On the Condensed Matter Analog of Baryon Chiral Perturbation Theory

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Abstract. It is shown that baryon chiral perturbation theory, i.e., the low-energy effective theory for pions and nucleons in quantum chromodynamics, has its condensed matter analog: A low-energy effective theory describing magnons as well as holes (or electrons) doped into antiferromagnets. We briefly present a symmetry analysis of the Hubbard and t-J-type models, and review the construction of the leading terms in the effective Lagrangian. As a nontrivial application we study different phases of hole- and electron-doped antiferromagnets - in particular, we investigate whether a so-called spiral phase with an inhomogeneous staggered magnetization (order parameter) may be stable. We would like to emphasize that the effective theory is universal and makes model-independent predictions for a large class of systems, whereas the material-specific properties enter the effective theory only through the numerical values of a few low-energy parameters.

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MOTIVATION

Chiral perturbation theory (CHPT), the effective theory of quantum chromodynamics, is well-established by now [1, 2]. It operates in the Goldstone boson sector of QCD, taking into account the octet of the pseudoscalar mesons that emerge due to the spontaneously broken chiral symmetry. Baryon chiral perturbation theory [3, 4, 5] is an extension of CHPT as it also includes heavy particles such as the baryon octet or decuplet, i.e. those degrees of freedom that remain massive in the chiral limit.

Spontaneous symmetry breaking is also a very common phenomenon in condensed matter physics: Magnons e.g. are the Goldstone bosons of a spontaneously broken spin symmetry $SU(2)_s \to U(1)_s$, while phonons emerge due to a spontaneously broken translation symmetry, to name just a few. For these systems effective Lagrangians have been constructed and various applications have been considered [6, 7, 8].

However, the incorporation of heavy degrees of freedom in the condensed matter domain – in a transparent and systematic manner – has only been achieved recently: An effective theory for weakly hole- and electron-doped antiferromagnets, the analog of baryon chiral perturbation theory, has been constructed in Refs.[9, 10, 11]. The choice of this system is well-motivated as it corresponds to one of the hot topics in condensed matter physics: Quasi-two-dimensional antiferromagnets that turn into high-temperature superconductors upon doping. Although this phenomenon has been discovered more than twenty years ago [12], a thorough theoretical understanding is still missing.

The reason may be readily identified: Due to the nonperturbative nature of the problem, analytic studies usually suffer from uncontrolled approximations, while numerical simulations of the microscopic Hubbard and t-J-type models suffer from a severe fermion sign problem away from half-filling. As we will see, the effective theory approach allows us to gain insight into the physics of these insulating precursors of high-temperature superconductors in a systematic and unambiguous manner.

SYMMETRY ANALYSIS OF THE HUBBARD MODEL

The effective Lagrangian framework is based on a symmetry analysis of the underlying theory, i.e. the Hubbard or t-J-type models in our case, which are believed to be minimal models for high-temperature superconductors [13].

The Hubbard Hamiltonian defined on a square lattice is given by

$$\begin{aligned} H = \quad & -t \sum_{x,i} (c^\dagger_{x\uparrow} c_{x+\hat{i}\uparrow} + c^\dagger_{x+\hat{i}\uparrow} c_{x\uparrow} + c^\dagger_{x\downarrow} c_{x+\hat{i}\downarrow} + c^\dagger_{x+\hat{i}\downarrow} c_{x\downarrow}) \\ & + U \sum_x c^\dagger_{x\uparrow} c_{x\uparrow} c^\dagger_{x\downarrow} c_{x\downarrow} - \mu \sum_x (c^\dagger_{x\uparrow} c_{x\uparrow} + c^\dagger_{x\downarrow} c_{x\downarrow} - 1), \end{aligned} \tag{1}$$

where t is the hopping parameter between nearest neighbors, U is the screened on-site Coulomb repulsion and μ is the chemical potential for fermion number. This Hamiltonian exhibits the following global and local symmetries:

$$\begin{aligned} SU(2)_s \quad &: \quad \text{Global spin rotation} \\ U(1)_Q \quad &: \quad \text{Fermion number conservation} \\ D_i \quad &: \quad \text{Displacement by one lattice spacing in the i-direction} \\ O \quad &: \quad \text{90 degrees rotation} \\ R \quad &: \quad \text{Reflection on a lattice axis} \\ T \quad &: \quad \text{Time reversal} \end{aligned} \tag{2}$$

Whereas the global spin symmetry $SU(2)_s$ is spontaneously broken, giving rise to two antiferromagnetic magnons, the fermion number symmetry remains unbroken until one reaches the superconducting phase.

CONSTRUCTION OF EFFECTIVE FIELD THEORIES

In the effective theory framework one writes down the most general Lagrangian, formulated in terms of the effective fields, consistent with all the symmetries of the underlying theory, order by order in a systematic derivative expansion [1, 2].

Effective field theory for antiferromagnetic magnons

In the pure Goldstone boson sector, the two magnons living in the coset $S^2 = SU(2)_s/U(1)_s$ may be parametrized by a three-dimensional unit-vector

$$\vec{e}(x) = \big(e_1(x), e_2(x), e_3(x)\big)\,, \tag{3}$$

with $x = (x_1, x_2, t)$. The low-energy physics is then described by the nonlinear-σ model, which represents the leading order effective Lagrangian for an antiferromagnet

$$\mathscr{L}_{magnon} = \frac{\rho_s}{2}(\partial_i \vec{e} \cdot \partial_i \vec{e} + \frac{1}{c^2}\partial_t \vec{e} \cdot \partial_t \vec{e}) + \cdots. \tag{4}$$

The low-energy parameters ρ_s (spin stiffness) and c (spin-wave velocity) are not determined by the symmetries and have to be extracted from numerical simulation or experiment. Note that, at leading order in the derivative expansion, antiferromagnetic magnons display a relativistic dispersion law.

Another important ingredient that will allow us to couple the Goldstone bosons to holes and electrons is the nonlinear realization of the spontaneously broken $SU(2)_s$ symmetry – for the detailed construction we refer the reader to Refs.[9, 10]. The essential point is that this construction is completely systematic and fully determined by the structure of the two groups $G = SU(2)_s$ and $H = U(1)_s$ involved in the spontaneous symmetry breaking. The outcome is that the holes and electrons are "charged" under the local $U(1)_s$ symmetry and transform with the nonlinear transformation $h(x)$

$$u(x)' = h(x)u(x)g^\dagger, \qquad h(x) \in U(1)_s, \qquad u_{11}(x)' \geq 0\,. \tag{5}$$

The field $u(x) \in SU(2)_s$ is a matrix-valued parametrization of the two magnons and serves to define yet another representation $v_\mu(x)$ of the magnon field which is the one that couples to the charge carriers. This anti-Hermitean field is defined by

$$v_\mu(x) = u(x)\partial_\mu u(x)^\dagger = i\begin{pmatrix} v^3_\mu(x) & v^+_\mu(x) \\ v^-_\mu(x) & -v^3_\mu(x) \end{pmatrix}, \qquad \mu \in \{1, 2, t\}\,, \tag{6}$$

and decomposes into an Abelian "gauge" field $v^3_\mu(x)$ and two "charged" vector fields $v^\pm_\mu(x)$.

Effective field theory for holes and electrons

The structure of the effective Lagrangian to be constructed here crucially depends on the location of the hole and electron pockets in the first Brillouin zone. Angle-resolved photoemission spectroscopy as well as numerical simulations reveal that the location of the hole pockets is different from the one of the electron pockets: While holes reside in pockets centered at lattice momenta $\vec{k} = \pm(\frac{\pi}{2a}, \pm\frac{\pi}{2a})$, electrons appear at $\vec{k} = (\frac{\pi}{a}, 0)$ and $\vec{k} = (0, \frac{\pi}{a})$. Interestingly, hole pockets show up in two independent "flavors" α and β, whereas for electrons no such distinction arises.

Among the transformation laws of the Grassmann fields $\psi^f_\pm(x)$ for holes of flavor f and spin projection $s = \pm$ (parallel or antiparallel with respect to the local staggered magnetization), the transformation under 90 degrees rotation is of particular interest,

$$O: \quad {}^O\psi^\alpha_\pm(x) = \mp\psi^\beta_\pm(Ox)\,, \qquad {}^O\psi^\beta_\pm(x) = \psi^\alpha_\pm(Ox)\,, \tag{7}$$

as it allows an interaction term in the effective hole-magnon Lagrangian that is absent in the case of electron doping. The leading order effective Lagrangian for magnon and hole fields, respecting all the symmetries (2) of the underlying Hubbard model, reads

$$\begin{aligned}\mathscr{L} = \mathscr{L}_{magnon} + \sum_{f=\alpha,\beta;s=+,-} \Big[& M\psi^{f\dagger}_s\psi^f_s + \psi^{f\dagger}_s D_t\psi^f_s \\ & + \tfrac{1}{2M'}D_i\psi^{f\dagger}_s D_i\psi^f_s + \sigma_f\tfrac{1}{2M''}\big(D_1\psi^{f\dagger}_s D_2\psi^f_s + D_2\psi^{f\dagger}_s D_1\psi^f_s\big) \\ & + \Lambda\big(\psi^{f\dagger}_s v^s_1\psi^f_{-s} + \sigma_f\psi^{f\dagger}_s v^s_2\psi^f_{-s}\big) \\ & + N_1\psi^{f\dagger}_s v^s_i v^{-s}_i\psi^f_s + \sigma_f N_2\big(\psi^{f\dagger}_s v^s_1 v^{-s}_2\psi^f_s + \psi^{f\dagger}_s v^s_2 v^{-s}_1\psi^f_s\big)\Big]\,,\end{aligned} \tag{8}$$

with

$$D_\mu\psi^f_\pm(x) = \Big[\partial_\mu \pm iv^3_\mu(x)\Big]\psi^f_\pm(x)\,, \qquad \sigma_\alpha = +1, \quad \sigma_\beta = -1\,. \tag{9}$$

Remember that the field $v^\pm_i(x)$ describing the magnons contains a spatial derivative: Indeed, the magnons are derivatively coupled to the holes. The various low-energy constants are the rest mass M and the two kinetic masses M' and M'' of the hole, the hole-one-magnon coupling Λ, and the two hole-two-magnon couplings N_1 and N_2.

Contrary to the hole fields, electron fields do not exhibit a flavor index and the leading order effective Lagrangian for magnon and electron fields is given by

$$\begin{aligned}\mathscr{L} = \mathscr{L}_{magnon} + \sum_{s=+,-} \Big[& M\psi^\dagger_s\psi_s + \psi^\dagger_s D_t\psi_s + \frac{1}{2M'}D_i\psi^\dagger_s D_i\psi_s + N\psi^\dagger_s v^s_i v^{-s}_i\psi_s \\ & + iK\big(D_1\psi^\dagger_s v^s_1\psi_{-s} - \psi^\dagger_s v^s_1 D_1\psi_{-s} - D_2\psi^\dagger_s v^s_2\psi_{-s} + \psi^\dagger_s v^s_2 D_2\psi_{-s}\big)\Big]\,.\end{aligned} \tag{10}$$

Note the difference with respect to the effective Lagrangian for holes: In the absence of a flavor index, a Λ-like term is no longer allowed, such that the leading interaction terms in derivative expansion, proportional to the coupling constants K and N, contain two derivatives. Already at this point we may anticipate that the interaction between electrons and magnons is thus weaker than the one between holes and magnons.

SPIRAL PHASES IN WEAKLY DOPED ANTIFERROMAGNETS

As a nontrivial application we now investigate whether so-called spiral phases in the staggered magnetization can occur upon hole- or electron-doping. While this problem has already been studied before using microscopic theories [14, 15, 16] for the first time has it become accessible to a systematic analysis using effective Lagrangians.

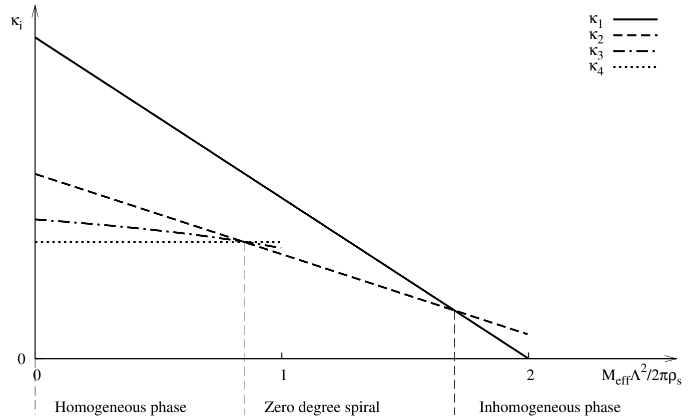

FIGURE 1. *The compressibilities κ_i in the presence of weak repulsive four-fermion couplings as functions of $M_{eff}\Lambda^2/2\pi\rho_s$ (with $M_{eff} = M'M''/\sqrt{M''^2 - M'^2}$) determine the stability ranges of the various phases. The index i denotes the number of hole pockets that are populated.*

In our study, we assumed that the fermions are doped into the system homogeneously and thus we limited ourselves to configurations of the staggered magnetization that are either homogeneous themselves or generate a constant background field $v_i(x)$ for the charge carriers. We also assumed that four-fermion contact interactions – not displayed in Eqs.(8) and (10) – are weak and took them into account perturbatively – for details see Refs.[11, 17].

First we showed that the configuration of the staggered magnetization $\vec{e}(x)$, leading to a uniform background field $v_i(x)$, is either homogeneous itself or a spiral. Indeed, in the case of hole doping a rich phase diagram emerges as seen in Fig.1: A homogeneous phase, a zero-degree spiral oriented along a lattice axis (see Fig.2) and a 45^o spiral oriented along a lattice diagonal are energetically favorable for large, intermediate and small values of ρ_s, respectively. At two isolated points, separating the three regimes, yet another 45^o spiral is degenerate with the other phases. For very small values of ρ_s the system is unstable towards a yet-unidentified inhomogeneous phase. In contrast to hole doping, spiral phases are not energetically favorable in electron-doped systems: The electron-magnon interaction is too weak to stabilize a spiral.

CONCLUSIONS

The effective theory for the insulating antiferromagnetic precursors of high-temperature superconductors is the condensed matter analog of baryon chiral perturbation theory: Magnons play the role of the pseudoscalar mesons, while the holes and electrons are the analog of the baryon octet. We analyzed the symmetries of the underlying Hubbard model and outlined the systematic construction of the effective low-energy field theory for weakly hole- and electron-doped antiferromagnets. The procedure is fully systematic order by order in a derivative expansion of the magnon and charge carrier fields.

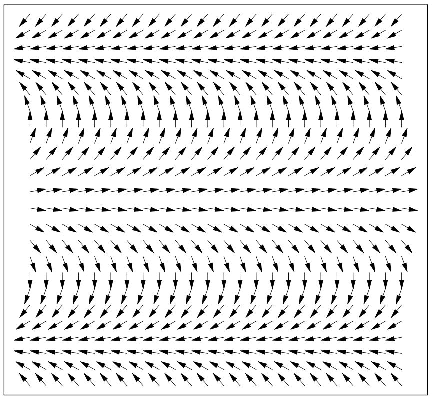

FIGURE 2. *A zero-degree spiral oriented along a lattice axis.*

As a nontrivial application we have considered the possible existence of spiral phases and found that, in some parameter regime, spirals of different orientation do exist for hole-doped antiferromagnets, whereas spiral phases are absent for electron doping. Remarkably, this result unambiguously follows from the very few basic assumptions of the systematic low-energy effective theory, such as symmetry, locality, and unitarity.

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