Phase transition dynamics and gravitational waves

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Outline

Motivation

Phase transition dynamics

Gravitational waves from a first-order phase transition

Phase transition dynamics and gravitational waves

First-order phase transitions

Phase transition dynamics

- supercooling
- nucleation and expansion of bubbles
- bubble collisions
- departure form equilibrium

Possible consequences

- topological defects, magnetic fields
- baryogenesis, inhomogeneities
- cosmological constant
- gravitational waves (GWs)

Gravitational waves

from first-order phase transitions

- Since GWs propagate freely, they may provide a direct source of information about the early Universe.

The spectrum

- The characteristic wavelength of the gravitational radiation is determined by the characteristic length of the source.
- The characteristic length is the size of bubbles, which depends on the phase transition dynamics and the Hubble length H^{-1}.
- For the electroweak phase transition, the characteristic frequency, *redshifted to today*, is $\sim$ milli-Hertz.
- This is within the sensitivity range of the planned Laser Interferometer Space Antenna (LISA).

Motivation

Phase transition dynamics

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Phase transition dynamics and gravitational waves

Thermodynamics

The free energy

Thermodynamic quantities (ρ, p, s,...) are derived from the free energy density (finite-temperature effective potential).

Example:

A theory with a Higgs field and particle masses $m_i(\phi)$

$$\mathcal{F}(\phi, T) = V_0(\phi) + V_{\text{1-loop}}(\phi, T),$$

$V_0(\phi) =$ tree-level potential

$V_{\text{1-loop}}(\phi, T) =$ zero-temperature corrections
$+$ finite-temperature corrections

The effective potential

$$\mathcal{F}(\phi, T) = V_0(\phi) + V_{\text{1-loop}}(\phi, T),$$

where

$$V_0(\phi) = -\tfrac{1}{2}\lambda v^2\phi^2 + \tfrac{1}{4}\lambda\phi^4$$

$$V_{\text{1-loop}}(\phi) = \sum \tfrac{\pm g_i}{64\pi^2}\left[m_i^4(\phi)\left(\log\left(\tfrac{m_i^2(\phi)}{m_i^2(v)}\right) - \tfrac{3}{2}\right) + 2m_i^2(\phi)m_i^2(v)\right]$$
$$+ \sum \tfrac{g_i T^4}{2\pi^2} I_{\mp}\left[\tfrac{m_i(\phi)}{T}\right]$$

with $\quad I_{\mp}(x) = \pm\int_0^\infty dy\, y^2 \log\left(1 \mp e^{-\sqrt{y^2+x^2}}\right)$

First-order phase transition

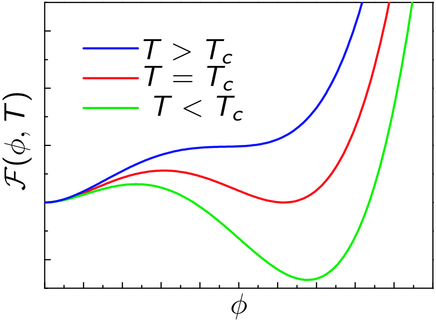


Figure: The free energy $\mathcal{F}(\phi, T)$ around the critical temperature

High T: $\phi = 0$ (*false vacuum*)
Low T: $\phi = \phi_m(T)$
(*true vacuum*)
$T_c =$ critical temperature

First-order phase transition

Thermodynamic quantities are different in each phase

$$T > T_c: \;\Rightarrow\; \mathcal{F}(\phi = 0, T) \equiv \mathcal{F}_+(T) \;\Rightarrow \rho_+, s_+, p_+, \ldots$$
$$T < T_c: \;\Rightarrow\; \mathcal{F}(\phi_m(T), T) \equiv \mathcal{F}_-(T) \;\Rightarrow \rho_-, s_-, p_-, \ldots$$

High-temperature phase $\phi = 0$

- Energy density: $\rho_+(T) = \rho_\Lambda + g_* \pi^2 T^4/30$
  $=$ false vacuum $+$ radiation

Low-temperature phase $\phi = \phi_m(T)$

- $\rho_-(T)$ depends on the effective potential

First-order phase transition

Discontinuities at $T = T_c$

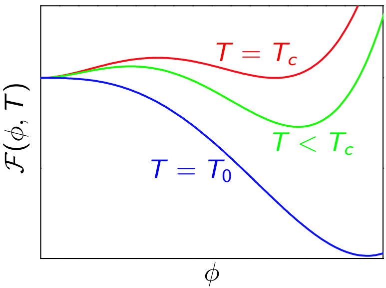
- At the critical temperature, $\mathcal{F}_+(T_c) = \mathcal{F}_-(T_c)$, but $\rho_+(T_c) > \rho_-(T_c)$.
- $L \equiv \rho_+(T_c) - \rho_-(T_c) =$ **latent heat**.

The latent heat

- L is released during bubble expansion.
- Should not be confused with ρ_Λ or $\Delta\mathcal{F}$.

Bubble nucleation

- During the adiabatic cooling of the Universe, the temperature T_c is reached.
- The system is in the $\phi = 0$ phase [i.e., $\phi(x) \equiv 0$].



- At $T < T_c$ bubbles of the stable phase [i.e., with $\phi = \phi_m$ inside] begin to nucleate in the supercooled $\phi = 0$ phase.
- At $T = T_0$ the barrier disappears. ($T_0 \sim T_c$.)

Bubble nucleation

Nucleation rate

Thermal tunneling probability per unit volume per unit time:

$$\Gamma \simeq T^4 e^{-S_3(T)/T}$$

$S_3\,(T)$ = three-dimensional instanton action
= free energy of the critical bubble

Γ is extremely sensitive to temperature:

- At $T = T_c, \quad \Gamma = 0 \qquad (S_3 = \infty)$
- At $T = T_0, \quad \Gamma \sim T^4 \quad (S_3 = 0)$
  - Nucleation becomes important as soon as $\Gamma \sim H^4$, and
  - $H^4 \sim (T^2/M_{\mathrm{Planck}})^4 \ll T^4$.

Bubble growth

- Once nucleated, bubbles expand until they fill all space.
- The velocity of bubble walls depends on several parameters.
  - **Pressure difference** $\Delta p = p_- - p_+$
    Depends on supercooling. (At $T = T_c$, $p_- = p_+$).
  - **Friction** of bubble wall with plasma
    Depends on microphysics (particles-Higgs interactions).
  - **Latent heat** $L = \rho_+ - \rho_-$ injected into the plasma.
    Causes reheating and fluid motions.
- Hydrodynamics allows two propagation modes:
  detonations and deflagrations.

Hydrodynamics

Detonations

- The phase transition front (bubble wall)
  moves faster than the speed of sound: $v_w > c_s$.
- No signal precedes the wall.
  It is followed by a rarefaction wave.
- A bubble wall does not influence other bubbles,
  except in the collision regions

Deflagrations

- The deflagration front is subsonic ($v_w < c_s$).
- The wall is preceded by a *shock wave* which moves
  at a velocity $v_{sh} \approx c_s$.
- Thus, it will influence other bubbles.

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Possible mechanisms

Bubble collisions

- The walls of expanding bubbles provide thin energy concentrations that move rapidly.

Turbulence

- In the early Universe, the Reynolds number is large enough to produce turbulence when energy is injected.

Magnetohydrodynamics (turbulence in a magnetized plasma)

- It develops in an electrically conducting fluid, in the presence of magnetic fields.

Cosmological turbulence

Kolmogoroff-type turbulence

- Energy is injected by a **stirring source** at a length scale L_S.
- Eddies of each size L break into smaller ones.
- When turbulence is fully developed, a cascade of energy is established from larger to smaller length scales.
- The cascade begins at the **stirring scale** L_S and stops at the **dissipation scale** $L_D \ll L_S$.
- Energy in the cascade is transmitted with a constant rate ε.
- For stationary turbulence, the **dissipation rate** ε equals the power that is **injected by the source**.

Cosmological turbulence

The energy spectrum

- Consider the velocity correlation tensor $\langle v_i(\mathbf{x}) v_j(\mathbf{y})\rangle$, where
  - $\mathbf{v}(\mathbf{x})$ = velocity of the fluid,
  - $\langle\cdots\rangle$ = statistical average.
- For stationary, homogeneous, isotropic turbulence, we have for the Fourier transform of v_i:

$$\langle v_i(\mathbf{k}) v_j^*(\mathbf{q})\rangle \propto \delta^3(\mathbf{k}-\mathbf{q}) \frac{E(k)}{k^2}\left(\delta_{ij} - \frac{k_i k_j}{k^2}\right),$$

  - $E(k)$ = turbulent energy density spectrum.
- For Kolmogoroff turbulence, $E(k) \propto \varepsilon^{2/3} k^{-5/3}$ for $L_D < L < L_S$ (with $k = 2\pi/L$).

Gravitational waves from turbulence

- The source for the tensor metric perturbation h_{ij} is the transverse and traceless piece of the stress-energy tensor T_{ij}.
- The relevant part of the stress-energy tensor for the relativistic fluid is

$$T_{ij}(\mathbf{x}) \propto v_i(\mathbf{x})\, v_j(\mathbf{x})\,.$$

- The energy density in GWs is

$$\rho_{GW} \sim \langle T_{ij} T_{ij} \rangle \sim \langle v_i v_j v_i v_j \rangle.$$

- The spectrum can be related to $\langle v_i v_j \rangle \sim E(k)$ (Kolmogoroff).

Gravitational waves from turbulence

The expansion of the Universe

- Can be neglected in the *production* of GWs
- Once produced, their wavelength scales
  with the scale factor a and their amplitude decays like a^{-1}.

The GW spectrum

- The spectrum is characterized by

$$\Omega_{GW}(f) = \frac{1}{\rho_c} \frac{d\rho_{GW}}{d \log f},$$

where $\rho_c =$ critical density.

- Peak frequency:

$$f_p = 1.6 \times 10^{-5} Hz \frac{T_*}{100\,GeV} \left(\frac{g_*}{100}\right)^{1/6} \frac{L_S^{-1}}{H_*}.$$

- Peak amplitude:

$$\Omega_{GW}(f_p) \approx \Omega_R \left(\frac{L_S}{H_*^{-1}}\right)^{10/3} \left(\frac{\varepsilon}{H_*}\right)^{4/3}$$

where $\Omega_R =$ radiation.
[Caprini & Durrer, PRD 74, 063521 (2006)]

GWs and phase transition dynamics

The spectrum $\Omega_{GW}(f)$ depends on:

- The temperature $T \sim T_c =$ critical temperature
- The stirring scale $L_S \sim$ size of bubbles
- The dissipation rate $\varepsilon =$ Injected power
  $\sim$ latent heat $\times$ bubble wall velocity

- The parameters depend on hydrodynamics
  (How bubble walls propagate in the fluid).

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GWs from detonations and deflagrations

Detonations (supersonic walls)

- The injected energy is concentrated in a thin region near the bubble wall. (Simpler calculations).
- The wall velocity is $v_w = v_w(\alpha)$, where $\alpha = L/\rho_{th}$ = *(latent heat)/(thermal energy)*
- The nucleation rate $\Gamma = e^{-S_3(T)/T}$ increases as temperature decreases with time.
- A Taylor expansion of the exponent gives $\Gamma = \Gamma_0 e^{\beta t}$.
- β^{-1} is the only time scale in the problem.
- It determines the duration of the phase transition $\Delta t \sim \beta^{-1}$ and the bubble size $d \sim v_w \beta^{-1}$.
- As a consequence, the spectrum of gravitational waves depends only on two parameters, α and β.

GWs from detonations and deflagrations

Deflagrations (subsonic walls)

- Calculations are more difficult:
- $v_w \sim \Delta p/\eta$
  - $\eta =$ friction coefficient
  - $\Delta p =$ pressure difference (depends on supercooling)
- Shock waves distribute the latent heat, causing reheating and bulk motions of the fluid far from the wall.
- The phase transition should be treated *globally*.

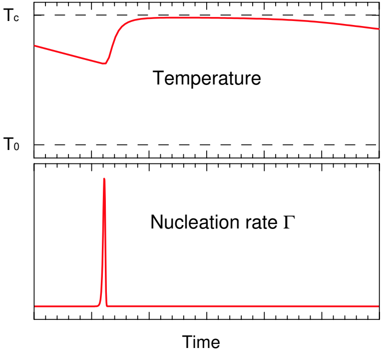
GWs from detonations and deflagrations

- Due to the difficulties of the deflagration case, calculations of the GW spectrum in specific models often **assume** that bubble walls propagate as **detonations**.
- The formulas for the detonation case (which depend only on α, β) are used.
  - For instance, to investigate GWs in the electroweak phase transition for different extensions of the Standard Model.
- However, the bubbles expand in general as **deflagrations**.
  - It is known that in the electroweak phase transition, $v_w \sim 10^{-2} - 10^{-1}$,
  - i.e., the walls are deflagrations ($v_w < c_s \approx 0.6$).

Global treatment of deflagration bubbles

Approximation for slow bubble walls:

- If $v_w \ll c_s$, the quick distribution of latent heat causes a *homogeneous* reheating (T depends only on t).
- Equations for $T(t), v_w(t), \ldots$ can be solved numerically. In general:



Relevant features:

- All bubbles nucleate in a short interval δt_Γ around the "initial" time $t \equiv t_\Gamma$.
- The bubble number density at $t = t_\Gamma$ determines the bubble size $d \sim n_b^{-1/3}$.
- Soon after $t = t_\Gamma$ the *shock waves* collide and turbulence starts.

Results

- Taking into account the general features of the dynamics, we can derive relations between ε, d, v_w,...
- and obtain analytical expressions [A.M., PRD 78, 084003 (2008)]

$$f_p \sim 10^{-2} mHz \left(\frac{T_c}{100 GeV} \right) \left(\frac{d}{H^{-1}} \right)^{-1},$$

$$\Omega_{GW}\big|_{\mathrm{peak}} \sim 10^{-4} (\alpha v_w)^{8/3} \left(\frac{d}{H^{-1}} \right)^{2}.$$

- For the electroweak phase transition at $T_c \sim 100 GeV$, we would need $d/H^{-1} \sim 10^{-2}$ so that $f_p \sim mHz$.
  - (In general, $10^{-5} \lesssim d \lesssim 10^{-1}$)

Results

- Then, for $d/H^{-1} \sim 10^{-2}$, $v_W \sim 0.1$ and $\alpha \sim 1$, we have $\Omega_{GW} \sim 10^{-11}$.

Detecting electroweak GWs at LISA

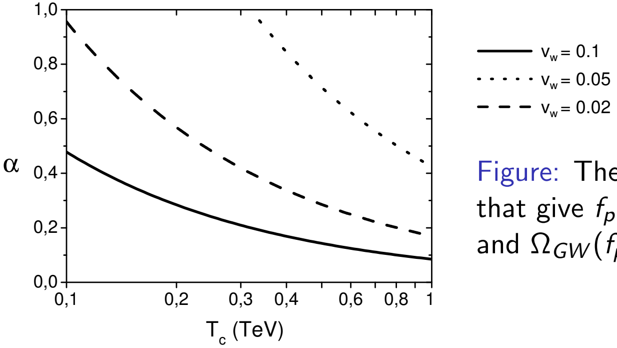


Figure: The values of α and T_c that give $f_p = 1mHz$ and $\Omega_{GW}(f_p) = 10^{-11}$.

Summary

- It is important to consider deflagrations as a source of GWs.
- The resulting amplitude may be comparable to the detonation case.

- Outlook
  - A complete numerical calculation is necessary to evaluate the quantities v_w, d, ... in specific models.
    [A.M. and A. Sánchez, work in progress]