A possible grand unification theory with 331 models

R. Gaitán* and R. Martinez†

*Centro de Investigaciones Teóricas, FES-Cuautitlán, UNAM, Apartado Postal 142, Cuautitlán Izcalli, Estado de Mexico, Código Postal 54700 México
†Universidad Nacional de Colombia, Departamento de Física, Bogotá,Colombia

Abstract. The 331 models with three families appear in a natural way by using the anomaly cancellations. In the present work we want to study the possibility to unify this class of models with three families into a $SU(7)$ gauge group. The fermion contents are given by 7^*, 21 and 35^* irreducible representations.

Since the birth of the Standard Model (SM) many attempts have been done to go beyond it, and solve some of the problems of the model such as the charge quantization, the number of families and the unification of the gauge couplings. In some cases the unification is done by taking a simple group of grand unification, arising the so called Grand Unification Theories (GUT), where the three interactions described by SM are treated as only one [1], the most common GUT's are $SO(10)$ and E_6. The first condition for these kind of theories is an equal value for the three couplings at certain scale of energy, M_U. This condition cannot be fulfilled by the simplest grand unification schemes with the minimal SM particle content and taking the precision low-energy data. However, the Minimal Supersymmetric Standard Model (MSSM) can achieve this scenario for the coupling constants [2]. Of course, there are other possibilities for the unification. Another challenge is to unify the color and electroweak interactions with the three families of the SM in a GUT. And by symmetry breaking get an ansatz for the mass matrices at low energy in order to predict masses and mixing angles of the quarks and leptons.

In particular, the model based on the $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ gauge group (hereafter 331 models) is an interesting choice that could address problems like the charge quantization [3] and the existence of three families based on cancellation of anomalies [4]. In the present work, we consider the possibility of embedding 331 models [5] into a grand unified theory (GUT) SU(7)[6].

We define the operator of electromagnetic charge as the linear combination $Q = T_3 + aY$, where a parameter is the normalization of the hypercharge. In this way, the a parameter becomes free, and we could reverse the problem in a certain way, since we have in many cases an allowed region for a and we could ask what groups of grand unification (if any) could lead to values of a admitted by our scheme. Let us elaborate about this possibility. After working some 331 models with one or three families, we can see that in some of them is possible to find a scale M_X and a normalization factor

a that gives unification of the coupling constants (UCC) [7]. Notwithstanding, UCC imposes some restrictions on a, and by using these values we can look for simple groups containing a 331 group and fixing an a belonging to the allowed interval mentioned above. Some good examples of GUT are E_6, and $SU(7)$. The group E_6 can be broken into $SU(3)_C \otimes SU(3)_L \otimes SU(3)_R$ or $SU(6)_L \otimes SU(2)$. But they are models of one family. On the other hand, the 331 models of three families can be embedded into $SU(7)$, thus, we shall study the later scenario in more detail.

The combination of irreducible representations free of anomalies, permits to accomodate those 331 models with three families taking into account the branching rules of the scheme $SU(7) \to SU(3)_C \otimes SU(3)_L \otimes U(1)$. There are different combinations of irreps anomaly free using $\Psi^{\alpha}_{(1)}[7]$, $\Psi^{\alpha\beta}_{(3)}[21]$ and $\Psi^{\alpha\beta\gamma}_{(2)}[35]$, where the subindex means the anomaly coefficient, the bracket coefficient means the dimension and the labels $\alpha, \beta, \gamma = 1, \cdots, 7$ are $SU(7)$ indices. Models can be classified if no irrep appears more than once, i.e., a linear combination like $\Psi_{(-1)\alpha} \oplus \Psi^{\alpha\beta}_{(3)} \oplus \Psi_{(-2)\alpha\beta\gamma}$; but the same irrep can be repeated such as $5 \times \Psi_{(-1)\alpha} \oplus \Psi^{\alpha\beta}_{(3)} \oplus \Psi^{\alpha\beta\gamma}_{(2)}$. The branching rules according to $(SU(3)_c, SU(2)_L)$ are given by

$$
\begin{aligned}
\Psi^{\alpha} \oplus \Psi^{\alpha\beta} \oplus \Psi^{\alpha\beta\gamma} \;=\;& [(3,1)+(1,2)+(1,1)+(1,1)]_7 \oplus [(3^*,1)+(3,2)\\
+\;& (3,1)+(3,1)+(1,1)+(1,2)+(1,2)+(1,1)]_{21}\\
\oplus\;& [(1,1)+(3,2)+(3,1)+(3,1)+(3^*,1)+(3^*,2)\\
+\;& (3^*,2)+(3^*,1)+(1,1)+(1,1)+(1,2)]_{35}
\end{aligned}
\tag{1}
$$

The electromagnetic charge of the particles can be chosen by defining the hypercharge which is a linear combination of the $U(1)$ factors of the $SU(7)$. This will be also done by imposing the condition that the singlet of color $(3,1)$ of the 7 irrep has electric charge $q = \pm 1/3, \pm 2/3, \pm 4/3, \pm 5/3$ or that the electric charge of the leptonic singlets are ± 1, 0.

Assuming an unification scheme with the simple group $SU(7)$ and passing through a 331 model with three families to SM, we get the scheme

$$SU(7) \to SU(3)_c \otimes SU(3)_L \otimes U(1)_X \to SU(3)_c \otimes SU(2)_L \otimes U(1)_Y.$$

The assignment of the electromagnetic charge is of the form

$$
\begin{aligned}
Q &= T_{3L} + \alpha Y^a + \beta Y^b + \gamma Y^c \equiv T_{3L} + aY\\
a^2 &\equiv \alpha^2 + \beta^2 + \gamma^2
\end{aligned}
$$

where Y^a, Y^b, Y^c have the same normalization as the T_{3L}, Y generators, and they correspond to the $U(1)$ abelian subgroups induced from different subalgebras of $SU(7) \supset SU(n)_L \otimes SU(m)_C \otimes U(1)^a$, where $SU(n)_L \supset SU(2)_L \otimes U(1)^b$ and $SU(m)_C \supset SU(3)_c \otimes U(1)^c$. If the fundamental representation 7 is decomposed as in Eq. (1), the most general assignment is

$$Q = diag(q,q,q,b,c,c-1,1-3q-2c-b). \tag{2}$$

And we can choose the charge of $(3,1)$ to be $-1/3$ and those of $(1,2)$ as $(1,0)$. Then

$$Q = diag(-1/3,-1/3,-1/3,b,1,0,-b) \tag{3}$$

From the following possible $SU(7)$ maximal subalgebras

$$\begin{array}{rl} \mathit{Model\ I} & SU(4)_C \otimes SU(3)_L \otimes U(1)^a \\ \mathit{Model\ II} & SU(3)_C \otimes SU(4)_L \otimes U(1)^a \\ \mathit{Model\ III} & SU(6)_L \otimes U(1)^a \rightarrow SU(3)_C \otimes SU(3)_L \otimes U(1)^c \otimes U(1)^a \end{array}$$

we can settle three different assignments for the hypercharge according to the scheme in Eqs. (2) and (3). For example, Model I is known in the literature as the Pati Salam model.

In particular, we find that some 331 models with three families, can be properly embedded in a grand unification scenario with $SU(7)$. This scheme will be depend of the normalization of the hypercharge or choosing the electric charge of the singlet particles of the fundamental representation.

Acknowledgements R. Martinez thank to Colciencias and Fundación Banco de la República for its financial support. R. Gaitán wish to thank the support from PAPIIT project No. IN104208.

REFERENCES

1. H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32, 438 (1974); H. Fritzsch and P. Minkowski, Ann. Phys. 93, 193 (1975); P. Langaker, Physics Reports 72, 185 (1981).
2. U. Amaldi, W. de Boer, and H. Fürstenau, Phys. Lett B260, 447 (1991); P. Langaker and M. X. Luo, Phys. Rev. D44, 817 (1991); J. Ellis, S. Kelley, y D. V. Nanopuolos, Phys. Lett. B287, 95 (1992).
3. C. A. de S. Pires, O. P. Ravinez, Phys. Rev. D58, 35008 (1998); C. A. de S. Pires, Phys. Rev. D60, 075013 (1999).
4. J. S. Bell, R. Jackiw, Nuovo Cim. A60, 47 (1969); S. L. Adler, Phys. Rev. 177, 2426 (1969); H. Georgi and S. L. Glashow, Phys. Rev. D6, 429 (1972); D. J. Gross, R. Jackiw, Phys. Rev. D6, 477 (1972); J. Banks and H. Georgi, Phys. Rev. 14, 1159 (1976); S. Okubo, Phys. Rev. D16, 3528 (1977).
5. F. Pisano and V. Pleitez, Phys. Rev. D46, 410 (1992); Rodolfo A. Diaz, R. Martinez, F. Ochoa, Phys. Rev. D72, 035018 (2005); P. Frampton, Phys. Rev. Lett. 69, 2889 (1992); L. Sanchez, W. Ponce and R. Martínez, Phys. Rev. D64, 075013 (2001); L. Sanchez, W. Ponce and R. Martínez, Phys. Rev. D65, 055013 (2002).
6. Jihn E. Kim, Phy. Rev. D23, 2706 (1981); Jihn E. Kim, Phy. Rev. Lett. D45, 1916 (1980); H. Chavez and J. A. Martins Simoes, Nucl. Phys. B783, 76 (2007).
7. Rodolfo A. Diaz, D. Gallego, R. Martinez, Int. J. Mod. Phys. A22, 1849 (2007).