New Non-Trivial Vacuum Structures in Supersymmetric Field Theories [1]

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Abstract. In this talk, we present three examples of new non-trivial vacuum structures that can occur in supersymmetric field theories, along with explicit models in which they arise. The first forms the basis of a metastable SUSY-breaking scenario in which no flat directions or runaways appear in the classical potential and no non-perturbative physics is required for vacuum stability. The second consists of large (and even infinite) towers of metastable vacua which exhibit a rich set of instanton-induced vacuum tunneling dynamics. The third consists of an infinite number of degenerate vacua and gives rise to a Bloch-wave ground state and a vacuum "band" structure.

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INTRODUCTION

The vacuum structure of any physical theory plays a significant and often crucial role in determining the physical properties of that theory. In this talk, we describe three hitherto-unexplored and highly non-trivial vacuum structures which can arise in supersymmetric field theories. The first scenario we present [1] is an example of a metastable SUSY-breaking model in which all relevant features arise at tree-level in a completely calculable, perturbative framework. These include a supersymmetric, R-symmetry-preserving ground state; a metastable state in which both SUSY and R-symmetry are broken; and a vacuum energy barrier between the two of a sort that results in a long lifetime for the metastable vacuum. Neither the ground state nor the metastable vacuum involve runaways or flat directions; moreover, the salient features of the vacuum potential are perturbative and robust against quantum corrections, and the lifetimes of metastable vacua can be calculated reliably. As far as we are aware, the model we present is the first such model with these properties presented in the literature. The second vacuum structure [2] we present consists of large (and even infinite) towers of metastable vacua, each with a distinct vacuum energy and particle spectrum. We present a series of models which realize this vacuum structure explicitly. As the number of vacua grows towards infinity in such models, the energy of the highest vacuum stays fixed while the energy of the ground state tends towards zero. The instanton-induced tunneling dynamics associated with such vacuum towers results in a variety of distinct decay patterns; these include not only regions of vacua experiencing direct collapses and/or tumbling cascades, but also

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TABLE 1. The classical vacuum structure of the metastable SUSY-breaking model with $(\lambda, m, \xi_a, \xi_b, g) = (1, 1, 5, 0, 1)$. Here Solutions A and B correspond to stable vacua, while C corresponds to a saddle point. For each of these solutions, we have listed the corresponding field VEVs (v_1, v_3, v_5) (each solution has $v_2 = v_4 = 0$), along with the value of the scalar potential, the stability, SUSY, and R-symmetry properties of that extremum, and the surviving (unbroken) gauge group. All dimensionful quantities are quoted in dimensionless units.

Label	(v_1, v_3, v_5)	V	**Stability**	**SUSY**	**R-symmetry**	**Gauge Group**
A	$(\sqrt{5}, 0, 0)$	0	Stable	Yes	Yes	$U(1)_b$
B	$(0, 2, 2)$	$9/2$	Metastable	No	No	None
C	$(\sqrt{3}/2, \sqrt{7}/2, \sqrt{5/2})$	$45/8$	Unstable	No	No	None

other regions of vacua whose stability is protected by "great walls" as well as regions of vacua populating "forbidden cities" into which tunneling cannot occur. The third vacuum structure we discuss [2, 3] arises as a limiting case of the previous scenario. In this limit, all of the metastable vacua in a given vacuum tower become degenerate, and a shift symmetry emerges relating one vacuum to the next. The true ground states of such theories are Bloch waves across these degenerate ground states, with energy eigenvalues approximating a continuum and giving rise to a vacuum "band" structure.

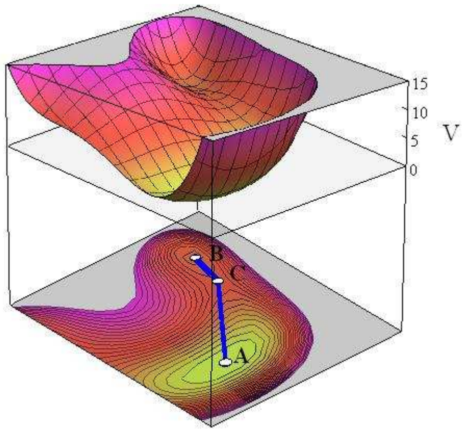
In this talk, we shall merely present the three structures and briefly sketch some of these applications; further details can be found in Refs. [1, 2, 3] and references therein.

I. TREE-LEVEL METASTABLE SUPERSYMMETRY-BREAKING

The first model we discuss is a simple one which can be used as a "kernel" for developing more complete models of metastable SUSY-breaking. This model consists of two $U(1)$ gauge groups, $U(1)_1$ and $U(1)_2$ with couplings $g_1 = g_2 = g$, and five chiral superfields Φ_i, $i = 1, \ldots, 5$, with charge assignments $(-1, 0)$, $(1, -1)$, $(0, 1)$, $(1, 1)$, and $(-1, -1)$ respectively. Given these charges, the most general renormalizable superpotential is given by $W = \lambda\,\Phi_1\Phi_2\Phi_3 + m\Phi_4\Phi_5$. We shall also generally permit non-zero Fayet-Iliopoulos terms ξ_a for each $U(1)_a$ gauge group. The scalar potential of the theory includes both F-term and D-term contributions and can be written in the form

$$V \;=\; \tfrac{1}{2}\sum_a g_a^2 D_a^2 + \sum_i |F_i|^2 \tag{1}$$

where $D_a = \xi_a + \sum_i Q_{ai}|\phi_i|^2$ (with ϕ_i the scalar component of Φ_i), where $F_i \equiv -\partial W^*/\partial\phi_i^*$, and where Q_{ai} is the $U(1)_a$-charge of Φ_i. For concreteness here, however, let us focus on the special case $(\lambda, m, \xi_a, \xi_b, g) = (1, 1, 5, 0, 1)$. We then find that the scalar potential V has three relevant critical points, which are shown in Table 1. Solution A represents a stable, supersymmetric, R-symmetry-preserving ground state with vanishing vacuum energy, while Solution B represents a metastable minimum in which SUSY and R-symmetry are both broken. Solution C represents the saddle-point solution through which the classical path between the two vacua passes. This vacuum structure is shown explicitly in Fig. 1.



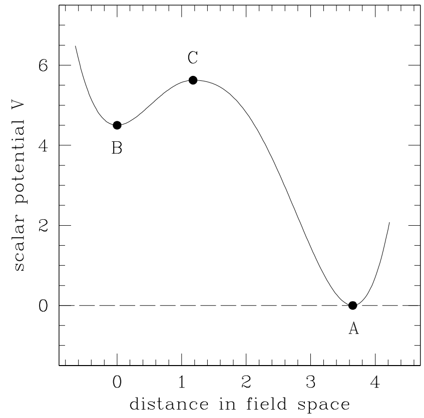


FIGURE 1. *Left figure*: A surface plot of the scalar potential V evaluated on the unique two-dimensional plane within the three-dimensional (v_1, v_3, v_5) field space which simultaneously contains the true vacuum solution A the metastable vacuum solution B, and the saddle-point solution C between them. Projected below the surface plot is a contour plot for V, showing the shortest path (blue) in field space connecting these three solutions. *Right figure*: The scalar potential V evaluated along this shortest path. Field-space distances are quoted relative to the metastable vacuum B along this path in dimensionless units.

In order to be of phenomenological interest for model-building, the lifetime of any given metastable vacuum must be at least on the order of the present age of the universe. This lifetime can be determined using standard instanton methods, and it can be shown that the metastable minimum in our model is stable on cosmological time scales over a large region of parameter space, including the point $(\lambda, m, \xi_a, \xi_b, g) = (1, 1, 5, 0, 1)$ discussed above. These calculations are discussed more fully in Ref. [1].

II. METASTABLE VACUUM TOWERS

The second scenario [2] we discuss involves a class of models which give rise to towers of non-degenerate, metastable vacua. The underlying structure of these models is that of an N-site orbifold Abelian moose consisting of N different $U(1)$ gauge groups with a common coupling g and $N+1$ chiral superfields Φ_i. To this structure, we then add three ingredients, each of which is vital for the emergence of our metastable vacuum towers. The first is a single Wilson-line operator $W = \lambda \, \Phi_1 \Phi_2 \Phi_3 \ldots \Phi_{N+1}$, which represents the most general superpotential that can be formed from the fields of the theory. The second is a pair of non-zero Fayet-Iliopoulos terms ξ_1 and ξ_N for the gauge groups $U(1)_1$ and $U(1)_N$ respectively. The third is to permit kinetic mixing among the various $U(1)$ factors in the theory, thereby modifying the gauge-kinetic part of the Lagrangian to include a mixing matrix X_{ab}:

$$\mathscr{L} \ni \frac{1}{32} \int d^2\theta \;\; W_{a\alpha} X_{ab} W_b^{\alpha} \qquad \text{where} \;\; X_{ab} \equiv \begin{pmatrix} 1 & -\chi_{12} & \cdots & -\chi_{1N} \\ -\chi_{12} & 1 & \cdots & -\chi_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ -\chi_{1N} & -\chi_{2N} & \cdots & 1 \end{pmatrix} . \quad (2)$$

To simplify our analysis, we will focus on the case in which mixing occurs only between nearest-neighbor sites on the moose, with a common mixing parameter χ (*i.e.*, $\chi_{ab} = \chi\delta_{a,b+1}$) in the range $0 < \chi < 1/2$. We shall also take $\xi_1 = \xi_N \equiv \xi$ for simplicity.

We find that the vacuum structure of this model contains $N-1$ stable vacua, which we label with an index $n = 1, \cdots, N-1$ in order of decreasing vacuum energy. The vacuum energy of the n-vacuum is given by

$$V_n \;=\; \tfrac{1}{2}\left(\frac{1}{\chi R_n}\right) \qquad \text{where } \; R_n \;\equiv\; \left(\frac{1}{\chi}-2\right)n+2\,, \tag{3}$$

and corresponds to the solution with

$$v_j^2 = \begin{cases} 1+1/R_n & \text{for } j=1 \\ 1/R_n & \text{for } 2\le j\le N-n \\ 0 & \text{for } j=N-n+1 \\ (R_{j-N+n-1}-1)/R_n & \text{for } N-n+2\le j\le N \\ 0 & \text{for } j=N+1 \end{cases} \tag{4}$$

where we continue to list all quantities in dimensionless units and where all dependence on the overall coupling g has been rescaled away. This vacuum tower is illustrated for the $N = 20$ case in Fig. 2.

As required, these vacua are separated from one another by saddle-point solutions in field space. The general expressions for the field-space coordinates and barrier heights associated with these saddle points are generally quite complicated, but they are controlled primarily by the Wilson-line coefficient λ; consequently, so is the stability of any given vacuum state in the tower. We find that for any N, the n-vacuum is stable as long as λ exceeds the critical value

$$\lambda^{*2}_{N,n} \;=\; y^n \,\frac{\Gamma(y)}{\Gamma(n+y)}\,\frac{R_n^{N-2}}{\chi(1+R_n)}\,. \tag{5}$$

Here $y \equiv \chi/(1-2\chi) = n/(R_n-2)$ and $\Gamma(z)$ is the Euler Γ-function. Consequently, all of the vacua in the tower will be stable as long as λ exceeds the maximum value of $\lambda^*_{N,n}$. Moreover, there is nothing which prevents us from taking $N\to\infty$ in all of our results. As a result, we see that we can achieve a vacuum structure containing literally an infinite tower of metastable vacua.

Instanton-induced tunneling can produce a variety of highly non-trivial vacuum decay patterns in such vacuum towers. These include regions of vacua experiencing direct "collapses" (in which a given metastable vacuum decays directly to the ground state) and/or tumbling "cascades" (in which a given metastable vacuum decays to another metastable vacuum, and so forth). There are also other regions of vacua whose stability is protected by "great walls" as well as regions of vacua populating "forbidden cities" into which tunnelling cannot occur. One example [2] which illustrates all of these features simultaneously is shown in Fig. 3. In this specific example, there are four independent potential cascade trajectories, each of which unfolds with increasing speed. It is only an initial condition that determines which trajectory a given system ultimately follows.

Clearly, there are a number of possible applications for a vacuum structure of this sort, one of which concerns a potential solution to the cosmological-constant problem. Over

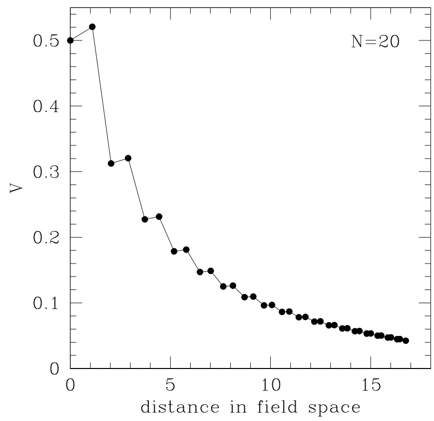


FIGURE 2. The vacuum structure of the $N = 20$ model, plotted for $\chi = 1/5$. This model gives rise to a tower of 18 metastable vacua above the true ground state. Vacuum energy is plotted on the vertical axis, while the horizontal axis indicates the cumulative distances in field space along a trajectory which begins at the $n = 1$ vacuum and then proceeds along straight-line path segments to the $(1,2)$ saddle point, then to the $n = 2$ vacuum, then to the $(2,3)$ saddle point, and so forth.

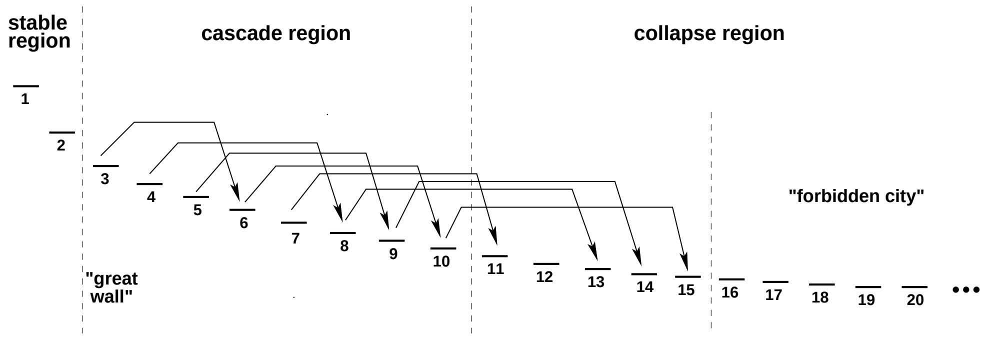


FIGURE 3. A schematic of the vacuum cascade dynamics that arises in a model with $N = 5000$ and $\chi = 2.8 \times 10^{-4}$. Vacua in the stable region to the left of the "Great Wall" have lifetimes exceeding the age of the universe, while vacua in the cascade region decay to other (lower) metastable vacua in the vacuum tower. Vacua in the collapse region decay directly to the ground state of the vacuum tower. Vacua which populate the "Forbidden City" cannot be reached from outside and can be populated only as an initial condition.

the past decade, several scenarios have been proposed in which a small cosmological constant emerges as a consequence of a large number of vacua (for references, see [2]). Such scenarios tend to posit a "landscape" of vacua with certain gross properties, including a vacuum state whose energy is nearly vanishing. It is our hope that the model we have presented here might provide an explicit field-theoretic realization of such a scenario. Another potential implication of such a vacuum structure concerns statistical studies of the string landscape, and in particular the proper definition of a *measure* across that landscape. The most naïve definition of measure is to count each string model equally, interpreting each as contributing a single vacuum state to the landscape as a whole. However, moose theories of the sort we have been discussing here often appear as the low-energy (deconstructed) limits of flux compactifications, and as we have seen, such theories give rise to infinite towers of metastable vacua. Thus, if the true underlying landscape measure is based on *vacua* rather than *models*, then a theory with infinite towers of vacua is likely to dominate any statistical study of the string landscape.

III. DEGENERATE VACUA AND BLOCH WAVES

The results presented in Sect. II concerning our N-site moose are applicable in the range $0 < \chi < 1/2$, for which each successive vacuum in the resulting vacuum tower has a lower energy than the previous one, and there exists a net direction for dynamical flow. However, for $\chi \to 1/2$, a new behavior develops: in this case, $R_n = 2$ for all n, and the expressions for the vacuum energies V_n become independent of n. As a result, the vacuum structure consists of $N-1$ *degenerate* vacua separated from each other by a set of equivalent saddle-point potential barriers of uniform height. The field-space distances between all pairings of vacua also become equal in this case. As a result, the true ground state of such a theory is no longer any of the individual n-vacua by itself. Instead, what emerges is an infinite set of *Bloch waves* across the entire set of degenerate vacua, the vacuum energies of which fill out a continuous band.

The vacuum energy of the true ground state of the theory is consequently smaller than that of any individual n-vacuum. It is interesting to speculate that the vacuum energy of this true ground state might actually vanish (thereby *restoring* SUSY) or alternatively merely *approach* zero as $N \to \infty$, yielding a a very small cosmological constant. Furthermore, regardless of the ground-state energy, our Bloch vacuum states are linear combinations of our individual n-vacua; thus a system originally populating a given n-vacuum will experience non-trivial *time-dependent oscillations* across the set of n-vacua as a whole — a consequence that will be explored more fully in Ref. [3].

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