Lattice QCD advances in baryon physics

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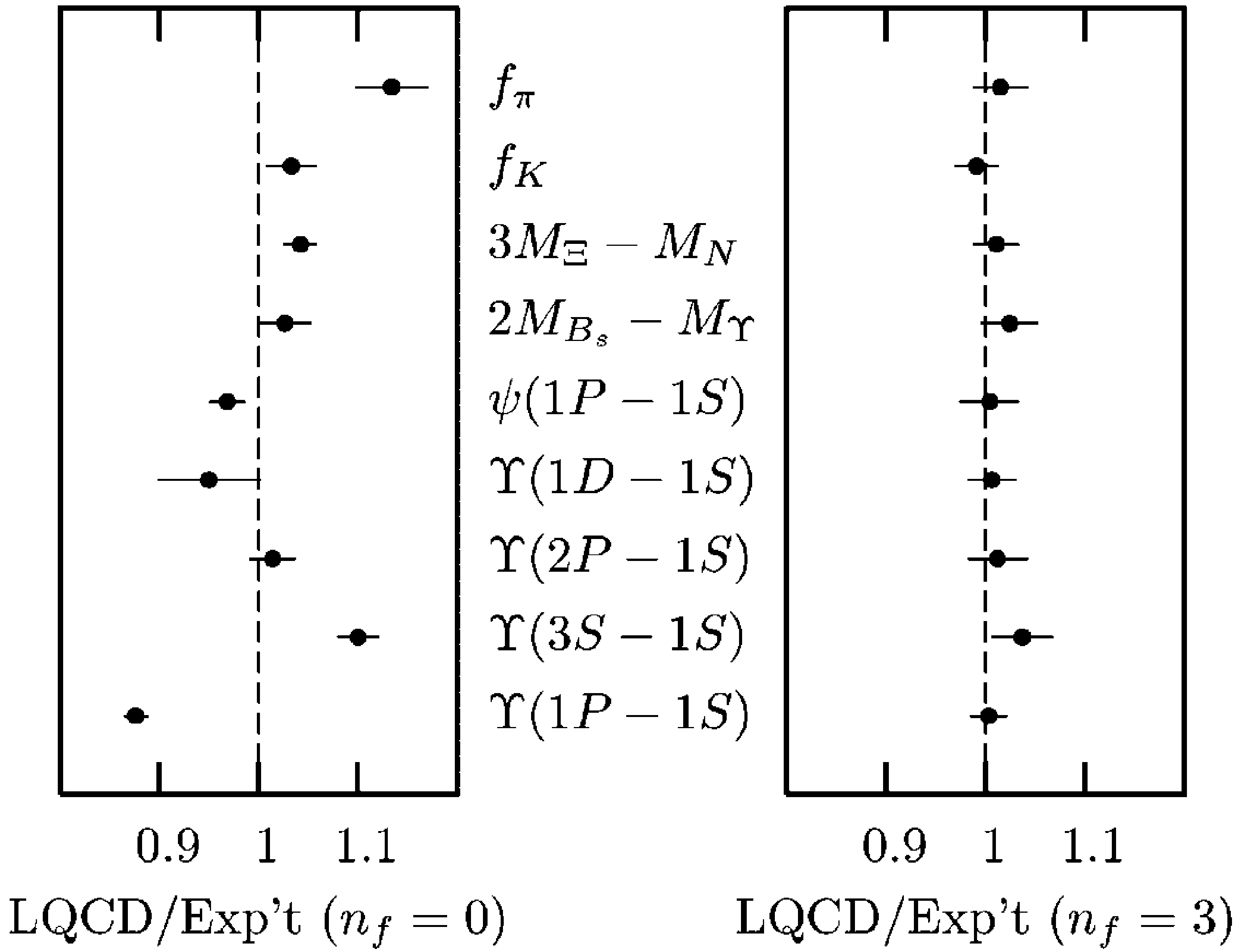
in collaboration with R. Carlini, J. Roche, D. Leinweber & A. Thomas

Outline

- Major advances in lattice QCD: precision tool
  - Particularly in heavy-quark physics
  - Remain challenges in the light-quark and particularly baryons
- Introduction to lattice QCD
- Recent results in 2+1-flavour dynamical simulations
  - Baryon spectrum
- SU(3) chiral extrapolation to physical quark masses

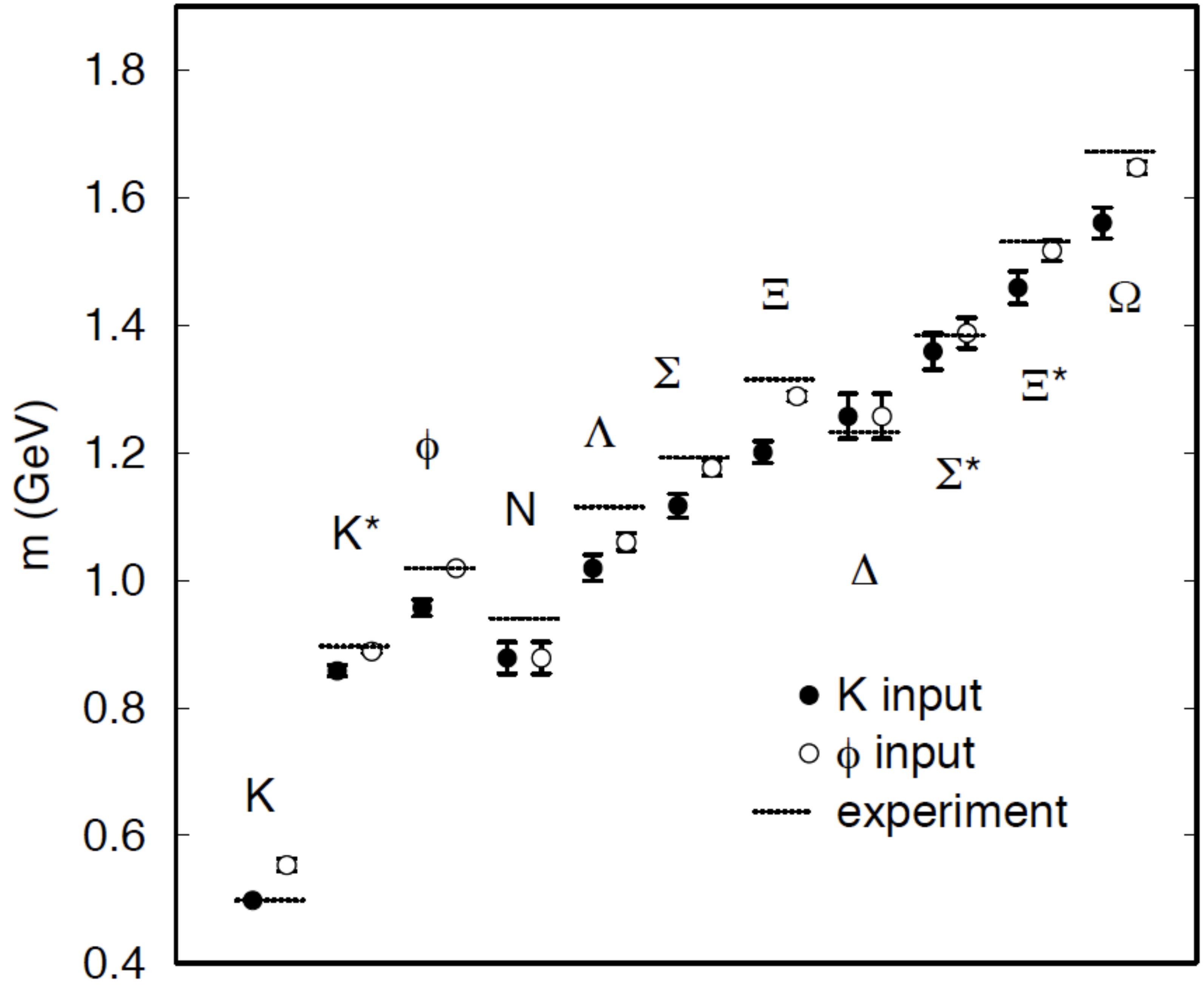
2003

"High-Precision Lattice QCD Confronts Experiment"



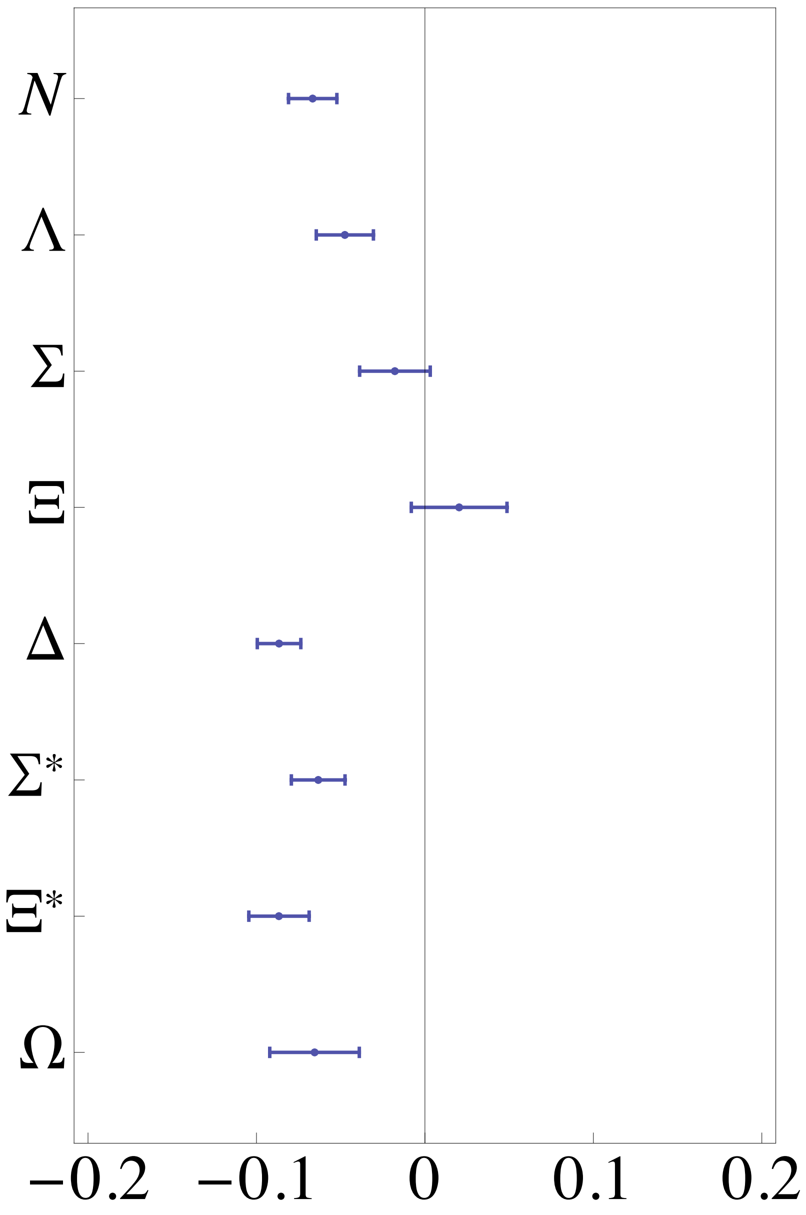
HPQCD / UKQCD / MILC / Fermilab, PRL92,022001(2004)

1999
"Quenched Light Hadron Spectrum"



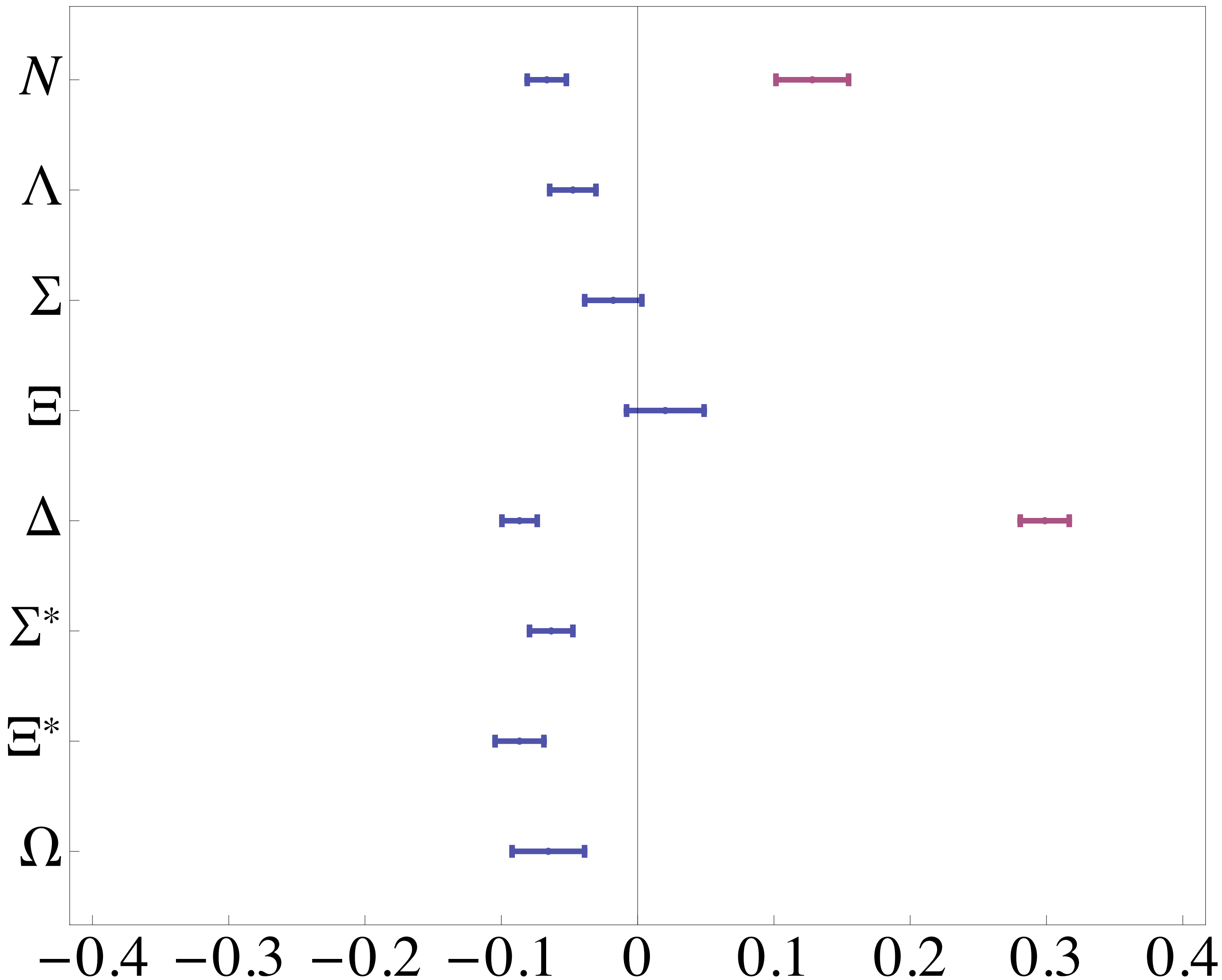
CP-PACS, PRL84,238(2000)

Ratio plot - Quenched QCD



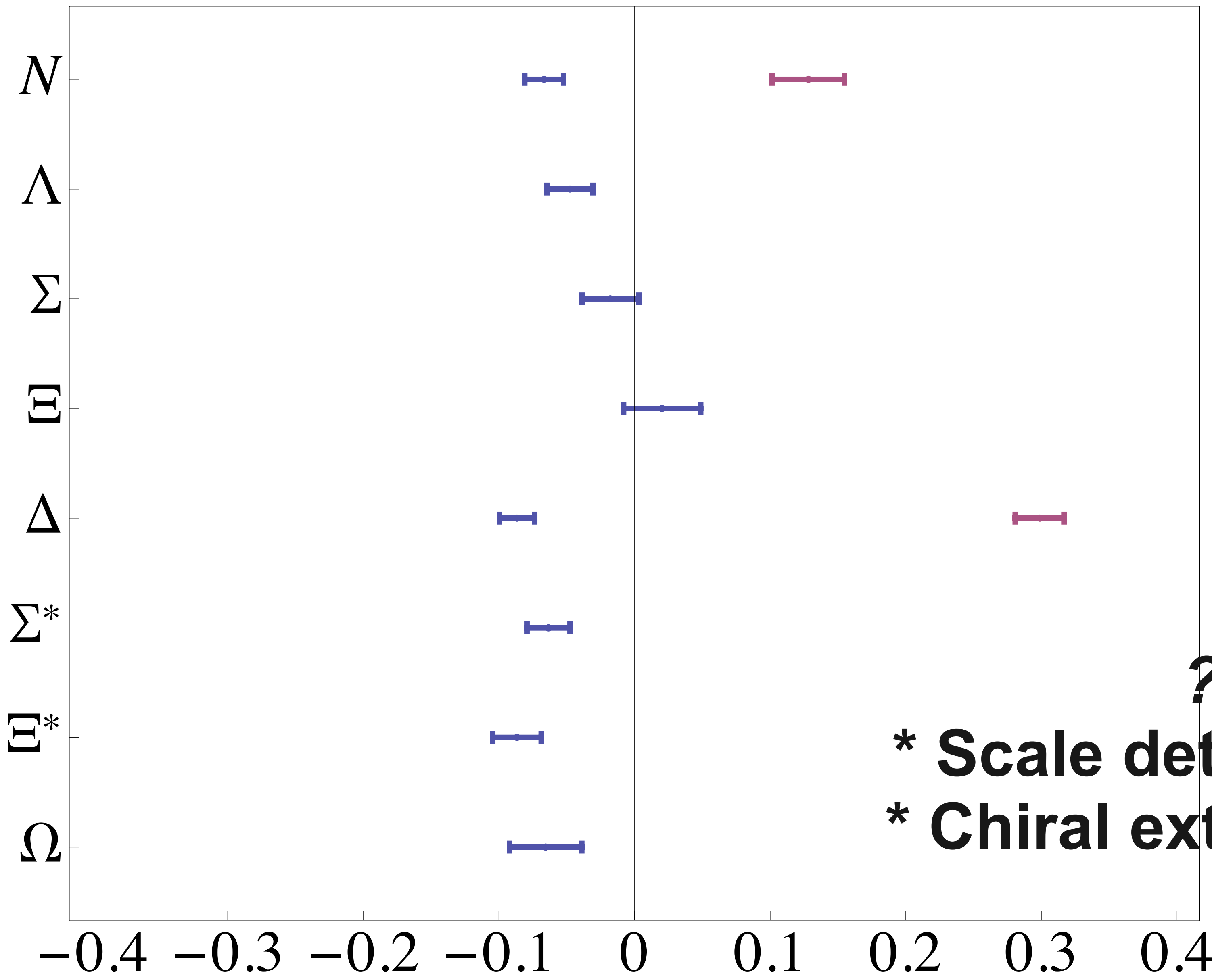
Ratio plot - Quenched QCD

New (2004) FLIC fermion results



Ratio plot - Quenched QCD

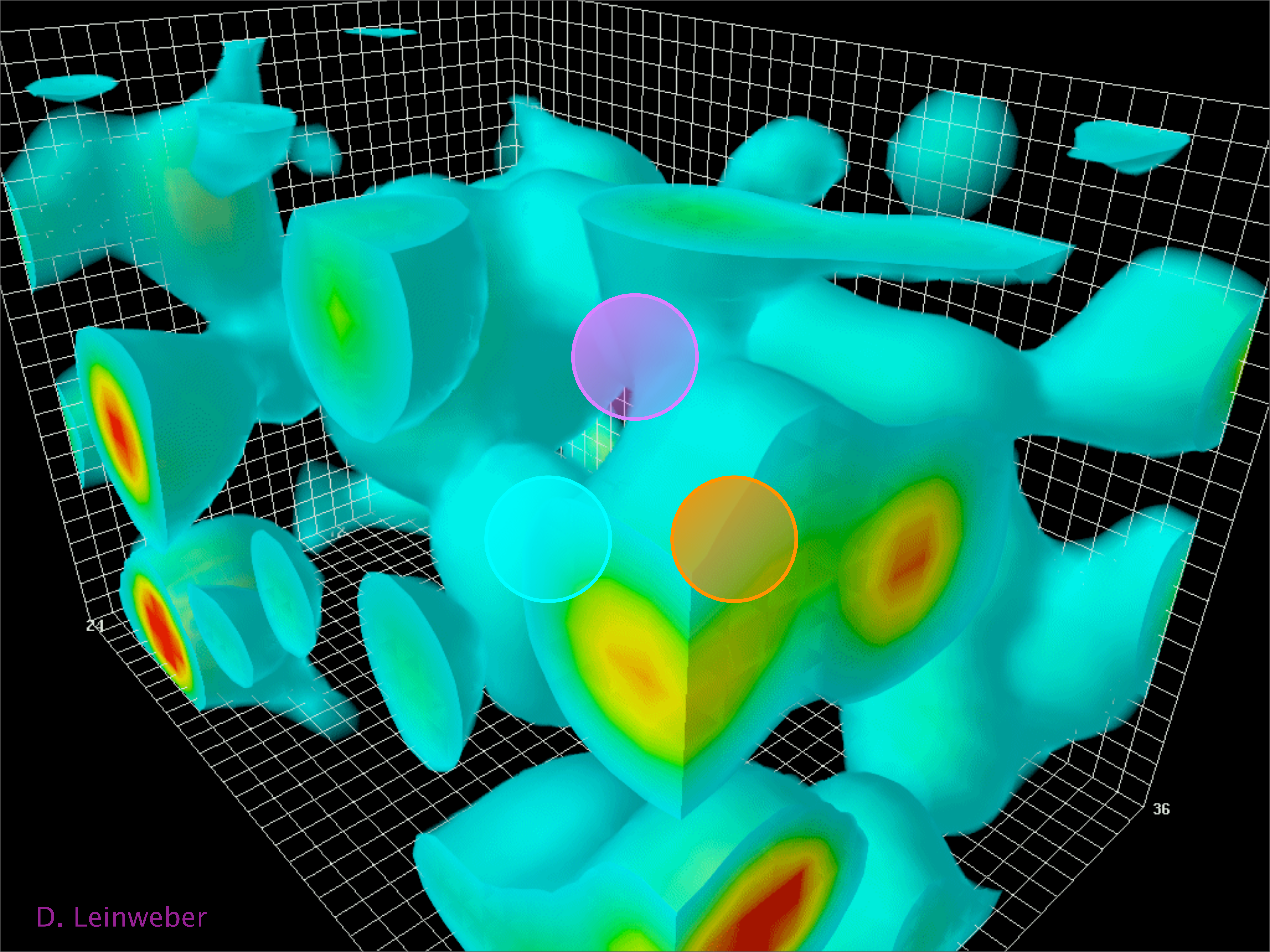
New (2004) FLIC fermion results



??

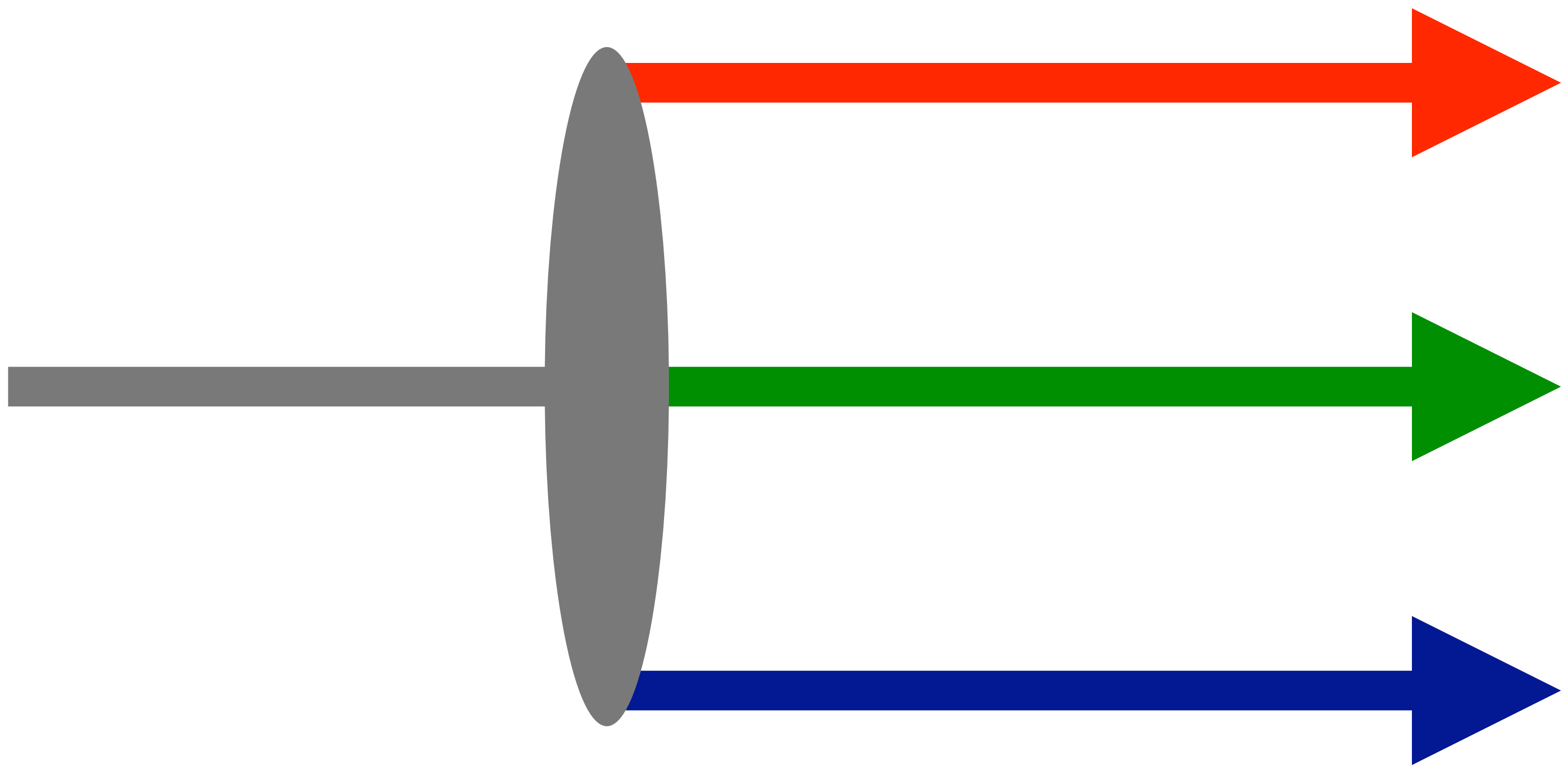
*** Scale determination**

*** Chiral extrapolation!**

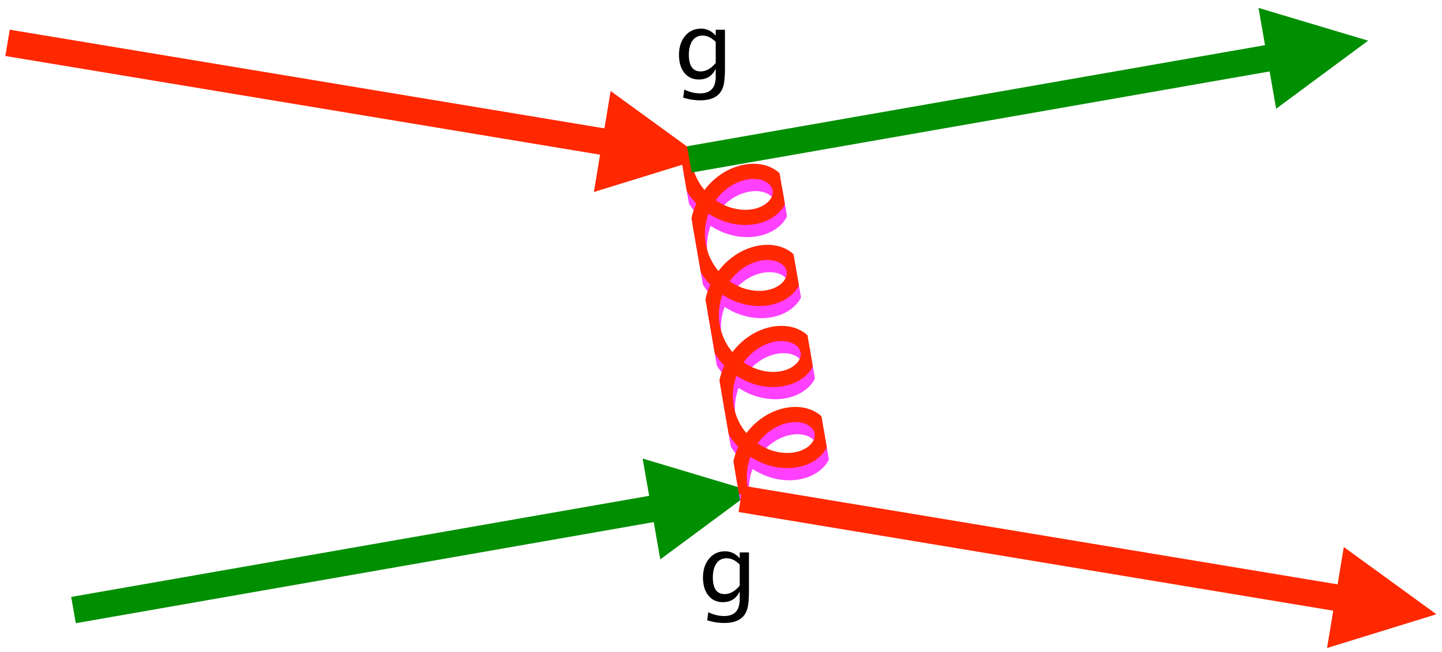

24
36
D. Leinweber

Perturbative Limit

- At high-energies (or short distances) quarks become essentially "free"
- Quarks in a highly relativistic nucleon interact weakly

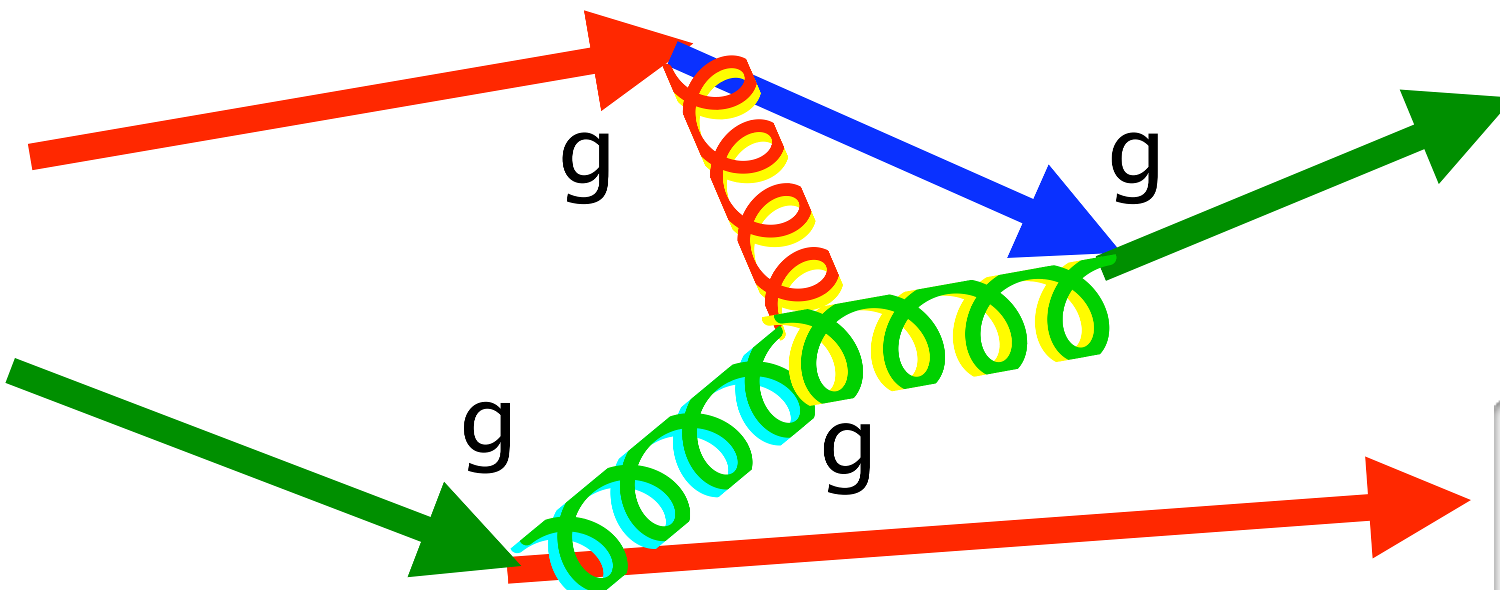


Small coupling - perturbative expansion reliable



Each vertex gives coupling factor "g"

$g^2 \sim$ small

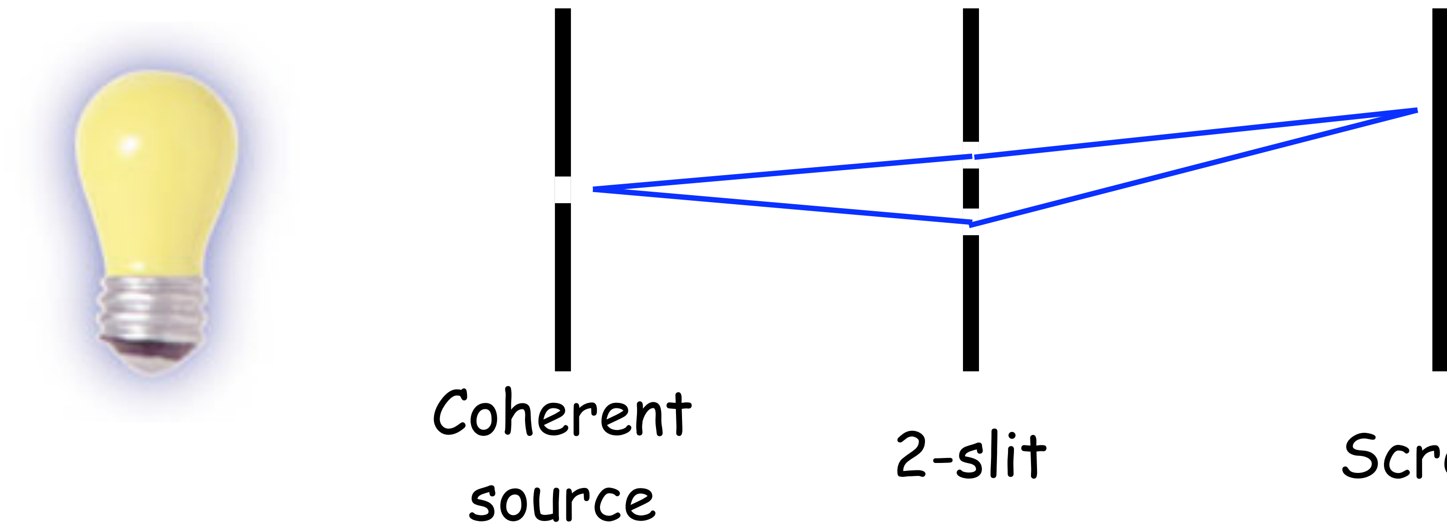


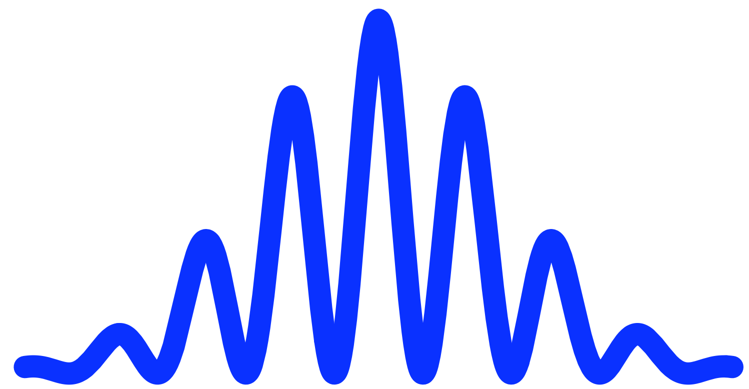
$g^4 \sim$ much smaller

At low-energy, large distances, coupling becomes strong.
Perturbative series meaningless!
$g^2 \sim g^4$

Path Integral

- Quantum Mechanics: Young's Double-slit

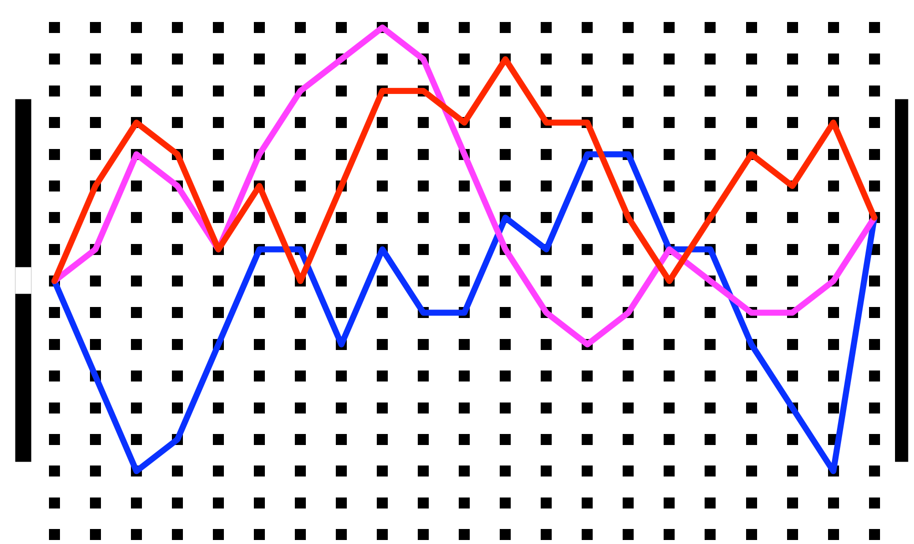




Interference fringes

Infinite # of slits

Infinite # of screens



Superposition of all paths

What is the weight for each path?

QCD Partition Function

$$Z = \int \mathcal{D}A_\mu \mathcal{D}\bar{\Psi}\mathcal{D}\Psi \exp(iS_{\mathrm{QCD}})$$

Gluons (8)

Antiquarks (6)

Quarks (6)

Functional integration

$$\mathcal{D}A_\mu \equiv \prod_x dA_\mu(x)$$

Down, Up, Strange, Charm, Bottom, Top

Dimension of integration:

$$8 \times 4 \times 6 \times 12 \times 6 \times 12 \times \#\text{points in space!}$$

$$= 165888 \quad \#\text{points in space!}$$

vector potential

colour x spin

Lattice QCD

- Rotate to imaginary time: Euclidean space

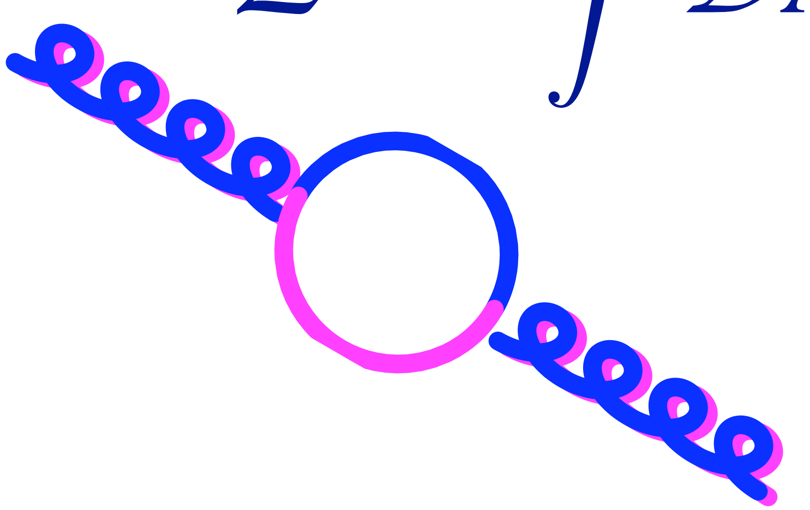
$$Z^E = \int \mathcal{D}A\,\mathcal{D}\bar{\Psi}\,\mathcal{D}\Psi\,\underline{\exp\left(-S^E_{\mathrm{QCD}}\right)}$$

Real, positive weight

PROBABILITY!

Integrate fermion fields (Gaussian integral):

$$Z^E = \int \mathcal{D}A\,\underline{\det M_f[A]}\,\exp\left(-S^E_{\mathrm{gluon}}\right)$$



Determinant difficult to calculate

Neglect heavy quarks: charm, bottom, top

Sometimes neglect light: Quenched Approx.

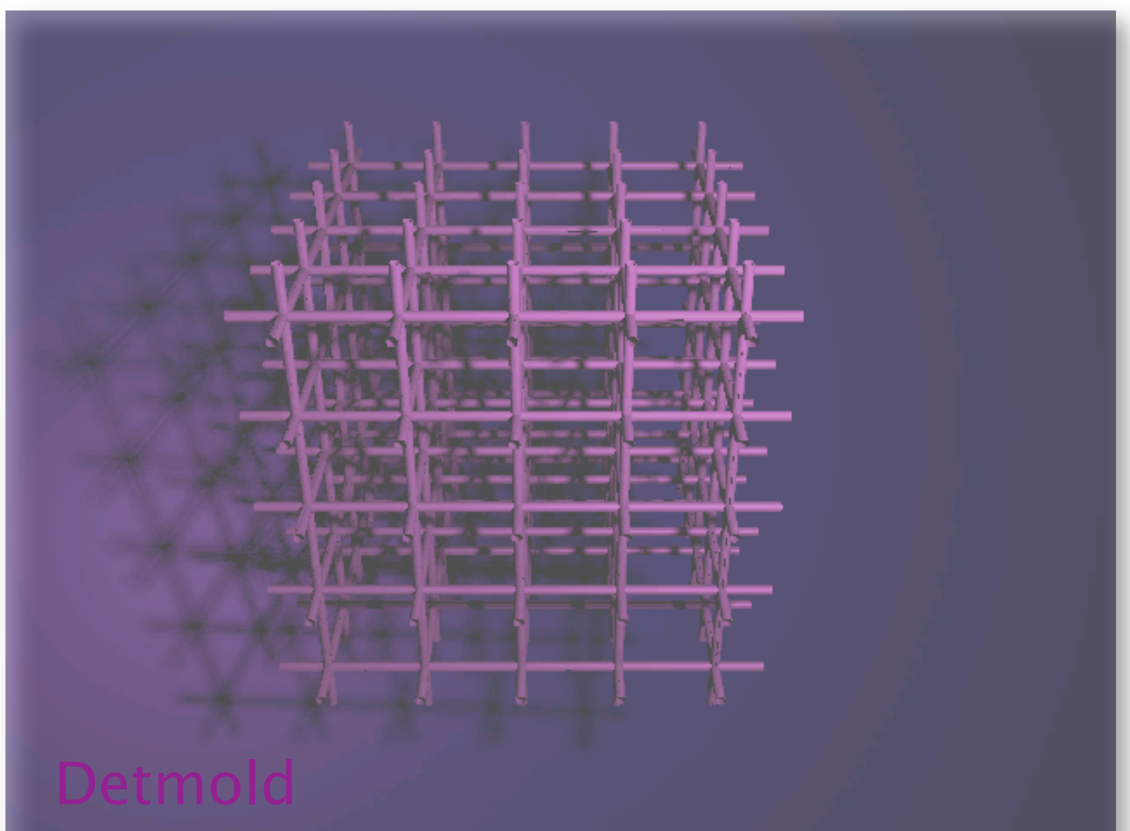
Discretise QCD Action

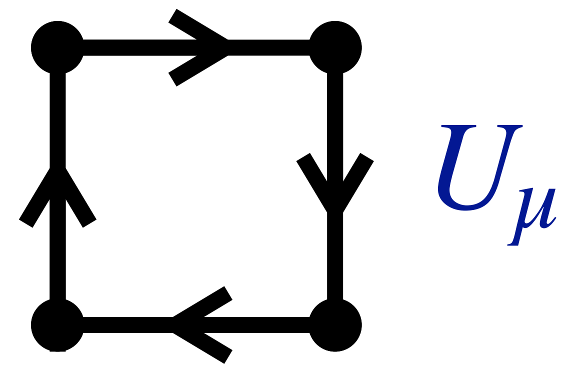
Derivatives -> Finite-difference eqns.

Integrals -> Sums

Gluon field -> "link variable":

Path-ordered exponential between neighbour sites





Plaquette

Sum over all **plaquettes** reduces to continuum action as lattice spacing approaches zero

QCD Green's Functions

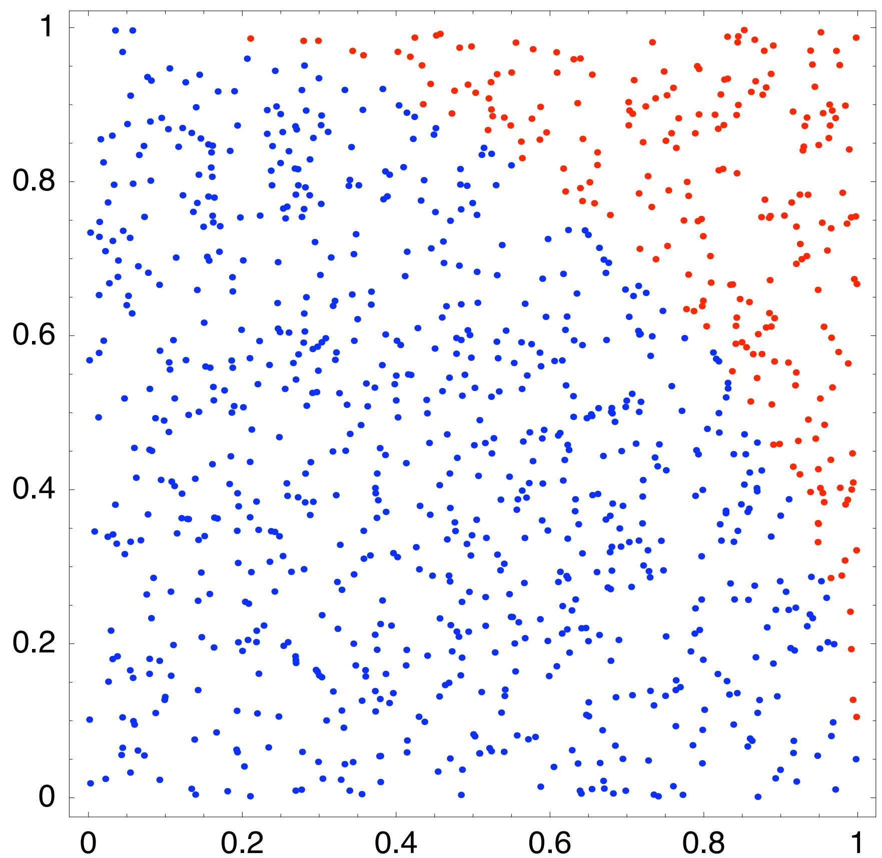
$$Z = \int \mathcal{D}U \, \det M_f[U] \exp\left(-S_{\text{gluon}}^{\text{LAT}}\right)$$

$$\langle \hat{O} \rangle = \frac{1}{Z} \int \mathcal{D}U \, \hat{O} \underbrace{\det M_f[U] \exp\left(-S_{\text{gluon}}^{\text{LAT}}\right)}_{\text{PROBABILITY!}}$$

For moderate lattice size still require $>10^7$-dimensional integration!

Monte Carlo Integration

- Estimating π



1000 random points:

$$\pi = 3.11 \pm 0.05$$

$$\langle \hat{O} \rangle = \frac{1}{Z} \int \mathcal{D}U \, \hat{O} \det M_f[U] \exp\left(-S_{\mathrm{gluon}}^{\mathrm{LAT}}\right)$$

Generate ensemble of gauge configurations according to Boltzmann weight

$$\langle \hat{O} \rangle = \sum_{\{U\}} \hat{O}[U]$$

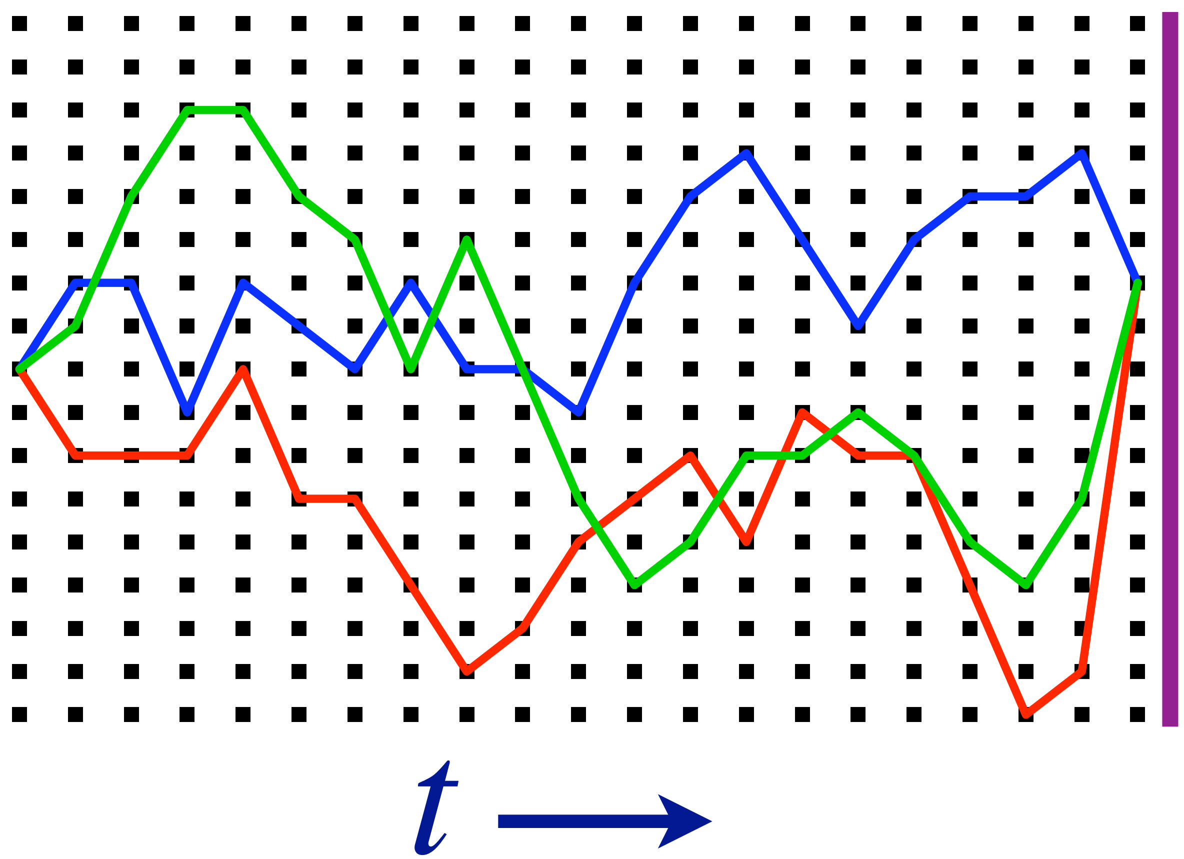
Can reuse same list of gauge configurations for many observables

Extracting a Mass

- A low energy observable of QCD!
- Each gauge configuration, calculate quark propagator: Invert fermion matrix

$$M = (\not{D} + m)$$

Choose quark configuration with desired quantum numbers



Sum over all spatial sites at sink: Projects out states of zero momenta

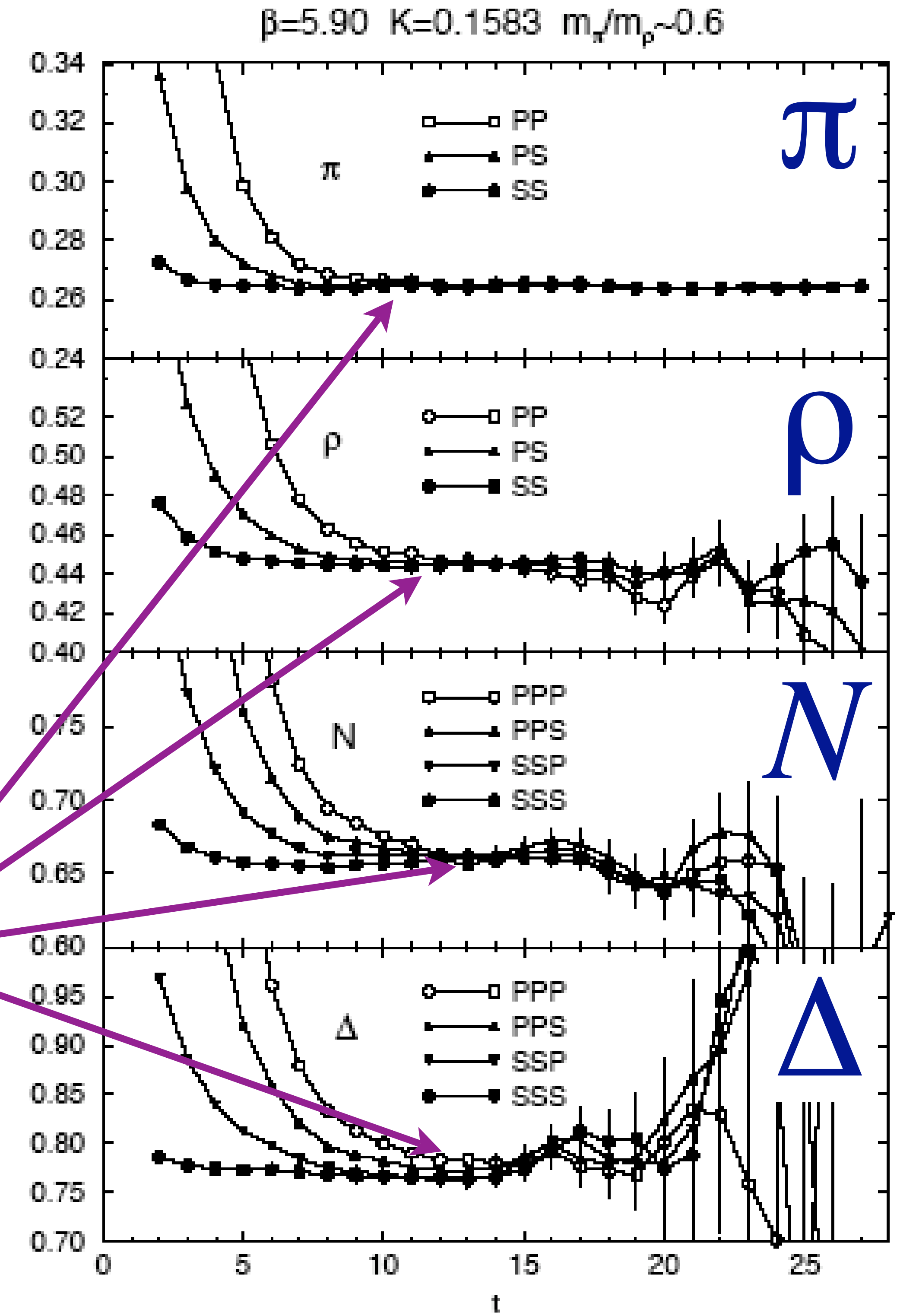
Euclidean Evolution

Correlation function $$C_N(t,\vec{p}) = \sum_{\vec{x}} e^{i\vec{p}.\vec{x}} \langle \Omega | \chi_N(x) \overline{\chi}_N(0) | \Omega \rangle$$

$$\begin{aligned}
C_N(t,\vec{p}) &= \sum_{\alpha,\vec{q}} \sum_{\vec{x}} e^{-i\vec{p}.\vec{x}} \langle \Omega | \chi_N(x) | \alpha(\vec{q}) \rangle \langle \alpha(\vec{q}) | \overline{\chi}_N(0) | \Omega \rangle \,, \\
&= \sum_{\alpha,\vec{q}} \sum_{\vec{x}} e^{-i\vec{p}.\vec{x}} \langle \Omega | e^{-i\hat{q}.x} \chi_N(0) e^{+i\hat{q}.x} | \alpha(\vec{q}) \rangle \langle \alpha(\vec{q}) | \overline{\chi}_N(0) | \Omega \rangle \,, \\
&= \sum_{\alpha,\vec{q}} \sum_{\vec{x}} e^{-i(\vec{p}-\vec{q}).\vec{x}} e^{-E_\alpha t} \langle \Omega | \chi_N(0) | \alpha(\vec{q}) \rangle \langle \alpha(\vec{q}) | \overline{\chi}_N(0) | \Omega \rangle \,, \\
&= \sum_{\alpha,\vec{q}} \delta(\vec{p}-\vec{q}) e^{-E_\alpha t} \langle \Omega | \chi_N(0) | \alpha(\vec{q}) \rangle \langle \alpha(\vec{q}) | \overline{\chi}_N(0) | \Omega \rangle \,, \\
&= \sum_{\alpha} e^{-E_\alpha(\vec{p})t} \langle \Omega | \chi_N(0) | \alpha(\vec{p}) \rangle \langle \alpha(\vec{p}) | \overline{\chi}_N(0) | \Omega \rangle \,.
\end{aligned}$$

Ground States

Effective mass plots



[Aoki et al. [CP-PACS] PRD67 (2003), 034503]

Correlation function as $t \longrightarrow \infty$

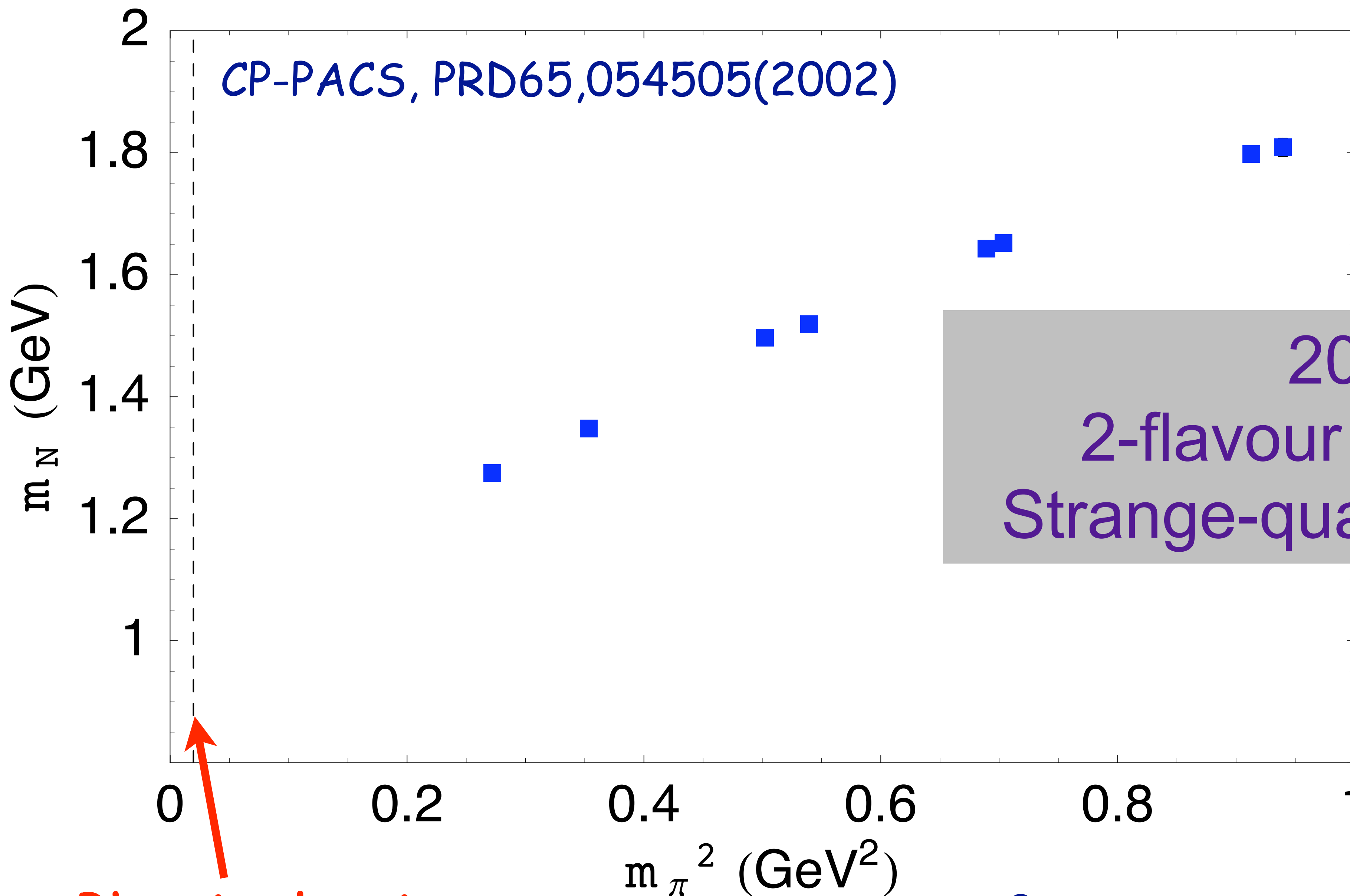
$$\lambda_0 e^{-m_0 t} + \lambda_1 e^{-m_1 t} + \ldots$$

Euclidean time evolution suppresses excited states

Ground-state mass from plateau

$$M_{\text{eff}} = \log \frac{C(t)}{C(t+1)}$$

Nucleon Mass

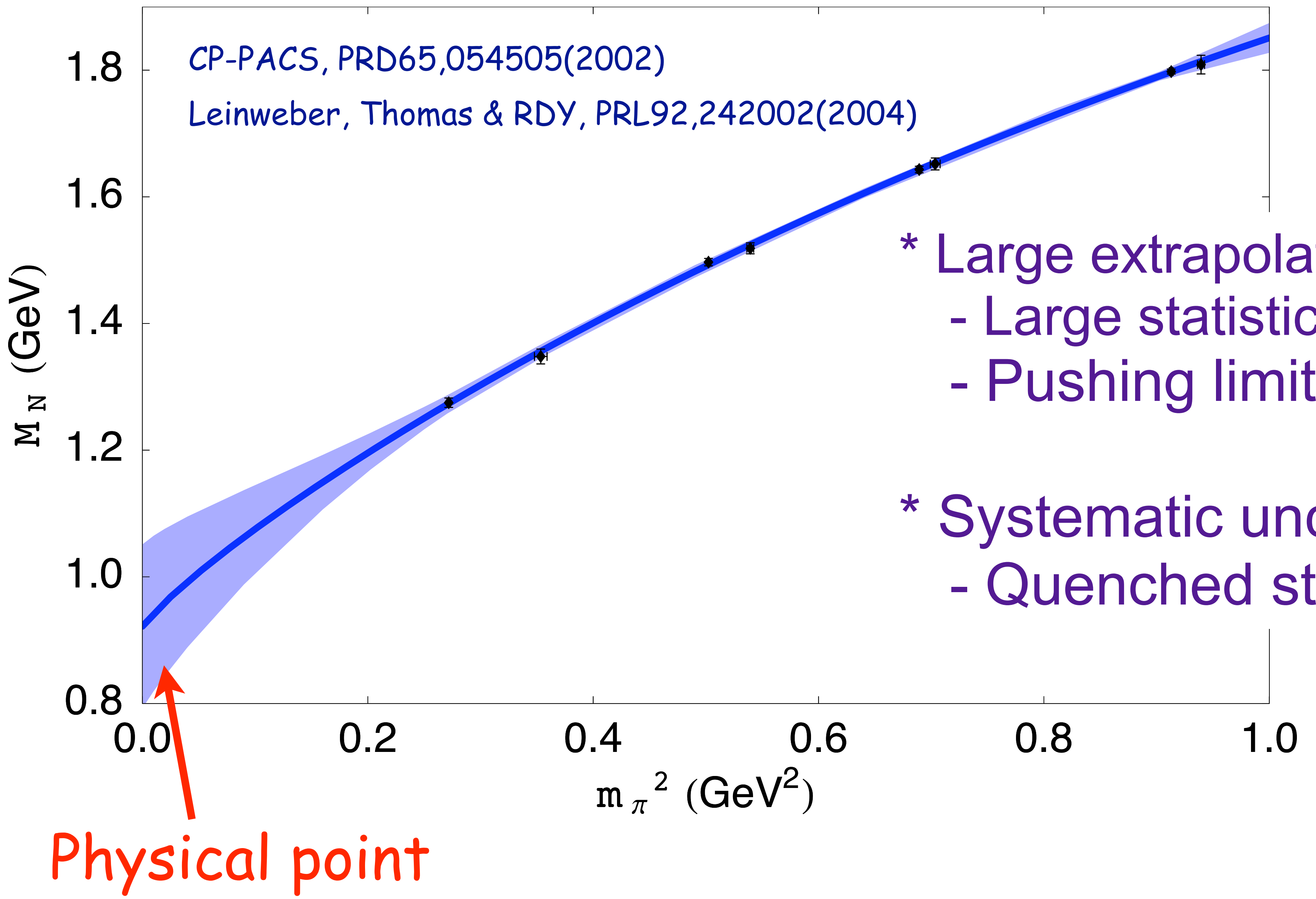


2002:
2-flavour dynamical
Strange-quark quenched

Physical point

$$m_\pi^2 \propto m_q$$

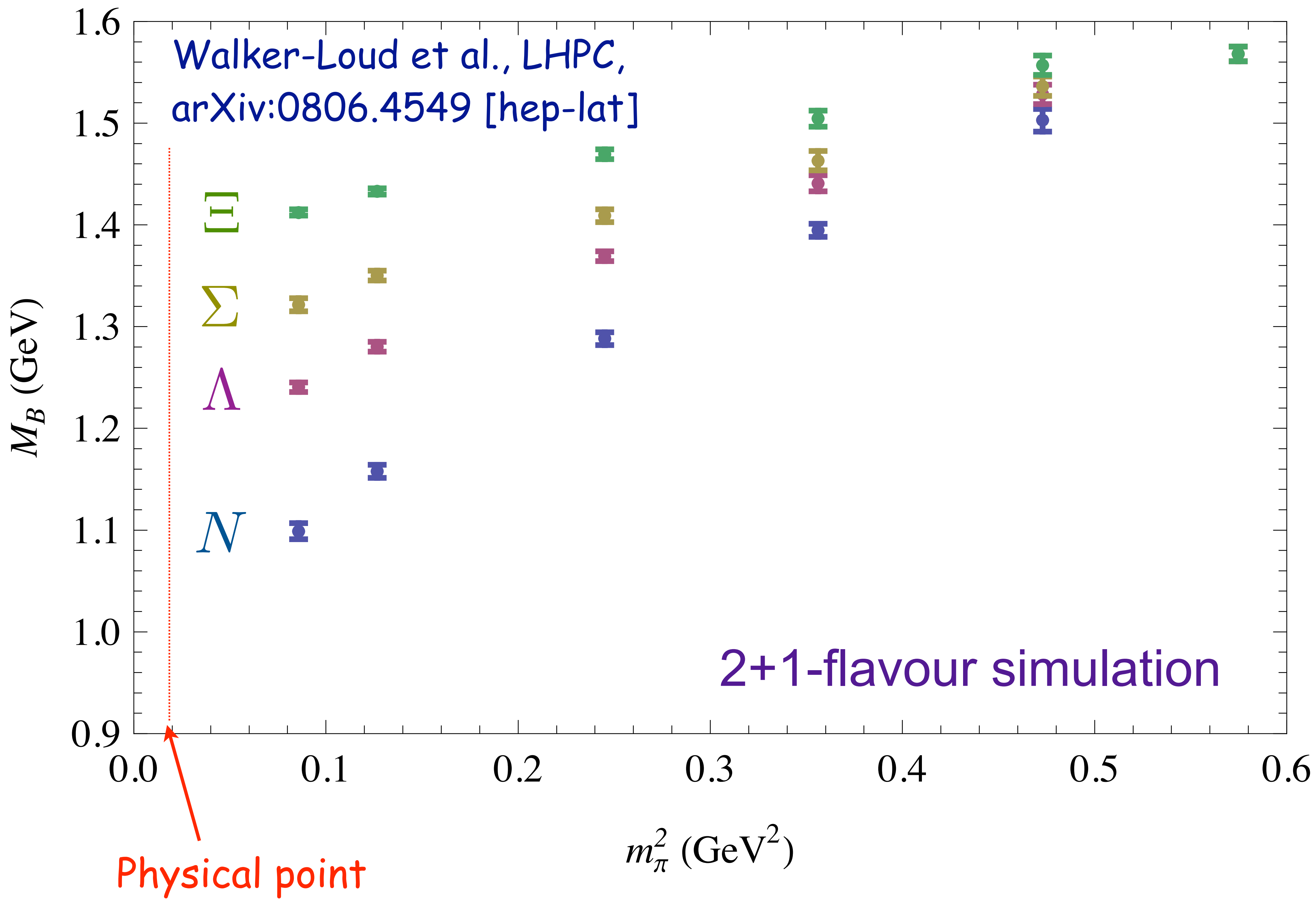
Nucleon Mass



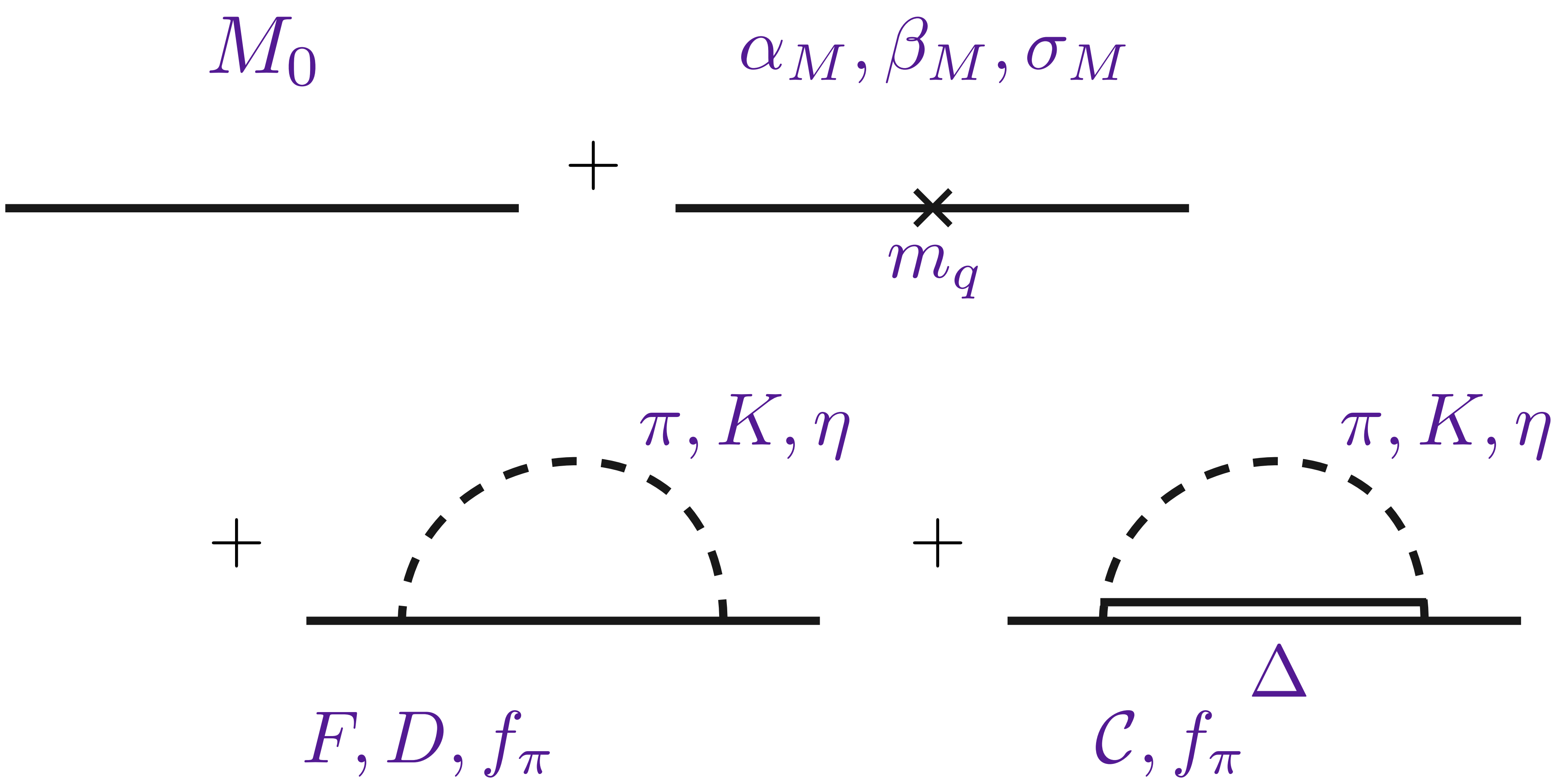
* Large extrapolation distance
- Large statistical uncertainty
- Pushing limits of EFT

* Systematic uncertainty
- Quenched strange quark?

New Simulation Results - LHPC



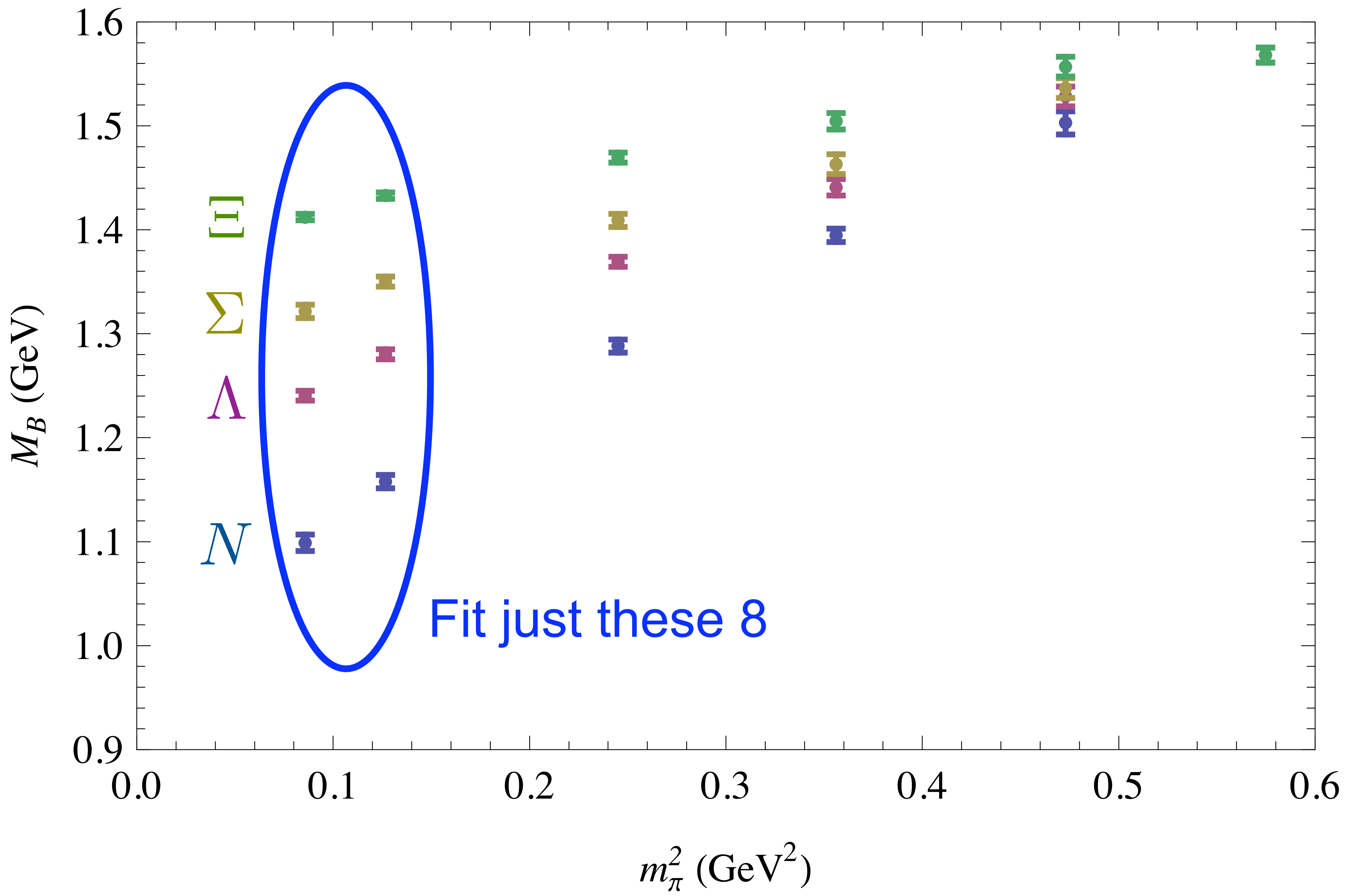
Everything to O(3/2)



Inputs: $g_A = 1.267,\ D = \frac{3}{5} g_A,\ F = \frac{2}{3} g_A,\ \mathcal{C} = -2D,\ f_\pi = 0.087\,\mathrm{GeV}$

$$M_B = M_0 + \delta M^{(1)} + \delta M^{(3/2)} + \mathcal{O}(2)$$

Lattice Simulation Results: LHPC



Power counting estimate for O(2)

- If we adopt conventional wisdom "4 pi fpi"
  - Physical point

$$\mathcal{O}(2) \sim \frac{m_\eta^4}{(4\pi f_\pi)^4} \sim 5\%$$

  - Lattice masses

$$\mathcal{O}(2) \sim \frac{m_\eta^4}{(4\pi f_\pi)^4} \sim 11\%$$

Best fit to lightest 2 quark masses

- Poor fit

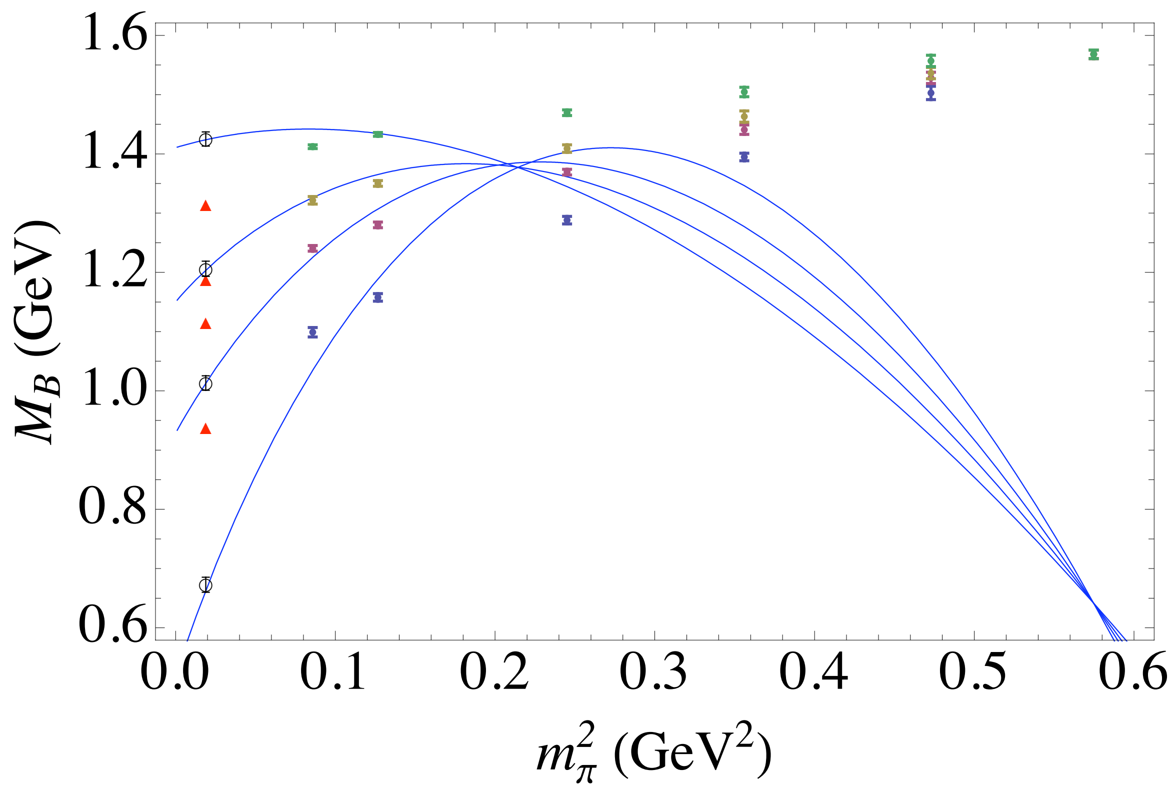
$$\chi^2/\text{dof} \sim 40$$

$$m_\pi \lesssim 0.35\,\text{GeV}$$
$$m_K \lesssim 0.6\,\text{GeV}$$

- "Best" fit $M_0 \sim 0.27\,\text{GeV}$
- *Empirical suggestion*

$$\mathcal{O}(2) \sim \left(\frac{m_\eta}{\Lambda_B}\right)^4 \sim 300\%$$

$$\Lambda_B \sim 0.6\,\text{GeV}$$



What about Finite-Range Regularisation (FRR)?

- Introduce a resummation of higher-order terms with a single parameter
- Chiral loop integrals modified to cut-off divergences

$$\int_0^\infty dk \, \frac{k^4}{k^2+m^2} \left(\frac{\Lambda^2}{\Lambda^2+k^2} \right)^4$$

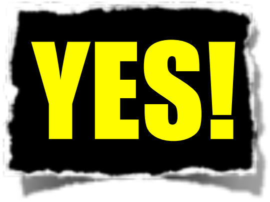
- Upon renormalisation gives identical expansion to *O*(3/2)

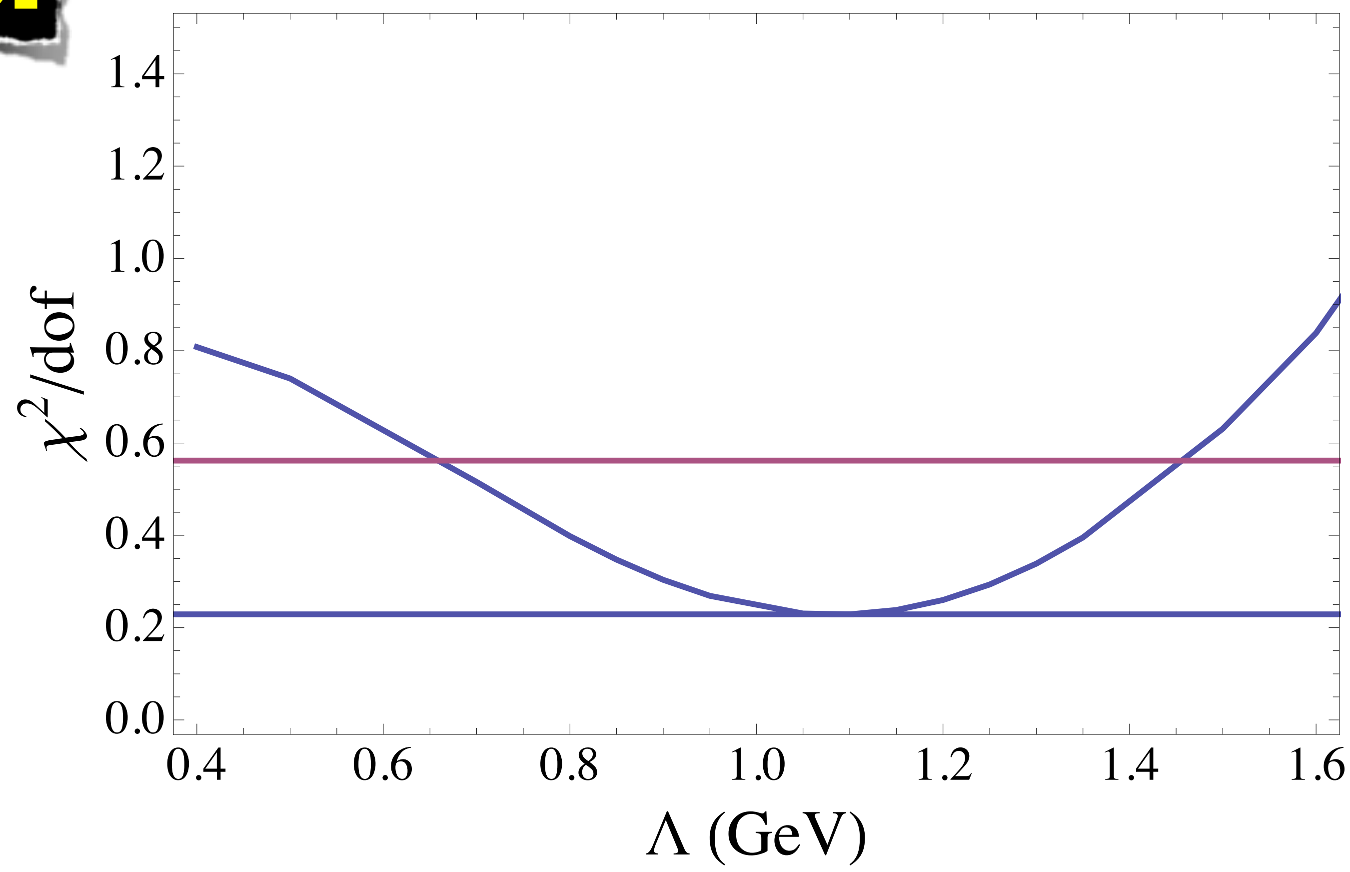
Text book: $M_B^{(3/2)} = M_0 + \delta M^{(1)} + \delta M^{(3/2)} + 0$

FRR: $M_B^{(3/2)} = M_0 + \delta M^{(1)} + \delta M^{(3/2)} + \mathcal{O}(\frac{m_{PS}^4}{\Lambda})$

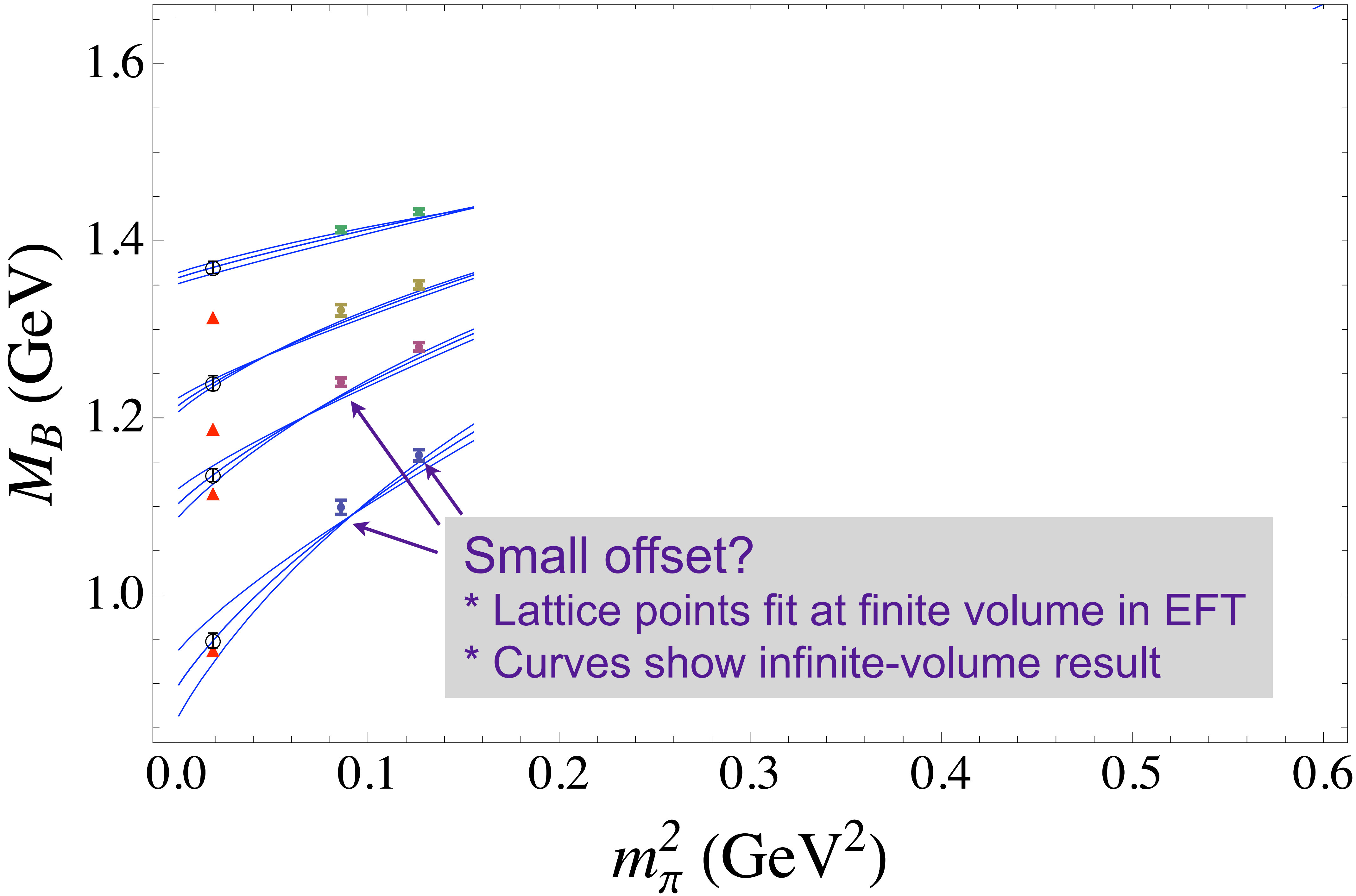
Regularisation parameter?

- Model-indepence of EFT only exists if results independent of this cutoff
- Can the lattice results select a preferred scale to regularise the EFT?

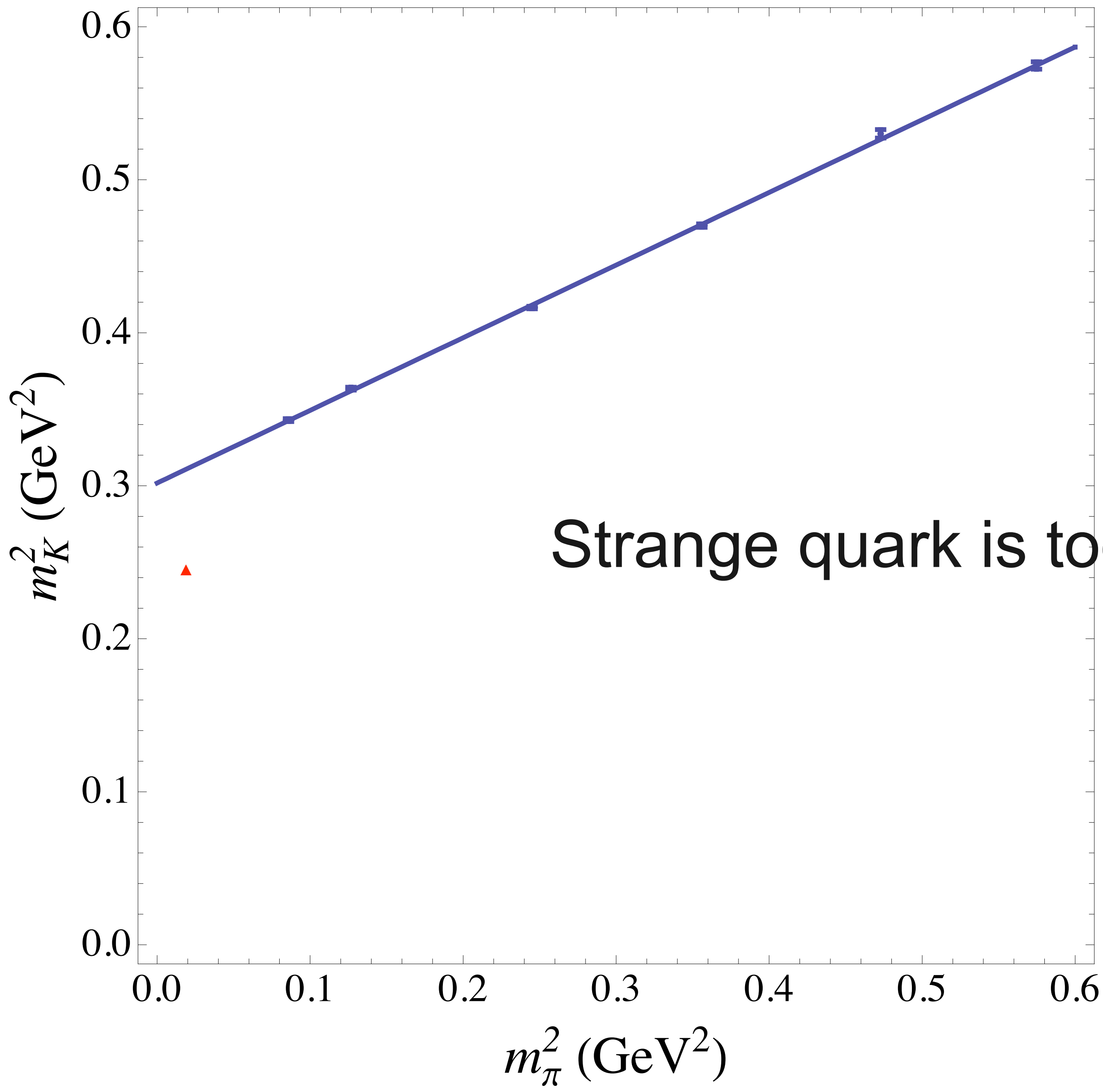




Fits to 2 lightest LHPC points

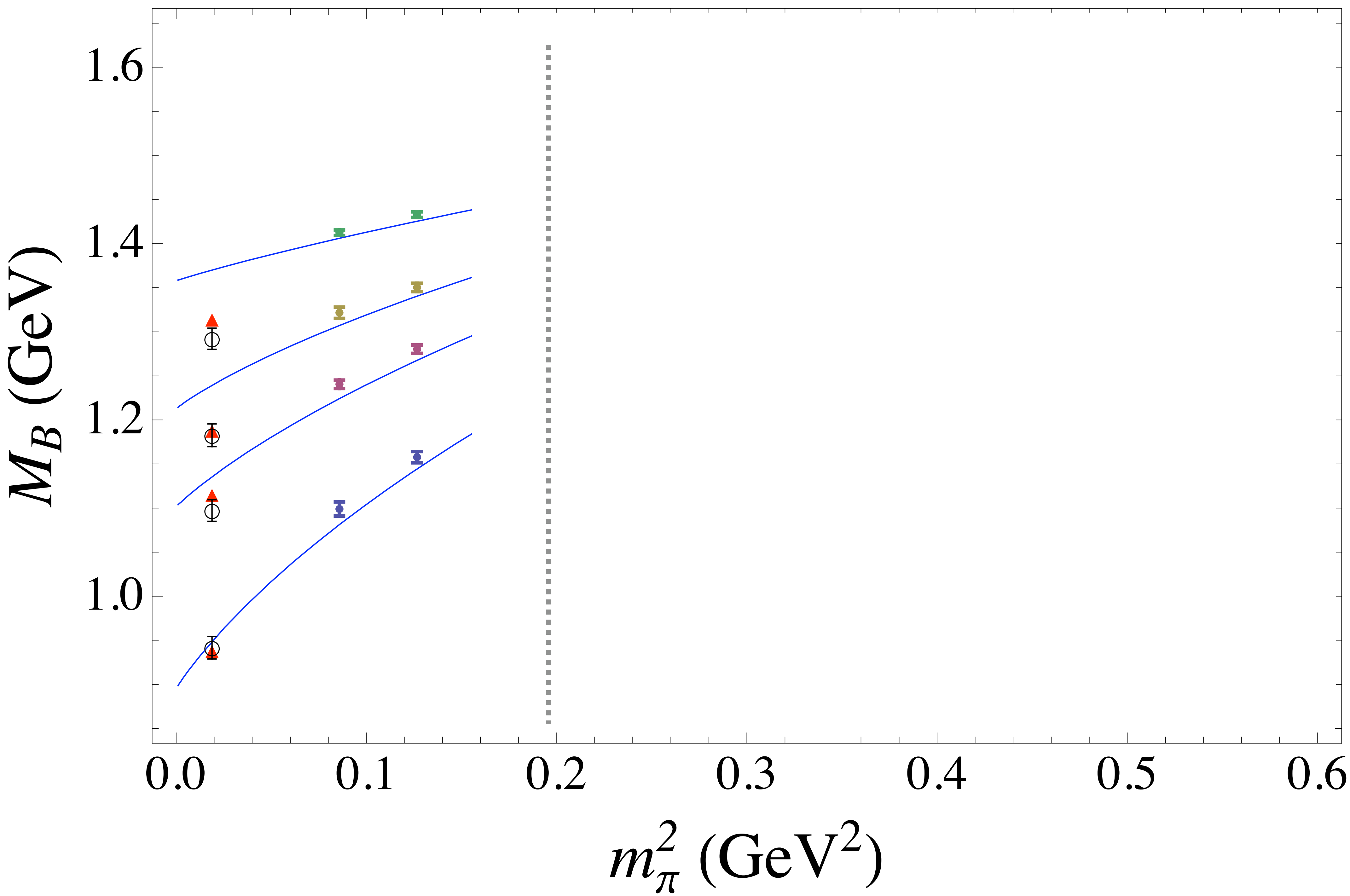


Meson masses - LHPC

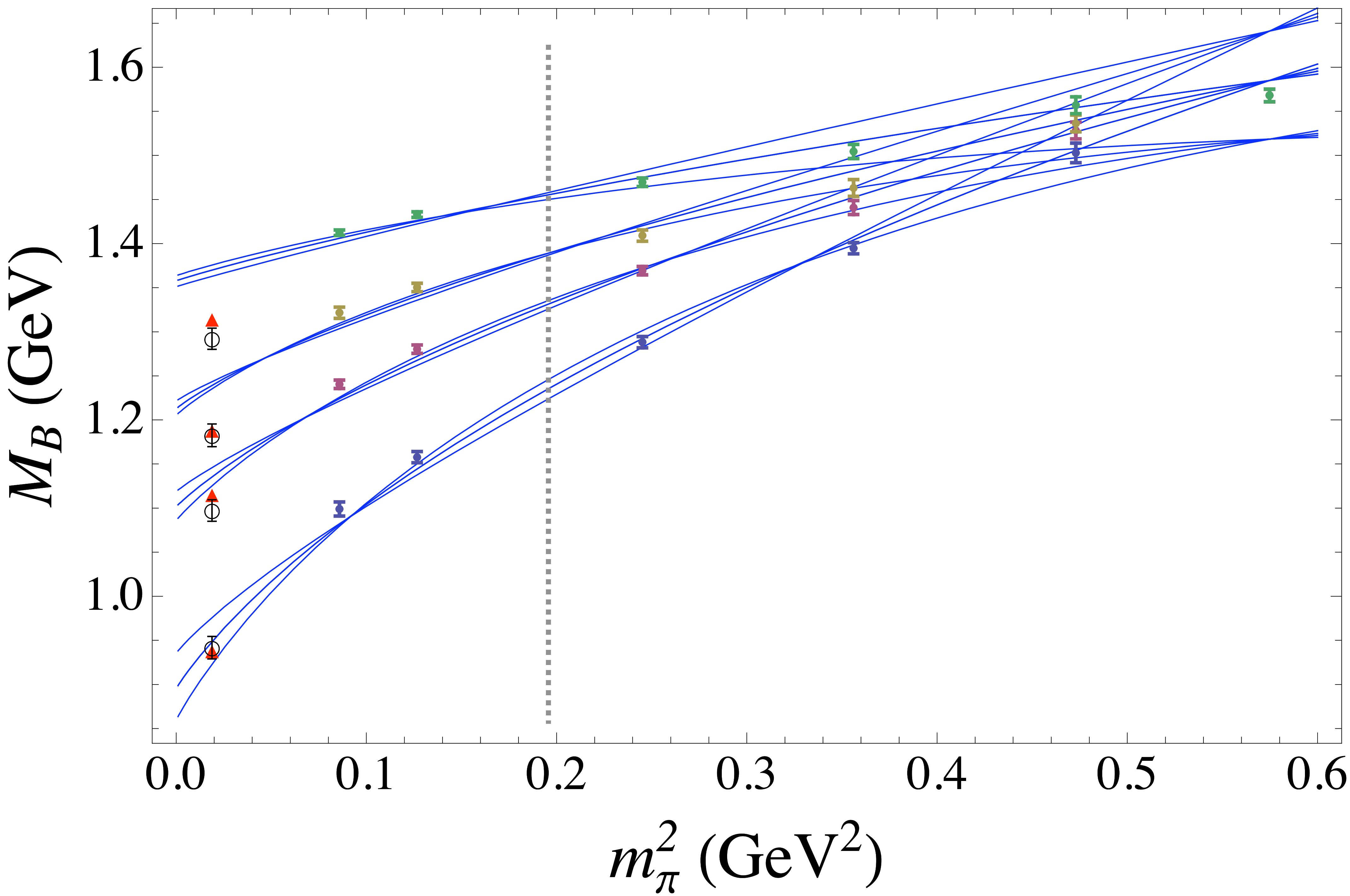


Strange quark is too heavy!

Fits to 2 lightest LHPC points

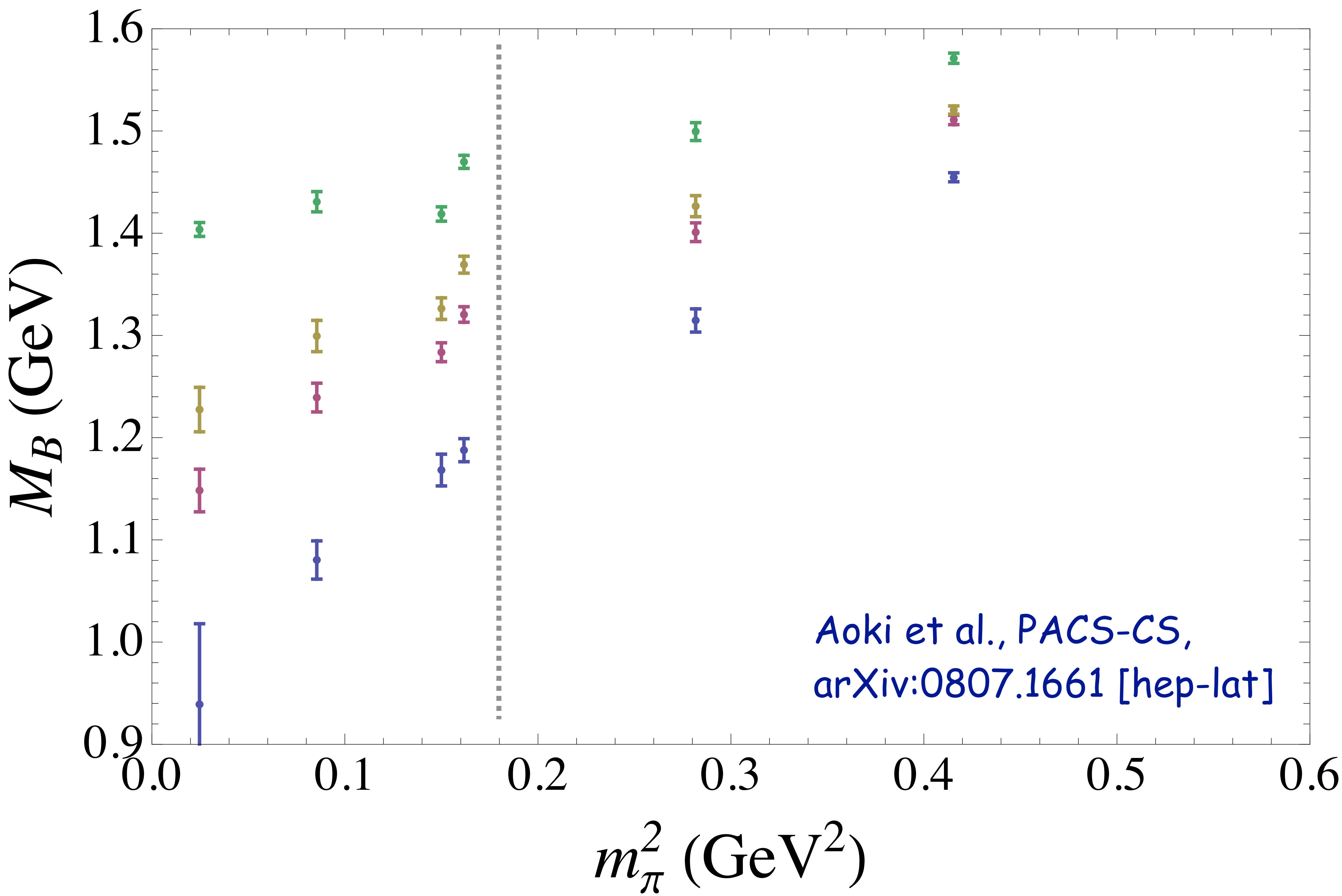


Fits to 2 lightest LHPC points

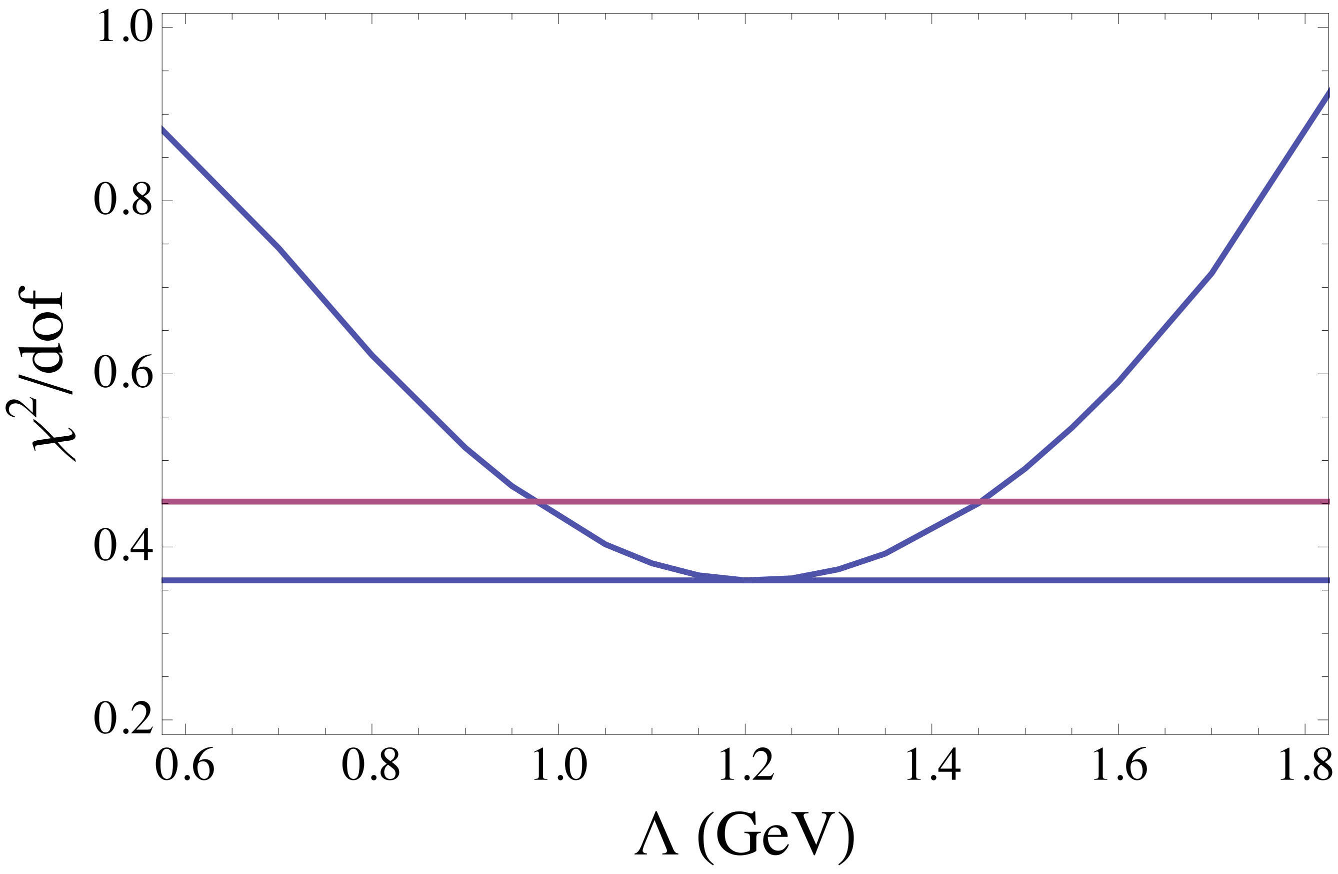


Argonne NATIONAL LABORATORY

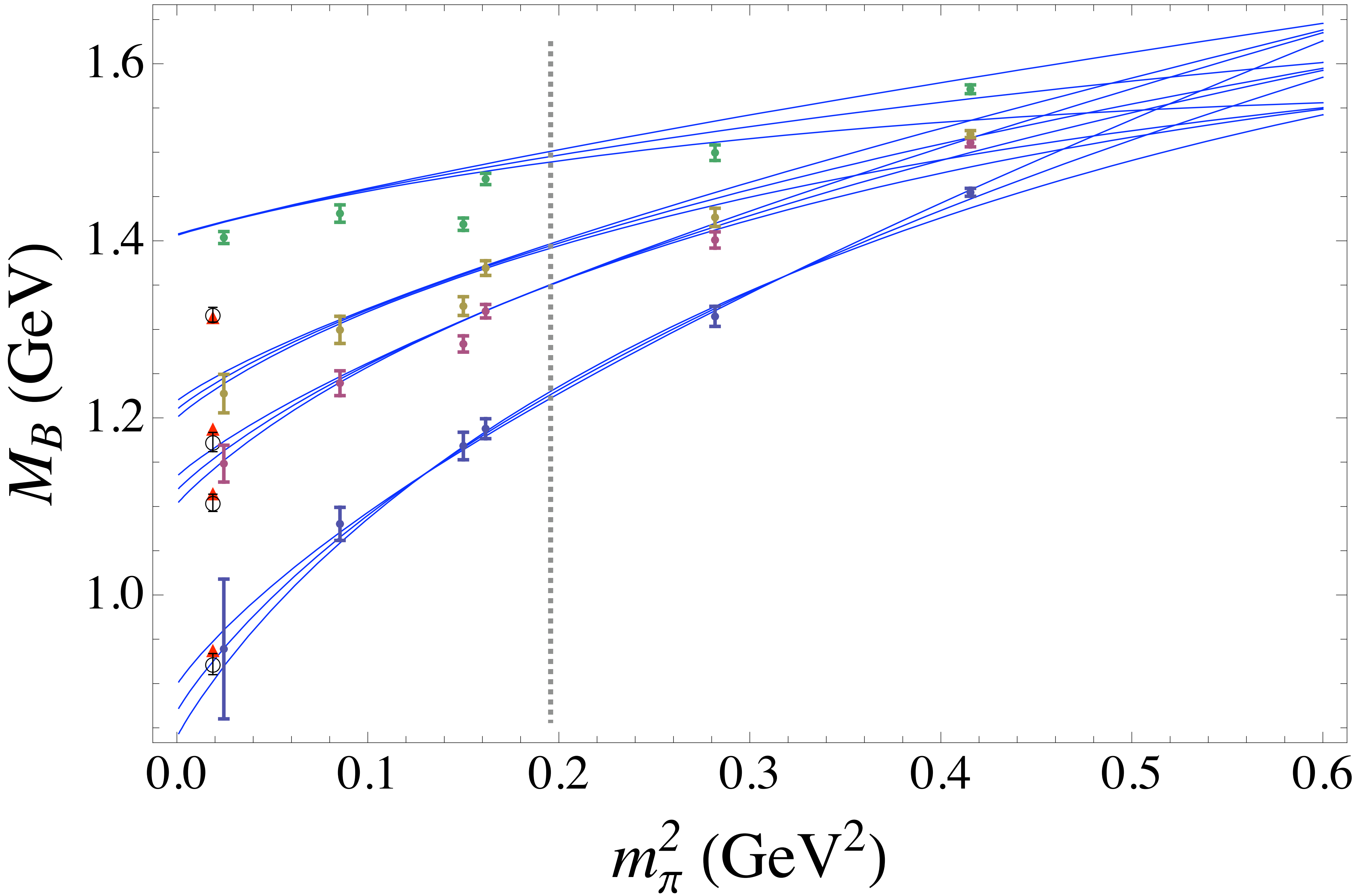
More new lattice results: PACS-CS



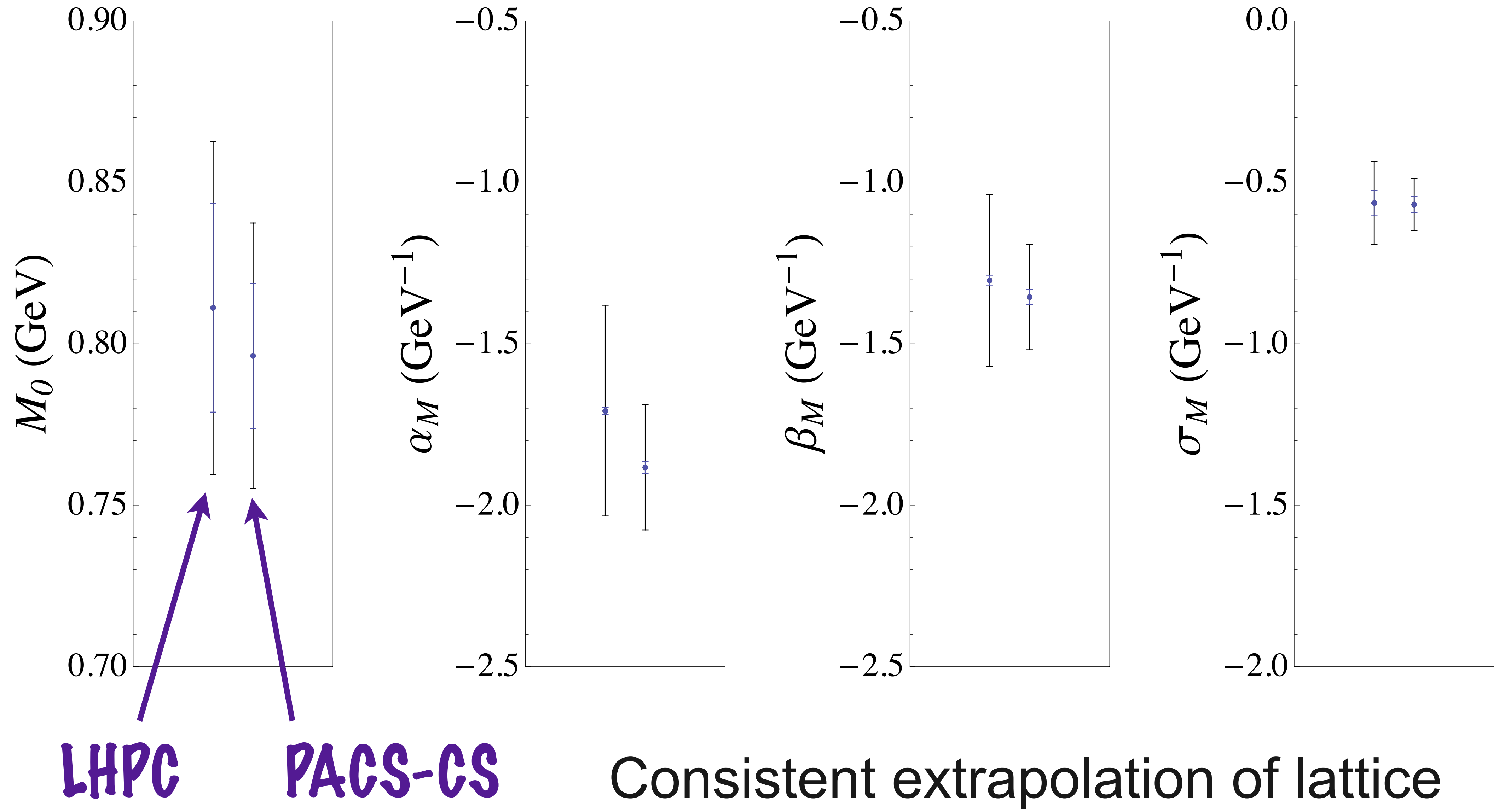
Same regularisation scale



Fit to 4 lightest PACS-CS points



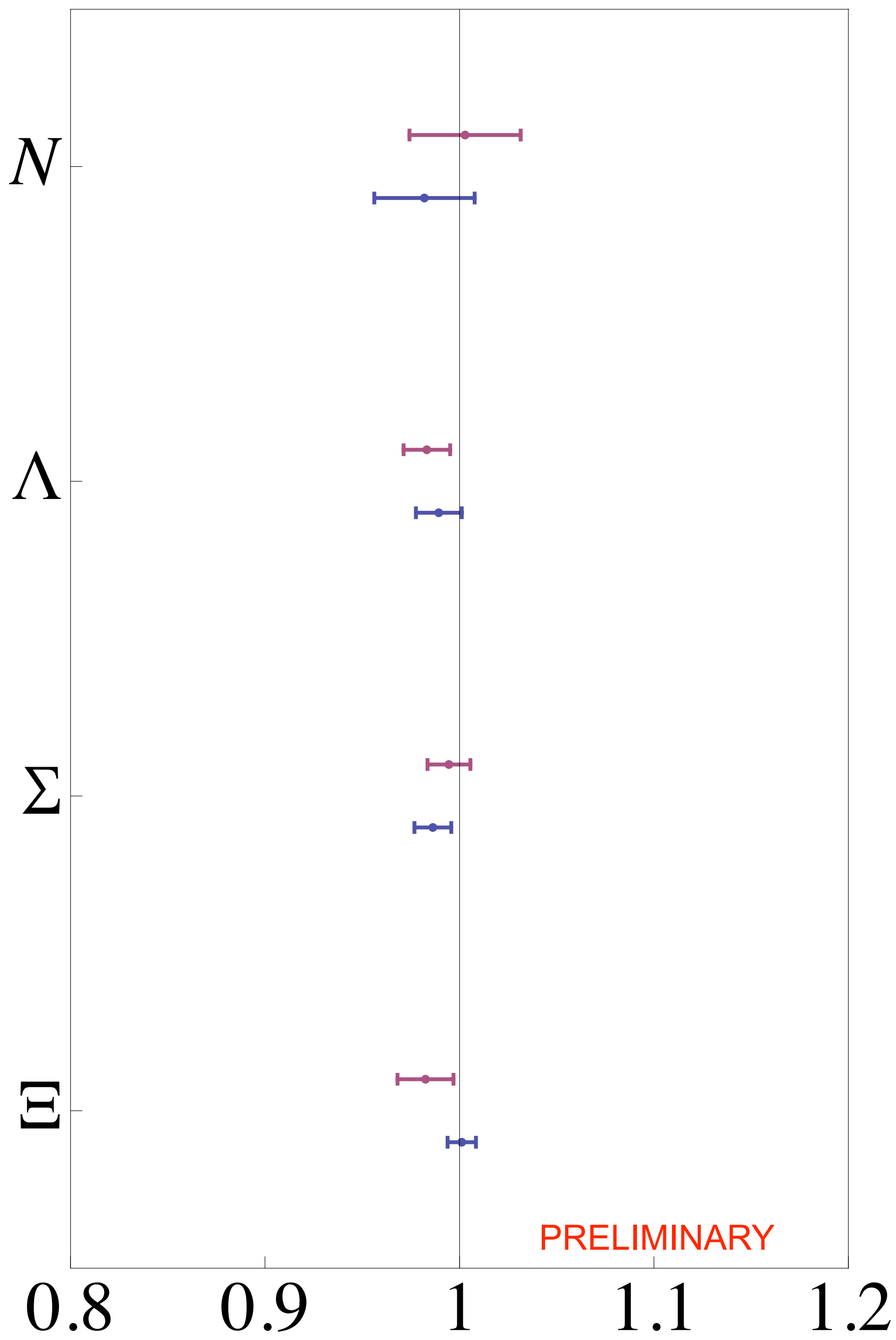
Consistency in LECs?



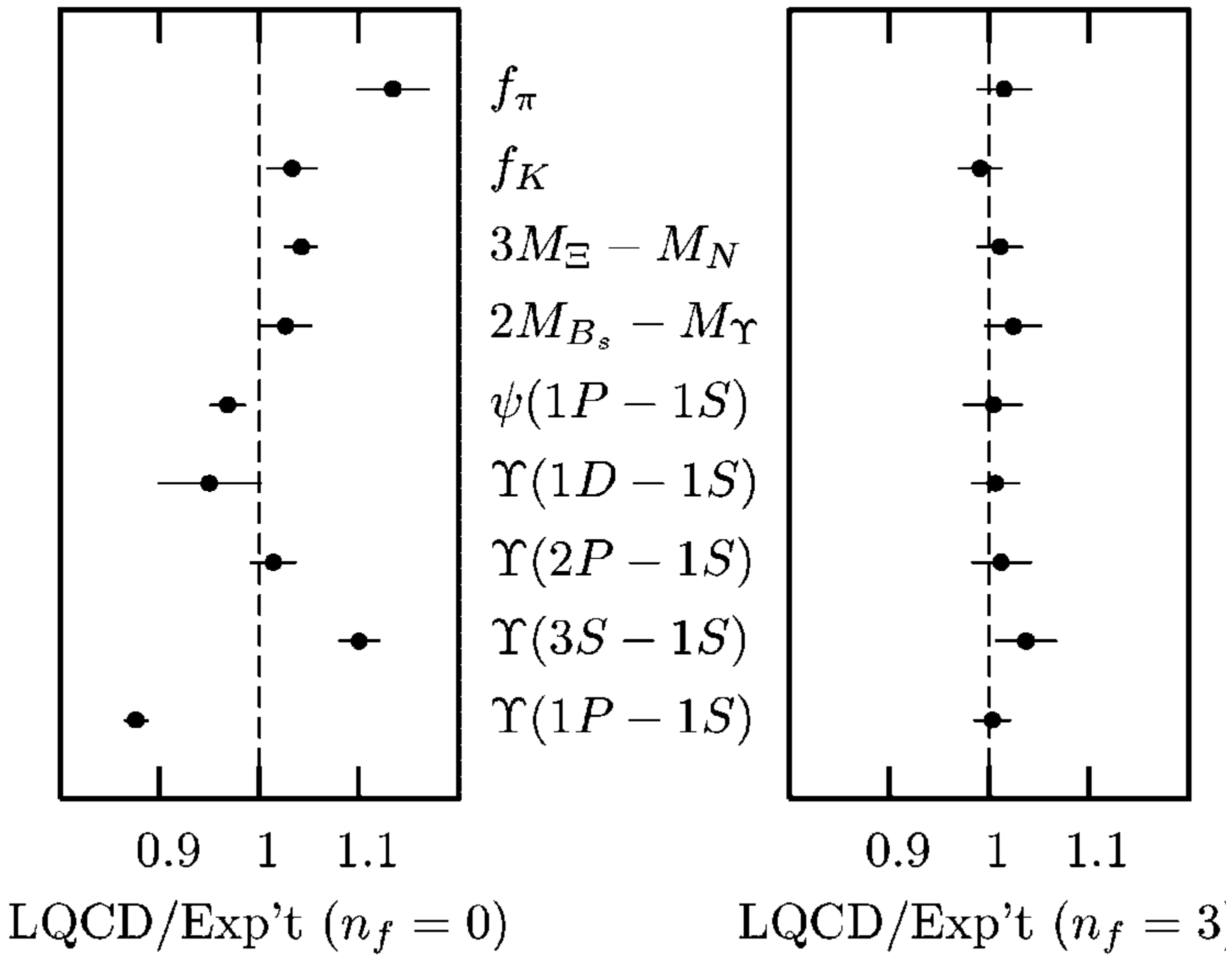
Consistent extrapolation of lattice results to SU(3) chiral limit

Consistency of lattice at finite "*a*"

Precision comparison with experiment

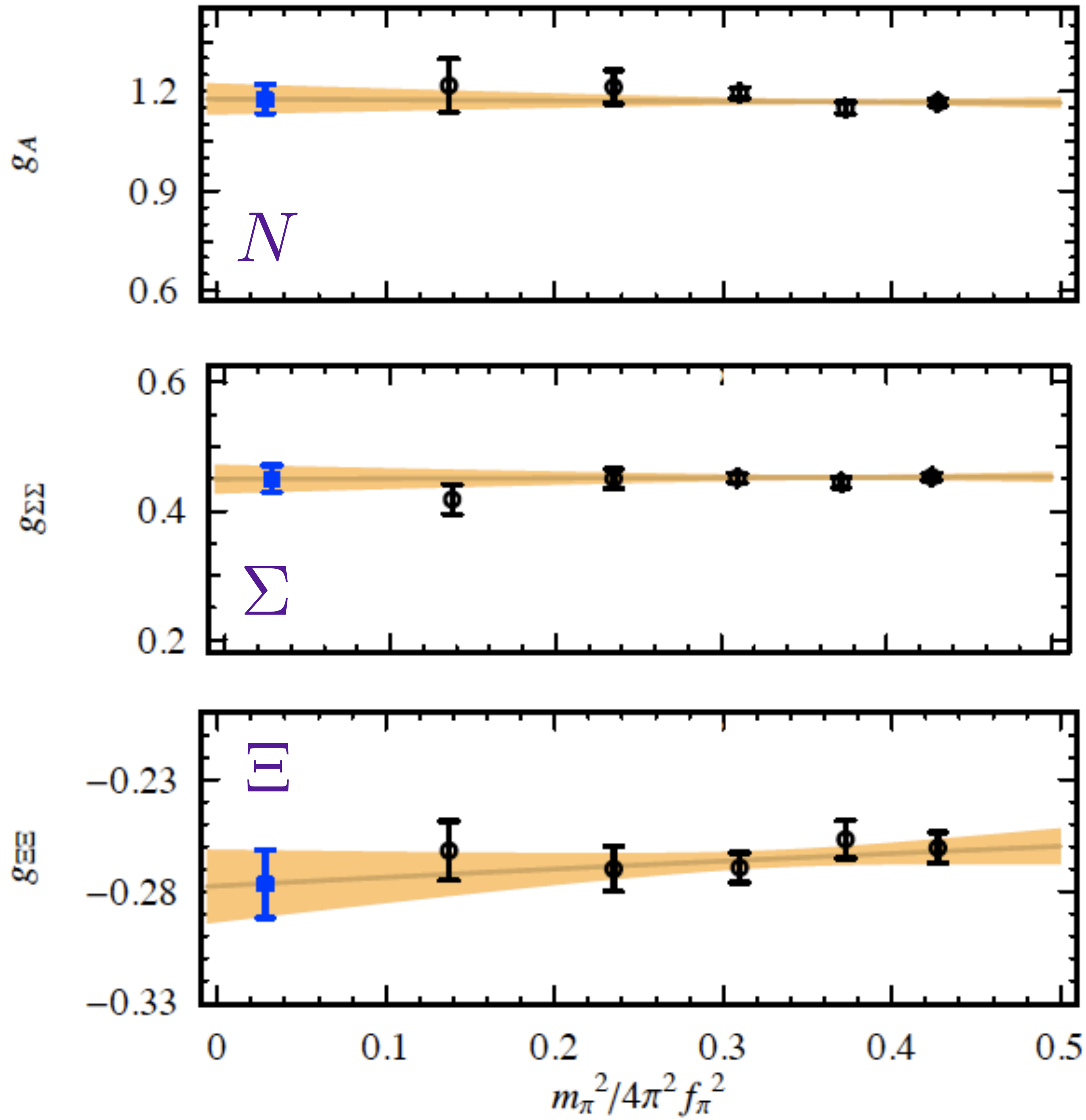


Baryon masses competitive precision with heavy-meson systems!



Beyond Masses - Hyperon Axial Charges

Lin & Orginos, arXiv:0712.1214 [hep-lat]



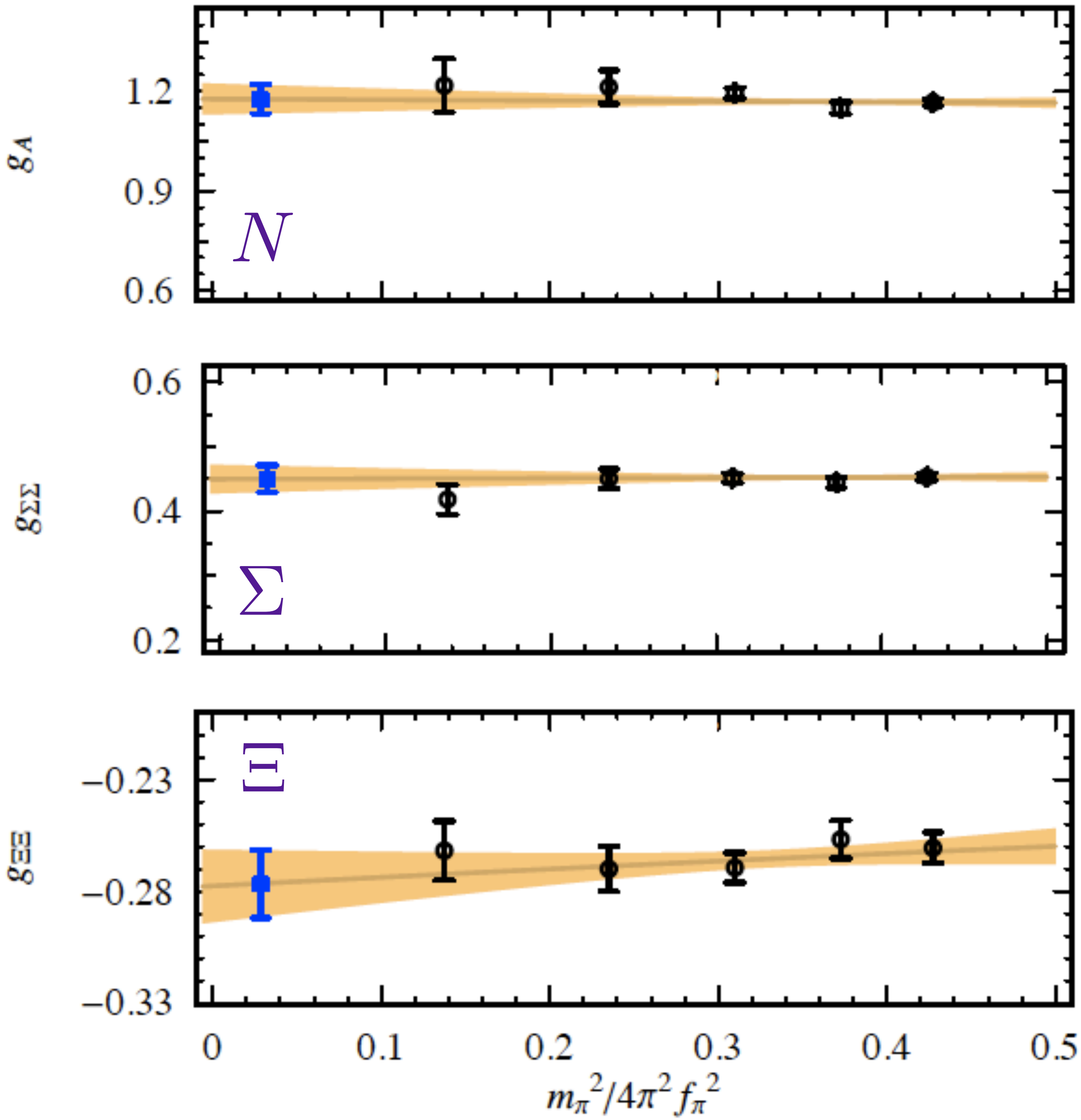
* Linear chiral extrapolations

* SU(3) (*naive*) chiral fits

$$\chi^2/dof \sim 100$$

Beyond Masses - Hyperon Axial Charges

Lin & Orginos, arXiv:0712.1214 [hep-lat]

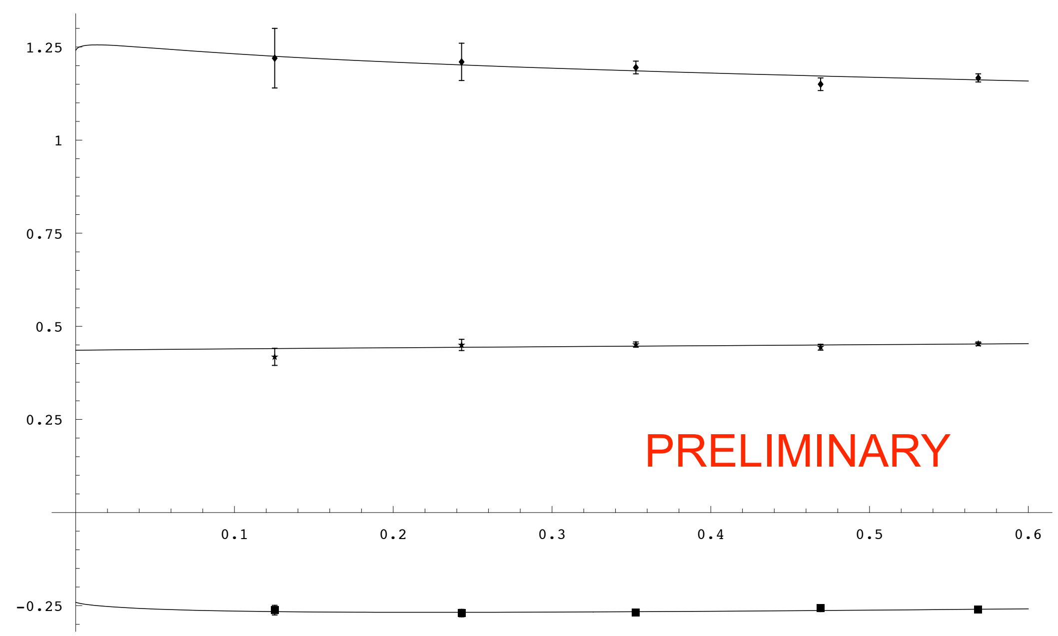


* Linear chiral extrapolations

* SU(3) *(naive)* chiral fits

$$\chi^2/dof \sim 100$$

* FRR fits $\chi^2/dof \sim 0.7$
  - Linear: "good" approximation



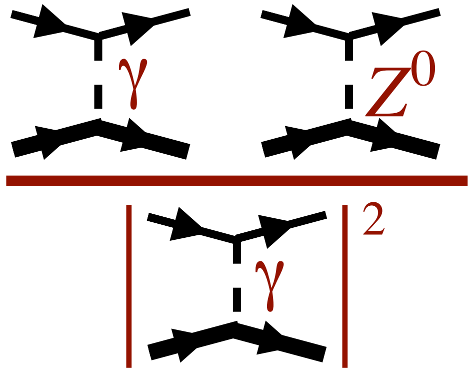
Quark-mass dependence of axial charges

- Gives information on SU(3) breaking in nucleon structure functions
  - Important for separation *u*, *d* & *s* contributions to nucleon spin

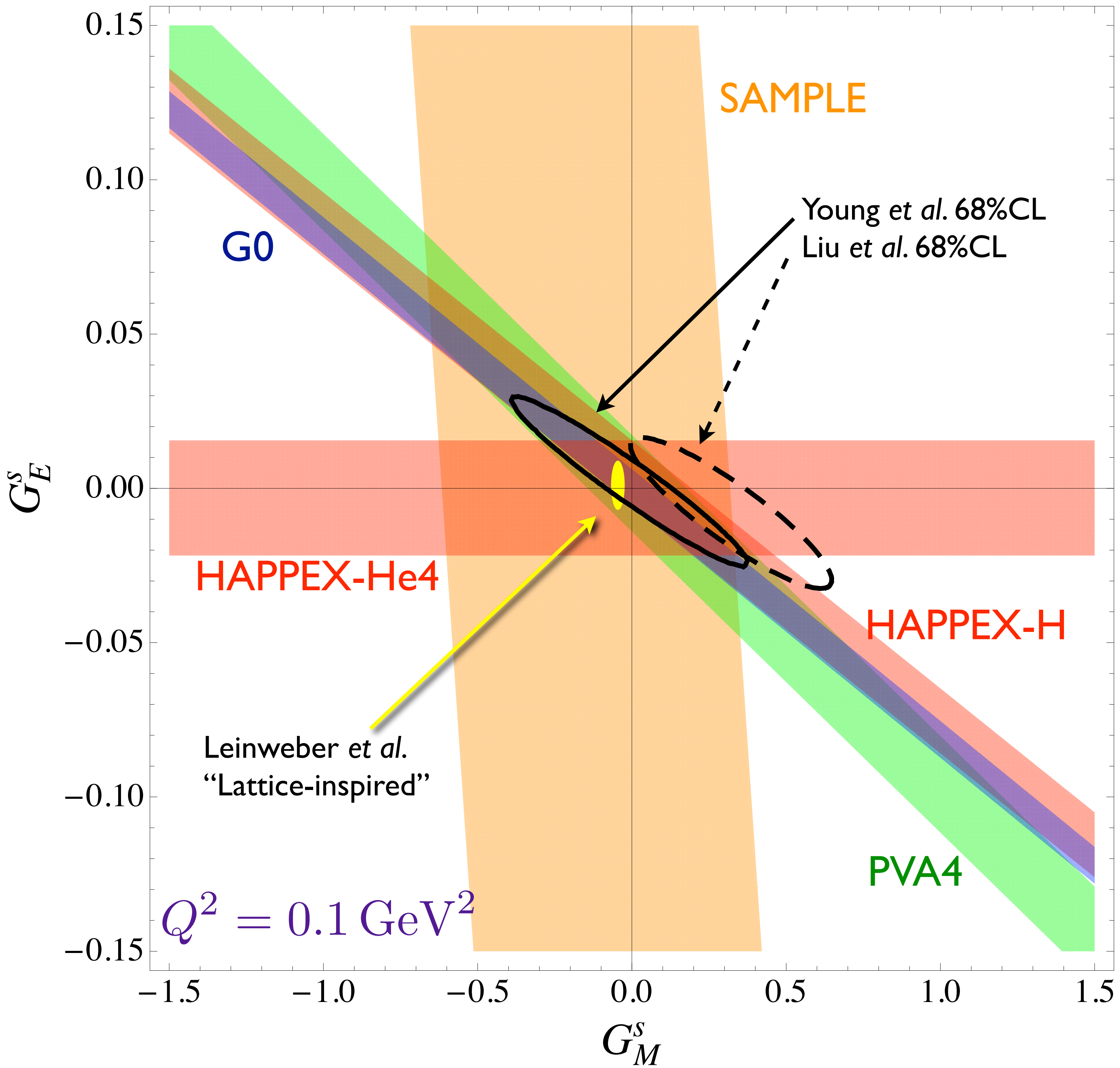
- Further can investigate charge-symmetry violations in nucleon structure functions
  - Could provide important constraint for future low-energy precision electroweak searches for new physics: PV-DIS@JLab

Precision Electroweak

$$A_{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} \sim$$

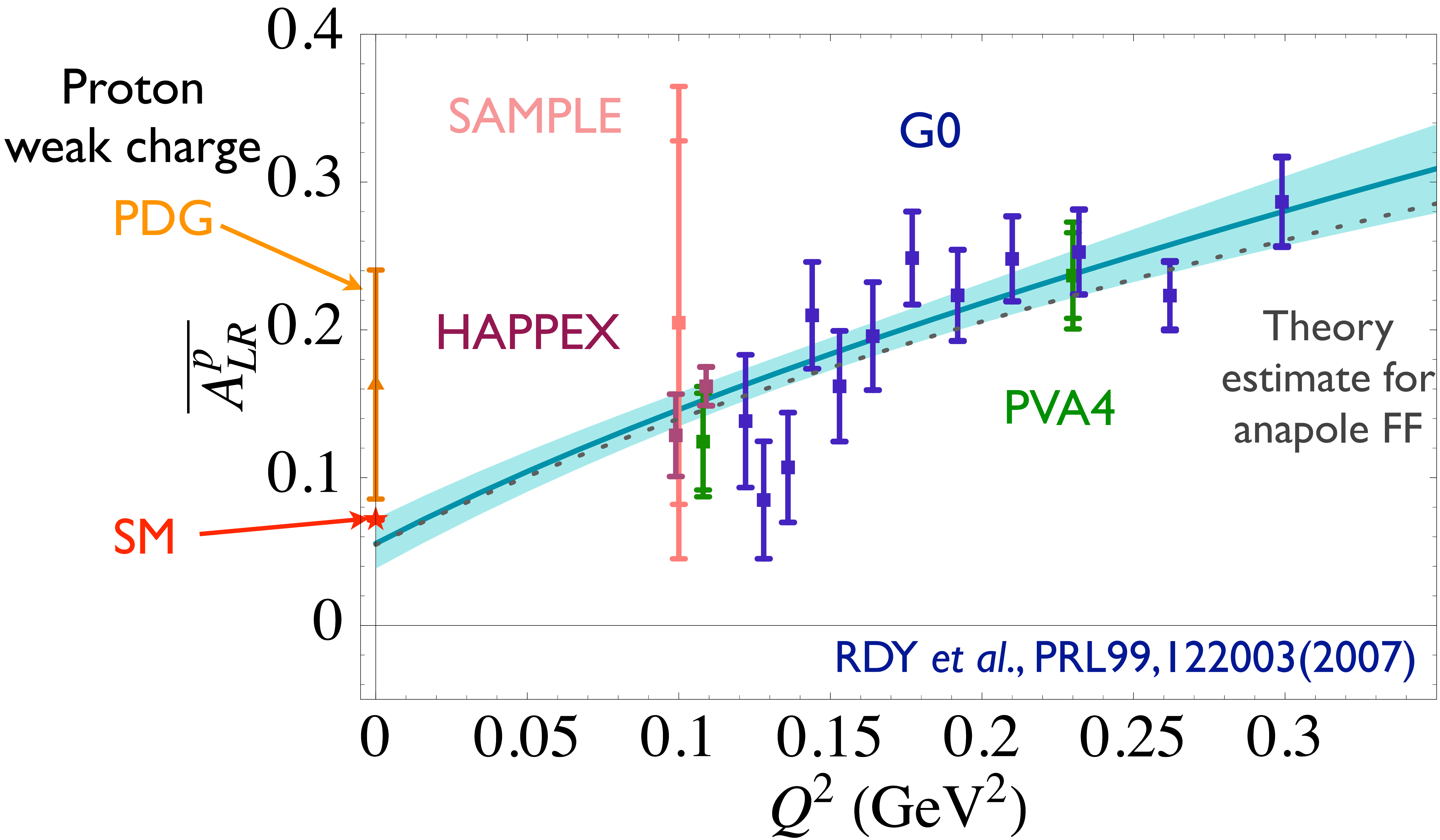


$$\sim \frac{Q^2}{M_Z^2}$$

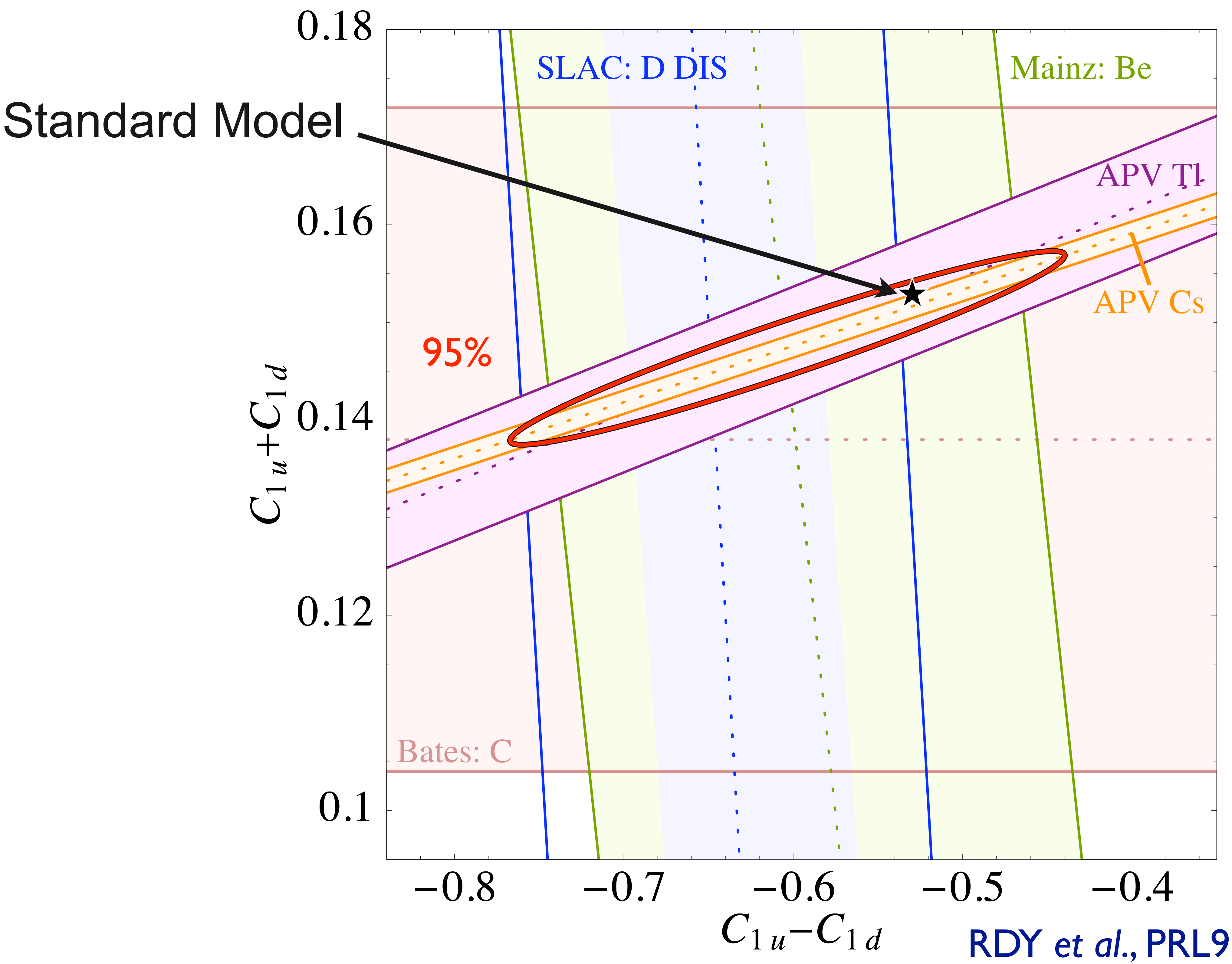


Measure strange quark content in nucleon electromagnetic form factors

Weak Charge of the proton



New limits on low energy EW parameters



RDY *et al.*, PRL99,122003(2007)

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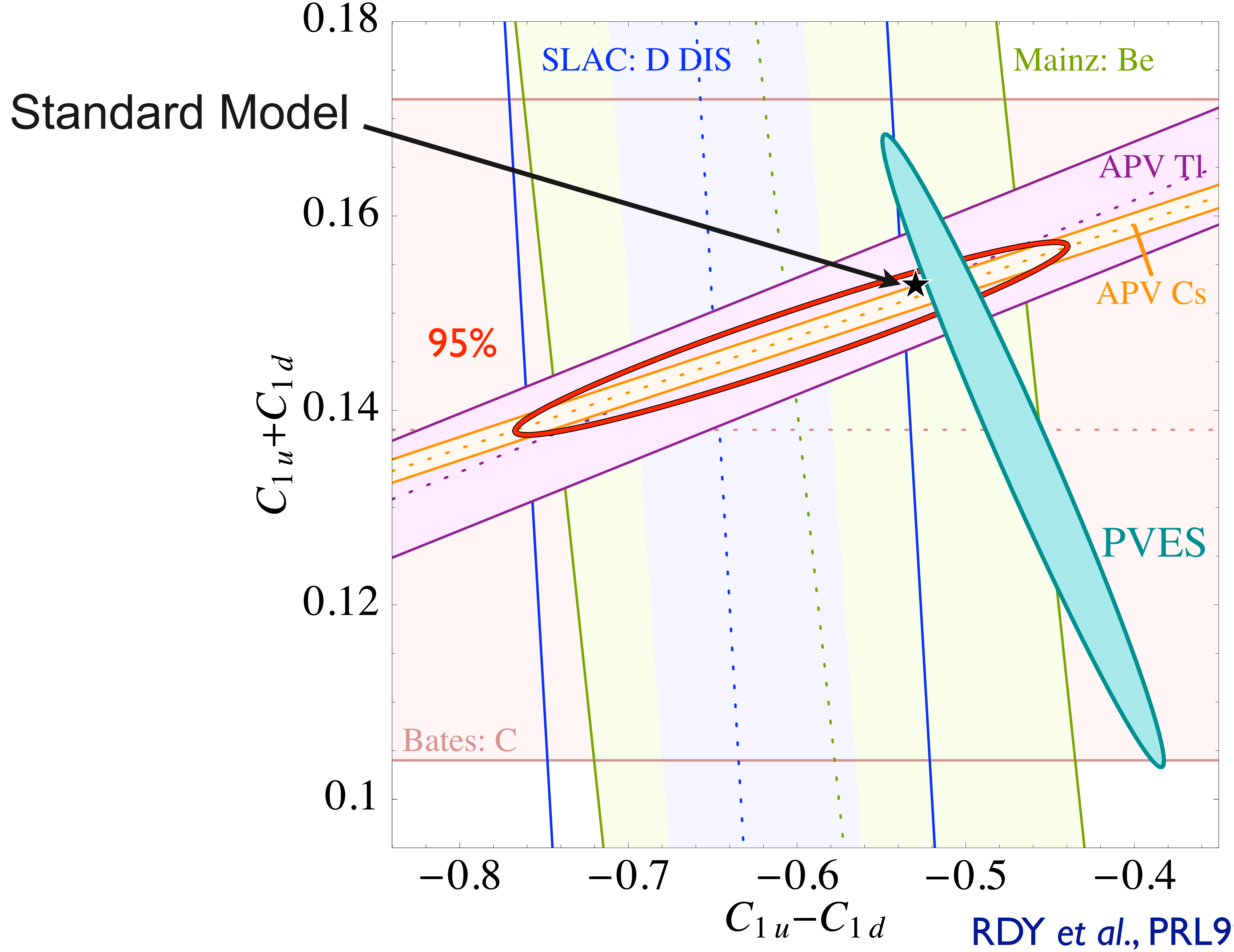

RDY *et al.*, PRL99,122003(2007)

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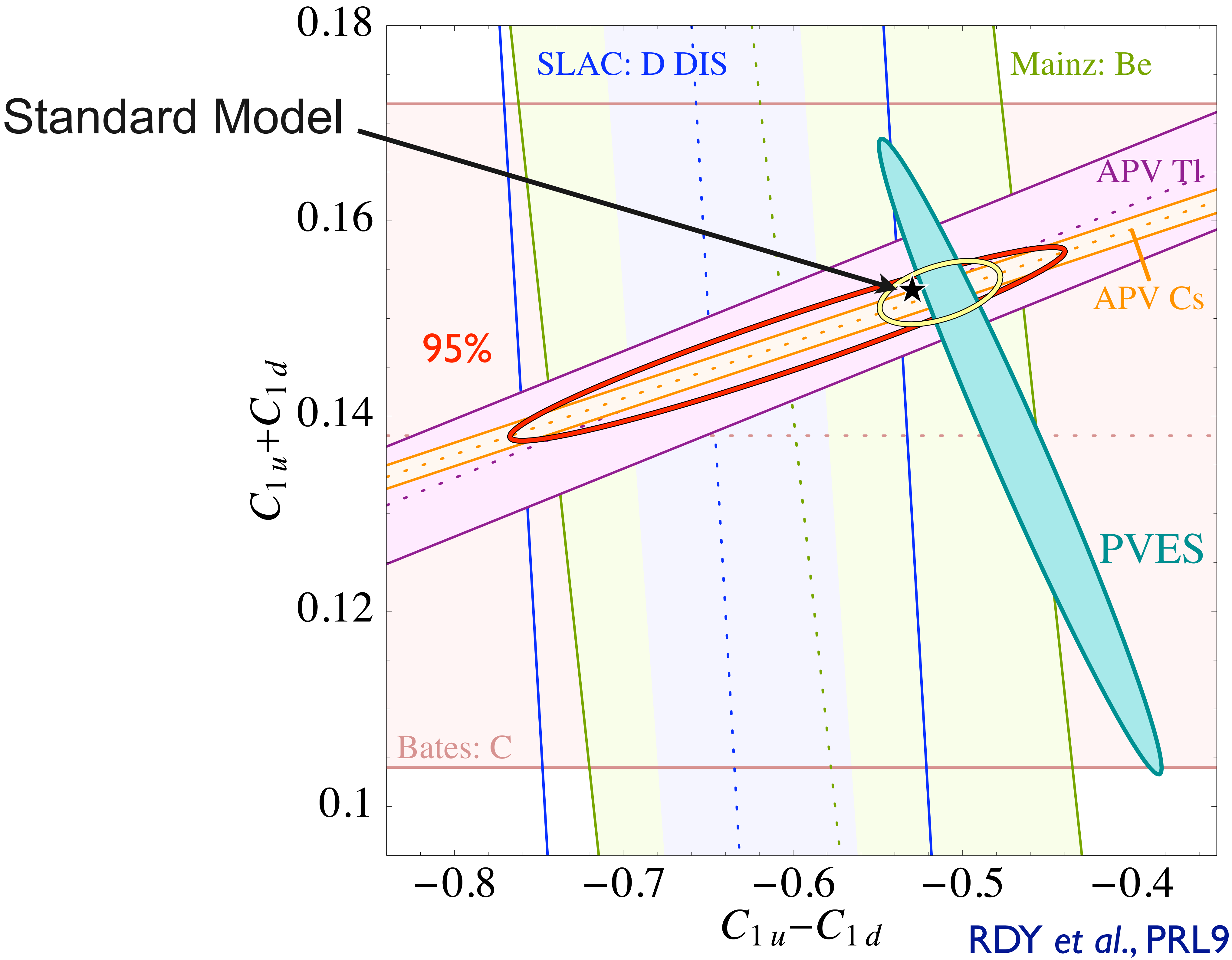
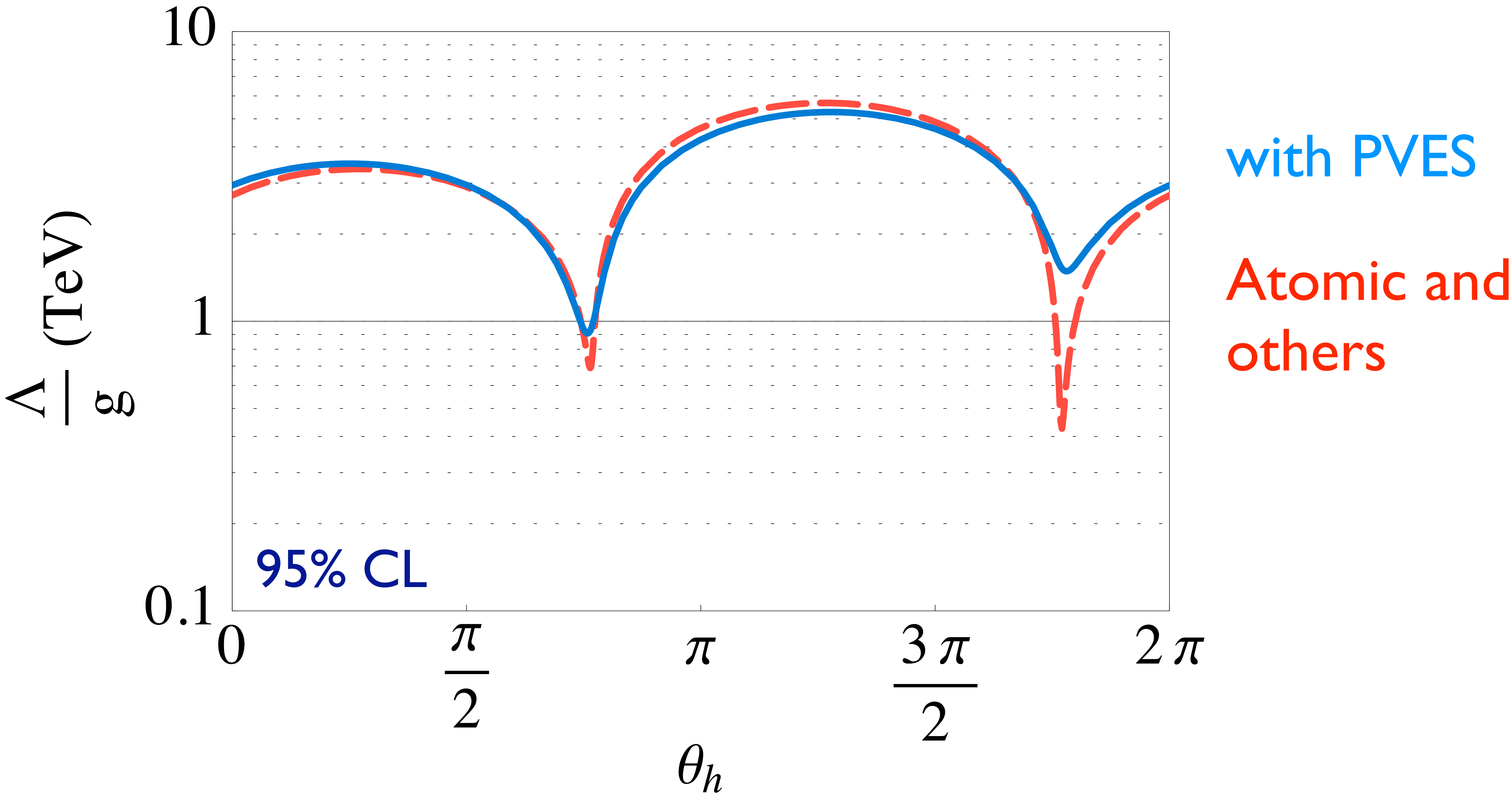


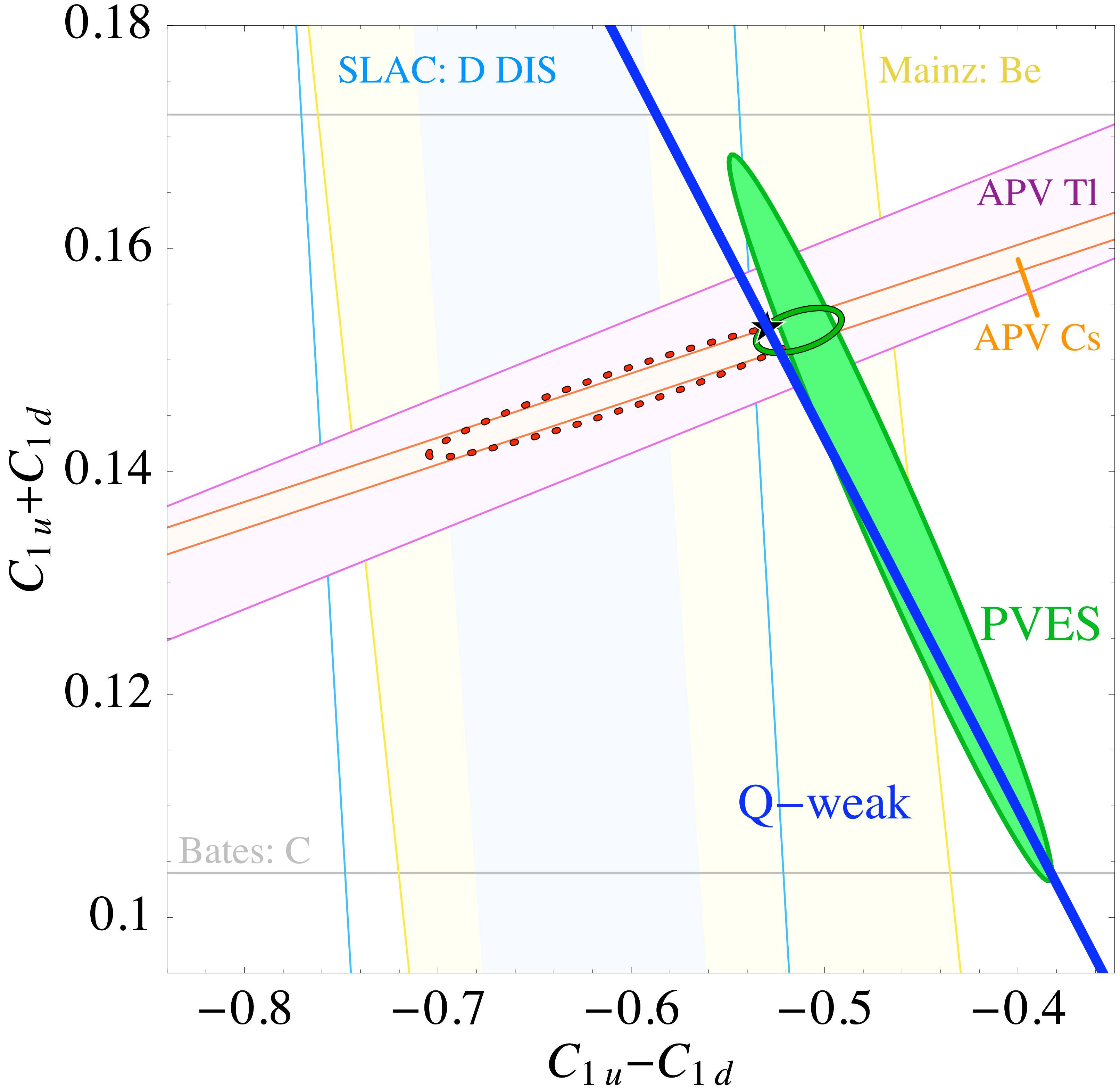
RDY *et al.*, PRL99,122003(2007)

Lower bound on "New Physics" energy scale

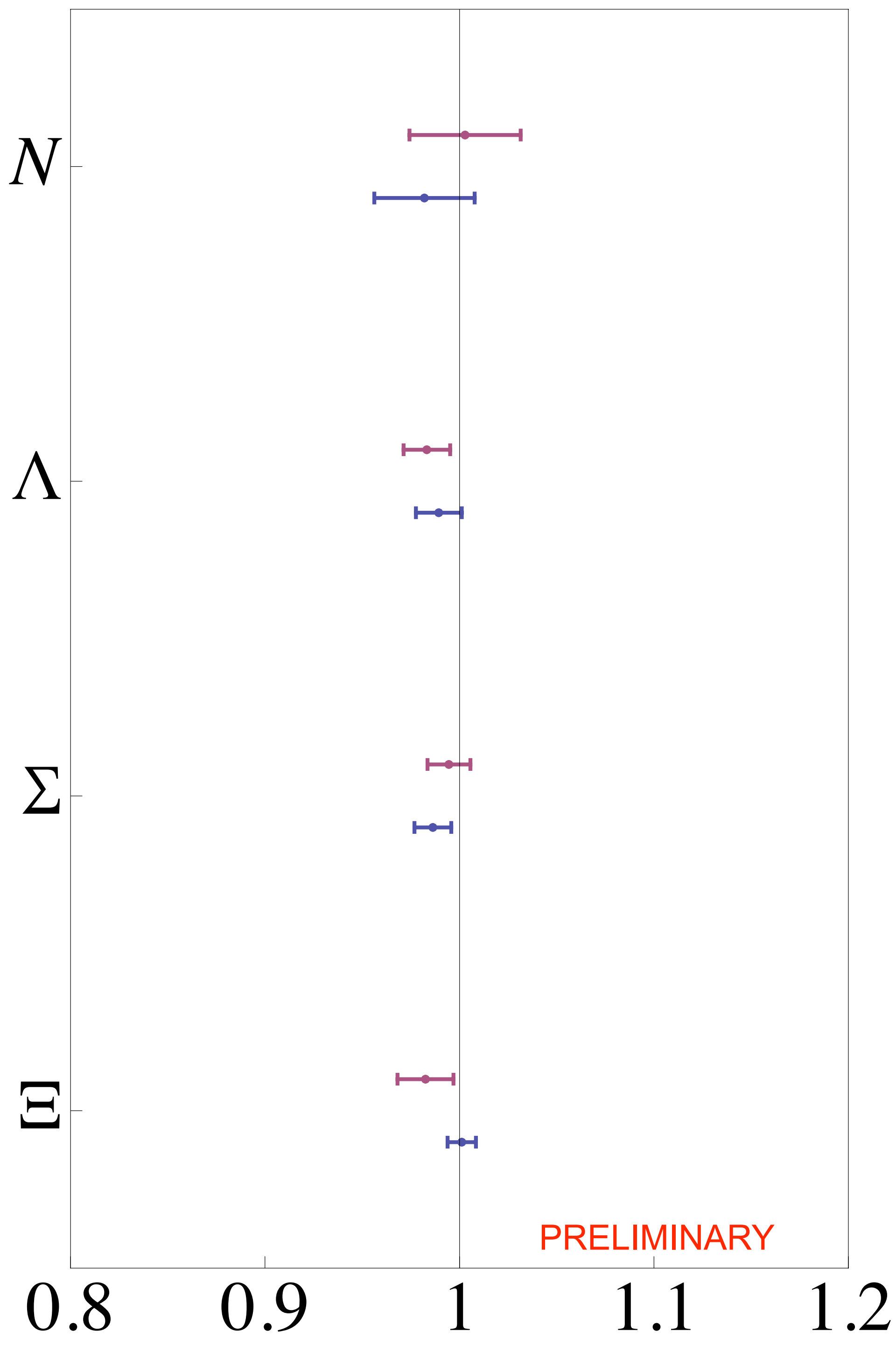


New physics scale >0.9 TeV! (from 0.4 TeV)

Future Q-weak measurement



Conclusions



- Baryon precision competing with meson sector
- Further extensions to investigate problems that are less well known experimentally
- Potential to obtain lattice QCD constraints for low-energy precision measurements