A(\mathbf{45}) + fA(\bar{\mathbf{5}}) + f'A(\mathbf{10}) = 0$, $A(\mathbf{45}) + A(\mathbf{15}) + 3A(\mathbf{5}) = 0$, where f and f' are positive integers and satisfy $f + f' = 6$. These are clearly non-equivalent models.

4. GAUGE COUPLING UNIFICATION

The one loop β functions for a general SUSY theory with gauge group G and matter field appearing in chiral supermultiplets are given by $\beta_1 = \sum_R T(R) - 3C_A$, where $T(R)$

denotes the index for the representation R, and C_A the cuadratic Casimir invariant for the adjoint representation. For SU(N) type gauge groups $C_A = N$.

From MSSM β-functions, assuming $M_{SUSY} \approx M_t$, one obtains that the gauge coupling is aproximately $g(M_{GUT}) = \frac{1}{24}$ [13], and unification occurs at $M_G = 2 \times 10^{16}$ GeV.

Now we are interested in evaluating the effect of the different representations in the running from M_{GUT} up to the Planck scale. Besides finding an effect of the different anomaly free combinations, we are also interested in finding which representations are perturbatively valid up to the Planck scale, as it is shown in Figure 1.

The β functions for some interesting anomaly-free combinations are: $\beta^X(\mathbf{45}+\overline{\mathbf{45}}) = 24$, $\beta^X(\mathbf{45}+6(\overline{\mathbf{5}})) = 15$, $\beta^X(\mathbf{45}+6(\mathbf{10})) = 21$, $\beta^X(\mathbf{45}+\mathbf{15}+2(\mathbf{10})+\mathbf{5}) = 19$, $\beta^X(\mathbf{50}+\mathbf{40}+\mathbf{5}) = 29$, where $\beta^X = \sum_\Phi T(\Phi)$ are the contributions of the extensions of the MSSM and the sum is over all SU(5) additional multiplets Φ.

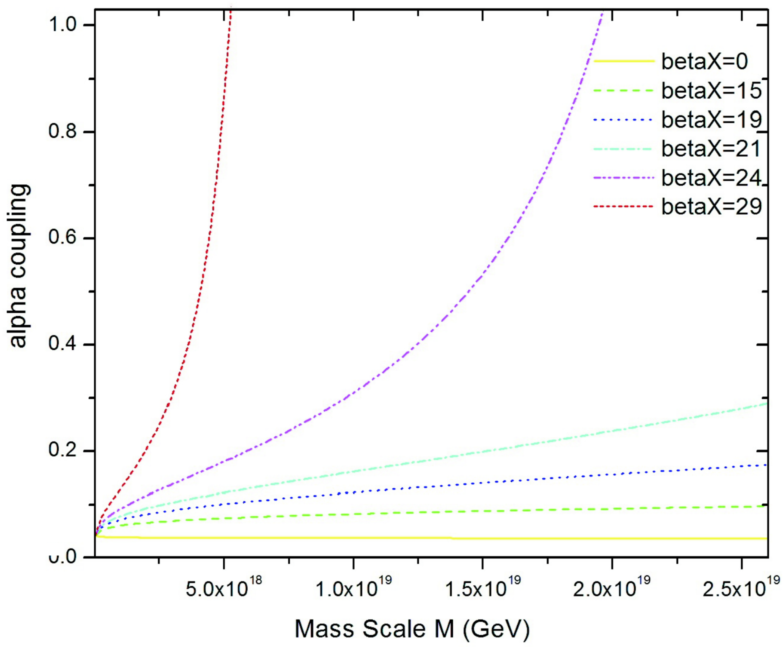

FIGURE 1. Evolution of the gauge coupling for free anomaly combinations listed above, up to the Planck scale .

It tourns out that the model with $\beta^X = 29$ induces a running of the gauge coupling that blows at the scale $M = 6.61 \times 10^{18}$, while for $\beta^X = 24$ this happens at $M = 2.63 \times 10^{19}$. The models with $\beta^X = 15, 19, 21$ are found to evolve safely even up to the Planck scale.

5. CONCLUSIONS

We have studied the problem of anomalies in SUSY gauge theories, in order to search for alternatives to the usual vector -like representations used in extended Higgs sector. We have succed in identifying ways to replace the $\mathbf{45}+\overline{\mathbf{45}}$ models within SU(5) SUSY GUTs. Then, we have studied the beta functions for all the alternatives , and we find that they are not equivalent in terms of the values of their β functions. These results have important implications for the perturbative validity of the GUT models at scales higher than the unification scale.

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