Electrodynamics in a 6D warped geometry

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Abstract. We obtain the effective 4D action that arises from a 6D free gauge action in the space-time metric RSI-1. Solving explicitly the 6D equations of motion we obtain the Kaluza-Klein decomposition of the 6D gauge field. This work constitutes the first step towards the discussion of the Gauge-Higgs Unification scenario in this background.

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INTRODUCTION

It is well known that the Higgs sector is still a lacking piece of the Standard Model. This sector governs the electroweak symmetry breaking and gives masses of quarks and leptons. Furthermore, the quadratic divergent correction to the Higgs mass strongly suggest the existence of new physics at the TeV scale. For this purpose a lot of scenarios beyond the Standard Model have been proposed, among them the so-called Gauge-Higgs Unification (GHU) scenario, which predicts various interesting properties in the Higgs couplings to the gauge and the fermion fields. The basic idea in the GHU is that the Higgs arises from the internal components of a higher dimensional gauge field. A crucial property of this scenario is that higher dimensional gauge invariance provides a protection to the Higgs mass from quadratic divergence. When the extra coordinate is not-simple connected, there are Wilson line phases associated with the extra dimensional component of the gauge field. Their 4D fluctuation is identified with the Higgs.

In the case that the 5D spacetime is flat, it has been shown that the Higgs mass is too small to satisfy the experimental lower bound and, trilinear couplings among the W and the Z bosons substantially deviate from the standard model values, which is inconsistent with experiments. The warped Randall-Sundrum spacetime [1] ameliorates these problems predicting a enhanced Higgs mass by a factor $\kappa\pi r \simeq 35$, compared to the case of the flat spacetime and the couplings among the W and Z bosons are in good agreement with those in the standard model [2].

Randall-Sundrum spacetime is a 5D theory compactified on S^1/Z_2, however although all the nice properties of the GHU scenario in this background, it does not naturally contain quartic couplings for the scalars in the gauge fields. Therefore one is compelled to look at 6D theories where the quartic scalar couplings are generated by the higher dimensional gauge interactions [3, 4, 5].

The 6D analysis in the GHU scenarios, have been considered for different types of orbifolds, but not in the context of the Randall-Sundrum model. For these reasons it would be interesting to investigate the GHU problem in a scenario that could mix both properties, the 6D issue of the higher dimensional theory and the warped geometry. The simplest model of this kind is the Randall-Sundrum scenario extended by one spatial extra coordinate with S^1 topology. This scenario is called in the literature the RSI-1 scenario. Because it is 6D, it must give origin to a quartic scalar coupling and because it is of the Randall-Sundrum type, it must also contain all the nice properties discussed above.

In this contribution we present the first step to discuss the GHU scenario in the RSI-1 background. We obtain the effective 4D action that arises from the 6D free gauge action in this background. Generalization of these results to the Yang-Mills case are under research [6].

FREE GAUGE ACTION IN A 6D WARPED GEOMETRY

The metric

The metric of the RSI-1 scenario describes a 6D non-factorizable geometry, based on a slice of AdS_6 spacetime

$$ds^2 = G_{\hat{\mu}\hat{\nu}} dx^{\hat{\mu}} dx^{\hat{\nu}} = e^{-2\sigma(x^6)} \left(\eta_{\mu\nu} dx^{\mu} dx^{\nu} - (dx^5)^2 \right) - (dx^6)^2, \tag{1}$$

where $e^{-2\sigma(x^6)} \equiv e^{-2\kappa|x^6|}$ is the warping factor. Hatted indexes denote the 6D space-time coordinates $x^{\hat{\mu}} = (x^{\mu}, x^5, x^6)$, while Greek indexes denote the 4D coordinates $x^{\mu} = (x^0, x^1, x^2, x^3)$, in a flat space-time of signature $\eta_{\mu\nu} = \mathrm{diag}(+,-,-,-)$. This metric considers two spatial extra dimensions, both of them compact but with different topology. The x^6 coordinate has a $\mathrm{S}^1/\mathrm{Z}_2$ orbifold topology. This means it satisfies two properties, $x^6 = x^6 + 2\pi r$, and $-x^6 \to x^6$. The x^5 coordinate is associated to the compact extra dimension which has topology S^1, therefore it only satisfies the periodicity condition, $x^5 = x^5 + 2\pi R$. As can be noticed in (1), the x^5 extra dimension together with the coordinates describing flat 4D spacetime are multiplied by the warping factor.

This metric can be obtained as an asymptotic solution to the 6D Einstein equations with negative bulk cosmological constant and two 4-branes, one being ‘visible’ with the other being ‘hidden’, with opposite tensions rigidly reside at $\mathrm{S}^1/\mathrm{Z}_2$ orbifold fixed points, taken to be $x^6 = 0$, and, $x^6 = \pi r$.

The action

We start our analysis considering the 6D free gauge field action

$$S=-\frac{1}{4}\int d^4x\int_0^{2\pi R}dx^5\int_{-\pi r}^{\pi r}dx^6\sqrt{-G}G^{\hat\mu\hat\rho}G^{\hat\nu\hat\delta}F_{\hat\mu\hat\nu}F_{\hat\rho\hat\delta}+\mathscr{L}_{g.f.}, \tag{2}$$

in the background metric (1). Here $G=\det(G_{\hat\mu\hat\nu})$, $F_{\hat\mu\hat\nu}$ is the 6D field strength tensor given by $F_{\hat\mu\hat\nu}=\partial_{\hat\mu}A_{\hat\nu}-\partial_{\hat\nu}A_{\hat\mu}$ and $\mathscr{L}_{g.f.}$ is the gauge fixing action[1], given by

$$\mathscr{L}_{g.f.}=-\frac{1}{2}\int d^6x\left(e^{-\sigma}\left(\partial_\alpha A^\alpha\right)^2+2\partial_\alpha A^\alpha\partial_6\left(e^{-3\sigma}A^6\right)+e^{\sigma}\left(\partial_6\left(e^{-3\sigma}A^6\right)\right)^2\right). \tag{3}$$

Applying the Hamilton principle to the action (2) we obtain as usual both boundary conditions and equations of motion. Regarding boundary conditions, it can be shown that the conditions for vanishing boundary terms are related to the Z_2-orbifold projections of the gauge fields. These conditions are of two types, for the A_μ and A_5 components of the gauge field, we obtain Neumann-type conditions [7]

$$\begin{aligned}\partial_6A_\mu(x^\nu,x^5,x^6=0)=\partial_6A_\mu(x^\nu,x^5,x^6=\pi r) &= 0, &(4)\\ \partial_6A_5(x^\nu,x^5,x^6=0)=\partial_6A_5(x^\nu,x^5,x^6=\pi r) &= 0, &(5)\end{aligned}$$

whereas for the A_6 component we obtain Dirichlet-type conditions [7]

$$A_6(x^\nu,x^5,x^6=0)=A_6(x^\nu,x^5,x^6=\pi r)=0. \tag{6}$$

The analysis of the boundary terms implies also that the Z_2-orbifold projection of A_μ and A_5 should be different from the one for A_6. This conclusion can be obtained from the analysis of the gauge transformations as well. It can be shown that in order to preserve gauge invariance A_6 must have an opposite sign relative to A_μ and A_5 under parity transformations. It is usual [8, 7] to assume that A_6 is Z_2-odd whereas A_μ and A_6 are Z_2 even

$$\begin{gathered}A_\mu(x^\nu,x^5,-x^6)=+A_\mu(x^\nu,x^5,x^6),\quad A_5(x^\nu,x^5,-x^6)=+A_5(x^\nu,x^5,x^6)\\ A_6(x^\nu,x^5,-x^6)=-A_6(x^\nu,x^5,x^6).\end{gathered} \tag{7}$$

This choice of Z_2 parity ensures that A_6 does not have a zero mode in the effective 4D theory whereas A_μ and A_5 do have. We stress at this point that the similar properties of the gauge components A_μ and A_5 are a direct consequence of the metric (1), where the coordinates x^μ and x^5 are considered by the warping factor at the same level.

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[1] The gauge fixing term resembles Lorentz gauge fixing, but it has a different structure in the x^6 dimension. This term is necessary in order to decouple the equations of motion associated to the gauge action in a covariant gauge.

The 6D field equations that follows from the action (2) are

$$\frac{1}{\sqrt{-G}}\partial_{\hat{\mu}}\left(\sqrt{-G}G^{\hat{\mu}\hat{\rho}}G^{\hat{\nu}\hat{\delta}}F_{\hat{\rho}\hat{\delta}}\right)+\delta\mathscr{L}_{g.f.}=0. \tag{8}$$

The standard form to solve these equations of motion is to apply the separation of variables method. One proposes a separable solution of the gauge fields in the form $A_{\hat{\mu}}(x^{\nu},x^5,x^6)=A_{\hat{\mu}}(x^{\nu})\Phi(x^5)\Psi(x^6)$. Since we separate the 6D equations of motion in three parts, in principle we should have 2 separation constants, but due to the fact that the component A_6 behave in a different way respect to the components A_{μ} and A_5, we shall have tree different separation constants. The equations of motion for the fields depending on the x^6 coordinate are

$$\frac{d}{dx^6}\left(e^{-3\sigma(x^6)}\frac{d}{dx^6}\chi(x^6)\right)+m_{\alpha}^2e^{-\sigma(x^6)}\chi(x^6) = 0, \tag{9}$$

$$\frac{d}{dx^6}\left(e^{\sigma(x^6)}\frac{d}{dx^6}\left(e^{-3\sigma(x^6)}\xi(x^6)\right)\right)+m_6^2\xi(x^6) = 0, \tag{10}$$

where m_{α}^2 and m_6^2 are separation constants. The equations of motion for the fields depending on the fifth coordinate all have the same form and correspond to the ones of a scalar field in a circle

$$\left(\partial_5^2+\frac{m^2}{R^2}\right)\Phi(x^5)=0, \quad \text{subject to the condition} \quad \Phi(x^5+2\pi R)=\Phi(x^5). \tag{11}$$

In this equation m is also a separation constant. For the 4D fields the equations of motion are

$$\left(\partial_{\mu}\partial^{\mu}+\frac{m^2}{R^2}+m_{\alpha}^2\right)A_{\rho}(x^{\nu})=0, \qquad \left(\partial_{\mu}\partial^{\mu}+\frac{m^2}{R^2}+m_{\alpha}^2\right)A_5(x^{\nu})=0. \tag{12}$$

and

$$\left(\partial_{\mu}\partial^{\mu}+\frac{m^2}{R^2}+m_6^2\right)A_6(x^{\nu})=0. \tag{13}$$

Kaluza-Klein expansion

Let's discuss briefly the solution to the different equations of motion. By performing the change of variable $z\equiv\frac{m}{\kappa}e^{\sigma(x^6)}$ in equations (9) and (10) and $f(z)\equiv e^{-\frac{3}{2}\sigma(z)}\chi(z)$ in equation (9) and $g(z)\equiv e^{-\frac{5}{2}\sigma(z)}\xi(z)$ in equation (10), they can be rewritten as Bessel equations

$$z^2\frac{d^2f(z)}{dz^2}+z\frac{df(z)}{dz}+\left(z^2-\left(\frac{3}{2}\right)^2\right)f(z)=0, \tag{14}$$

$$z^2\frac{d^2g(z)}{dz^2}+z\frac{dg(z)}{dz}+\left(z^2-\left(\frac{1}{2}\right)^2\right)g(z)=0, \tag{15}$$

whose solutions are respectively Bessel and Neumann functions of order 3/2 and 1/2. In terms of the original functions χ and ξ, the solutions for $n \neq 0$ [2] are

$$\begin{aligned}\chi^n(x^6) &= e^{\frac{3}{2}kx^6}\left[a_1 J_{\frac{3}{2}}\left(\frac{m_{\alpha n}}{\kappa}e^{\kappa x^6}\right) + a_2 Y_{\frac{3}{2}}\left(\frac{m_{\alpha n}}{\kappa}e^{\kappa x^6}\right)\right], &(16)\\ \xi^n(x^6) &= e^{\frac{5}{2}kx^6}\left[b_1 J_{\frac{1}{2}}\left(\frac{m_{6n}}{\kappa}e^{\kappa x^6}\right) + b_2 Y_{\frac{1}{2}}\left(\frac{m_{6n}}{\kappa}e^{\kappa x^6}\right)\right]. &(17)\end{aligned}$$

From the Sturm-Liouville theory we know that the eigenfunctions χ^n and ξ^n form complete sets and satisfy the orthonormality relations

$$\frac{1}{\pi r}\int_0^{\pi r} dx^6 e^{-kx^6}\chi^n(x^6)\chi^m(x^6) = \delta_{nm}, \tag{18}$$

$$\frac{1}{\pi r}\int_0^{\pi r} dx^6 e^{-3kx^6}\xi^n(x^6)\xi^m(x^6) = \delta_{nm}. \tag{19}$$

We want to stress here that the solution to the differential equation (9) is given in terms of Bessel functions of order $3/2$, in contrast to the standard 5D situation, where the order of the analogous Bessel functions is 1 [8, 9, 7]. Also notice that solutions to the differential equation (10) are Bessel functions of order 1/2, instead of order 0, as is the case for the gauge action in the standard 5D RSI model [7, 10]. The reason for this difference is the extra compact dimension x^5. This situation is very similar to the one occurring for a scalar field, where each warped extra compact dimension changes the order of the Bessel functions by $1/2$ [11, 12].

The solution to the equations of motion (11) are well known, they correspond to

$$\Phi(x^5) \approx e^{i\frac{m}{R}x^5} \quad \text{with } m \text{ an integer number.} \tag{20}$$

These solutions satisfy the orthogonality conditions

$$\frac{1}{2\pi R}\int_0^{2\pi R} dx^5 e^{i\frac{x^5}{R}(n-m)} = \delta_{nm}. \tag{21}$$

Putting the solutions together, the Kaluza-Klein expansion of the 6D gauge fields $A_{\hat{\mu}}$ are given by

$$\begin{aligned}A_\mu(x,x^5,x^6) &= \sum_{n=0}^{\infty}\sum_{m=-\infty}^{\infty} A_\mu^{(m,n)}(x)\, e^{i\frac{m}{R}x^5}\,\chi^{(n)}(x^6),\\ A_5(x,x^5,x^6) &= \sum_{n=0}^{\infty}\sum_{m=-\infty}^{\infty} A_5^{(m,n)}(x)\, e^{i\frac{m}{R}x^5}\,\chi^{(n)}(x^6),\\ A_6(x,x^5,x^6) &= \sum_{n=1}^{\infty}\sum_{m=-\infty}^{\infty} A_6^{(m,n)}(x)\, e^{i\frac{m}{R}x^5}\,\xi^{(n)}(x^6),\end{aligned}$$

where the 4D fields $A_\mu^{(m,n)}(x)$ and $A_5^{(m,n)}(x)$ satisfy the equations of motion (12) with m_α replaced by the eigenvalues $m_{\alpha n}$ and $A_6^{(m,n)}(x)$ satisfies the equation of motion (13)

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[2] The zero mode χ^0 can be obtained easily from equation (9) setting $m_\alpha = 0$.

with m_6 replaced by the eigenvalues m_{6n}. Substituting the Kaluza-Klein expansion of the gauge fields into the action (2) we obtain the 4D effective action

$$\begin{aligned} S_{4D} &= -\frac{1}{4}(\pi r)(2\pi R)\sum_{n=0}^{\infty}\sum_{m=-\infty}^{\infty}\int d^4x \left[F_{\mu\nu}^{(n,m)}F^{\mu\nu(n,-m)} + 2\partial_\mu A^{\mu(n,m)}\partial_\nu A^{\nu(n,-m)}\right. \\ & \qquad \left. -2\left(\frac{m^2}{R^2}+m_{\alpha n}^2\right)A_\mu^{(n,m)}A^{\mu(n,-m)}\right] \\ &+ \frac{1}{2}(\pi r)(2\pi R)\sum_{n=0}^{\infty}\sum_{m=-\infty}^{\infty}\int d^4x\left[\partial_\mu A_5^{(n,m)}\partial^\mu A_5^{(n,-m)} - \left(\frac{m^2}{R^2}+m_{\alpha n}^2\right)A_5^{(n,m)}A_5^{(n,-m)}\right] \\ &- \frac{1}{2}(\pi r)(2\pi R)\sum_{n=1}^{\infty}\sum_{m=-\infty}^{\infty}\int d^4x\left[\partial_\mu A_6^{(n,m)}\partial^\mu A_6^{(n,-m)} - \left(\frac{m^2}{R^2}+m_{6n}^2\right)A_6^{(n,m)}A_6^{(n,-m)}\right] \end{aligned}$$

CONCLUSIONS

In this work we have obtained the 4D action that arises from the 6D free gauge action in a Randall-Sundrum metric extended by one compact extra dimension. We obtained the Kaluza-Klein spectrum. Regarding the eigenfunctions in the coordinate direction of the S^1/Z_2 orbifold, we obtained a different order for the Bessel and Neumann functions respect to the same problem in the standard 5D Randall-Sundrum scenario. The obtained results here, can be directly generalized to the Yang-Mills case and therefore to the Gauge-Higgs Unification scenario in this background [6].

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