Steps toward Dyson-Schwinger equations for equal-time correlation functions

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Outline

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Gauge fixing

(Euclidean) QCD Lagrangian $\mathcal{L} = \bar{q}\,(-\gamma_\mu D_\mu + m)\,q + \frac{1}{2}\,\mathrm{tr}\,(F_{\mu\nu}F_{\mu\nu})$,

$$D_\mu = \partial_\mu - igA_\mu \;, \quad F_{\mu\nu} = \frac{i}{g}\,[D_\mu, D_\nu]$$

- $\mathcal{L}$ invariant under gauge transformations

$$q \longrightarrow q^U = U\,q \;, \qquad U = e^{ig\omega} = e^{ig\omega^a\sigma^a/2}$$

$$A_\mu \longrightarrow A_\mu^U = U\left(A_\mu + \frac{i}{g}\,\partial_\mu\right)U^\dagger = \frac{i}{g}\,UD_\mu U^\dagger$$

- infinitesimal transformation:

$$\delta q = ig\omega q \;, \qquad \delta A_\mu = D_\mu\omega = \partial_\mu\omega - ig\,[A_\mu, \omega]$$

- (covariant) gauge fixing:

$$\partial_\mu A_\mu = 0$$

Ghosts

- change of variables: $A_\mu = \bar{A}^U_\mu \,, \quad \partial_\mu \bar{A}_\mu = 0$

$$\Rightarrow \int D[A] = \int D[U] \int D[\bar{A}] \det \mathcal{F}$$

$$= \int D[U] \int D[A] \, \delta(\partial_\mu A_\mu) \det \left(\frac{1}{g} \partial_\mu D_\mu \right)$$

$$= \lim_{\alpha \to 0} \int D[U] \int D[A] \exp \left(-\frac{1}{\alpha} \int d^4x \, \mathrm{tr}(\partial_\mu A_\mu)^2 \right) \int D[c, \bar{c}] \exp \left(2 \int d^4x \, \mathrm{tr}(\bar{c} \, \partial_\mu D_\mu c) \right)$$

- ghosts $c, \bar{c}$: spin zero fermions

$$\int D[A] \exp \left(- \int d^4x \, \mathcal{L} \right) \propto \int D[A, c, \bar{c}] \exp \left(- \int d^4x \, \mathcal{L}_{gf} \right) ,$$

$$\mathcal{L}_{gf} = \mathcal{L} + \mathrm{tr} \left[\frac{1}{\alpha} (\partial_\mu A_\mu)^2 - 2\bar{c} \, \partial_\mu D_\mu c \right] ,$$

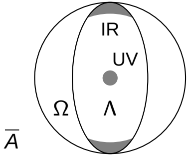
limit $\alpha \to 0$: Landau gauge

Gribov copies

- problem: $\bar{A}^U_\mu \neq \bar{A}_\mu$ with $\partial_\mu \bar{A}^U_\mu = 0 = \partial_\mu \bar{A}_\mu$ exists, "Gribov copy"
- Gribov (1978): $\bar{A}$ should only be integrated over the first Gribov region Ω, where $(-\partial_\mu D_\mu)$ is positive definite
- Zwanziger (1994): more precisely, $\bar{A}$ should be integrated over the fundamental modular region

$$\Lambda = \left\{ \bar{A}_\mu \,\middle|\, \int d^4x \, \mathrm{tr}(\bar{A}_\mu \bar{A}_\mu) = \min_U \int d^4x \, \mathrm{tr}(\bar{A}^U_\mu \bar{A}^U_\mu) \right\} \subset \Omega$$

Gribov horizon $\partial\Omega$:
$\mathrm{FP}(A) \equiv \det(-\partial_\mu D_\mu) = 0$



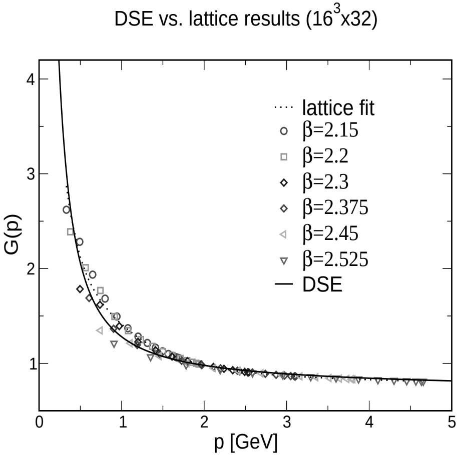
- ghost propagator

$$\int D[A, c, \bar{c}] \, c^a(x) \bar{c}^b(y) \, \exp\left(- \int d^4x \, \mathcal{L}_{gf} \right)$$
$$= \int D[A, c, \bar{c}] \, \langle x, a | (-\partial_\mu D_\mu)^{-1} | y, b \rangle \, \exp\left(- \int d^4x \, \mathcal{L}_{gf} \right)$$

IR enhanced

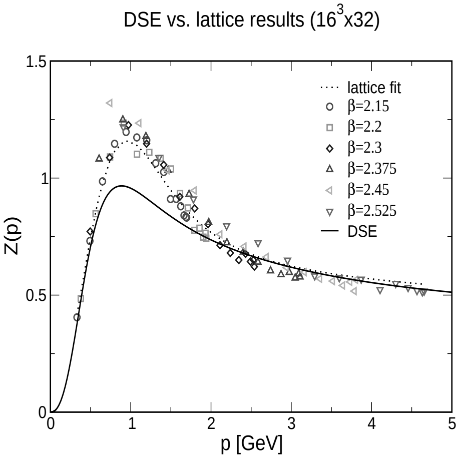
- Kugo-Ojima (1979) confinement criterion fulfilled $\Rightarrow$ color confinement

Numerical results



ghost propagator $\frac{G(p^2)}{p^2}$

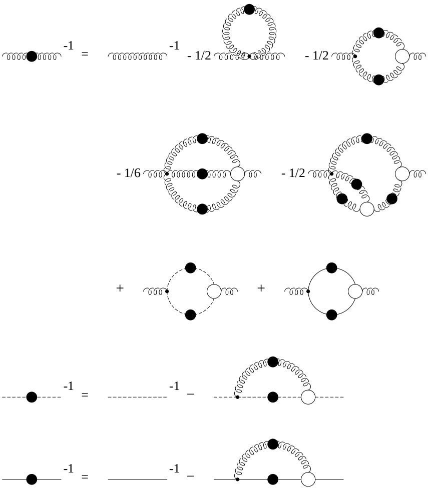
Dyson-Schwinger equations: Fischer, Alkofer (2002); Lattice: Langfeld (2002)



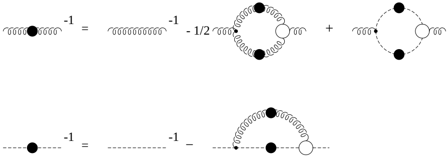
gluon propagator

$$\frac{Z(p^2)}{p^2}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)$$

Dyson-Schwinger equations



- approximation in Yang-Mills theory (no quarks)



numerical solution: von Smekal, Hauck, Alkofer (1997); Fischer, Alkofer (2002)

- in addition, ghost dominance + non-renormalization of the ghost-gluon vertex $\Rightarrow$ ($N_c = 3$)

$$G(p^2) \propto (p^2)^{-\kappa} \,, \quad Z(p^2) \propto (p^2)^{2\kappa} \,, \quad \kappa = 0.595$$

$$\alpha_s(\mu^2) = \frac{g^2}{4\pi} Z(\mu^2) G^2(\mu^2) \longrightarrow 2.97 \,, \quad \mu^2 \longrightarrow 0$$

analytical solution: Lerche, von Smekal (2002); Zwanziger (2002);
Pawlowski, Litim, Nedelko, von Smekal (2004); Schleifenbaum, Leder, Reinhardt (2006)

Outline

Coulomb gauge

- consider Yang-Mills theory in canonical quantization, Weyl gauge: $A_0^a(x) \equiv 0$
- states $\psi(\mathbf{A}) = \psi(\mathbf{A}^U)$ for gauge transformations $U(\mathbf{x})$

$$\langle \phi | \psi \rangle = \int D[\mathbf{A}]\, \phi^*(\mathbf{A}) \psi(\mathbf{A})$$

- Coulomb gauge: $\nabla \cdot \mathbf{A} = 0$

$$\Rightarrow \int D[\mathbf{A}] = \int D[U] \int D[\mathbf{A}]\, \delta(\nabla \cdot \mathbf{A})\, \mathrm{FP}(\mathbf{A})\,, \quad \mathrm{FP}(\mathbf{A}) = \det\left(-\nabla \cdot \mathbf{D}(\mathbf{A})\right)$$

$$\Rightarrow \langle \phi | \psi \rangle \propto \int D[\mathbf{A}]\, \delta(\nabla \cdot \mathbf{A})\, \mathrm{FP}(\mathbf{A})\, \phi^*(\mathbf{A}) \psi(\mathbf{A})$$

- Gribov copies $\Rightarrow$ restrict $\overline{\mathbf{A}}$-integration to the fundamental modular region $\Lambda \subset \Omega$

Christ-Lee Hamiltonian

Hamiltonian in Weyl gauge

$$H = \frac{1}{2} \int d^3x \left[- \frac{\delta}{\delta \mathbf{A}^a(\mathbf{x})} \cdot \frac{\delta}{\delta \mathbf{A}^a(\mathbf{x})} + \mathbf{B}^a(\mathbf{x}) \cdot \mathbf{B}^a(\mathbf{x}) \right]$$

$$B_j^a = \frac{1}{2} \epsilon_{jk\ell} F_{k\ell}^a = \left[\nabla \times \mathbf{A}^a + \frac{g}{2} f^{abc} \mathbf{A}^b \times \mathbf{A}^c \right]_j , \qquad \frac{1}{i} \frac{\delta}{\delta A_j^a(\mathbf{x})} = -E_j^a = F_{0j}^a$$

elimination of the longitudinal component $\mathbf{E}_L^a$ (classical):

$$\mathbf{E}^a = \mathbf{E}_L^a + \mathbf{E}_T^a , \quad \mathbf{E}_L^a = -\nabla \phi^a , \quad \phi^a \text{ the color Coulomb potencial}$$

$$\text{Gauss' law: } \mathbf{D}(\mathbf{A}_T) \cdot \mathbf{E} = \mathbf{D}(\mathbf{A}_T) \cdot (\mathbf{E}_L + \mathbf{E}_T) = -\mathbf{D}(\mathbf{A}_T) \cdot \nabla \phi + ig \left[\mathbf{A}_T , \mathbf{E}_T \right] = \rho_q$$

$$\Rightarrow -\nabla \cdot \mathbf{D}(\mathbf{A}_T) \, \phi = \rho_q - ig \left[\mathbf{A}_T , \mathbf{E}_T \right]$$

$$\Rightarrow \phi^a(\mathbf{x}) = \int d^3y \, \langle \mathbf{x}, a | (-\nabla \cdot \mathbf{D})^{-1} | \mathbf{y}, b \rangle \left[\rho_q^b(\mathbf{y}) + g f^b{}_{cd} \, \mathbf{A}_T^c(\mathbf{y}) \cdot \mathbf{E}_T^d(\mathbf{y}) \right]$$

$\Rightarrow$ Coulomb gauge Hamiltonian, hermitian with respect to the scalar product in Coulomb gauge: Christ-Lee Hamiltonian

$$H = \frac{1}{2}\int d^3x \left(-\frac{1}{\mathrm{FP}(\mathbf{A}_T)} \frac{\delta}{\delta A^a_{T,j}(\mathbf{x})} \mathrm{FP}(\mathbf{A}_T) \frac{\delta}{\delta A^a_{T,j}(\mathbf{x})} + B^a_j(\mathbf{x}) B^a_j(\mathbf{x}) \right)$$

$$+ \frac{1}{2}\int d^3x\, d^3y\, \frac{1}{\mathrm{FP}(\mathbf{A}_T)}\, \rho^a(\mathbf{x})\, \mathrm{FP}(\mathbf{A}_T)$$

$$\times \langle \mathbf{x}, a | (-\nabla \cdot \mathbf{D})^{-1} (-\nabla^2) (-\nabla \cdot \mathbf{D})^{-1} | \mathbf{y}, b \rangle\, \rho^b(\mathbf{y}) \,,$$

$$\rho^a(\mathbf{x}) = \rho^a_q(\mathbf{x}) - g f^{abc} A^b_{T,j}(\mathbf{x}) \frac{1}{i} \frac{\delta}{\delta A^c_{T,j}(\mathbf{x})}$$

(taking into account that $|\mathbf{E}_L|^2 = |\nabla(-\nabla \cdot \mathbf{D})^{-1}\rho|^2$)

singularity of $(-\nabla \cdot \mathbf{D})^{-1}$ at $\partial\Omega$ causes the growth of $\phi^a(\mathbf{x})$ for large $|\mathbf{x}|$ (small $\mathbf{p}$) and hence the confinement
$\Rightarrow$ direct access to the confining potential in Coulomb gauge, cf. Kugo-Ojima confinement criterion in Landau gauge

Numerical results

- variational approach: minimize

$$\langle H \rangle = \int D[\mathbf{A}]\, \delta(\nabla \cdot \mathbf{A})\, \mathrm{FP}(\mathbf{A})\, \psi^*(\mathbf{A}) H \psi(\mathbf{A})$$

  (properly normalized); *ansatz* for the vacuum functional:

$$\psi(\mathbf{A}) \propto \exp\left[-\frac{1}{2} \int \frac{d^3p}{(2\pi)^3} \mathbf{A}_T^a(-\mathbf{p}) f_2(\mathbf{p}) \mathbf{A}_T^a(\mathbf{p}) \right]$$

  $\Rightarrow$ Dyson-Schwinger-type equations for $f_2(\mathbf{p}) \equiv Z(\mathbf{p}^2)/(2|\mathbf{p}|)$ and the "ghost propagator" $G(\mathbf{p}^2)/\mathbf{p}^2$ [expectation value of $(-\nabla \cdot \mathbf{D})^{-1}$]
- two possible results in the IR

$$G(\mathbf{p}^2) \propto (\mathbf{p}^2)^{-\kappa}\,, \quad Z(\mathbf{p}^2) \propto (\mathbf{p}^2)^{2\kappa}$$
$$\text{with}\, \kappa = 0.398 \quad \text{or} \quad \kappa = 0.5$$

  numerical solution: Szczepaniak, Swanson (2001); Feuchter, Reinhardt (2004); Epple, Reinhardt, Schleifenbaum (2007); analytical solution: Zwanziger (2004); Schleifenbaum, Leder, Reinhardt (2006)
- inclusion of form factor for the Coulomb potential inconsistent
- latest lattice results point to $\kappa = 0.5$ Burgio, Quandt, Reinhardt (2008)
- suggestion: try to derive Dyson-Schwinger equations for equal-time correlation functions

Correlation functions in Coulomb and Landau gauge

- equal-time n-point correlation function in Coulomb gauge

$$\langle A_i^a(\mathbf{p}_1, t=0) A_j^b(\mathbf{p}_2, t=0) \cdots A_r^f(\mathbf{p}_n, t=0)\rangle$$
$$= \int D[\mathbf{A}]\,\delta(\nabla\cdot\mathbf{A})\,\mathrm{FP}(\mathbf{A})\,A_i^a(\mathbf{p}_1) A_j^b(\mathbf{p}_2)\cdots A_r^f(\mathbf{p}_n)\,|\psi_0(\mathbf{A})|^2 \,,$$

$\psi_0(\mathbf{A})$ vacuum wave functional

- similar to (Euclidean) functional integral in Landau gauge with $(A_\mu^a) \longrightarrow (A_i^a)$, $p_\mu \longrightarrow p_i$, $e^{-S} \longrightarrow |\psi_0(\mathbf{A})|^2$

  $\Rightarrow$ functional integral techniques applicable with $|\psi_0(\mathbf{A})|^2 = e^{-S'}$

- can we built on this analogy to derive Dyson-Schwinger equations for the equal-time correlation functions in Coulomb gauge?

Outline

Ansatz in ϕ^4 theory

- consider $\lambda\phi^4$ theory for simplicity (the basic formalism should be the same for QCD):

$$H = \frac{1}{2}\int \frac{d^3p}{(2\pi)^3}\left(-(2\pi)^3 \frac{\delta}{\delta\phi(\mathbf{p})}(2\pi)^3 \frac{\delta}{\delta\phi(-\mathbf{p})} + \phi(-\mathbf{p})(m^2+\mathbf{p}^2)\phi(\mathbf{p})\right)$$
$$+ \frac{\lambda}{4!}\int \frac{d^3p_1}{(2\pi)^3}\cdots\frac{d^3p_4}{(2\pi)^3}\,\phi(\mathbf{p}_1)\cdots\phi(\mathbf{p}_4)(2\pi)^3\delta(\mathbf{p}_1+\ldots+\mathbf{p}_4)$$

- exponential ansatz for the vacuum wave functional

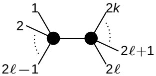
$$\psi_0(\phi) = \exp\Bigg(-\sum_{k=1}^{\infty}\frac{1}{(2k)!}\int \frac{d^3p_1}{(2\pi)^3}\cdots\frac{d^3p_{2k}}{(2\pi)^3}\, f_{2k}(\mathbf{p}_1,\ldots,\mathbf{p}_{2k})$$
$$\times\,\phi(\mathbf{p}_1)\cdots\phi(\mathbf{p}_{2k})(2\pi)^3\delta(\mathbf{p}_1+\ldots+\mathbf{p}_{2k})\Bigg)$$

Equations for coefficient functions

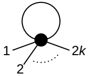
insert ansatz in $H\psi_0(\phi) = E_0\psi_0(\phi)$ and equate coefficients of $\phi^{2k}\psi_0(\phi)$ on both sides

$\Rightarrow$ infinite set of equations for the $f_{2k}(\mathbf{p}_1, \ldots, \mathbf{p}_{2k})$:

$$\sum_{\ell=1}^{k} 2\ell[2(k-\ell)+2]\Big[f_{2\ell}(\mathbf{p}_1, \ldots, \mathbf{p}_{2\ell-1}, -\mathbf{p}_1 - \ldots - \mathbf{p}_{2\ell-1}) \times f_{2(k-\ell)+2}(\mathbf{p}_{2\ell}, \ldots, \mathbf{p}_{2k}, \mathbf{p}_1 + \ldots + \mathbf{p}_{2\ell-1})\Big]_{\text{symm. in } (\mathbf{p}_i \leftrightarrow \mathbf{p}_j)}$$



$$= \frac{1}{2}\int \frac{d^3q}{(2\pi)^3} f_{2k+2}(\mathbf{p}_1, \ldots, \mathbf{p}_{2k}, -\mathbf{q}, \mathbf{q}) + \left\{ \begin{array}{ccl} m^2 + \mathbf{p}_1^2 & , & \text{for } 2k = 2 \\ \lambda & , & \text{for } 2k = 4 \\ 0 & , & \text{for } 2k \geq 6 \end{array} \right\}$$



$$[f_2(\mathbf{p}_1)]^2 \equiv [f_2(\mathbf{p}_1, \mathbf{p}_2)]^2$$
$$= m^2 + \mathbf{p}_1^2 + \frac{1}{2} \int \frac{d^3q}{(2\pi)^3}\, f_4(\mathbf{p}_1, \mathbf{p}_2, -\mathbf{q}, \mathbf{q}) \qquad (\mathbf{p}_2 = -\mathbf{p}_1)\,,$$

$$\Big[f_2(\mathbf{p}_1) + \ldots + f_2(\mathbf{p}_4)\Big] f_4(\mathbf{p}_1, \ldots, \mathbf{p}_4)$$
$$= \lambda + \frac{1}{2} \int \frac{d^3q}{(2\pi)^3}\, f_6(\mathbf{p}_1, \ldots, \mathbf{p}_4, -\mathbf{q}, \mathbf{q})\,,$$

$$\big[f_2(\mathbf{p}_1) + \ldots + f_2(\mathbf{p}_6)\big] f_6(\mathbf{p}_1, \ldots, \mathbf{p}_6)$$
$$= -\big[f_4(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, -\mathbf{p}_1 - \mathbf{p}_2 - \mathbf{p}_3) f_4(\mathbf{p}_4, \mathbf{p}_5, \mathbf{p}_6, \mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3) + \text{perms.}\big]$$
$$+ \frac{1}{2} \int \frac{d^3q}{(2\pi)^3}\, f_8(\mathbf{p}_1, \ldots, \mathbf{p}_6, -\mathbf{q}, \mathbf{q})\,,$$

$$\ldots$$

Outline

1. Motivation
   - QCD in Landau gauge
   - QCD in Coulomb gauge
2. Vacuum wave functional
   - Exponential ansatz
   - Iterative perturbative solution
3. Correlation functions
   - Perturbative expansion
   - Coulomb gauge Yang-Mills theory
   - E-operator
4. Conclusions

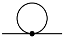
Perturbative expansion

$$f_2(\mathbf{p}) = \sqrt{m^2 + \mathbf{p}^2} + \mathcal{O}(\lambda) \equiv \omega_{\mathbf{p}} + \mathcal{O}(\lambda) \,,$$

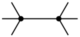
$$f_4(\mathbf{p}_1, \dots, \mathbf{p}_4) = \frac{\lambda}{\omega_{\mathbf{p}_1} + \dots + \omega_{\mathbf{p}_4}} + \mathcal{O}(\lambda^2) \equiv$$  $$+ \mathcal{O}(\lambda^2) \,,$$

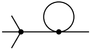
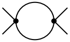
$$[f_2(\mathbf{p})]^2 = \omega_{\mathbf{p}}^2 + \frac{\lambda}{2} \int \frac{d^3q}{(2\pi)^3} \frac{1}{2\omega_{\mathbf{p}} + 2\omega_{\mathbf{q}}} + \mathcal{O}(\lambda^2)$$

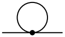
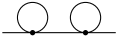
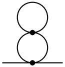
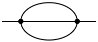
$$\Rightarrow \quad f_2(\mathbf{p}) = \omega_{\mathbf{p}} + \frac{\lambda}{2} \frac{1}{2\omega_{\mathbf{p}}} \int \frac{d^3q}{(2\pi)^3} \frac{1}{2\omega_{\mathbf{p}} + 2\omega_{\mathbf{q}}} + \mathcal{O}(\lambda^2)$$

$$\equiv (\text{——})^{-1} -$$  $$+ \mathcal{O}(\lambda^2) \,,$$

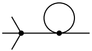
$$f_6(\mathbf{p}_1, \ldots, \mathbf{p}_6) = -\frac{\lambda^2}{\omega_{\mathbf{p}_1} + \ldots + \omega_{\mathbf{p}_6}} \left(\frac{1}{\omega_{\mathbf{p}_1} + \omega_{\mathbf{p}_2} + \omega_{\mathbf{p}_3} + \omega_{\mathbf{p}_1+\mathbf{p}_2+\mathbf{p}_3}} \right.$$
$$\left. \times \frac{1}{\omega_{\mathbf{p}_4} + \omega_{\mathbf{p}_5} + \omega_{\mathbf{p}_6} + \omega_{\mathbf{p}_1+\mathbf{p}_2+\mathbf{p}_3}} + (9 \text{ perms.}) \right) + \mathcal{O}(\lambda^3)$$

$\equiv$  $+ (9 \text{ perms.}) + \mathcal{O}(\lambda^3)$,

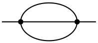
$f_4(\mathbf{p}_1, \ldots, \mathbf{p}_4) =$  $- \Big($  $+ 3 \text{ perms.}\Big) - \Big($  $+ 2 \text{ perms.}\Big) + \mathcal{O}(\lambda^3)$,

$f_2(\mathbf{p}) = (\text{—})^{-1} -$  $-$  $-$  $-$  $+ \mathcal{O}(\lambda^3)$

Examples



$$= \frac{1}{2} \frac{\lambda^2}{(\omega_{\mathbf{p}_1} + \ldots + \omega_{\mathbf{p}_4})^2} \int \frac{d^3q}{(2\pi)^3} \sum_{\ell=1}^{4} \frac{1}{2\omega_{\mathbf{p}_\ell} + 2\omega_{\mathbf{q}}} \times \left(\frac{1}{2\omega_{\mathbf{p}_\ell}} + \frac{1}{\omega_{\mathbf{p}_1} + \ldots + \omega_{\mathbf{p}_4} + 2\omega_{\mathbf{q}}} \right) ,$$



$$= \frac{\lambda^2}{2\omega_{\mathbf{p}}} \int \frac{d^3q}{(2\pi)^3} \frac{d^3q'}{(2\pi)^3} \frac{1}{2\omega_{\mathbf{p}} + 2\omega_{\mathbf{q}}} \times \frac{1}{2\omega_{\mathbf{p}} + 2\omega_{\mathbf{q}} + 2\omega_{\mathbf{q}'}} \frac{1}{(\omega_{\mathbf{p}} + \omega_{\mathbf{q}} + \omega_{\mathbf{q}'} + \omega_{\mathbf{p}+\mathbf{q}+\mathbf{q}'})^2}$$

Outline

Equal-time and covariant correlation functions (with F. Astorga)

- equal-time n-point correlation function

$$\langle \phi(\mathbf{p}_1, t=0) \cdots \phi(\mathbf{p}_n, t=0) \rangle = \int D[\phi]\, \phi(\mathbf{p}_1) \cdots \phi(\mathbf{p}_n)\, e^{-S'} \,,$$

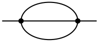
  with

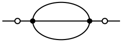
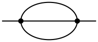
$$e^{-S'} = |\psi_0(\phi)|^2 \,, \qquad S' = \sum_{k=1}^{\infty} \frac{2}{(2k)!} f_{2k}(\mathbf{p}_1, \ldots, \mathbf{p}_{2k})\, \phi(\mathbf{p}_1) \cdots \phi(\mathbf{p}_{2k})$$

- use common perturbation theory with “bare” propagators $(2\omega_{\mathbf{p}})^{-1}$ and vertices $(2f_2(\mathbf{p}) - 2\omega_{\mathbf{p}})$ and $2f_{2k}(\mathbf{p}_1, \ldots, \mathbf{p}_{2k})$, $2k \geq 4$
- relation to covariant correlation functions:

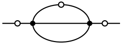
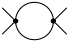
$$\langle \phi(\mathbf{p}_1, t=0) \cdots \phi(\mathbf{p}_n, t=0) \rangle = \int \frac{dp_1^0}{2\pi} \cdots \frac{dp_n^0}{2\pi} \langle \phi(\mathbf{p}_1, p_1^0) \cdots \phi(\mathbf{p}_n, p_n^0) \rangle$$

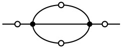
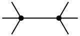
Example

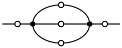
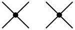
different contributions to the 2-point correlation function with topology 

-  $=$ contraction of  from $f_2(\mathbf{p}_1, \mathbf{p}_2)$ with

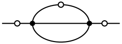
   $\equiv (2\omega_{\mathbf{p}})^{-1}$

-  $=$ contraction of  from $f_4(\mathbf{p}_1, \ldots, \mathbf{p}_4)$ with 

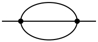
-  $=$ contraction of  from $f_6(\mathbf{p}_1, \ldots, \mathbf{p}_6)$ with 

-  $=$ contraction of  from $f_4(\mathbf{p}_1, \ldots, \mathbf{p}_4)$ with 

for example,

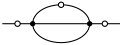


$$= \frac{1}{2}\,\frac{(2\lambda)^2}{(2\omega_{\mathbf{p}})^2}\int\frac{d^3q}{(2\pi)^3}\,\frac{d^3q'}{(2\pi)^3}\,\frac{1}{2\omega_{\mathbf{q}'}}\,\frac{1}{2\omega_{\mathbf{p}}+2\omega_{\mathbf{q}'}}$$
$$\times\,\frac{1}{2\omega_{\mathbf{p}}+2\omega_{\mathbf{q}}+2\omega_{\mathbf{q}'}}\,\frac{1}{(\omega_{\mathbf{p}}+\omega_{\mathbf{q}}+\omega_{\mathbf{q}'}+\omega_{\mathbf{p}+\mathbf{q}+\mathbf{q}'})^2}$$

sum of all diagrams with topology  :

$$\frac{1}{3!}\,\frac{(2\lambda)^2}{(2\omega_{\mathbf{p}})^3}\int\frac{d^3q}{(2\pi)^3}\,\frac{d^3q'}{(2\pi)^3}\,\frac{2\omega_{\mathbf{p}}+\omega_{\mathbf{q}}+\omega_{\mathbf{q}'}+\omega_{\mathbf{p}+\mathbf{q}+\mathbf{q}'}}{2\omega_{\mathbf{q}}\,2\omega_{\mathbf{q}'}\,2\omega_{\mathbf{p}+\mathbf{q}+\mathbf{q}'}\,(\omega_{\mathbf{p}}+\omega_{\mathbf{q}}+\omega_{\mathbf{q}'}+\omega_{\mathbf{p}+\mathbf{q}+\mathbf{q}'})^2}$$

remarkable simplification; all factors in the denominator (except for one extra factor $2\omega_{\mathbf{p}}$) are

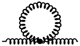
provided by  and , contrary to ; similarly for all topologies

Outline

Vacuum wave functional

$$\psi_0(\mathbf{A}_T) = \exp\Bigg(-\sum_{k=2}^{\infty} \frac{1}{k!} \int \frac{d^3p_1}{(2\pi)^3} \cdots \frac{d^3p_k}{(2\pi)^3} \sum_{i,j,\ldots,r} \sum_{a,b,\ldots,f} f^{a,b,\ldots,f}_{k;i,j,\ldots,r}(\mathbf{p}_1,\ldots,\mathbf{p}_k) \\ \times A^a_{T,i}(\mathbf{p}_1) \cdots A^f_{T,r}(\mathbf{p}_k)(2\pi)^3 \delta(\mathbf{p}_1 + \ldots + \mathbf{p}_k) \Bigg)$$

$\Rightarrow$ coefficient functions

$f^{ab}_{2,ij}(\mathbf{p}_1,\mathbf{p}_2) = \omega_{\mathbf{p}_1} \delta^T_{ij}(\mathbf{p}_1)\delta^{ab} -$  $-$  $-$  $-$  $+\, \mathcal{O}(g^4)$

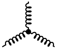
with

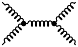
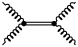
$$\omega_{\mathbf{p}} \equiv |\mathbf{p}|$$

 = ghost loop (from the Faddeev-Popov determinant)

 = “bare” Coulomb potential $1/(4\pi|\mathbf{x}-\mathbf{x}'|)$

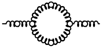
$$f^{abc}_{3,ijk}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) = -\frac{g}{\omega_{\mathbf{p}_1} + \omega_{\mathbf{p}_2} + \omega_{\mathbf{p}_3}} f^{abc} i \big[(p_{1,k} - p_{2,k})\delta_{ij} + (p_{2,i} - p_{3,i})\delta_{jk} + (p_{3,j} - p_{1,j})\delta_{ki} \big] + \mathcal{O}(g^3)$$

$$\equiv$$  $$+ \mathcal{O}(g^3)$$

$$f^{abcd}_{4,ijkl}(\mathbf{p}_1, \ldots, \mathbf{p}_4) =$$  $$-$$  $$-$$  $$+ \mathcal{O}(g^4)$$

Equal-time 2-point correlation function

$$\langle A^a_{T,i}(\mathbf{p}_1) A^b_{T,j}(\mathbf{p}_2)\rangle$$

=  +  +  +  $+\,\mathcal{O}(g^4)$

$$\equiv \Bigg[\frac{1}{2\omega_{\mathbf{p}_1}} - \frac{4}{3}\frac{2N_c g^2}{(2\omega_{\mathbf{p}_1})^3}\int\frac{d^3q}{(2\pi)^3}\frac{1}{2\omega_{\mathbf{q}}}$$

$$+\frac{1}{2}\frac{2N_c g^2}{(2\omega_{\mathbf{p}_1})^3}\int\frac{d^3q}{(2\pi)^3}\frac{1+(\hat{\mathbf{p}}_1\cdot\hat{\mathbf{q}})^2}{2\omega_{\mathbf{q}}}\frac{\omega^2_{\mathbf{p}_1}-\omega^2_{\mathbf{q}}}{(\mathbf{p}_1-\mathbf{q})^2}$$

$$+2\frac{2N_c g^2}{(2\omega_{\mathbf{p}_1})^3}\int\frac{d^3q}{(2\pi)^3}\frac{\left[1-(\hat{\mathbf{p}}_1\cdot\hat{\mathbf{q}})^2\right](2\omega_{\mathbf{p}_1}+\omega_{\mathbf{q}}+\omega_{\mathbf{p}_1+\mathbf{q}})}{2\omega_{\mathbf{q}}\,2\omega_{\mathbf{p}_1+\mathbf{q}}\,(\omega_{\mathbf{p}_1}+\omega_{\mathbf{q}}+\omega_{\mathbf{p}_1+\mathbf{q}})^2}$$

$$\times\left(2(\mathbf{p}_1^2+\mathbf{q}^2)+\frac{\mathbf{p}_1^2\mathbf{q}^2+(\mathbf{p}_1\cdot\mathbf{q})^2}{(\mathbf{p}_1+\mathbf{q})^2}\right)\Bigg]\,\delta^T_{ij}(\mathbf{p}_1)\delta^{ab}(2\pi)^3\delta(\mathbf{p}_1+\mathbf{p}_2)+\mathcal{O}(g^4)$$

- ghost loop cancels against FP($\mathbf{A}_T$) from the integration measure in the scalar product
- checked against a calculation with straightforward Rayleigh-Schrödinger perturbation theory
  Campagnari, unpublished
- *consistent* with the results for $\langle A^a_{T,i}(p_1) A^b_{T,j}(p_2) \rangle$ from covariant perturbation theory
  Watson, Reinhardt (2007, 2008)

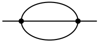
Summary

- equal-time correlation functions can be derived from a generating functional given in terms of the vacuum wave functional (as it must be)
- organization of the corresponding perturbative series: in order to determine a certain *n*-point function to a given order, not only the corresponding term in the vacuum wave functional has to be known to the same order, but also higher (and lower) terms to the corresponding orders
- here exemplified for ϕ^4 theory; same rules apply to Coulomb gauge Yang-Mills theory, at least for the two-point functions to one-loop order
- for practical calculations of equal-time correlation functions, the method using the vacuum wave functional will generally be *much* quicker than integrating the covariant correlation functions over the zero- or energy-components
- from the representation of the generating functional, Dyson-Schwinger equations can be derived for the equal-time correlation functions: not only an infinite tower of equations, but also an infinite number of terms in every single equation

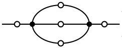
Outline

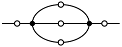
Outlook

observation for $\lambda\phi^4$ theory:

contribution to $\langle\phi(\mathbf{p}_1)\phi(\mathbf{p}_2)\rangle$ with topology 

$$\frac{1}{3!}\,\frac{(2\lambda)^2}{(2\omega_{\mathbf{p}})^3}\int\frac{d^3q}{(2\pi)^3}\,\frac{d^3q'}{(2\pi)^3}\,\frac{2\omega_{\mathbf{p}}+\omega_{\mathbf{q}}+\omega_{\mathbf{q}'}+\omega_{\mathbf{p}+\mathbf{q}+\mathbf{q}'}}{2\omega_{\mathbf{q}}\,2\omega_{\mathbf{q}'}\,2\omega_{\mathbf{p}+\mathbf{q}+\mathbf{q}'}\,(\omega_{\mathbf{p}}+\omega_{\mathbf{q}}+\omega_{\mathbf{q}'}+\omega_{\mathbf{p}+\mathbf{q}+\mathbf{q}'})^2}$$

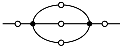
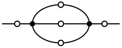
"E-diagram", notation E[];

compare with the "F-diagram" 

$$=\frac{1}{3!}\,\frac{(2\lambda)^2}{(2\omega_{\mathbf{p}})^2}\int\frac{d^3q}{(2\pi)^3}\,\frac{d^3q'}{(2\pi)^3}\,\frac{1}{2\omega_{\mathbf{q}}\,2\omega_{\mathbf{q}'}\,2\omega_{\mathbf{p}+\mathbf{q}+\mathbf{q}'}\,(\omega_{\mathbf{p}}+\omega_{\mathbf{q}}+\omega_{\mathbf{q}'}+\omega_{\mathbf{p}+\mathbf{q}+\mathbf{q}'})^2}$$

from the Feynman rules with propagator  $\equiv (2\omega_{\mathbf{p}})^{-1}$ and vertex

 $= \dfrac{-2\lambda}{\omega_{\mathbf{p}_1}+\ldots+\omega_{\mathbf{p}_4}}$

- the E-operator maps  to E[] by
  - multiplying with $\omega_{\mathbf{k}}$ for every propagator (with momentum **k**) and adding
  - dividing through the corresponding sum restricted to the external propagators
- applying the E-operator to every “independent factor” (independent momentum flow) of every (linked) F-diagram appears to generate all equal-time correlation functions
- it may be possible to establish Dyson-Schwinger-type equations for the equal-time correlation functions (E-diagrams), mixing the latter with F-diagrams

Conclusions

- a simple analytical approximation to Dyson-Schwinger equations gives a successful description of Landau gauge QCD in the deep infrared; one would like to have a similar approach to Coulomb gauge QCD where the color Coulomb potential gives a direct description of confinement; our suggestion is to try and derive Dyson-Schwinger equations for equal-time correlation functions to this end
- we show in detail how equal-time correlation functions can be derived from a generating functional given in terms of the vacuum functional; the latter can be determined order by order in a perturbative expansion; the determination of a given correlation function to all orders involves the complete vacuum wave functional
- a simpler way of calculating equal-time correlation functions seems to exist in terms of the E-operator; it appears possible to derive Dyson-Schwinger-type equations directly for the equal-time correlation functions