Phase transition dynamics and gravitational waves

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Abstract. During a first-order phase transition, gravitational radiation is generated either by bubble collisions or by turbulence. For phase transitions which took place at the electroweak scale and beyond, the signal is expected to be within the sensitivity range of planned interferometers such as LISA or BBO. We review the generation of gravitational waves in a first-order phase transition and discuss the dependence of the spectrum on the dynamics of the phase transition.

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INTRODUCTION

An essential feature of the dynamics of first-order phase transitions is *supercooling*: the temperature must decrease below the critical temperature T_c for the phase transition to begin. Then, bubbles nucleate, expand and collide until they fill all space, causing a departure from the condition of (approximate) thermodynamic equilibrium that predominates throughout most of the history of the Universe. An important consequence of this dynamics is the formation of cosmological relics, in particular, gravitational waves (GWs). Once generated, gravitational radiation propagates freely (in contrast to electromagnetic radiation). Therefore, GWs may provide information about the early Universe.

The characteristic wavelength of the gravitational waves is determined by the characteristic length of the source. In the case of a phase transition, this is the size of bubbles, which depends on the dynamics of the phase transition and is a fraction of the Hubble length. For a given model, one can estimate the characteristic frequency of the GWs. It is interesting that, for the case of the electroweak phase transition, such estimations give a milli-Hertz frequency, which is within the sensitivity range of the planned interferometer LISA.

PHASE TRANSITION DYNAMICS

The most important quantity in finite-temperature field theory is the free energy density, or *finite-temperature effective potential*. All the thermodynamical quantities (e.g., the energy density, pressure, entropy, etc.) are derived from it. As an example, consider a theory with a single Higgs field ϕ and particles with ϕ-dependent masses $m_i(\phi)$.

Then, the free energy $\mathscr{F}(\phi, T)$ is given by the tree-level potential plus, say, one-loop corrections. These corrections include the usual zero-temperature contributions, plus finite temperature contributions. Both contributions depend on the masses $m_i(\phi)$.

A first-order phase transition occurs when the free energy has two minima separated by a barrier. Usually, at high temperatures the value $\phi = 0$ is the global minimum. This corresponds in general to a symmetric phase. A second minimum at $\phi = \phi_m(T)$ becomes the global one at low temperatures. This value corresponds to a spontaneously broken symmetry. The critical temperature $T = T_c$ is that at which the two minima are degenerate. At a smaller temperature $T_0 \lesssim T_c$ the barrier between minima disappears (see e.g. Ref. [1]).

Hence, above T_c the field is in the high-temperature minimum and the free energy is a certain function of temperature, $\mathscr{F}_+(T) = \mathscr{F}(\phi = 0, T)$. Below T_c, the field is in the low-temperature minimum and the free energy is a different function of temperature, $\mathscr{F}_-(T) = \mathscr{F}(\phi_m(T), T)$. All the thermodynamical quantities will be different functions of T, too. By definition, the functions $\mathscr{F}_+$ and $\mathscr{F}_-$ match at $T = T_c$, since the critical temperature is that at which the two minima have the same free energy. However, several quantities are discontinuous. This is the case, e.g., of the energy density ρ, which is smaller in the stable phase. The difference $L \equiv \rho_+(T_c) - \rho_-(T_c)$ is called latent heat. This heat is released in the phase transition interfaces, which are the walls of expanding bubbles.

During the adiabatic cooling, the Universe reaches the critical temperature. At this moment the system is in the high-temperature phase, with $\phi(\mathbf{x}) \equiv 0$. As the temperature decreases from T_c, this phase becomes metastable. The system becomes supercooled, since now the stable phase is the low-T one, with $\phi = \phi_m$. Therefore, bubbles begin to nucleate.

A bubble is a configuration in which the field has the value ϕ_m inside a certain radius and falls to $\phi = 0$ outside. The nucleation rate $\Gamma \simeq T^4 e^{-S_3(T)/T}$ is dominated by a Boltzmann factor corresponding to the probability of thermal activation. The three-dimensional instanton action S_3 coincides with the free energy that is necessary to form a "critical" bubble in unstable equilibrium between expansion and contraction. Roughly, nucleation becomes important when Γ is comparable to H^4, where H is the Hubble rate. The nucleation rate is extremely sensitive to temperature. At the critical temperature we have $S_3 = \infty$ and $\Gamma = 0$, whereas at the temperature T_0 we have $S_3 = 0$ and $\Gamma \sim T^4$. This is a huge rate in comparison with the expansion rate, which is suppressed by factors of the Planck mass, $H^4 \sim (T^2/M_{\text{Planck}})^4 \ll T^4$.

Once bubbles nucleate, they begin to expand until they fill all space. The velocity of bubble walls depends on several parameters (see e.g. Ref. [2]). The pressure difference Δp between phases pushes the walls towards the false vacuum. Δp depends on the amount of supercooling, since at the critical temperature we have $p_+ = p_-$. The plasma exerts a friction on the bubble wall, which depends on the interactions of the particles with the Higgs field that makes up the wall configuration. The latent heat that is injected into the plasma causes reheating and bulk motions of the fluid.

According to Hydrodynamics, a phase transition front (bubble wall) can propagate either as a detonation or as a deflagration. For a detonation, the bubble wall moves faster than the speed of sound in the plasma, $c_s = 1/\sqrt{3}$. As a consequence, no signal precedes the wall, which is followed by a rarefaction wave. Thus, in this case a bubble cannot influence other bubbles, except in the regions where bubble walls meet and collide. On the contrary, a deflagration front is slower than the sound. In this case the wall is preceded by a *shock wave* which moves at $v_{sh} \approx c_s$. Therefore, in the case of deflagrations, bubbles will influence each other.

GENERATION OF GRAVITATIONAL WAVES

Gravitational waves are produced in a first-order phase transition, either by bubble collisions (due to the motion of thin energy concentrations), or by the turbulence that the moving walls generate in the relativistic fluid [3]. In general, turbulence is a stronger source of GWs than bubble collisions. It can be seen that the Reynolds number is large enough to produce turbulence when a source injects energy into the plasma. If the phase transition takes place in an electrically conducting fluid in the presence of magnetic fields, then turbulence develops in a completely different way. This gives a third mechanism for generation of GWs, called magnetohydrodynamics. Here I will consider only the case of turbulence without magnetic fields.

The simplest model of turbulence is Kolmogoroff-type turbulence. In this model, energy is injected by a stirring source which has a characteristic length L_S. Turbulent eddies of this size are produced, which then break into smaller ones. The same happens at any size scale $L < L_S$. When turbulence is fully developed, a cascade is established, in which energy is transmitted from larger to smaller length scales, beginning at the stirring scale L_S and finishing at a scale L_D at which viscosity dissipates the energy into heat. An important feature is that the energy in the cascade is transmitted with a constant rate ε. For stationary turbulence, this rate equals the power that is injected by the source.

Consider the velocity correlation tensor of the fluid, $\langle v_i(\mathbf{x}) v_j(\mathbf{y}) \rangle$, where the brackets mean a statistical average. For stationary, homogeneous, and isotropic turbulence, the two-point velocity correlator in momentum space has a simple form,

$$\langle v_i(\mathbf{k}) v_j^*(\mathbf{q}) \rangle \propto \delta^3(\mathbf{k}-\mathbf{q}) \frac{E(k)}{k^2} \left(\delta_{ij} - \frac{k_i k_j}{k^2} \right), \tag{1}$$

where $E(k)$ is the energy density spectrum of the turbulent fluid. For Kolmogoroff turbulence, we have $E(k) \propto \varepsilon^{2/3} k^{-5/3}$.

It is well known that the source of GWs is the spatial, transverse and traceless piece of the stress-energy tensor. In our case, the relevant part is $T_{ij}(\mathbf{x}) \propto v_i(\mathbf{x}) v_j(\mathbf{x})$. The energy density in GWs involves a statistical average, $\rho_{GW} \sim \langle T_{ij} T_{ij} \rangle \sim \langle v_i v_j v_i v_j \rangle$. To calculate this quantity, the theory of turbulence must be used. In the present case the energy density can be related to the Kolmogoroff spectrum through Eq. (1). The expansion of the Universe can be neglected during the *production* of the gravitational radiation, since the duration of the phase transition is shorter than the Hubble time. Once the GWs have been generated, their wavelengths scale with the scale factor a, and their amplitudes scale with a^{-1}.

The GW spectrum is characterized by the quantity $\Omega_{GW}(f) = \frac{1}{\rho_c} \frac{d\rho_{GW}}{d\log f}$, where ρ_c is the critical energy density today. The spectrum has been recently calculated in Refs.

[4, 5] (with different approaches and results). The peak frequency is given by

$$f_p = 1.6 \times 10^{-5} Hz \frac{T_*}{100GeV} \left(\frac{g_*}{100}\right)^{1/6} \frac{L_S^{-1}}{H_*}, \tag{2}$$

Using the results from Ref. [4], the peak amplitude can be written as

$$\Omega_{GW}(f_p) \approx \Omega_R \left(\frac{L_S}{H_*^{-1}}\right)^{10/3} \left(\frac{\varepsilon}{H_*}\right)^{4/3}, \tag{3}$$

where Ω_R corresponds to radiation today and H_* is the Hubble rate at the time of turbulence. The relevant parameters that appear in the energy density spectrum are the stirring scale L_S and the dissipation rate in the turbulent cascade, ε. In the case of a phase transition, L_S is roughly the size of the largest bubbles, and the dissipation rate equals the injected power, so it is proportional to the latent heat and the velocity of the bubble wall, $\varepsilon \propto L v_w$.

PHASE TRANSITION DYNAMICS AND GRAVITATIONAL WAVES

The dynamics is completely different for detonations or deflagrations. A detonation wall is supersonic and the injected energy is concentrated in a thin region behind the wall. This makes the calculations simpler. Furthermore, the detonation is usually assumed to satisfy the Chapman-Jouget condition; in this case, the wall velocity depends on a single parameter $\alpha = L/\rho_{th} =$ *latent heat/thermal energy density*. The nucleation rate $\Gamma = e^{-S_3(T)/T}$ increases as temperature decreases from T_c. As we have seen, Γ is extremely sensitive to temperature; in spite of that, it is usually approximated by Taylor expanding the exponent around the "nucleation" temperature T_*, which is estimated by comparing the nucleation and expansion rates. This gives $\Gamma = \Gamma_0 e^{\beta t}$. The parameter β^{-1} is thus the only time scale in the problem. It determines, in particular, the duration of the phase transition, $\Delta t \sim \beta^{-1}$, and the size of bubbles, $d \sim v_w \beta^{-1}$. As a consequence of these approximations, the GW spectrum depends only on the two parameters α and β.

On the other hand, a deflagration wall is subsonic. In this case the calculations are more involved. Firstly, the wall velocity v_w does not depend only on the latent heat as in the previous case; it can be approximated by $v_w \sim \Delta p/\eta$, where Δp is the pressure difference between the two phases and η is a friction coefficient. Since Δp depends on the amount of supercooling, v_w depends nontrivially on the dynamics of the transition. Secondly, shock fronts distribute the latent heat away from the wall. Hence, bubbles influence each other. This influence must be taken into account in the treatment of the phase transition.

To calculate the GW spectrum in specific models it is usual to assume that bubble walls propagate as detonations. Then, the formulas for the detonation case (which depend only on α and β) can be used. Thus, given a model (e.g., the electroweak phase transition for different extensions of the Standard Model) it is only necessary to calculate the parameters α and β to obtain the spectrum. However, it is known that

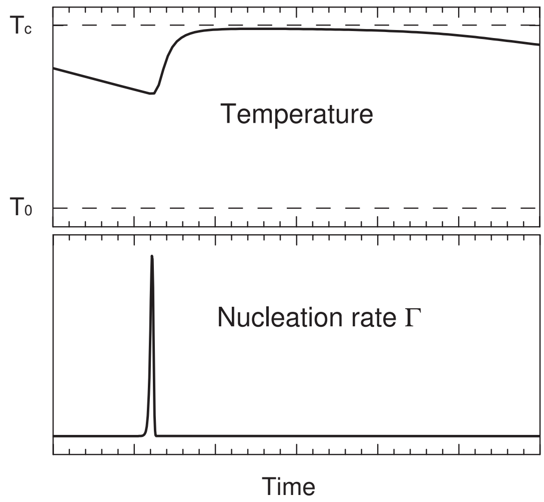

FIGURE 1. Development of a phase transition with slow deflagration bubbles.

bubbles in general expand as *deflagrations*. Indeed, the wall velocity has been calculated for the electroweak phase transition in the context of electroweak baryogenesis, finding velocities $v_w \sim 10^{-2} - 10^{-1}$ [8], which are much smaller than the speed of sound $c_s \approx 0.6$.

For deflagrations, the released latent heat is quickly distributed by shock waves with velocity $v_{sh} \approx c_s$. In the limit $v_w \ll c_s$, one can assume that the only effect on the dynamics of bubble expansion is to cause a homogeneous reheating. Thus, assuming that the temperature depends only on time, one obtains a set of equations that can be calculated numerically [7]. In general (see Fig. 1), the temperature decreases below T_c until bubbles become noticeable. The released latent heat reheats the universe back to a temperature close to T_c. Then, the temperature remains almost constant until the phase transition finishes. The nucleation rate has a sharp peak in a short time interval δt_Γ and vanishes outside. This is because Γ first grows quickly because of the adiabatic cooling, but then it quickly turns-off as a consequence of reheating.

Since all bubbles are formed in the short time δt_Γ, the peak of Γ defines an "initial" time t_Γ for bubble expansion. The bubble number density n_b is set at this time, as well as the mean distance between centers of nucleation, $d \sim n_b^{-1/3}$. The latter gives the final size of bubbles. Turbulence begins before bubble percolation, since shock waves propagate at $v_{sh} \gg v_w$ and collide before bubble walls.

These general features of the dynamics can be taken into account to derive relations between the parameters and obtain analytical approximations for the GW spectrum, just like in the case of detonations. For the peak frequency and amplitude we find [6]

$$f_p \sim 10^{-2} mHz \, (T_c/100GeV) \left(d/H^{-1}\right)^{-1}, \tag{4}$$

and

$$\Omega_{GW}|_{\text{peak}} \sim 10^{-4} (\alpha v_w)^{8/3} \left(d/H^{-1}\right)^2. \tag{5}$$

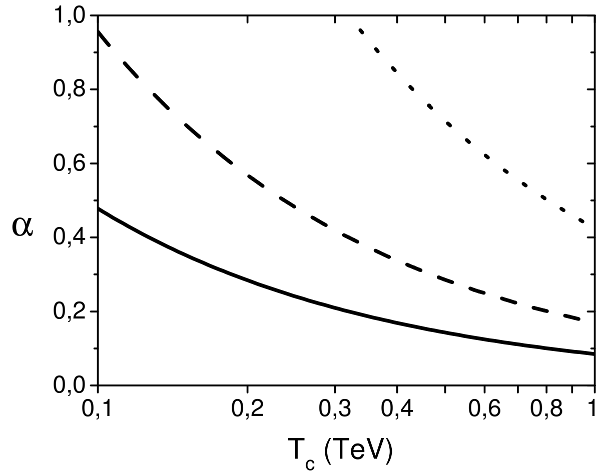

FIGURE 2. The values of α and T_c that give $f_p = 1mHz$ and $\Omega_{GW}(f_p) = 10^{-11}$, for $v_w = 0.1$ (solid line), $v_w = 0.05$ (dashed line) and $v_w = 0.02$ (dotted line).

The distance d may vary in a wide range for different models. In order to obtain mHz frequencies (corresponding to the peak sensitivity of LISA) we would need, e.g., for the electroweak phase transition with $T_c = 100GeV$, a value of $d/H^{-1} \sim 10^{-2}$. For such values of d and reasonable values of the latent heat and the wall velocity, we may obtain large enough amplitudes $\Omega_{GW} \gtrsim 10^{-11}$, i.e., above LISA's detection threshold. The figure shows the values of the parameters which give electroweak GWs that may be detected at LISA. For different wall velocities, the allowed regions are those above the curves.

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