Radion stabilization from the vacuum

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Abstract. When translational invariance is spontaneously broken the volume stabilization in models with flat extra dimension could follow from vacuum energy residing in the extra dimensional space. We study a simple toy model that exemplifies this mechanism which considers a massive scalar field with non trivial boundary conditions at the end points of the compact space, and includes contributions from brane and bulk cosmological constants. We perform our analysis in the conformal frame where the radion field, associated to volume variations, is defined, and present a general strategy for building stabilization potentials out of those ingredients. We also provide working examples for the interval and the T^n/Z_2 orbifold configuration.

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INTRODUCTION

Models with space-like extra dimensions may have a non trivial configuration and topology, and be characterized by a variety of sizes, that, according to some speculations [1], may even be as large as few micrometers, in contrast with the much smaller Planck length, $\ell_P \sim 10^{-33}$ cm. The idea seems to find some motivation from the study of the non-perturbative regime of the $E_8 \times E_8$ theory by Witten and Horava [2], where one of these extra dimensions appears to be larger than the naively expected Planck size for quantum gravity physics. The possibility that there could be such extra dimension has renewed the interest in a class of models once inspired by the works of Kaluza and Klein [3]; and lately suggested by several authors [4, 5].

Of particular interest are the so called brane models, in which our observable world is constrained to live on a four dimensional hypersurface (the brane) embedded in a flat higher dimensional space (the bulk), such that the extra dimensions can only be tested by gravity, and perhaps standard model singlets, a set up that resembles D-brane theory constructions. These models have the extra feature that they may provide an understanding of the large difference among Planck, M_P, and electroweak, m_{ew}, scales almost by construction. Here M_P is replaced by the truly fundamental gravity scale, M_*, associated to quantum gravity in the $4+n$ dimensional theory. Both scales are then related by the volume of the compact manifold, vol_n, through out the expression [1]

$$M_P^2 = M_*^{n+2} vol_n \ , \tag{1}$$

which indicates that the so far unknown value for M_* could lay anywhere within m_{ew} and M_P. If it happens to be in the TeV range there would be no big hierarchy, but a rather large volume is required.

Most phenomenological models built on this scenario usually assume that the extra dimensions are stable, which typically becomes a fundamental requirement since most effects of extra dimensions on low energy physics depend either on the effective size of the compact space $b_0 \sim vol_n^{1/n}$ or the effective Planck scale. However, since the compact space is dynamical those become time dependent in general, against observations [6]. Understanding the stability of the compact space can be seeing as finding the mechanism that provides the force which keeps the radion, a modulous field associated to the extra volume, fixed at its zero value. Thus, in order to have a stable bulk volume, there has to be a potential which provides such a force. The last can provided purely from vacuum energy [7], as we shall discuss in this work.

THE RADION IN THE EINSTEIN FRAME

We start the discussion by assuming that Einstein gravity holds in the complete $(4+n)$D theory, and proceed with dimensional reduction to introduce the definition of the radion field and its couplings. Thus we first write down the Einstein-Hilbert action

$$S = \frac{M_*^{2+n}}{2} \int d^4x \, d^n y \, \sqrt{|g_{(4+n)}|} \, R_{(4+n)} \tag{2}$$

where $R_{(4+n)}$ stands for the $4+n$D dimensional scalar curvature, and $|g_{(4+n)}|$ is the absolute determinant of the $(4+n)$D metric. We then consider the background metric parameterization $ds^2 = g_{AB}dx^A dx^B = g_{\mu\nu}dx^\mu dx^\nu - h_{ab}dy^a dy^b$, that is conformally consistent with 4D Poincarè invariance, and describes a compact and flat extra space. So we assume y^a as dimensionless coordinates on a unitary and closed manifold. Thus, h_{ab} has length dimension two. Here we use for the indices the convention $A, B = \mu, a$ where $\mu = 0, \ldots, 3$ and $a = 5, \ldots, 4+n$. Notice we are not considering the usual vector-like A_μ^a connection pieces. This is so because we want to concentrate only on the variations of the metric along the transverse directions for the rest of our discussion.

Upon dimensional reduction and with in order to get a proper gravity action one has to go to a different frame. Thus, we perform the conformal transformation $g_{\mu\nu} \to e^{2\varphi} g_{\mu\nu}$ with $e^{2\varphi} = vol_n/\sqrt{|h|}$, to obtain the 4D gravity in canonical form

$$S = \frac{M_P^2}{2} \int d^4x \, \sqrt{|g_{(4)}|} \left\{ R_{(4)} - \frac{1}{4}\partial_\mu h^{ab} \, \partial^\mu h_{ab} + \frac{1}{8} h^{ab}\partial_\mu h_{ab} \cdot h^{cd}\partial^\mu h_{cd} \right\}, \tag{3}$$

that we shall refer as the conformal (or Einstein) frame. Next, $g_{\mu\nu}$ can be assumed to be the standard metric for a Poincarè invariant brane Universe or the Friedmann-Robertson-Walker metric for cosmology. Nevertheless, to simplify, we shall take $h_{ab} = b^2 \delta_{ab}$, such that b represents the actual size of the compact space.

In the desired stable configuration, the physical size of the extra dimension would be given by $b = b_0$. So, in the conformal frame the effective Planck scale is well defined

and constant. Volume variation effects appear as the radion field (see for instance [6])

$$\sigma(t) = M_P \sqrt{n(n+2)/2} \ln(b/b_0) \ . \tag{4}$$

An active radion means a variable bulk, a potetially unwanted and harmful scenario.

Radion couplings to other fields are fixed from te structure of the higher dimensional action that describe the last. Consider for instance a bulk scalar field, $\phi(x,y)$. The corresponding action, before performing the conformal transformation on the metric, goes as

$$S_\phi = \int d^4x \, d^n y \sqrt{|g_{(4)}|} \sqrt{|h|} \left[\frac{1}{2} G^{AB} \partial_A \phi \, \partial_B \phi - U(\phi) \right] \ . \tag{5}$$

Without lost of generality, we can always assume that ϕ has a proper Kaluza Klein (KK) mode decomposition, which should be defined for each given topology of the compact space. In the efective four dimensional theory, and in the Einstein frame the general form of the potential for the KK modes has the form $e^{-\alpha \sigma/M_P} U_{eff}(\phi_{\vec{n}})$ where $\alpha = \sqrt{2n/(n+2)}$ and

$$U_{eff} = vol_n \cdot \int d^n y \left(\vec{\nabla}_y \phi \cdot \vec{\nabla}_y \phi / (2b^2) + U(\phi) \right) \ , \tag{6}$$

with $\vec{\nabla}_y$ the gradient on the compact space coordinates. Last expression actually corresponds to the potential energy, U_{ini}, one calculates in the initial frame, but for the global factor vol_n instead of the physical volume $\sqrt{|h|}$. In the case of a bulk cosmological constant, the action in Einstein frame becomes

$$S_\Lambda = \int d^4x \sqrt{|g_{(4)}|} \, \Lambda_n \, e^{-\alpha \sigma/M_P} \ , \tag{7}$$

with the effective cosmological constant $\Lambda_n = vol_n \cdot \Lambda$. In contrast for a 3-brane cosmological constant, λ, one has

$$\int d^4x \sqrt{|g_{(4)}|} e^{-2\alpha \sigma/M_P} \lambda \ . \tag{8}$$

RADION STABILIZATION BY VACUUM ENERGY

As we already mentioned, some ideas on how to generate an stabilization potential for the radion can be found already in the literature (see for instance Refs. [8, 9, 10]). In particular, for a single extra dimension, it is has been pointed out [9, 10] that a radion potential can be produced if translational invariance is broken in the bulk by the vacuum expectation value of a scalar field. Here, we will further explore this idea for flat extra dimensions on the conformal frame where the radion has been identified. The basics of the mechanism we are exploring are rather simple. We consider bulk energy densities that break translational invariance along bulk coordinates, which provide a radious potential energy, $U_{rad}(b)$. Of course, if there is a non trivial minimum for $U_{rad}(b)$, this would be identified as b_0. Curiously enough, such a potenial can be build out of only cosmological constants, as we discuss next.
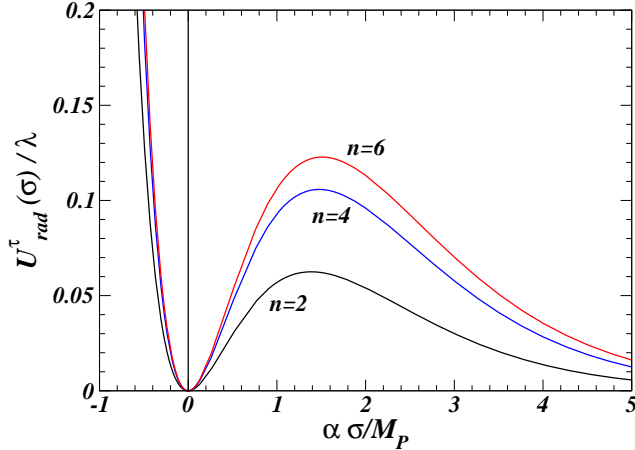
FIGURE 1. Radion potential profile generated by cosmological constants for n as indicated.

For any individual cosmological constant the radion potential is just an exponentially decaying function without non trivial minimum. However, the combination of both, brane and bulk csmological constants, do work. From Eqs. (7) and (8), the most general radion potential one can build with a minimum in this case is [7] $U_{rad}^\lambda(\sigma) = \lambda\,e^{-\alpha\sigma/M_P}(e^{-\alpha\sigma/M_P} - 2)$, where $\Lambda_n + 2\lambda = 0$ fixes the stable radius to $b_0 = (-2\lambda/\Lambda)^{1/n}$. From this potential one gets a Planck suppressed effective radion mass at the minimum, $m_\sigma = \alpha\sqrt{2\lambda}/M_P$, which may also imply a too light radion potential, against observational limits on gravity strength coupled scalars, that indicate $m_\sigma > 10^{-3}\,eV$, which would require that $\lambda > TeV^4$. This mechanism can be extended to more dimensions [7] as it is shown in figure 1.

Next simplest example one can provide is a y-dependent vacuum. That arises in models where non trivial boundary conditions are imposed on a bulk scalar field configuration. Consider a massive scalar field, ϕ, described by the action given in Eq. (5) for $U(\phi) = \frac{1}{2}m^2\phi^2$. Therefore, the vacuum configuration in the initial frame, with a given volume of size b, should be a solution to the equation of motion $[-\nabla_y^2 + \kappa^2]\langle\phi\rangle(y) = 0$, where $\kappa = mb$. Boundary conditions for $\langle\phi\rangle$ induce a non trivial profile for the vev along the bulk. By setting the vev back into the Lagrangian, $\mathscr{L}$, at any given radius b, one formally gets, in the Einstein frame, the radion potential contribution [7]:

$$U_{rad}^\phi(b) = (b_0/b)^{2n}\,U_{ini}(b)\,, \tag{9}$$

here written in terms of the radius, and where we have used the potential as it is read in the initial frame $U_{ini}(b) = -b^n \cdot \int d^n y\,\mathscr{L}(\langle\phi\rangle)$. By written the potential this way, it becomes clear that in general a minimum for U_{ini} is not a minimum of U_{rad}^ϕ. The conformal factor deforms the potential, and may even compromise stabilization in some cases. Let us discuss next some examples.

On the interval it is straightforward to calculate the stabilization potential, which on the initial frame gets the form

$$U_{ini}(b) = \frac{m}{2} \frac{\left(v_0^2 + v_1^2\right) \cosh \kappa - 2v_0 v_1}{\sinh \kappa}, \tag{10}$$

where v_i are the boundary conditons. This potential has a sizable minimum at $mb_i = arccosh(v_0^2 + v_1^2)/(2v_0 v_1)$. Particularly, for large v_0/v_1 ratios one gets the approximate expression $mb_0 \approx [\ln(v_0/v_1)]^2$. At the minimum we get $U_{ini}(b_i) = \frac{m}{2}\|v_0^2 - v_1^2\|$, and so the potential is always positive. Notice also that the potential goes asymptotically to a constant: $U_{ini}(b \to \infty) = m(v_0^2 + v_1^2)/2$, and for small b behaves like $\sim (v_0 - v_1)^2/2b$, provided $b_0 \neq 0$. Clearly, $v_0 = v_1$ is not a favored scenario. First of all, it implies $b_i = 0$, where the potential vanishes. The $1/b^2$ squared modulation on the actual potential removes this minimum and kills the asymptotic behavior, such that the only possible minimum in the Einstein frame becomes $b \to \infty$. Ths problem is cured by adding a proper combination of bulk and brane cosmological constants as it is shown in Ref. [7]. As the first approximation we add a brane cosmological constant $\lambda = -m(v_0^2 - v_1^2)/2$, and take $\Lambda = 0$. Thus, the resulting radion potential, $U_{rad}(b) = (b_0/b)^2[U_{ini}(b) + \lambda]$, keeps the minimum at b_0, and fixes $U_{rad}(b_0)$ to zero.

Next, we consider a T^n/Z_2 orbifold, where $Z_2 : \vec{y} \to -\vec{y}$ on the symmetric T^n torus with common radii b. We analize only the volumetric radion, such that the metric on the compact space remains of the form $ds_{compact}^2 = b^2 \delta_{ij} dy^i dy^j$. In this orbifold, there are 2^n fixed points which correspond to the vertices of the unitary hypercube $\mathscr{I}_0^n = \mathscr{I}_0 \times \cdots \times \mathscr{I}_0$, where $\mathscr{I}_0 = [0, 1]$. This orbifold has a residual discrete symmetry $\mathscr{R}_{\pi/2}^n$, given as the set of rotations by $\pi/2$ around any y_i coordinate axis, which indicates that only $n+1$ boundary conditons on equal classes of fixed points can be allowed to be different. Thus we can work out our analysis considering only the contribution of the vacuum that resides on the hypercubic slice $\mathscr{I}_0^n$. Total potential energy on the orbifold shall be just a 2^{n-1} multiple of this. The vacuum can be factored as $\phi(\vec{y}) = \Pi_i^n \varphi_i(y_i)$, where $\varphi_i(y_i) = \varphi(y_i) = Ae^{ky_i} + Be^{-ky_i}$, with the global constants $A = (v_1 - v_0 e^k)/\sinh k$ and $B = (v_0 e^{-k} - v_1)/\sinh k$, and $nk^2 = \kappa^2 = m^2 b^2$, with the boundary conditions $\varphi_i(0) = v_0$ and $\varphi_i(1) = v_1$, are indepent of the index due to the $\mathscr{R}_{\pi/2}^n$ symmetry. Thus, in this scenario different directions along any coordinate axis look alike for the scalar field. That is the reason why volume varies as a whole while the basic geometry stands still. The potential energy from this vacuum, as calculated in the initial frame is given now by the general expression $U_{ini}^n(b) = 2^{n-1} \times \frac{1}{2} b^{n-2} \int_0^1 dy \left[n(\varphi'(y))^2 + \kappa^2 \varphi^2(y)\right] \cdot \left[\int_0^1 dy\, \varphi^2(y)\right]^{n-1}$. After some algebra, one gets the rather complicated expression [7]

$$U_{ini}^n(b) = \frac{n^{n/2}}{2m^{n-2}} \left(\frac{\left(v_0^2 + v_1^2\right) \cosh k - 2v_0 v_1}{\sinh k}\right) \times$$

$$\left(\frac{2v_0 v_1 \left(k \cosh k - \sinh k\right) + \left(v_0^2 + v_1^2\right) \left(\cosh k \sinh k - k\right)}{\sinh^2 k}\right)^{n-1}, \tag{11}$$
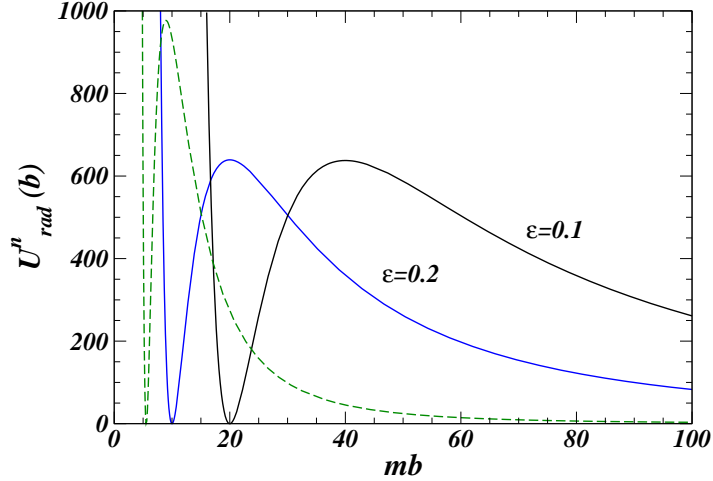
FIGURE 2. Radion stabilization potentials generated for the T^n/Z_2 orbifold, in arbitrary units.

A minimum exist only for $n = 1$, which reduces to the case we discussed above. Nevertheless in the Einstein frame the addition of cosmological constants do provide a way to generate the desired minimum, through out the potential

$$U_{rad}^n = \left(\frac{b_0}{b}\right)^n \left[\left(\frac{b_0}{b}\right)\left[\left(\frac{b_0}{b}\right)^{n-1} U_{ini}^n(b) + \tau_n\right] + \Lambda_n\right] , \qquad (12)$$

where a proper selection of constants has to be made [7]. Some example results are ploted in figure 2.

CONCLUSIONS

Summarizing, our present work pin points a clear conclusion: the combination of cosmological constant configurations, with bulk vacuum energy from scalar fields, do provides successful and manageable scenarios for the understanding of the stabilization of the radion field, within the context of the four dimensional effective theory, in flat extra dimension models. We have developed some basic strategies to handle and build radion potentials, with local minima and zero effective cosmological constant, out of these two minimal ingredients. The analysis has to be done in the Einstein frame, where the radion is defined, showing that conformal factors play an key role on defining the minimum of the potentials. Our results are an indication that it is well possible to built phenomenological stabilization potentials, out of the most common ingredients that any bulk-brane theory could have: brane and bulk cosmological constants, and bulk scalar degrees of freedom with non trivial bulk configurations. As a note we observe that since cosmological constants contribute non trivially to the radion potential, the problem of managing its quantum corrections may be worth to study.

ACKNOWLEDGMENTS

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