High Spin Baryons in Quantum Mechanical Chromodynamics

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Abstract. A framework of quantum mechanical chromodynamics (QMCD) is developed with the aim to place the description of the nucleon on a comparable footing with Schrödinger's quantum mechanical treatment of the hydrogen atom. Such indeed turns out to be possible upon replacing the $(e^- - p)$ by a $(q - qq)$ system, on the one hand, and the Coulomb potential by the recently reported by us exactly solvable trigonometric extension of the Cornell (TEC) potential, on the other. The TEC potential translates the inverse distance potential in ordinary flat space to a space of constant positive curvature, the 3D hypersphere, a reason for which both potentials have the $SO(4)$ and $SO(2,1)$ symmetries in common. In effect, the nucleon spectrum, inclusive its Δ branch, acquire the degeneracy patterns of the electron excitations with spin in ^1H without copying them, however. There are two essential differences between the $N(\Delta)$ and H atom spectra. The first concerns the parity of the states which can be unnatural for the N and Δ excitations due to compositeness of the diquark, the second refers to the level splittings in the baryon spectra which contain besides the Balmer term also its inverse of opposite sign. Our scheme reproduces the complete number of states (except the hybrid $\Delta(1600)$), predicts a total of 33 new resonances, and explains the splittings of the N and Δ levels containing high-spin resonances. It also describes accurately the proton electric charge form factor. We here calculate the potential in momentum space (instantaneous effective gluon propagator) as a Fourier transform of the TEC potential and show that the concept of curvature allows to avoid the integral divergences suffered by schemes based on power potentials. We find a propagator that is finite at origin, likely to produce confinement. The advocated new potential picture allows for deconfinement too as effect of space flattening in the limit of infinite radius of the 3D hypersphere. The potential's $SO(4)/SO(2,1)$ symmetries reflect AdS_5/CFT correspondence.

1 INTRODUCTION

A master piece of a quantum mechanical treatment of a two-body system is the Schrödinger equation with the Coulomb potential in its application to the hydrogen atom. The signifi cance of this example is manifold but perhaps its most important facet is the adequacy of the Coulomb potential for the $(e^-–p)$ system. From the perspective of contemporary knowledge, the success of the Coulomb potential in getting the observed positions of the H atom emission(absorption) lines right is due to the fact that quantum electrodynamics is the correct fundamental gauge theory of electromagnetic interaction. This is reflected by the property of the Coulomb potential of being the Fourier transform of the instantaneous photon propagator from momentum to position space, and vice versa. Fortunately, the Coulomb law has been fi gured out correctly before the rise of QED and the form of the potential used in the Schrödinger equation for the H atom has never been a subject of speculations and debates. Had this not been so, had one to handle the H atom without previous knowledge on the empirical Coulomb law, one most probably would have tried to employ the harmonic oscillator with the justification that any interaction is approximately harmonic at small distances. As a result of such inadequate treatment, the H atom spectrum would have evolved to a big enigma and one would have ended up puzzled and debating why several predicted lines are "missing" from the spectrum, why others, unpredicted, are there, why the line splittings are at odds with the "theory" etc. One would have been forced to invent a variety of interactions to compensate for the insufficiency of the harmonic oscillator picture and invoked a plethora of free parameters to fit data. Considerations of the type are alerting about the importance of being aware of the correct link between the potential model and the underlying fundamental gauge theory and about the danger in creating fictitious enigmas in effect of employing wrong potentials. Unfortunately, the power potentials of common use in quark models do not qualify as related to all three QCD regimes. A big deal of the enigmas of the baryon spectra might quite be, as emphasized above, fictitious and due to the employment of inadequate potentials. Our point is that the correct QM limit of QCD is hitted by the trigonometric potential that (i) satisfies the Laplace-Beltrami equation on the space of constant positive curvature, the 3D hypersphere, (ii) captures the essentials of QCD quark-gluon dynamics, and (iii) is the adequate one in reproducing

- the nucleon spectrum (including its Δ branch),
- the proton electric charge form factor,
- an effective gluon propagator suited for confinement,
- the deconfinement as flattening of space.

The contribution is organized as follows. In the next section we present the TEC potential. Section 3 is a brief review of our results on the solutions of the Schrödinger equation for the quark-diquark (q–qq) system with the TEC potential in terms of Romanovski polynomials. It also illuminates coincidence between predicted and empirical spectra. In section 4 we show how our calculation reproduces accurately the proton electric charge form factor and highlight the construction of the instantaneous effective gluon propagator. Finally, the scenario of deconfinement as shutting down the curvature of the hypersphere is outlined in section 5. The presentations closes with concise conclusions.

2 THE TEC QUARK CONFINEMENT POTENTIAL

Significant progress in understanding hadron properties has been reached through the elaboration of the connection between the QCD Lagrangian and the potential models as deduced within the framework of effective field theories [1], and especially through the non-perturbative methods such as lattice simulations, the most prominent outcome being the linear plus Coulomb confinement potential [2], [3]. The potentials derived from the QCD Lagrangian have been most successful in the description of heavy quarkonia and heavy baryon properties [4]. On the other hand, light flavor baryons are ordinarily treated in terms of quark models based on $SU(6)_{SF} \times O(3)_L$ in combination with the harmonic oscillator potential, a cumbersome scheme overcrowded with parameters, "missing" resonances, and other peculiarities. Although the Cornell potential has found applications also in nucleon and Δ quark models [5], the provided level of quality in the description of the non-strange sector stays below the one reached for the heavy flavor sector. This behavior indicates that the one gluon exchange (giving rise to the Coulomb(like) term) and the flux-tube interaction (associated with the linear part) do not fully account for the complexity of the dynamics of three light quarks. Various improvements have been under consideration in the literature such as screening effects in combination with spin-spin forces (see [5] and references therein).

Very recently, the Cornell potential has been upgraded by us in [6] through its embedding in an exactly solvable (in the sense of the Schrödinger equation, or, the Klein-Gordon equation with equal scalar and vector potentials) trigonometric quark confinement potential which takes its origin from early work by Schrödinger. To be specific, around 1941 Schrödinger had the idea to solve the quantum mechanical Coulomb problem in the cosmological context of Einstein's closed universe, i.e. on the three dimensional (3D) hypersphere, S^3_R, of a constant radius R [7]. In order to construct on S^3_R the counterpart of the flat-space inverse distance potential Schrödinger had to solve the corresponding four-dimensional Laplace-Beltrami equation. In result, after some straightforward algebra, he found the following potential

$$\mathcal{V}(\chi) \quad = \quad c\cot\chi + \kappa\frac{\hbar}{2\mu}\frac{l(l+1)}{\sin^2\chi} + \text{const}, \quad \kappa = \frac{1}{R^2}. \tag{1}$$

Here χ stands for the second polar angle in E_4, and the $\csc^2$ term describes the centrifugal barrier on S^3_R. Schrödinger's prime result, the presence of curvature provokes that the orbiting particle appears confined within a trigonometric potential of infinite depth and the hydrogen spectrum shows only bound states. An especially interesting observation was that the $O(4)$ degeneracy of the levels observed in the flat space H atom spectrum was respected by the curved space spectrum too in the sense that also there the levels could be labeled by the standard atomic indices n, l, and m, and the energy depended on n alone. An especially simple and convenient parametrization of the χ variable in terms of r, also used by Schrödinger [7] is given by

$$\chi = \frac{r}{R}\pi \equiv \frac{r}{d}, \quad d = \frac{R}{\pi}, \quad \frac{r}{R} \in [0,1], \quad \kappa \to \widetilde{\kappa} = \frac{1}{d^2}. \tag{2}$$

Here, the length parameter d assumes the róle of rescaled hyper-radius. Correspondingly, the place of the genuine curvature, $\kappa = 1/R^2$, is taken by the rescaled one, $\widetilde{\kappa} = 1/d^2$. Setting now $c = -2G\sqrt{\widetilde{\kappa}}$, and const= $-\frac{\hbar\sqrt{\widetilde{\kappa}}}{2\mu} + \frac{2\mu G}{\hbar}$, amounts to the following radial Schrödinger equation,

$$\left[H_0 - \widetilde{\kappa}\frac{\hbar}{2\mu}\frac{\mathrm{d}^2}{\mathrm{d}\left(r\sqrt{\widetilde{\kappa}}\right)^2} + \mathcal{V}\left(r\sqrt{\widetilde{\kappa}},\widetilde{\kappa}\right)\right] X\left(r\sqrt{\widetilde{\kappa}},\widetilde{\kappa}\right) = \left(\varepsilon(\widetilde{\kappa}) + \frac{\hbar}{2\mu}\widetilde{\kappa}\right) X\left(r\sqrt{\widetilde{\kappa}},\widetilde{\kappa}\right),$$

$$\mathcal{V}\left(r\sqrt{\widetilde{\kappa}},\widetilde{\kappa}\right) = \widetilde{\kappa}\frac{\hbar}{2\mu}\frac{l(l+1)}{\sin^2\left(r\sqrt{\widetilde{\kappa}}\right)} - 2G\sqrt{\widetilde{\kappa}}\cot\left(r\sqrt{\widetilde{\kappa}}\right) - \frac{\hbar\sqrt{\widetilde{\kappa}}}{2\mu} + \frac{2\mu G}{\hbar}. \tag{3}$$

A similarly shaped potential is managed by SUSYQM [8] under the name of Rosen-Morse I. The essential difference between Rosen-Morse I and $\mathcal{V}(r\sqrt{\widetilde{\kappa}},\widetilde{\kappa})$ is the suppression of the curvature in the former and its treatment as a central potential in ordinary flat space.

It is now quite instructive to expand $\mathscr{V}(r\sqrt{\widetilde{\kappa}},\widetilde{\kappa})$ in a Taylor series. In so doing, one finds the following approximation,

$$-2G\sqrt{\widetilde{\kappa}}\cot r\sqrt{\widetilde{\kappa}}+\widetilde{\kappa}\frac{\hbar^2}{2\mu}\frac{l(l+1)}{\sin^2\left(r\sqrt{\widetilde{\kappa}}\right)} \quad \approx \quad -\frac{2G}{r}+\frac{2G\widetilde{\kappa}}{3}r+\frac{\hbar^2}{2\mu}\frac{l(l+1)}{r^2}, \qquad (4)$$

with $\widetilde{\kappa}=\frac{1}{d^2}=\frac{\pi^2}{R^2}$.

> The finite range potential, $\mathscr{V}(r\sqrt{\widetilde{\kappa}},\widetilde{\kappa})$, is the exactly solvable *trigonometric extension to the infinite range Cornell* potential, and will be referred to as TEC potential. The latter captures remarkably well the essentials of QCD quark-gluon dynamics as it interpolates between the Coulomb potential (associated with one gluon exchange in the perturbative regime) and the infinite well (describing free though trapped quarks as in the regime of the asymptotic freedom) while passing through a region of essentially linear growth (associated with gluon flux-tube interactions in the non-perturbative regime).

Besides Schrödinger, eq. (3) has been solved by various authors using different schemes. The solutions obtained in [9] are built on top of Jacobi polynomials of imaginary arguments and parameters that are complex conjugate to each other, while ref. [10] expands the wave functions of the interacting case in the free particle basis. The most recent construction in our previous work [11] instead relies upon real Romanovski polynomials. In the χ variable in eq. (2) our solutions to eq. (3) take the form,

$$X_{(Kl)}(\chi,\widetilde{\kappa})=N_{(Kl)}\sin^{K+1}\chi e^{-\frac{b\chi}{K+1}}R_{K-l}^{(\frac{2b}{K+1},-(K+1))}(\cot\chi), \qquad b=\frac{2\mu G}{\sqrt{\widetilde{\kappa}}\hbar^2}.$$
$$K=0,1,2,\ldots, \quad l=0,1,\ldots,K, \qquad (5)$$

where $N_{(Kl)}$ is a normalization constant. The $R_n^{(\alpha,\beta)}(\cot\chi)$ functions are the non-classical Romanovski polynomials [12, 13] which are defined by the following Rodrigues formula,

$$R_n^{(\alpha,\beta)}(x) \quad = \quad e^{\alpha\cot^{-1}x}(1+x^2)^{-\beta+1}\times\frac{\mathrm{d}^n}{\mathrm{d}x^n}e^{-\alpha\cot^{-1}x}(1+x^2)^{\beta-1+n}, \qquad (6)$$

where $x=\cot r\sqrt{\widetilde{\kappa}}$ (see ref. [14] for a recent review).

The energy spectrum of $\mathscr{V}\left(r\sqrt{\widetilde{\kappa}},\widetilde{\kappa}\right)$ is calculated as

$$E_K(\widetilde{\kappa})=-\frac{G^2}{\frac{\hbar^2}{2\mu}}\frac{1}{(K+1)^2}+\widetilde{\kappa}\frac{\hbar^2}{2\mu}((K+1)^2-1), \quad l=0,1,2,\ldots,K, \qquad (7)$$

where $E_K=\varepsilon_K+\frac{\hbar^2\sqrt{\widetilde{\kappa}}}{2\mu}-\frac{2\mu G}{\hbar^2}+H_0$. Giving $(K+1)$ the interpretation of a principal quantum number $n=0,1,2,\ldots$ (as in the H atom), one easily recognizes that the energy in eq. (7) is defined by the Balmer term and its inverse of opposite sign, thus revealing $SO(4)$ as dynamical symmetry of the problem. Stated differently, particular levels bound within different potentials (distinct by the values of l) carry same energies and align to levels (multiplets) characterized by the four dimensional angular momentum, K. The K-levels belong to the irreducible $SO(4)$ representations of the type $\left(\frac{K}{2},\frac{K}{2}\right)$. When the confined particle carries spin-1/2, as is the case of electrons in quantum dots, or quarks in baryons, one has to couple the spin, i.e. the $\left(\frac{1}{2},0\right)\oplus\left(0,\frac{1}{2}\right)$ representation, to the previous multiplet, ending up with the (reducible) $SO(4)$ representation

$$|Klm,s=\frac{1}{2}\rangle=\left(\frac{K}{2},\frac{K}{2}\right)\otimes\left[\left(\frac{1}{2},0\right)\oplus\left(0,\frac{1}{2}\right)\right]. \tag{8}$$

This representation contains K parity dyads and a state of maximal spin, $J_{\max}=K+\frac{1}{2}$, without parity companion and of either positive ($\pi=+$) or, negative ($\pi=-$) parity,

$$\frac{1}{2}^{\pm},\ldots,\left(K-\frac{1}{2}\right)^{\pm},\left(K+\frac{1}{2}\right)^{\pi}\in|Klm,s=\frac{1}{2}\rangle. \tag{9}$$

The parity dyads have underlying angular momenta differing by one unit (i.e. $\Delta l=1$), and can not be given the interpretation of parity doublets which reside in different Fock spaces built on top of either scalar or pseudo scalar vacuum. As we shall see below this scenario turns to be remarkably adequate for the description of non-strange baryon properties.

3 $SO(4)$ DEGENERACIES IN THE N AND Δ SPECTRA

The spectrum of the nucleon continues being under debate despite the long history of the respective studies (see refs. [15], [16] for recent reviews). Unprejudiced inspection of the data reported by the Particle Data Group [17] reveals a systematic degeneracy of the excited states of the baryons of the best coverage, the nucleon (N) and the $\Delta(1232)$. Our case is that

- levels and level splittings of the nucleon and Δ spectra match remarkably well the spectrum in eq. (7).

The N and Δ spectra: The measured nucleon resonances with masses below 2.5 GeV fall into the three $K=1,3,5$ levels in eq. (9) with only the two F_{17} and $H_{1,11}$ states still "missing", a systematics anticipated earlier by one of us (M.K.) in refs. [18] on the basis of pure algebraic considerations. Moreover, the level splittings follow with an amazing accuracy eq. (7). The nucleon spectrum in the quark-diquark picture of internal structure, shown in Fig. 1, is fitted by the following parameters of the "curved" potential in eq. (3),

$$\mu=1.06\ \mathrm{fm}^{-1}\,,\quad G=237.55\ \mathrm{MeV}\cdot\mathrm{fm}\,,\quad d=2.31\ \mathrm{fm}. \tag{10}$$

Almost same set of parameters, up to a modification of d to $d=3$ fm, fits the $\Delta(1232)$ spectrum, which exhibits exactly same degeneracy patterns, and from which only the

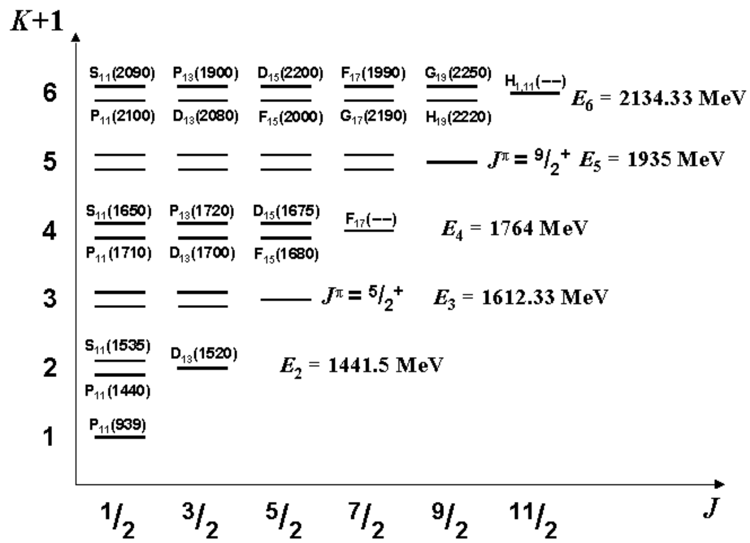

FIGURE 1. Assignments of the reported N excitations to the K levels of the S_R^3 potential, $\mathscr{V}(r\sqrt{\widetilde{\kappa}},\widetilde{\kappa})$, in eq. (3), taken as the quark-diquark confinement potential. The potential parameters are those from eq. (10). Double bars represent parity dyads, single bars the unpaired states of maximum spin. The notion $L_{2I,2J}(--)$ has been used for resonances "missing" from a level. The model predicts two more levels of maximal spins $J^\pi = 5/2^+$, and $J^\pi = 9/2^+$, respectively, which are completely "missing". In order not to overload the figure with notations, the names of the resonances belonging to them have been suppressed. The predicted degenerate energy (equal to the degenerate rest mass) of each level is given to its most right.

three P_{31}, P_{33}, and D_{33} states from the $K=5$ level are "missing". Remarkably, none of the reported states, with exception of the $\Delta(1600)$ resonance, presumably a hybrid, drops from the systematics. The unnatural parity of the $K=3,5$ levels requires a pseudo scalar diquark. For that one has to account for an 1^- internal excitation of the diquark which, when coupled to its maximal spin 1^+, can produce a pseudo scalar in one of the possibilities. The change of parity from natural to unnatural can be given the interpretation of a chiral phase transition in baryon spectra. Levels with $K=2,4$ have been attributed to entirely "missing" resonances in both the N and Δ spectra. To them, natural parities have been assigned on the basis of a detailed analysis of the $(1p\text{-}1h)$ Hilbert space of three quarks and its decomposition in the $|Klm, s=\frac{1}{2}\rangle$ basis [18]. We accommodate 37 from the reported 38 non-strange resonances and predict a total of 33 unobserved resonances of a dominant quark-diquark configurations in the N and $\Delta(1232)$ spectra with masses below ~ 2500 MeV, much less but any other of the traditional models. Some of the predicted states throughout might be suppressed in depending on the scale of chiral symmetry restoration.

In our previous work [6], the potential in eq. (3) has been considered in the spirit of SUSYQM as a *central two-strength parameter* potential in E_3 and without reference to S_R^3, a reason for which the values of the parameter accompanying the $\csc^2$ term had to be taken as integer *ad hoc* and for the only sake of a better fit to the spectra, i.e., without any deeper justification. Instead, in the present work, $\mathscr{V}(\chi,\kappa)$ in eq. (1), is identified as a *non-central one-strength parameter* potential in which the strength of the $\csc^2$ term has been uniquely fixed by the S_R^3 geometry.

4 MOMENTUM SPACE PHYSICS FROM TEC

In this section we shall test the potential parameters in eq. (10) and the wave functions in eqs. (3), (5) in the calculation of the proton electric charge form-factor, the touch stone of any spectroscopic model. As everywhere through the paper, the internal nucleon structure is approximated by a quark-diquark configuration. In conventional three-dimensional flat space the electric form factor, $G^{\rm p}_{\rm E}(|\mathbf{q}|)$, is defined in the standard way as the matrix element of the charge component, of the proton electric current between the states of the incoming, and outgoing, electrons in the dispersion process [6]. The mean square charge radius is then defined in terms of the slope of the electric charge form factor at origin as, $\langle \mathbf{r}^2\rangle = -6\frac{\partial G^{\rm p}_{\rm E}(|\mathbf{q}|)}{\partial |\mathbf{q}|^2}\Big|_{|\mathbf{q}|^2=0}$. On S^3_R, the three-dimensional radius vector, $\mathbf{r}$, has to be replaced by, $\overline{\mathbf{r}}$ with $|\overline{\mathbf{r}}| = R\sin\chi = \sin\chi/\sqrt{\kappa}$. The evaluation of the form-factor as four-dimensional Fourier transform requires the four-dimensional plane wave,

$$e^{iq\cdot\overline{x}} = e^{i|\mathbf{q}||\overline{\mathbf{r}}|\cos\theta} = e^{i|\mathbf{q}|\frac{\sin\chi}{\sqrt{\kappa}}\cos\theta}, \quad |\overline{\mathbf{r}}| = R\sin\chi = \frac{\sin\chi}{\sqrt{\kappa}}. \tag{11}$$

The latter refers to a z axis chosen along the momentum vector (a choice justified in elastic scattering where $q_0 = 0$), and a position vector of the confined quark having in general a non-zero projection on the extra dimension axis in E_4. The integration volume on S^3_R is given by $\sin^2\chi\sin\theta \mathrm{d}\chi\mathrm{d}\theta\mathrm{d}\varphi$. The explicit form of the nucleon ground state wave function obtained from eq. (5). The charge form factor obtained in this manner is displayed in Fig. 2 together with data.

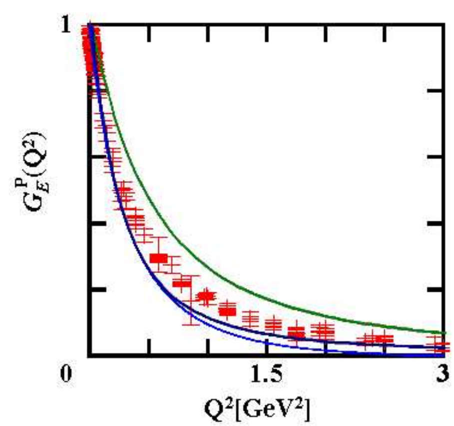

FIGURE 2. The G^p_E form factor as a function of $\mathbf{Q}^2 = -\mathbf{q}^2$ and calculated for various curvature parameters. The upper line corresponds to the curvature as fitted to the nucleon spectrum, the curvature leading to the middle line has been fitted to the experimental value of the mean square of the charge radius,$\sqrt{<\mathbf{r}^2>} = 0.87$ fm. The lowest line follows from a Bethe-Salpeter calculation based upon an instanton induced two-body potential and has been presented in ref. [19]. Data compilation taken from [19].

Next we calculate the potential in momentum space. As already mentioned in the introduction, it is a well known fact that in QED the Fourier transform of the (static) Coulomb potential, generates in momentum space the instantaneous $(q_0 = 0)$ photon propagator. We apply same concept to construct the Fourier transform of the TEC potential with the aim to obtain the potential in momentum space, equivalently, an instantaneous effective gluon propagator, as needed for example in Faddeev three-body calculations. In so doing we found that if treated as a central potential in ordinary flat position space, the Fourier integral is divergent and one needs to introduce a π range correlation function in order to get it finite. Power potentials are known to suffer same pathology. In contrast to this, on the hypersphere the Fourier transform is well defined and the result is displayed on Fig. 3.

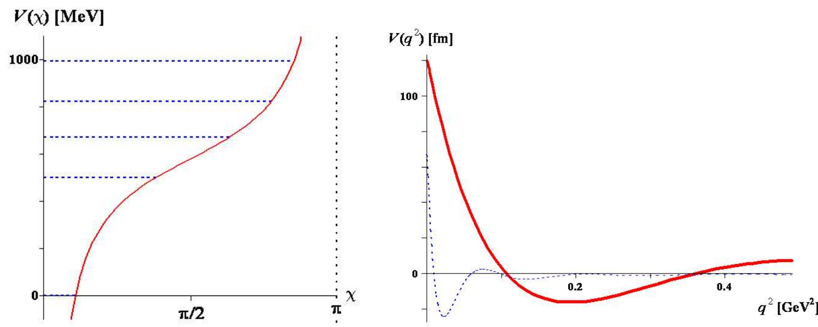

FIGURE 3. The TEC potential in position space (left) and its Fourier transform to momentum-space (right). Solid and dashed lines refer to the TEC potential treated as "curved" and "flat", respectively. The latter needs a π-range correlation taken as $(1 - \sin^{2N} r\sqrt{\widetilde{\kappa}})$ here.

5 CURVATURE SHUT-DOWN:THE DECONFINEMENT

Introducing the curvature parameter in the trigonometrically extended Cornell confinement potential opens an intriguing venue toward deconfinement as a S^3_R curvature shut-down. It can be shown that high-lying bound states from the TEC confinement potential approach scattering states of its underlying $\sim 1/r$ flat space piece. Stated differently, the TEC confinement gradually fades away with vanishing curvature and approaches deconfinement. In the direct $\kappa \to 0$ limit, the second term of the r.h.s. in eq. (7) vanishes and the spectrum becomes the one of H atom(like) bound states. In the softer $\sqrt{\kappa}K \to \mathrm{k}$ (small κ, big K) limit, where "k" is a constant, the term in question approaches the scattering continuum of the underlying Coulomb(like) piece. In effect, the $\mathcal{V}(\chi,\kappa)$ spectrum collapses down to the spectrum of its ordinary Coulomb(like) piece,

$$E_K(\kappa) \stackrel{\kappa\to 0}{\longrightarrow} -\frac{G^2}{\frac{\hbar^2}{2\mu}}\frac{1}{n^2} + \frac{\hbar^2}{2\mu}\mathrm{k}^2, \quad l = 0,1,2,\ldots,n-1. \tag{12}$$

The rigorous proof that also the wave functions of the complete TEC potential collapse to those of the corresponding Coulomb(like) problem for vanishing curvature is a bit more involved and can be found in [10].

Deconfinement as a gradual flattening of space has earlier been considered by Takagi [20]. Compared to [20], our scheme brings the advantage that the flattening of space is paralleled by a temperature driven regression of the TEC potential in eq. (3) to its underlying flat Coulomb(like) piece, and correspondingly, by the temperature driven regression of the TEC wave functions from the confined to the Coulomb(like) wave functions from the deconfined phases.

5. SUMMARY

We demonstrated importance of hitting the correct quantum mechanical limit of QCD in terms of a potential harmonic on the S^3_R hypersphere that captures the essentials of all three regimes of QCD. In effect of the $SO(4)/SO(2,1)$ symmetries of the potential (which link it to AdS_5/CFT correspondence [21] as shown in [22]) each N and Δ spectra acquire a well pronounced level structure in accord with data. All the observed resonances (except the hybrid $\Delta(1600)$) fit into the levels and the number of the "missing" states (with masses below 2500 MeV) is significantly reduced compared to other schemes. Correct proton charge electric form factor and an effective gluon propagator anticipating confinement count to the further merits of the scheme developed, not to forget the possibility for deconfinement as curvature shut-down.

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