Conformal hydrodynamics beyond supergravity approximation

Alex Buchel

(Perimeter Institute & University of Western Ontario)

Based on: arXiv:0804.3161, 0806.0788, 0808.1601, 0808.1837, and to appear

with: Rob Myers, Miguel Paulos, Aninda Sinha

Outline of the talk:

- Motivation
- Conformal hydrodynamics from the gauge theory perspective:

$\Longrightarrow$ first order hydrodynamics;

$\Longrightarrow$ consistency of hydrodynamic description;

$\Longrightarrow$ second order (causal) hydrodynamics;

$\Longrightarrow$ *boost-invariant expansion of a CFT plasma.

- $\mathcal{N}=4$ SYM gauge theory plasma as a toy model:

$\Longrightarrow$ non-equilibrium AdS/CFT correspondence beyond the supergravity approximation;

$\Longrightarrow$ universality of transport of CFT plasma beyond the supergravity approximation.

- Non-universal viscosity bound violation in CFT plasma with $c\neq a$ central charges
- Conclusions and future directions

Motivation

$\Longrightarrow$ One of the striking application of the gauge theory/string theory duality to study strongly coupled gauge theory plasma is the (conjectured) KSS bound:

$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_b}$$

The bound is saturated at infinitly strong coupling, and in the planar limit. Can this bound be violated? If so, under which conditions?

$\Longrightarrow$ Can we test gauge theory/string theory duality in the non-equilibrum setting?

$\Longrightarrow$ How do we formulate a causal relativistic hydrodynamics, and describe boost-invariant expansion of plasma (which could be of relevance to RHIC/LHC)?

First-order 4d conformal hydrodynamics (gauge theory perspctive)

$\Longrightarrow$ consider translationary invariant theory in flat space in equilibrium.

In local rest frame

$$T_{\mu\nu} = \begin{pmatrix} \epsilon & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{pmatrix}, \qquad [\mathrm{for\ CFT}: \qquad T_\mu^{\ \mu} = 0 \Rightarrow \epsilon = 3P]$$

Theory is characterized by conserved quantities, in particular the stress-energy tensor $T_{\mu\nu}$:

$$\partial_\mu T^{\mu\nu} = 0$$

$\Longrightarrow$ consider slow, macroscopic fluctuations

$$|\bar{q}|,\ \omega \qquad \ll \qquad \left\{ T,\ \mathrm{any\ other\ microscopic\ scale} \right\}$$

Effective description of such fluctuations is provided by macroscopic hydrodynamics

Hydrodynamics is based on two assumptions:

a: $T^{\mu\nu}[\text{fluctuations}]$ are conserved (as in equilibrium)

- fluctuations are always on-shell — expect to be a good approximation for

b: “Linear response theory is valid” — good approximation from small amplitudes

- linear response theory introduces phenomenological parameters into effective description of fluctuations

Let $u^\mu = (u^0, u^i)$ — fluid 4-velocity. Introduce a proper (rest) frame for the fluid element

$$u^0 = 1, \qquad u^i = 0, \qquad , \qquad [\,\partial_\mu u^\nu \neq 0 \qquad \mathrm{off-equilibrium}\,]$$

$$T_{\mu\nu} = \left\{(P+\epsilon)u_\mu u_\nu + P\eta_{\mu\nu}\right\} \qquad\qquad + \left\{\tau_{\mu\nu}\right\}$$

$$\Uparrow \qquad\qquad\qquad\qquad\qquad\qquad \Uparrow$$

equilibrium stress tensor — stress tensor due to velocity gradients

Definition of the rest frame: $\tau_{00}, \tau_{0i} = 0 \qquad \Rightarrow$

$$T_{00} = \epsilon \qquad ; \qquad T_{0i} = 0$$

"Constitutive" relation for remaining components:

$$\tau_{ij} = -\zeta \left\{ \delta_{ij}\, \partial_k u^k \right\} - \eta \left\{ \partial_i u_j + \partial_j u_i - \frac{2}{3} \delta_{ij}\, \partial_k u^u \right\}$$

ζ — couples to the trace of the velocity gradients — bulk viscosity [in CFT $\zeta = 0$]

η — couples to the traceless part of the velocity gradients — shear viscosity

$\Longrightarrow$ stress-energy conservation

$$\partial_0 \tilde{T}^{00} + \partial_i T^{0i} = 0 \qquad ; \qquad \partial_0 T^{0i} + \partial_j \tilde{T}^{ij} = 0$$

where $\tilde{T}^{00} \equiv T^{00} - \epsilon$, and

$$\tilde{T}^{ij} \equiv T^{ij} - P\delta^{ij} = -\frac{1}{\epsilon + P} \left[\eta \left(\partial^i T^{0j} + \partial^j T^{0i} - \frac{2}{3} \delta^{ij} \partial_k T^{0k} \right) + \zeta\, \delta^{ij} \partial_k T^{0k} \right]$$

$\Longrightarrow$ we would like to study on-shell fluctuation, i.e, eigenmodes of the above conservation laws

Here we have two types of eigenmodes:

a: the shear mode (transverse fluctuations of the momentum density T^{0i})

$$\omega = -\frac{i\eta}{\epsilon + P}\, q^2 = -i\, \frac{\eta}{Ts}\, q^2$$

where we used $\epsilon + P = Ts$

b: sound mode (simultaneous fluctuations of the energy density $\tilde{T}^{00}$ and longitudinal component of T^{0i})

$$\omega = c_s\, q - \frac{i}{2}\, \frac{4}{3}\, \frac{\eta}{Ts}\, \left[1 + \frac{3\zeta}{4\eta}\right]\, q^2$$

c_s— the speed of sound

η, ζ— shear and bulk viscosities

Dispersion relations for the fluctuations are realized (mostly) as poles in equilibrium correlation functions

I say 'mostly' because for the shear mode

$$\bar{v} = (0, v_y, 0), \qquad v_y = v_y(z), \qquad xy - \text{ is a shear plane}$$

$< T_{xy}(z) T_{xy}(0) >_R$ does not have a pole because it does not couple to energy or momentum fluctuations.

Rather, we have

Kubo formula **(sh.1)**

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt d\vec{x} e^{i\omega t} \ < [T_{xy}(\vec{x}), T_{xy}(0)] >$$
$$= \lim_{\omega \to 0} \frac{1}{2\omega i} \left[G^A_{xy,xy}(\omega, 0) - G^R_{xy,xy}(\omega, 0) \right]$$

Other correlation functions of $T_{\mu\nu}$ will have a diffusive pole **(sh.2)**

$$G^R_{xz,xz}(\omega, q_z) \qquad \sim \qquad \frac{1}{i\omega - Dq_z^2}, \qquad D = \frac{\eta}{Ts}$$

For the sound wave mode (ζ, η, c_s):

(sw.1) can be extracted from equilibrium 1-point correlation function $< T_{\mu\nu} >$

$$c_s^2 = \frac{\partial P}{\partial \epsilon}$$

Recall, for conformal theories: $\epsilon = 3P$, so $v_s^{CFT} = \frac{1}{\sqrt{3}}$

(sw.2)

$$< T_{00} T_{00} >_R \; \propto \; \frac{1}{\omega^2 - c_s^2 q^2 + i\Gamma\omega q^2}$$

there is a pole at $\omega = c_s q - i\frac{\Gamma}{2} q^2 + \mathcal{O}(q^3)$

Recall, for conformal theories: $\zeta = 0$

Consistency of hydrodynamic description

hydro mode	**computation**	**produces**
shear **(sh.1)**	$< T_{xy,xy} >_{R,A}$ +Kubo formula	η
shear **(sh.2)**	$< T_{xz,xz} >_R$ +pole	$D = \frac{\eta}{Ts}$
sound **(sw.1)**	$< T_{00} >, < T_{ii} >$	c_s
sound **(sw.2)**	$< T_{00,00} >_R$ +pole	c_s, Γ

$\Longrightarrow$ **(sh.1)** and **(sh.2)** produces η — must be consistent

$\Longrightarrow$ **(sw.1)** and **(sw.2)** produces c_s — must be consistent, also $\Gamma = \frac{4}{3}\frac{\eta}{Ts}\left[1 + \frac{3\zeta}{4\eta}\right]$ is sensitive to D, η

$\Longrightarrow$ First order hydrodynamics is acausal: the linearized equation for a diffusive mode is not *hyperbolic* (first order in temporal but second order in spatial derivatives) — discontinuity in initial conditions propagates at infinite speed. The acausality is a real problem in numerical simulations.

Second order causal hydrodynamics

$\Longrightarrow$ Motivated largely by AdS/CFT correspondence (though AdS/CFT strictly speaking was not needed for this), the effective field theory of conformal hydrodynamics was developed by **Braier et.al** and **Bhattacharyya et.al**

$\Longrightarrow$ First order hydrodynamics involves first-order gradients of the local 4-velocity $\nabla_\alpha u_\beta$; second order hydrodynamics includes 2-order gradients of the local 4-velocity. In principle, one can extend the theory to arbitrary order gravients at the expence of introducing new phenomenological parameters (suplumenting η, ζ at the first order). AdS/CFT provides a first-principle evaluation of ALL phenomenological parameters for a given CFT.

$\Longrightarrow$ The hydrodynamic equations is the familiar one:

$$\nabla_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + P\Delta^{\mu\nu} + \Pi^{\mu\nu}\,, \qquad \Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$$

$$\Pi^{\mu\nu} = -\eta\sigma^{\mu\nu} + (\text{2nd order terms})\,, \qquad u_\mu \Pi^{\mu\nu} = 0\,, \qquad g_{\mu\nu}\Pi^{\mu\nu} = 0$$

where $\sigma^{\mu\nu}$ is symmetric transverse tensor constructed of first derivatives.

$\Longrightarrow$ besides the shear viscosity η, the second-order conformal hydrodynamics is described by 5 additional phenomenological parameters:

$$\{\tau_\Pi\,, \kappa\,, \lambda_1\,, \lambda_2\,, \lambda_3\}$$

- τ_Π is the relaxation time that 'restores' causality in first-order hydro
- λ_1 is a coupling of a term bilinear in the velocities, which show up in boost-invariant expansion of the plasma
- $\lambda_{2,3}$ are not needed for irrotational flows

Consistency of the second order hydrodynamic description

$\Longrightarrow$ Second-order Kubo formular:

$$G_R^{xy,xy}(\omega, q) = P - i\eta\omega + \eta\tau_\Pi\omega^2 - \frac{\kappa}{2}\left(\omega^2 + q^2\right)$$

$\Longrightarrow$ Dispersion relation for the sound:

$$\omega = c_s q - \frac{i}{\Gamma}q^2 + \frac{\Gamma}{c_s}\left(c_s^2\tau_\Pi - \frac{\Gamma}{2}\right)q^3$$

where Γ is from the 1st-order hydrodynamics.

Notice that looking at q^2 dependence in the second order Kubo formular we can obtain τ_Π; the same phenomenological coefficient can be extracted from the $\mathcal{O}(q^3)$ sound wave dispersion relation

$\mathcal{N} = 4$ SYM gauge theory plasma as a toy model

gauge theory		string theory
$\mathcal{N} = 4 SU(N)\,\mathrm{SYM}$	$\Longleftrightarrow$	N-units of 5-form flux in type IIB string theory
g^2_{YM}	$\Longleftrightarrow$	g_s

$\Longrightarrow$ Consider the theory in the 't Hooft (planar limit), $N \to \infty$, $g^2_{YM} \to 0$ with $N g^2_{YM}$ kept fixed. SUGRA is valid $N g_s \to \infty$. In which case the background geometry is

$$AdS_5 \times S^5$$

$\Longrightarrow$ Beyong the SUGRA approximation

$$\frac{1}{N}\text{-corrections} \quad \Longleftrightarrow \quad g_s\text{-corrections}$$

$$\frac{1}{N g^2_{YM}}\text{-corrections} \quad \Longleftrightarrow \quad \alpha'\text{-corrections}$$

In the planar limit, but for a finite (large) ’t Hooft coupling Ng^2_{YM}:

$$S_{IIB} = \frac{1}{16\pi G_{10}} \int d^{10}x\sqrt{-g}\left[R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4\cdot 5!}(F_5)^2 + \cdots + \gamma e^{-\frac{3}{2}\phi} W + \cdots\right]$$

where ϕ is a dilaton, $\gamma = \frac{1}{8}\zeta(3)(\alpha')^3$, and W is constructed from the Weyl tensor C_{mnpq}

$$W \equiv C^{hmnk} C_{pmnq} C_h{}^{rsp} C^q{}_{rsk} + \frac{1}{2} C^{hkmn} C_{rqmn} C_h{}^{rsp} C^q{}_{rsk}$$

and $\cdots$ denote other SUGRA modes and higher order α' corrections

Some features of the α' corrected geometry at $T \neq 0$

$\alpha' = 0$	$\alpha' \neq 0$
$\phi = 0$	$\phi \neq 0,$ depends on r
size of S^5 is constant	size of S^5 depends on r
$S = \frac{\mathcal{A}_{horizon}}{4G_{10}}$	$S \neq \frac{\mathcal{A}_{horizon}}{4G_{10}}$ use Wald formula
$T_H \equiv T_0$	$T_H \equiv T_0(1 + 15\gamma)$

Non-equilibrium AdS/CFT correspondence beyong the spergravity approximation

To obtain retarded correlation function of the boundary stress energy tensor, we study scalar perturbations of the background geometry :

$$g_{5\mu\nu} \to g_{5\mu\nu} + h_{xy}(u, \boldsymbol{x})$$

It will be convenient to introduce a field $\varphi(u, \boldsymbol{x})$,

$$\varphi(u, \boldsymbol{x}) = \frac{u}{r_0^2}\, h_{xy}(u, \boldsymbol{x})$$

and use the Fourier decomposition

$$\varphi(u, \boldsymbol{x}) = \int \frac{d^4k}{(2\pi)^4} e^{-i\omega t + i\boldsymbol{k}\cdot\boldsymbol{x}} \varphi_k(u)$$

Finally, we introduce

$$\mathfrak{w} \equiv \frac{\omega}{2\pi T_0}\,, \qquad \boldsymbol{k} \equiv \frac{k}{2\pi T_0}$$

The effective action to order $\mathcal{O}(\gamma)$ for $\varphi_k(u)$ takes form:

$$S_{eff} = \frac{N_c^2}{8\pi^2} \int \frac{d^4k}{(2\pi)^4} \int_0^1 du \Big[A\, \varphi_k'' \varphi_{-k} + B\, \varphi_k' \varphi_{-k}' + C\, \varphi_k' \varphi_{-k}$$

$$+ D\, \varphi_k \varphi_{-k} + E\, \varphi_k'' \varphi_{-k}'' + F\, \varphi_k'' \varphi_{-k}' \Big]$$

where A, B, C, D, E, F are even functions of the momenta, and depend explicitly of the background geometry — the α'^3-corrected $AdS_5 \times S^5$ background.

Variation of S_{eff} leads to

$$\delta S_{eff} = \frac{N_c^2}{8\pi^2} \int \frac{d^4k}{(2\pi)^4} \left[\int_0^1 du\, (EOM)\, \delta\varphi_{-k} + \Big(\mathcal{B}_1 \delta\varphi_{-k} + \mathcal{B}_2 \delta\varphi_{-k}'\Big)\Big|_0^1 \right]$$

$\Longrightarrow$ To have a well-defined variational principle one needs to include the **generalized** Gibbons-Hawking term $\mathcal{K}$, involving the extrinsic curvature of the boundary:

$$\mathcal{K}_{generalized} \neq \mathcal{K}_{standard}\,, \qquad \mathcal{K}_{generalized} - \mathcal{K}_{standard} = \mathcal{O}(\gamma)$$

As necessary for a diffeo-invariant theory, the bulk action must be a total derivative on-shell. Indeed, we find

$$S_{eff} = \frac{N_c^2}{8\pi^2} \int \frac{d^4k}{(2\pi)^4} \int_0^1 du \left(\partial_u \mathcal{B} + \frac{1}{2} [EOM] \right)$$

Thus on-shell, it reduces to the sum of two boundary term: the horizon contribution (as $u \to 1$) and the boundary contribution (as $u \to 0$). In computing the two-point retarded correlation function of the boundary stress-energy tensor, the horizon contribution must be discarded; the boundary contribution is divergent as $u = \epsilon \to 0$ and must be supplemented by the counterterm action:

$$S_{ct} = -\frac{3N_c^2}{4\pi^2} \int_{u=\epsilon} d^4x \sqrt{-\gamma} \left(1 + \frac{1}{2} P - \frac{1}{12} \left(P^{kl} P_{kl} - P^2 \right) \ln \epsilon \right)$$

where γ_{ij} is the metric induced at the $u = \epsilon$ boundary, and

$$P = \gamma^{ij} P_{ij} \,, \qquad P_{ij} = \frac{1}{2} \left(R_{ij} - \frac{1}{6} R \gamma_{ij} \right) \,.$$

Altogether, the total renormalized boundary action takes the form

$$S_{tot}(\epsilon) = -\frac{N_c^2}{8\pi^2} \int \frac{d^4k}{(2\pi)^4}\, \mathcal{F}_k \Big|_{u=\epsilon}$$

$\Longrightarrow$ Having found the solution for a gravitational perturbation, we can compute the correlation function $G_{xy,xy}(\omega, q)$ by applying the Minkowski AdS/CFT prescription

$$G^R_{xy,xy}(\omega, q) = \lim_{u\to 0} \frac{2\mathcal{F}_q}{|\varphi_q|^2}\,.$$

Explicitly we find

$$G^R_{xy,xy}(\omega, q) = \frac{\pi^2 N_c^2 T^4 (1+15\gamma)}{4} \Bigg(\frac{1}{2} - i\hat{\mathfrak{w}} \Big[1 + 120\gamma \Big] + \Big[-\hat{\mathfrak{q}}^2 + \hat{\mathfrak{w}}^2 - \hat{\mathfrak{w}}^2 \ln 2$$

$$+\gamma \left(-120\hat{\mathfrak{w}}^2 \ln 2 + 25\hat{\mathfrak{q}}^2 + \frac{905}{2}\hat{\mathfrak{w}}^2 \right) \Big] + \mathcal{O}(\hat{\mathfrak{w}}^3, \hat{\mathfrak{w}}\hat{\mathfrak{q}}^2) \Bigg) + \mathcal{O}(\gamma^2)$$

In the hydrodynamic limit the retarded correlation function $G^R_{xy,xy}(\omega, q)$ takes form

$$G^R_{xy,xy}(\omega, q) = P - i\eta\omega + \eta\tau_\Pi\omega^2 - \frac{\kappa}{2}\left(\omega^2 + q^2\right) + \mathcal{O}(\omega^3, \omega q^2)$$

Comparing the hydro and the gravity results we conclude

$$P = \frac{\pi^2 N_c^2 T^4}{8}\left(1 + 15\gamma + \mathcal{O}(\gamma^2)\right), \qquad \frac{\eta}{s} = \frac{1}{4\pi}\left(1 + 120\gamma + \mathcal{O}(\gamma^2)\right)$$

$$\tau_\Pi T = \frac{2 - \ln 2}{2\pi} + \frac{375}{4\pi}\gamma + \mathcal{O}(\gamma^2), \qquad \kappa = \frac{\eta}{\pi T}\left(1 - 145\gamma + \mathcal{O}(\gamma^2)\right)$$

Morally similar (albeit technically quite different computations) has to be performed to extract the dispersion relation for the sound quasinormal mode:

$$\mathfrak{w}(\mathfrak{q}) = \frac{1}{\sqrt{3}}\mathfrak{q} - i\mathfrak{q}^2\left(\frac{1}{3} + \frac{105}{3}\gamma\right) + \mathfrak{q}^3\left(\frac{3-2\ln 2}{6\sqrt{3}}\right.$$

$$\left. + \frac{1}{24\sqrt{3}}\left(-2758 + 12z_{1,0}^{(2)} + 1705\ln 2\right)\gamma\right) + \mathcal{O}(\mathfrak{q}^4, \gamma^2)$$

We were unable to evaluate $z_{1,0}^{(2)}$ analytically; numerically, we find

$$z_{1,0}^{(2)} = 264.7598406$$

Second order relativistic hydrodynamics of conformal fluids implies the following dispersion relation for the sound mode

$$\omega = c_s q - i\Gamma q^2 + \frac{\Gamma}{c_s}\left(c_s^2\tau_\Pi - \frac{\Gamma}{2}\right)k^3 + \mathcal{O}(k^4)$$

Comparing gauge and gravity computations we find

$$c_s = \frac{1}{\sqrt{3}} + 0 \cdot \gamma + \mathcal{O}(\gamma^2)\,, \qquad \Gamma T = \frac{1}{6\pi}\left(1 + 120\gamma\right) + \mathcal{O}(\gamma^2)\,,$$

in agreement with the conformal equation of state at order $\mathcal{O}(\gamma)$, as well as in agreement with the ratio $\frac{\eta}{s}$. Additionally, we compute

$$\tau_\Pi T = \frac{2 - \ln 2}{2\pi} + \frac{1}{16\pi}\left(2425 \ln 2 - 3358 + 12 z^{(2)}_{1,0}\right)\gamma + \mathcal{O}(\gamma^2)$$

A required agreement between Kubo-fortumal and the quasinormal mode computations provides a prediction for $z^{(2)}_{1,0}$

$$z^{(2)}_{1,0}\Big|_{prediction} = \frac{2429}{6} - \frac{2425}{12}\ln 2\,,$$

which is in excellent agreement with the actually numerical result.

Universality of transport of CFT plasma beyond the supergravity approximation

Theorem-I: In the planar limit, and for infinite 't Hooft coupling $Ng^2_{YM} = \infty$ the ratio of shear viscosity to the enetropy density is universal under all conceivable considitions:

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Theorem-II: In the planar limit, and for large, but finite 't Hooft coupling $Ng^2_{YM} \gg 1$, the ratio of shear viscosity to the entropy density in conformal gauge theories in 4d and in the absence of chemical potentials for the conserved $U(1)$ charges is universal

$$\frac{\eta}{s} = \frac{1}{4\pi}\left(1 + \frac{15\zeta(3)}{\lambda^{3/2}} + \cdots\right)$$

Similarly, all other second- and higher-order hydrodynamic coefficients are universal.

Question: if we model QCD at RHIC scales as a conformal plasma, does it mean that we know what is its shear viscosity?

I would claim the answer is: **NO**

$\Longrightarrow$ The crucial word in the Theorem-II is 'planar limit'. Now, a given conformal gauge theory is characterized by two different central charges c and a, defining its conformal anomaly

$$\langle T^\mu_\mu \rangle = \frac{c}{16\pi^2} I_4 - \frac{a}{16\pi^2} E_4$$

where

$$E_4 = R_{\mu\nu\rho\lambda}R^{\mu\nu\rho\lambda} - 4R_{\mu\nu}R^{\mu\nu} + R^2\,, \qquad I_4 = R_{\mu\nu\rho\lambda}R^{\mu\nu\rho\lambda} - 2R_{\mu\nu}R^{\mu\nu} + \frac{1}{3}R^2$$

$\Longrightarrow$ In the planar limit

$$c = a$$

$\Longrightarrow$ In a conformal toy model of QCD we expect

$$c \neq a$$

because of the presence of fundamental matter.

Consider an effective higher-derivative model of gauge theory/string theory duality

$$S = \int d^5x\sqrt{-g}\left(\frac{1}{\kappa^2}R - \Lambda + c_1 R_{abcd}R^{abcd} + c_2 R_{ab}R^{ab} + c_3 R^2 + \mathcal{O}(R^4)\right)$$

where $\kappa^2 = 16\pi G_N$. The holographic conformal anomaly is

$$\langle T^\mu_\mu \rangle_{holographic} = \left(-\frac{l^3}{8\kappa^2} + c_2 l + 5c_3 l\right)(E_4 - I_4) + \frac{c_1 l}{2}(E_4 + I_4)$$

while Kats et.al and Brigante et.al found

$$\frac{\eta}{s} = \frac{1}{4\pi}\left(1 - \frac{8c_1\kappa^2}{\ell^2} + \cdots\right) = \frac{1}{4\pi}\left(1 - \frac{(c-a)}{c} + \cdots\right) = \frac{1}{4\pi}(1 - \Delta + \cdots)$$

- Notice that c_1 coefficient can come only form $R_{abcd}R^{abcd}$, and it is precisely the coefficient that corresponds to having in the dual CFT $c \neq a$. In particular R^4-terms, relevant for the universality Theorem-II does not effect $(c-a)$ anomaly of a CFT.

- The KSS viscosity bound is violated in a CFT whenever (c-a). The violation is under contrall, if $|c-a|/c \ll 1$.

Non-universal violation of the KSS bound Consider a superconformal gauge theory. The superconformal albegra implies the existance of an anomaly-free $U(1)_R$ symmetry. It was found in Anselmi et.al that

$$c - a = -\frac{1}{16}\left(\dim G + \sum_i (\dim R_i)\,(r_i - 1)\right)$$

$$c = \frac{1}{32}\left(4\,(\dim G) + \sum_i (\dim R_i)\,(1 - r_i)\,\big(5 - 9(1 - r_i)^2\big)\right)$$

where r_i denote the R-charge of a matter chiral multiplet in the representation R_i

$\Longrightarrow$ So all we need to do is to scan through the list of available CFT's and compute $(c - a)$.

- Superconformal gauge theories with exactly marginal gauge coupling

Consider $SU(N_c)$ supersymmetric gauge theory with n_{adj} χsf in the adjoint representation, n_f flavors in the fundamental representation, n_{sym} flavors in the symmetric representation and n_{asym} flavors in the anti-symmetric representation. It is easy now to enumerate all the models with $G = SU(N_c)$ and $\Delta \ll 1$ as $N_c \to \infty$:

	$(n_{adj}, n_{asym}, n_{sym}, n_f)$	$c - a$	Δ
(a)	(3,0,0,0)	0	0
(b)	(2,1,0,1)	$\frac{3N_c+1}{48}$	$\frac{1}{4N_c} + \mathcal{O}(N_c^{-2})$
(c)	(1,2,0,2)	$\frac{3N_c+1}{24}$	$\frac{1}{2N_c} + \mathcal{O}(N_c^{-2})$
(d)	(1,1,1,0)	$\frac{1}{24}$	$\frac{1}{6N_c^2} + \mathcal{O}(N_c^{-4})$
(e)	(0,3,0,3)	$\frac{3N_c+1}{16}$	$\frac{3}{4N_c} + \mathcal{O}(N_c^{-2})$
(f)	(0,2,1,1)	$\frac{N_c+1}{16}$	$\frac{1}{4N_c} + \mathcal{O}(N_c^{-2})$

For the $Sp(2N_c)$ supersymmetric gauge theories

	(n_{adj}, n_{asym}, n_f)	$c-a$	Δ
(a)	(3,0,0)	0	0
(b)	(2,1,4)	$\frac{6N_c-1}{48}$	$\frac{1}{4N_c} + \mathcal{O}(N_c^{-2})$
(c)	(1,2,8)	$\frac{6N_c-1}{24}$	$\frac{1}{2N_c} + \mathcal{O}(N_c^{-2})$
(d)	(0,3,12)	$\frac{6N_c-1}{16}$	$\frac{3}{4N_c} + \mathcal{O}(N_c^{-2})$

$\Longrightarrow$ The are no models in this class with orthogonal gauge groups

- $\mathcal{N} = 2$ superconformal fixed points from F-theory

Consider N D3-branes probing an F-theory singularity generated by n_7 coincident (p, q) 7-branes, resulting in a constant dilaton. As $N \to \infty$,

$$c - a = \frac{1}{4} N\,(\delta - 1) - \frac{1}{24}\,, \qquad \Delta = \frac{\delta - 1}{N\delta} + \mathcal{O}(N^{-2})$$

where δ is a definite angle characterizing an F-theory singularity with a symmetry group $\mathcal{G}$

$\mathcal{G}$	H_0	H_1	H_2	D_4	E_6	E_7	E_8
n_7	2	3	4	6	8	9	10
δ	6/5	4/3	3/2	2	3	4	6

Notice that in all cases $0 < \Delta \ll 1$ as $N \to \infty$.

$\Longrightarrow$ In all examples presented the KSS bound is violated since $(c-a) > 0$

$\Longrightarrow$ There many more CFT's with $c \neq a$. For them, however, $c - a \sim c$ and so we can not say anything reliable about KSS bound. Curiosly though, we did not find a single CFT with $c \neq a$ so that $(c-a) < 0$.

Conclusions and future directions

- I gave an orverview of transport properties in 4d conformal gauge theories;
- CFT's with $c = a$ have a universal transport properties at finite 't Hooft coupling;
- CFT's with $c \neq a$ generically violate KSS viscosity bound in a non-universal way;
- our computations provide a highly nontrivial check of holographic gauge theory/string theory correspondence in the non-equilibrium setting

In the future:

- what can we say about CFT tranport at finite 't Hooft coupling and with non-vanishing chemical petentials? is η/s a nontrivial function of baryon density?
- what is the finite 't Hooft coupling transport of non-CFT's?
- what is the second-order relativistic non-conformal hydrodynamics?
- $\cdots$

Consider <u>expansion of a CFT fluid (gauge theory plasma) in boost invariant frame</u>

$\Rightarrow$ Widely expected to be a correct description of central region of QGP produced in ultra-relativistic collisions of heavy nuclei

Convert Minkowski frame

$$ds_4^2 = -dx_0^2 + dx_\perp^2 + dx_3^2$$

into a frame with boost-invariance along x_3 direction

$$x_0 = \tau \ \cosh y \, , \qquad x_3 = \tau \ \sinh y$$

$$ds_4^2 = -d\tau^2 + \tau^2 \ dy^2 + dx_\perp^2$$

Assume

$$\epsilon = \epsilon(\tau) \, , \qquad P = P(\tau)$$

for local energy density ϵ and pressure p in the fluid

- Ideal CFT fluid

Stress energy tensor:

$$T_{\mu\nu} \equiv T_{\mu\nu}^{equilibrium} = (\epsilon + p) u_\mu u_\nu + P \eta_{\mu\nu}$$

where u^μ is local 4-velocity of the fluid, $u^2 = -1$.

From conformal invariance

$$T_\mu^\mu = 0 \qquad \Rightarrow \qquad \epsilon = 3P$$

Conservation law in boost-invariant frame:

$$\partial_\mu T^{\mu\nu} = 0 \qquad \Rightarrow \qquad \partial_\tau \epsilon = -\frac{4}{3} \frac{\epsilon}{\tau}$$

Scaling of ϵ, s (entropy density), η (shear viscosity), T (temperature), τ_π (relaxation time)

$$\epsilon \propto \tau^{-4/3}, \qquad T \propto \epsilon^{1/4} \propto \tau^{-1/3}, \qquad \eta \propto s \propto T^3 \propto \tau^{-1}$$

$$\tau_\pi \propto T^{-1} \propto \tau^{1/3}$$

- First-order dissipative CFT fluid dynamics:

Stress energy tensor:

$$T_{\mu\nu} = T_{\mu\nu}^{equilibrium} + \tau_{\mu\nu}\,, \qquad \tau_{\mu\nu} \propto \eta\,(\nabla_\mu u_\nu + \nabla_\nu u_\mu - \mathrm{trace})$$

$\Rightarrow$

$$\partial_\tau \epsilon = -\frac{4}{3}\frac{\epsilon}{\tau} + \frac{4\eta}{3\tau^2}$$

From scaling, viscous correction becomes subdominant as $\tau \longrightarrow \infty$:

$$\frac{\epsilon}{\tau} \sim \frac{\tau^{-4/3}}{\tau} \sim \tau^{-7/3}\,, \qquad \frac{\eta}{\tau^2} \sim \frac{\tau^{-1}}{\tau^2} \sim \tau^{-9/3}$$

Thus we expect approach to equilibrium in boost-invariant frame to correspond to late-time dynamics

- Second-order dissipative CFT fluid dynamics:

$$\frac{d\epsilon}{d\tau} = -\frac{4}{3}\frac{\epsilon}{\tau} + \frac{1}{\tau}\Phi$$

$$\tau_\Pi \frac{d\Phi}{d\tau} = \frac{4}{3}\frac{\eta}{\tau} - \Phi - \frac{4}{3}\frac{\tau_\Pi}{\tau}\Phi - \frac{1}{2}\frac{\lambda_1}{\eta^2}\Phi^2$$

where

$$\Phi \equiv -\Pi_\xi^\xi$$

From scaling, $\tau \longrightarrow \infty$ limit corresponds effectively to $\tau_\pi \longrightarrow 0$ and second-order hydro is reduced to a first order hydro

$\Rightarrow$Clearly, as in this limit relaxation is *instantaneous*, it is not surprising that causality is violated

- Second-order dissipative $\mathcal{N} = 4$ SYM plasma

$$\epsilon(\tau) = \frac{3}{8}\pi^2 N^2\, T(\tau)^4\,, \qquad p(\tau) = \frac{1}{3}\epsilon(\tau)\,, \qquad \eta(\tau) = \mathcal{C}\eta_0 \left(\frac{\epsilon}{\mathcal{C}}\right)^{3/4}$$

$$\tau_\pi(\tau) = \tau_\Pi^0 \left(\frac{\epsilon}{\mathcal{C}}\right)^{-1/4}\,, \qquad \lambda_1 = \mathcal{C}\lambda_1^0 \left(\frac{\epsilon}{\mathcal{C}}\right)^{1/2}$$

where $C, \eta_0, \tau_\Pi^0, \lambda_1^0$ are some constants.

From second order hydrodynamic equations as $\tau \longrightarrow \infty$:

$$\frac{\epsilon(\tau)}{\mathcal{C}} = \tau^{-4/3} - 2\eta_0\, \tau^{-2} + \left[\frac{3}{2}\eta_0^2 - \frac{2}{3}\left(\eta_0\tau^{\Pi} - \lambda_1^0\right)\right]\tau^{-8/3} + \mathcal{O}\left(\tau^{-10/3}\right)$$

Janik-Peschanki proposal for the SUGRA dual to boost-invariant $\mathcal{N} = 4$ SYM dynamics

Given symmetries of the problem, most general truncation of type IIB SUGRA takes form

$$ds_{10} = e^{-2\alpha(\tau,z)} \left\{ \frac{1}{z^2} \left[-e^{2a(\tau,z)} d\tau^2 + e^{2b(\tau,z)} \tau^2 dy^2 + e^{2c(\tau,z)} dx_\perp^2 \right] + \frac{dz^2}{z^2} \right\}$$

$$+ e^{6/5\alpha(\tau,z)} \left(dS^5 \right)^2$$

for the Einstein frame metric;

$$F_5 \;=\; \mathcal{F}_5 + \star\mathcal{F}_5 \,, \qquad \mathcal{F}_5 \;=\; -4Q\,\omega_{S^5} \,, \qquad \phi = \phi(\tau, z)$$

for the 5-form (Q is constant related to the rank of the gauge group) and the dilaton

$$Q = 1 \qquad \Leftrightarrow \qquad R_{AdS} = 1$$

Asymptotically as $z \longrightarrow 0$

$$\{a, b, c, \alpha, \phi\} \longrightarrow 0$$

however,

$$a(\tau, z) \sim \mathcal{O}\left(z^4\right) \neq 0$$

$\Rightarrow$We try to construct a nonsingular geometry everywhere in the bulk, subject to the above boundary conditions

$\Rightarrow$ evaluate stress-energy tensor one-point correlation function

$$\langle T_{\mu\nu}(\tau)\rangle = \frac{N_c^2}{2\pi} \lim_{z\to 0} \frac{g^{(5)}_{\mu\nu}(\tau) - \eta_{\mu\nu}}{z^4}$$

$\Rightarrow$ extract from $\langle T_{\mu\nu}(\tau)\rangle$

$$\epsilon(\tau)\,, \qquad p(\tau)$$

and interpret results in the framework of dissipative relativistic fluid dynamics

From the 1-point correlation function of the boundary stress-energy tensor in the expanding boost-invariant geometry at α'^3-level, we find that the energy density is given by

$$\epsilon(\tau) = -\frac{N^2}{2\pi^2}\lim_{v\rightarrow 0}\frac{2a(v,\tau)}{v^4\tau^{4/3}}\,, \qquad v \equiv \frac{z}{\tau^{1/3}}$$

Explicitly, we find:

$$\epsilon(\tau) = \frac{N^2(6+576\,\gamma+\gamma\,\delta_1)}{12\pi^2}\,\frac{1}{\tau^{4/3}} - \frac{N^2\,2^{1/2}\,3^{1/4}\,(1566\gamma+8+\gamma\delta_1)}{48\pi^2}\,\frac{1}{\tau^2}$$
$$+\frac{N^2 3^{1/2}}{864\pi^2}\left(12+24\ln 2+\gamma\,(2\delta_1\ln 2+\delta_1+7086+4212\ln 2)\right)\frac{1}{\tau^{8/3}}+\mathcal{O}(\tau^{-10/3})$$

where δ_1 is an arbitrary constant.

To match the string theory result with the second-order hydro expectations we need to recall the equation of state for the $\mathcal{N} = 4$ SYM plasma

$$\epsilon(T) = \frac{3}{8}\pi^2 N^2 T^4 \; (1 + 15\gamma)$$

and the $\mathcal{N} = 4$ SYM relaxation time τ_Π, computed from equilibrium correlation functions

$$\tau_\Pi T = \frac{2 - \ln 2}{2\pi} + \frac{375}{4\pi}\gamma$$

Ultimately, we find:

$$\frac{\eta}{s} = \frac{1}{4\pi}\,(1 + 120\gamma) \;, \qquad \frac{\lambda_1 T}{\eta} = \frac{1}{2\pi}\,(1 + 215\gamma)$$

Notice that the ratio of shear viscosity to the entropy density agrees with the results obtained from the equilibrium correlation functions.