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Trigonometrically Extended Cornell Potential, High Spins and Deconfinement

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1. Problems in high-spin description (N, Δ)
2. Upgrading the Cornell potential towards a trigonometric finite range potential
3. SO(4)/SO(2,1) symmetries of the potential and link to the AdS_5/CFT algebraic aspects
4. TEC at work: N and Δ spectroscopy
5. Curvature parameter and deconfinement
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1. Problems of high-spin description (N, Δ)

According to QCD baryons consist of three valence quarks, $3q$, + sea contributions while mesons are built from $\bar{q}q$ + sea contributions. Being composite objects, baryons and mesons can be excited by external probes to states of high spins which constitute their respective spectra. According to Particle Data Group the spins of the N excitations (resonances) range from $\frac{1}{2}$ to $\frac{13}{2}$. Observed integer (meson) spins range from 0 to 6.

Some of the *big questions* on high-spins are:

- their formation by quarks in a way consistent with the QCD Lagrangian,
- their consistent field-theoretic description.

2. Upgrading the Cornell potential towards a trigonometric finite range potential

The Cornell potential predicted by Lattice QCD is

$$V(r) = -\frac{A}{r} + \sigma\, r\ + \frac{l(l+1)}{r^2},$$

$$A = 0.27\,\mathrm{GeV{\cdot}fm}, \quad \sigma = 0.89\,\mathrm{GeV/fm}.$$

Farmer's question:

The Coulomb- plus linear terms coincide with the lowest order Taylor decomposition of $(-2B\cot r)$,

$$-2B\cot r \approx -\frac{2B}{r} + \frac{2B}{3}r,$$

while the centrifugal barrier is part of the Taylor expansion of

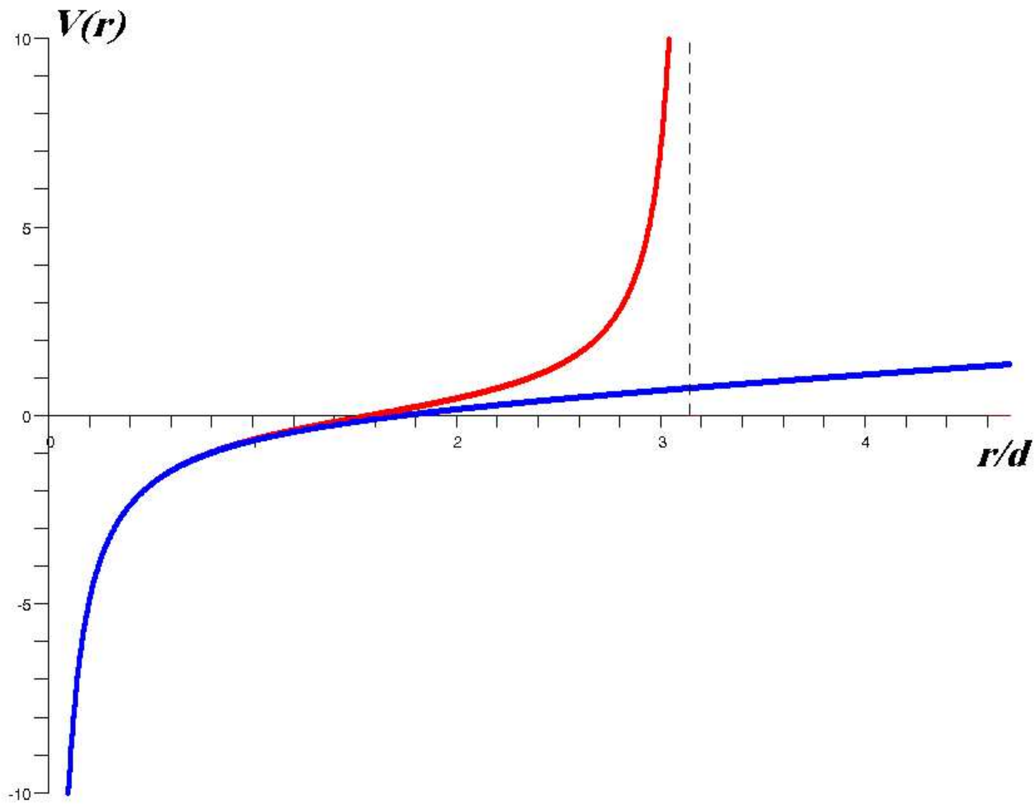
$$l(l+1)\csc^2 z \approx \frac{l(l+1)}{z^2} + \frac{l(l+1)}{15}z^2.$$

Is the Cornell potential an approximation to the finite range potential (here dimensionless)

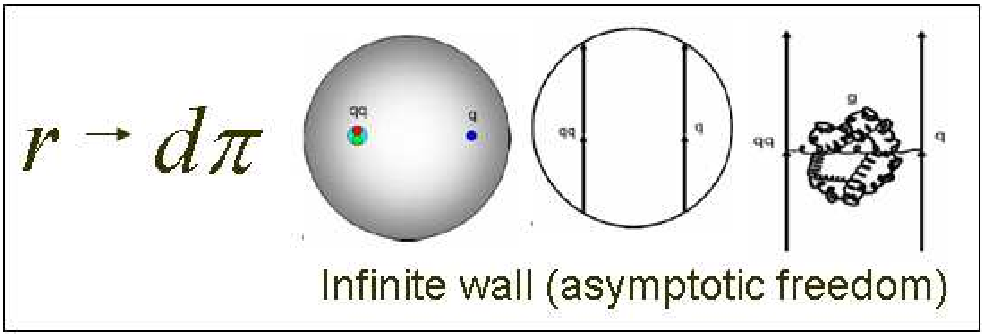
$$V_t(z) = -2b \cot z + l(l+1)\csc^2 z,$$

$$b = \frac{B}{\frac{\hbar^2}{2\mu d^2}}, \quad z = \frac{r}{d},$$

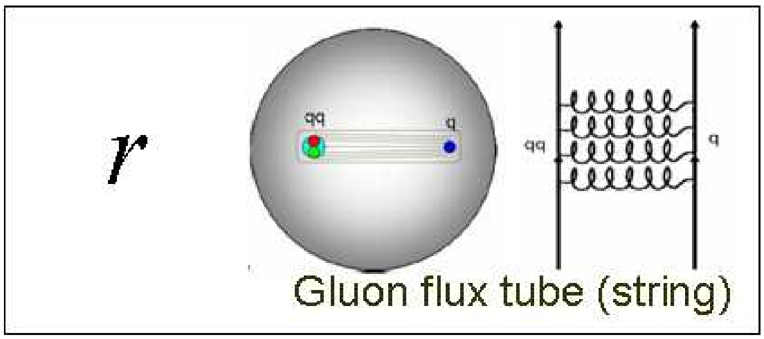
d length parameter, b and z dimensionless?



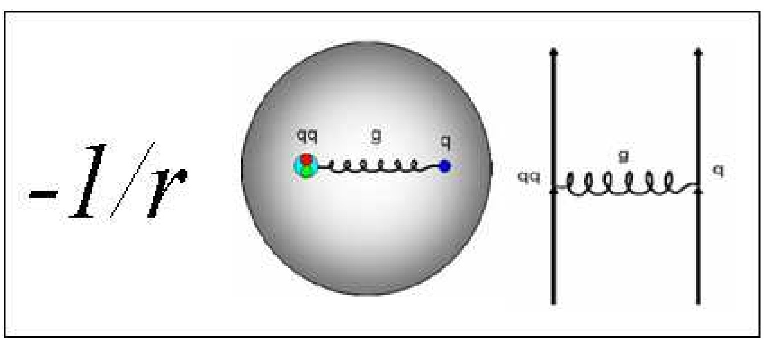
Trigonometric potential (red line)
Cornell pot. from the Taylor series (blue line)



Infinite wall piece $\Longrightarrow$ asymptotic freedom.



Linear piece$\Longrightarrow$ flux-tube interactions.



Coulomb piece $\Longrightarrow$ $1g$ exchange

Master Schrödinger's reply:

For z replaced by χ, the second polar angle in E_4, $V_t(\chi)$ solves in E_4 the Laplace-Beltrami equation on the hypersphere, S^3_R,

$$x_1 + x_2 + x_3 + x_4 = R^2, \text{ the curvature is } \quad \kappa = \frac{1}{R^2},$$

and acts as counterpart to $1/r$ in E_3, look at

[E. Schrödinger, Proc.Roy.Irish Acad. **46 A**, *9 (1940)]*

Our studies link $V_t(\chi)$ to QCD potentiology and put it at work in N and Δ spectroscopy:

- *[Compean, M.K., J. Phys. A:Math.Gen.* **39** *(2006)]*
- *[Compean, M.K., Bled Workshops in Physics, Letnik 7, 7 (2006): quant-phys/0610001]*
- *[M.K., Compean, hep-ph/0805.2404]*

Curved space aspects of $V_t(z)$:

$V_t(z)$ takes its origin from curved S^3_R space

- There, $z \rightarrow \chi$, $\cot\chi$, with $0 \leq \chi \leq \pi$, satisfies the E_4 Laplace-Beltrami eq.

$$\Box \cot\chi = 0, \quad \cot\chi = \frac{x_4}{r},$$

  meaning that $\cot\chi$ is a harmonic potential in E_4, same as is $1/r$ in E_3, which satisfies $\vec{\nabla}^2 \frac{1}{r} = 0$.

- $\frac{1}{R^2} l(l+1)\csc^2\chi$ is the centrifugal barrier on S^3_R:

$$\hat{\Box} = -\frac{1}{R^2}\mathcal{K}^2 = -\frac{1}{R^2}\left[\frac{1}{\sin^2\chi}\frac{\partial}{\partial\chi}\sin^2\chi\frac{\partial}{\partial\chi} - \frac{L^2}{\sin^2\chi}\right],$$

$$\vec{\nabla}^2 = -\frac{1}{a^2}L^2 = -\frac{1}{a^2}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} + \frac{L_z^2}{\sin^2\theta}\right]$$

$\mathcal{K}^2$, L^2, L_z^2 are the respective 4D-, 3D, and 2D angular momenta, $-\frac{1}{a^2}\frac{L_z^2}{\sin\theta^2}$ is the S^2_a centrifugal barrier. Common forms of the S^3_R, S^2_a barriers reflect general $O(n)$ properties. The $\chi = \frac{r}{R}\pi \equiv \frac{r}{d}$ parametrization with $d = \frac{R}{\pi}$ recovers $V_t(z)$.

Curved space news :

- in E_4 the róle of the radial coordinate of infinite range in ordinary flat space, $0 < r < \infty$, has been taken by the angular variable, χ, of finite range.
- While the harmonic potential in E_3 is central, in E_4 it is non-central.
- the inverse distance potential of finite depth in E_3 is converted to an infinite barrier and therefore to a confinement potential in E_4.
- Various parametrizations of χ such like $\chi = \frac{r}{R}\pi$, $\chi = \tan^{-1}\frac{r}{R}$ etc. give rise to a variety of potentials in ordinary three space. The first potential which is central and will be shown to embed the Cornell one, is in the focus of our study, while the second is a non-central gradient potential with position dependent reduced mass and is used in quantum dots.
- The power of the curvature concept is to act as the common prototype of confinement phenomena of different disguises.

Exact Wave Functions and Energies of the Trigonometrically Extended Cornell Potential

We define the TEC potential as:

$$\mathcal{V}_{TEC}(\chi)=\widetilde{\kappa}\frac{\hbar^2}{2\mu}\frac{l(l+1)}{\sin^2\chi}-2G\sqrt{\widetilde{\kappa}}\cot\chi,\quad \chi=r\sqrt{\widetilde{\kappa}}.$$

The TEC Schrödinger equation:

$$\Big[-\widetilde{\kappa}\frac{\hbar^2}{2\mu}\frac{\mathrm{d}^2}{\mathrm{d}\chi^2}+\mathcal{V}_{TEC}(\chi)\Big]X\left(\chi,\sqrt{\widetilde{\kappa}}\right)$$
$$=\left(E(\widetilde{\kappa})+\frac{\hbar^2}{2\mu}\widetilde{\kappa}\right)X\left(\chi,\sqrt{\widetilde{\kappa}}\right),$$

We solve anew the TEC Schrödinger eq. on S^3_R

Our respective wave functions:

$$X_{(Kl)}(\chi,\widetilde{\kappa})=N_{(Kl)}\sin^{K+1}\chi e^{-\frac{b\chi}{K+1}}R_{K-l}^{\left(\frac{2b}{K+1},-(K+1)\right)}(\cot\chi),$$

$$b=\frac{2\mu G}{\sqrt{\widetilde{\kappa}}\hbar^2},\quad K=0,1,2,...,\quad l=0,1,...,K$$

$N_{(Kl)}$ is a normalization constant

require the non-classical Romanovski polynomials with weight function

$$w^{(\beta,\alpha)}(x){=}(1+x^2)^{\beta-1}e^{-\alpha\cot^{-1}x}\,.\quad x=\cot z,$$

$w^{(0,0)}=(1+x^2)^{-1}$: Cauchy (Breit-Wigner) distrib.,
$w^{(t+1,0)}=(1+x^2)^{-t}$: Student's t distribution,
and built up from the Rodrigues formula as

$$\mathcal{R}_m^{(\beta,\alpha)}(x)=\frac{N_{nl}}{w^{(\beta,\alpha)}(x)}\frac{d^m}{dx^m}\left((1+x^2)^m w^{(\beta,\alpha)}(x)\right).$$

- *[Compean, M.K., J. Phys. A:Math.Gen.* **39**, *547 (2006)]*
- *[Raposo, Weber, Alvarez-Castillo, M.K., C. Eur. J. Phys.* **5**, *253 (2007)]*

The K-levels belong to $O(4)$ irreps $\left(\frac{K}{2},\frac{K}{2}\right)$.

For a particle with spin-1/2 within $\mathcal{V}_{TEC}$ one has to couple $(\frac{1}{2},0)\oplus(0,\frac{1}{2})$ to $(\frac{K}{2},\frac{K}{2})$ finding its exitations as part of

$$|Klm, s=\frac{1}{2}\rangle = \left(\frac{K}{2},\frac{K}{2}\right)\otimes\left[\left(\frac{1}{2},0\right)\oplus\left(0,\frac{1}{2}\right)\right].$$

TEC Degeneracy Patterns:

K parity dyads

$$\frac{1}{2}^{\pm}, ..., \left(K-\frac{1}{2}\right)^{\pm} \in |Klm, s=\frac{1}{2}\rangle.$$

and a state of maximal spin,

$$J_{\max}{=}\left(K+\frac{1}{2}\right)^{\pi} \in |Klm, s=\frac{1}{2}\rangle$$

without parity companion and of either natural $(\pi=(-1)^{L})$ or, unnatural $(\pi=(-)^{L+1})$ parity

3.SO(4)/SO(2,1) symmetries of the potential and its link to the AdS_5/CFT algebraic aspects

$$E_K(\widetilde{\kappa}) = -\frac{G^2}{\frac{\hbar^2}{2\mu}}\frac{1}{(K+1)^2} + \widetilde{\kappa}\frac{\hbar^2}{2\mu}((K+1)^2-1)$$

O(4) symmetry.

$$H(\widetilde{\kappa}) = H_0 - \frac{G^2}{\frac{\hbar^2}{2\mu}}\frac{1}{\mathcal{K}^2+1} + \widetilde{\kappa}\frac{\hbar^2}{2\mu}\mathcal{K}^2,$$

$$\mathcal{K}^2|Klm> = [(K+1)^2-1]|Klm> .$$

[M. K., Moshinsky, Smirnov, PRD **64***, 11405 (2001)].*

SO(2,1) symmetry for G restricted to $G = g(K+1)$:

$$H(\widetilde{\kappa}) = H_0 - g^2\frac{\hbar^2}{2\mu} + \widetilde{\kappa}\frac{\hbar^2}{2\mu}\left({J_z}^2 - 1\right),$$

$$\mathcal{J}^2 = J_x^2 + J_y^2 - J_z^2, \quad \mathcal{J}^2|jm'> = j(1-j)|jm'>,$$

$$J_z|jm'> = m'|jm'>, \quad m' = j+n, \quad j = l+1$$

[M.K., C. Compean, hep-ph/0805.2404]

AdS_5/CFT algebraic aspects:

$$AdS_5: \quad SO(3,2) \supset SO(2,2) \supset SO(2,1) \supset SO(2)$$
$$\qquad\qquad\qquad\qquad\qquad\qquad j \qquad\quad m'$$

$$CFT: \quad SO(4,2) \supset SO(1,4) \supset SO(4) \supset SO(3) \supset SO(2)$$
$$\qquad\qquad\qquad\qquad\qquad\qquad K \qquad\quad l \qquad\quad m.$$

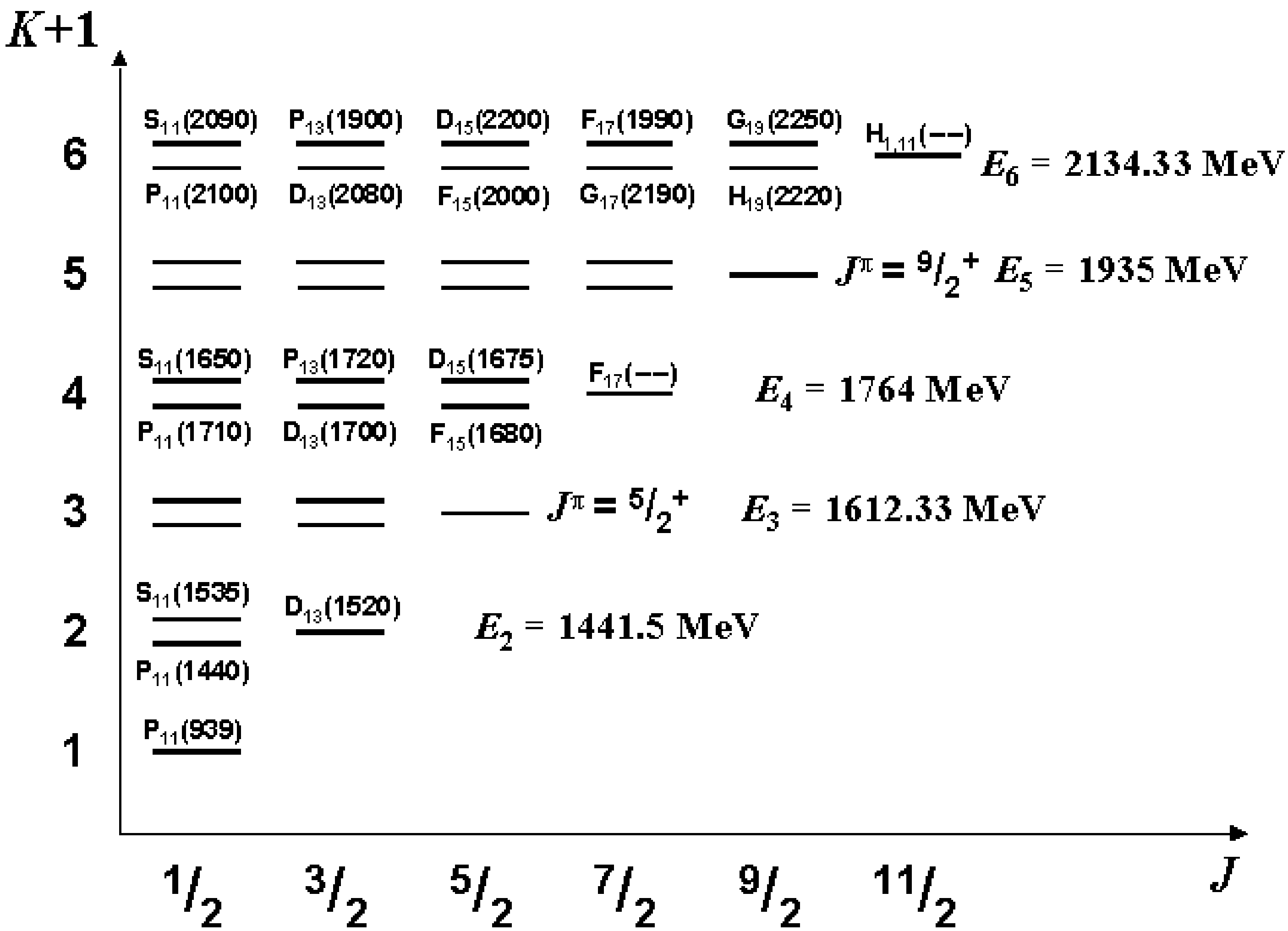
The TEC potential matches the

- QCD dynamics aspects,
- AdS_5/CFT algebraic aspects.

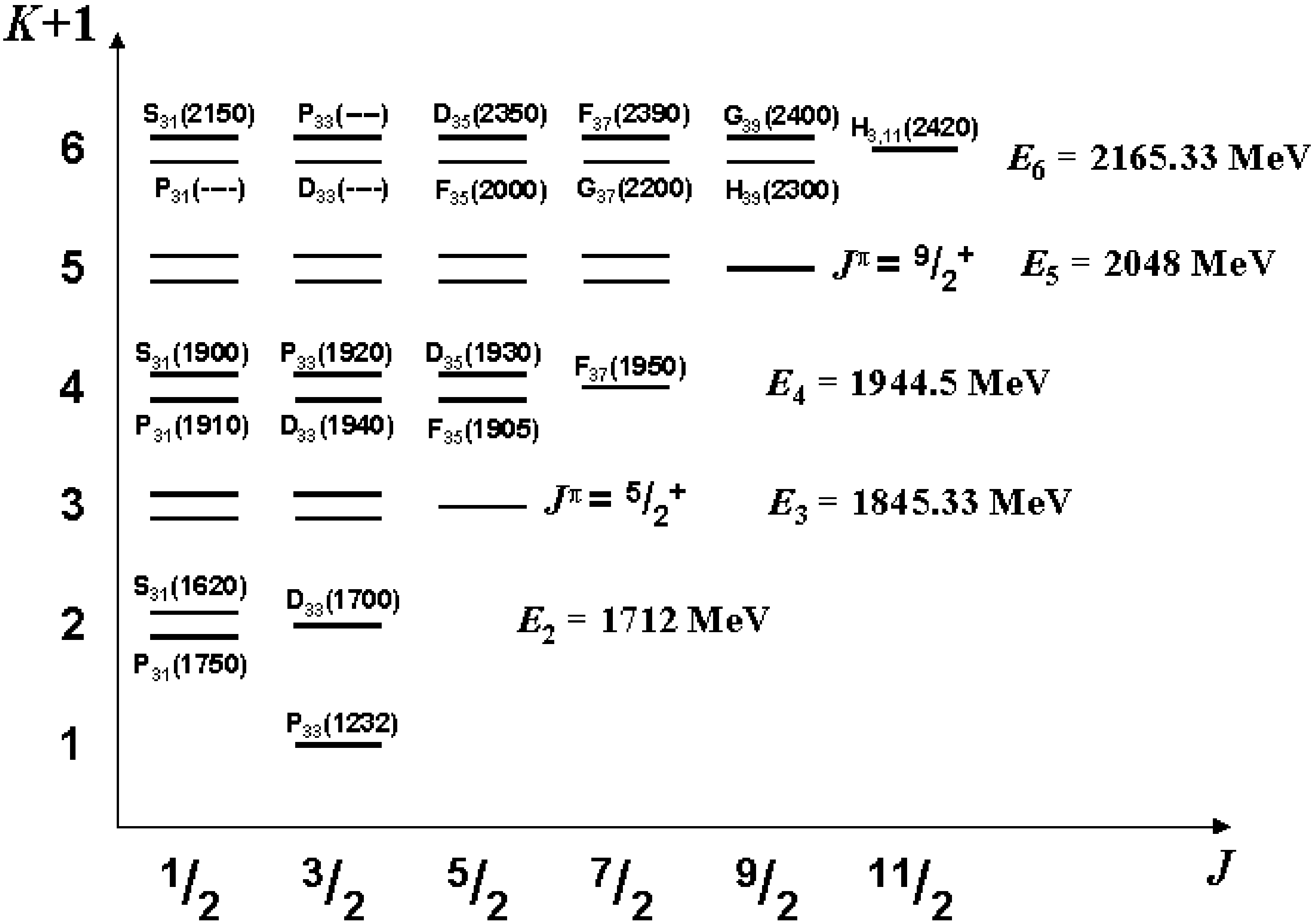
It links the algebraic aspects of AdS_5/CFT correspondence to QCD potentiology.

4. TEC at work: N and Δ spectroscopy.

Fitting potential parameters to the N spectrum gives $G = 237.55$ MeV$\cdot$ fm, $\kappa = 0.019$ fm $^{-2}$, $\mu = 1.057\,\mathrm{fm}^{-1}$.



N levels in the TEC potential and the $q-(qq)$ picture.
Complete data show prominent $SO(4)/SO(2,1)$ degeneracy.



Δ levels in the TEC potential and the $q-(qq)$ picture.

Complete data show prominent $O(4)/SO(2,1)$ degeneracy.

A total of 33 "missing" N and Δ resonances.

Nucleon electric charge form factor:

$$G_E^p = < \mathbf{p}_f | J_0(\mathbf{r}) | \mathbf{p}_i > , \quad \mathbf{Q}^2 = -q^2 = -(q_0^2 - \mathbf{q}^2),$$

$$\mathcal{R}_{10} = N_{10} \sin \frac{|\mathbf{r}|}{d} e^{-b\frac{|\mathbf{r}|}{d}},$$

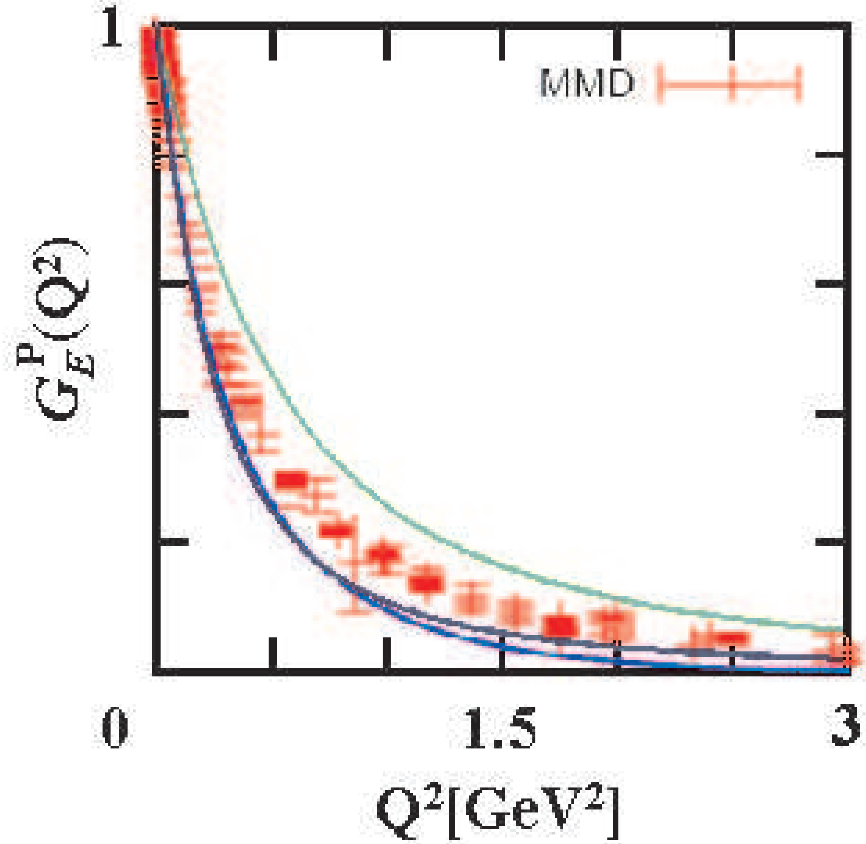
$$G_E^p = \int_0^{\pi d} d|\mathbf{r}| \frac{(\mathcal{R}_{10}(|\mathbf{r}|))^2 \sin |\mathbf{q}||\mathbf{r}|}{|\mathbf{q}||\mathbf{r}|},$$

$$\approx \int_0^{\infty} \ldots\ldots$$

$$= \frac{b(b^2+1)}{|\widetilde{\mathbf{q}}|} \tan^{-1} \frac{16b|\widetilde{\mathbf{q}}|}{\widetilde{\mathbf{q}}^4 + 4(2b^2-1)\widetilde{\mathbf{q}}^2 + 16b^2(b^2+1)},$$

with $\widetilde{\mathbf{q}} = \mathbf{q}d$; reminder: $b = \frac{2\mu G}{\hbar^2 \sqrt{\widetilde{\kappa}}}$, $d = \frac{\pi}{R} \equiv \sqrt{\widetilde{\kappa}}$

Quadratic denominator relates to the dipole



green line: our analytic formula from

[Compean, M.K., EPJA **39**, *1(2007)],[hep-ph:0805.2404]*

corresponding to $\kappa = 0.019$ fm^{-2} fitted to spectra.

middle blue line: fit to the proton mean square charge radius, $\sqrt{<\mathbf{r}^2>} = 0.87$ fm , $\kappa = 0.009$ fm^{-2}

lowest blue line: Bethe-Salpeter calculation by `Ch. Haupt, Ph.D. thesis,`

`www.itkp.uni-bonn.de/~haupt/talks/Internal/2005.pdf`

5. Curvature parameter and deconfinement

TEC is a two-parameter potential, the strenght G, and the curvature, $\widetilde{\kappa}=\frac{\pi^2}{R^2}$, as driver of the confinement-deconfinement transition

When curvature goes down, High-lying "curved" states approach "flat" scattering states of the $1/r$ piece for both the energy and wave functions.

Two limits:

- $\widetilde{\kappa} \longrightarrow 0$,
- $K\sqrt{\widetilde{\kappa}} \longrightarrow k, \quad k$ constant,

$$E_K(\widetilde{\kappa}) \stackrel{\widetilde{\kappa}\to 0}{\longrightarrow} -\frac{G^2}{\frac{\hbar^2}{2\mu}}\frac{1}{n^2},$$

$$E_K(\widetilde{\kappa}) \stackrel{K\sqrt{\widetilde{\kappa}}\to k}{\longrightarrow} -\frac{G^2}{\frac{\hbar^2}{2\mu}}\frac{1}{n^2}+\frac{\hbar^2}{2\mu}\mathrm{k}^2.$$

Possible because of the common $SO(4)/SO(2,1)$ symmetries shared by the TEC and $1/r$ potentials.

- Barut,Wilson, J.Phys.A:Math.Gen. **20**, 6271 (1987)
- Vinitsky et al., Phys. Atom. Nucl.**56**, 321 (1993)

As curvature goes down because of thermal dependence, confinement fades away, an observation that is suggestive of a deconfinement scenario controlled by the curvature parameter of the TEC potential.

Deconfinement as flattening of space considered by

- F. Takagi, PRD **35**, 2226 (1987)

within the context of a AdS_5 black hole universe as bag scenario.

Advantage of our scheme:

Temperature dependent space flattening paralleled by regression of the "curved" TEC– to the flat $1/r$ potential, and correspondingly, by regression of the TEC wave functions from the confined to the $1/r$ wave-functions from the deconfined phases.

6. Conclusions

The trigonometrically extended Cornell potential, $\mathcal{V}_{TEC}(\chi)$, with $\chi = \frac{r}{R}\pi$, succeeds in providing the

- practically complete description of N and Δ spectra (modulo the hybrid $\Delta(1600)$) explaining their prominent SO(4)/SO(2,1) degeneracy patterns, level splittings, number of the (so far) observed states,
- prediction of a minimum (compared to other models) of unobserved states (a total of 33), of leading quark-diquark configuration,
- description of the confinement-deconfinement transition in terms of a temperature dependent curvature parameter (curvature shut-down),
- knowledge on the "before" (confinement) and "after" (deconfinement) wave functions and consequently of the explicit temperature evolution of matrix elements,
- link between the algebraic AdS_5/CFT aspects to QCD potentiology.
- scenario of Quantum Mechanical CromoDynamics (QMCD)

Next goal: putting high-spins on equal footing with spin-$\frac{1}{2}$

TEC levels/ Rarita-Schwinger fields correspondence:

$$\left(\frac{K}{2},\frac{K}{2}\right)\otimes\Big[\left(\frac{1}{2},0\right)\oplus\left(0,\frac{1}{2}\right)\Big]\simeq A_{\mu_1\ldots\mu_K}\otimes\psi\equiv\Psi_{\mu_1\ldots\mu_K}$$

- First construct a consistent wave equation for a point-like Ψ_μ and get a fundamental $g_{\frac{3}{2}}=2$ value (done),
- Next calculate $g_{\frac{3}{2}}^{\mathrm{TEC}}$ of $D_{13}(1520)$ with the respective TQC quark wave function and extract the effect of the internal structure as $\left(2-g_{\frac{3}{2}}^{\mathrm{TEC}}\right)$ (pending).

4. Consistent high-spin Lagrangians.

High-spin description within the conventional Rarita-Schwinger approach is inconsistent:

- Electromagnetically interacting fields propagate acausally (Velo-Zwanziger problem),
- Cross sections depend on undermined off-shell parameters.

4.1 New ways. Poincaré covariant projectors.

Suggested in:
[M.K., Foundations of Physics, **33**, *781 (2003)].*

Employ the two Casimir invariants of the Poincaré group, P^2 , and $\mathcal{W}^2$ to pin down spin-3/2 in ψ_μ by means of the Poincaré covariant projector:

$$\mathcal{P}^{(m,3/2)}_{\mu\nu}\psi^{(m,3/2)\ \nu} = \psi^{(m,3/2)}_{\mu}\,,$$

$$\mathcal{P}^{(m,3/2)}_{\mu\nu} = -\frac{P^2}{m^2}\frac{1}{3}\left[\frac{1}{P^2}\mathcal{W}^2 + \frac{3}{4}1_4 \otimes 1_4\right]_{\mu\nu}\,.$$

Fully executed in

1. *[Napsuciale, M.K., Rodriguez, Eur. Phys. J. A* ***29****, 289 (2006)]* (single spin-3/2),

2. *[Napsuciale, Rodriguez, Delgado-Acosta, Kirchbach, Phys. Rev. D* ***77****, 014099 (2008)]* (single spin-1).

Principal results:

- Causal propagation of electromagnetically interacting spin-$\frac{3}{2}$ fields,
- Causality fixes the giromagnetic ratio to $g_{\frac{3}{2}} = 2$,
- Unitarity fixes $g_1 = 2$ for any vector particle, be it Abelian or non-Abelian,
- The massless spin-$\frac{3}{2}$ theory of is unique,
- The massive spin-$\frac{3}{2}$theory depends on two parameters, a, and b, which can be treated in terms of gauge fixing, meaning 't Hooft-like propagation of ψ_μ:

The electromagnetically gauged spin-$\frac{3}{2}$ wave equation that is consistent with causality, i.e. it is free from the Velo-Zwanziger pathology suffered by the Rarita-Schwinger framework reads:

$$\Big(\left(\pi^2 - m^2\right) g_{\alpha\beta} - i g_{\frac{3}{2}} \left(\frac{\sigma_{\mu\nu}\pi^\mu \pi^\nu}{2} g_{\alpha\beta} - e\, F_{\alpha\beta} \right)$$

$$+ \frac{1}{3}\left(\gamma_\alpha \not\pi - 4\pi_\alpha\right)\pi_\beta$$

$$+ \frac{1}{3}(\pi_\alpha \not\pi - \gamma_\alpha \pi^2)\gamma_\beta \Big)\psi^\beta = 0 \;, \quad g_{\frac{3}{2}} = 2.$$

<u>Spin-$\frac{3}{2}$ propagator in $\psi_\mu = A_\mu \otimes \left[\left(\frac{1}{2},0\right) \oplus \left(0,\frac{1}{2}\right)\right]$</u> :

Particular ($b \to \infty$) gauge,

$$\Pi(a,\infty) = \frac{-\mathbf{P}^{\left(\frac{3}{2}\right)} + \frac{p^2-m^2}{m^2}\mathbf{P}_{11}^{\left(\frac{1}{2}\right)} - a\,\frac{p^2-m^2}{p^2-a m^2}\mathbf{P}_{22}^{\left(\frac{1}{2}\right)}}{p^2-m^2+i\varepsilon}.$$

Compares with the

<u>spin-1 't Hooft propagator in $A_\mu = \left(\frac{1}{2},\frac{1}{2}\right)$</u>

$$\Pi^{'tHooft}(\xi) = \frac{-\mathbf{P}^{(1)} - \xi\,\frac{p^2-m^2}{p^2-\xi m^2}\mathbf{P}^{(0)}}{p^2-m^2+i\varepsilon}.$$

5. Summary...

UNDER CONTROL: Both the

- internal structure consistent with QCD and data,
- external space-time propagation consistent with relativity,

of high-spins as

$$\left(\frac{K}{2},\frac{K}{2}\right)\otimes\Big[\left(\frac{1}{2},0\right)\oplus\left(0,\frac{1}{2}\right)\Big]\simeq\Psi_{\mu_1\ldots\mu_K}$$

.... Perspectives

LOTS of WORK,...... si nos dejan!