Quark flavor mixing and mass matrices

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Abstract

Some implications of the textures of the mass matrices for the flavor mixing matrix V are reviewed. Constraints on the structure of the mass matrices are given using some of the experimently measured properties of V and the quark masses at $2\ GeV$ and M_Z energy scales. In addition, to the Fritzsch and Stech type mass matrices a new type of mass matrix (designated as "CGS") is considered. The CGS type gives much better fits in the physical basis. The fits at the two energy scales are similar, implying that our results are unaffected by the evolution of the quark masses from 2 to 91 GeV.

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Introduction

The study of weak interactions really began in 1932 with the discovery of the neutron n [1]. Its observed decay into $p + e^-$ seemed to open Pandora's box! The decay seemed to violate the conservation of energy and angular momentum!! Pauli suggested that if a spin 1/2 neutral particle with a tiny mass was also present (but not observed) then there would not be a problem with these sacred conservation laws. This 'little' particle, named the "neutrino" by Fermi, was finnaly discovered in 1956 [2]. Absence of $\mu^- \to e^- + \gamma$ suggested that the electron and muon neutrinos were different, this was proved in 1962 [3]. Paucity of ν_e from the sun, suggested flavor mixing between the two known neutrinos in the early-sixties [4]. The idea that hadrons had fractionally charged constituents (todays quarks) was mooted around the mid-sixties. The weak current was written in terms of the SU(3) flavour triplet (u,d,s). Many theoreticians realized that CP-violation [5] could not be accommodated even with four quarks. However, the idea that all the interactions have to be written in terms of leptons and quarks did not seem mandatory in those days. As late as 1973, Kobayashi and Maskawa [6] tried several representations of SU(2) weak for 4 quarks unsuccessfully and concluded that with three quark doublets (or a 3x3 mixing matrix [7]) there was no problem. It is interesting to note that if CP-violation had not been discovered in 1964, theoreticians might have "predicted" that discovery of CP-violation in the K-meson system would imply the existence of a third generation without needing a high energy accelerator! In other words, study of flavor mixing can be a low energy window for new physics [8]. It would be true to say that without flavor mixing, the study of weak interactions would be vapid!!

Currently, experiments are underway at Belle and Babar to check the 'unitarity triangle' for the 3x3 mixing matrices <u>as accurately as possible</u>. If there is a significant deviation then it would be a signal for more than 3 generations. Also, in the 3x3 mixing matrix there is only one CP-violating phase, thus implying that CP-violation in different processes are related. Violation of these relations would be a signal for more generations. For example, a 4x4 mixing matrix, in general, contains 3 CP-violating phases.

1 Standard model

In the standard model, the flavor mixing is due to the gauge basis being different from the physical basis in which the quark mass matrices are diagonal. The standard model Lagrangian relevant for us can be written as

$$\mathscr{L} = \sum_{k=u,d} \bar{q}_{kL} \hat{M}_k q_{kR} + \frac{g}{\sqrt{2}} \bar{q}_{uL} \gamma_u V q_{dL} W_u^+ + H.c., \tag{1}$$

where $q_u = (u, c, t)$, $q_d = (d, s, b)$ and the quark mixing (or CKM) matrix

$$V = V_u^\dagger V_d. \tag{2}$$

The diagonal form $\hat{M}_k$ of the hermitian mass matrix M_k is given by $\hat{M}_k = V_k^\dagger M_k V_k$, $k = u$ and d. So that in the up-quark diagonal basis the CKM matrix $V = V_d$ and

$V = V_u^\dagger$ in the down-quark diagonal basis. Physically, one can work in either of these basies or the physical basis.

2 Results for one mass matrix $M = V\hat{M}V^\dagger$

These would apply to either the up or down quark diagonal basis with appropriate identification of V with the CKM matrix. They also apply to the lepton case, since one always works there in basis in which the charged lepton mass matrix is diagonal. Two interesting properties of V namely, the asymmetry $\Delta(V)$ and the Jarlskog invariant *J(V)* [9] can be expressed directly in terms of the eigenvalues $m_i, i = 1, 2, 3$ and the matrix elements of M [10].

One obtains

$$\Delta(V) \equiv |V_{12}|^2 - |V_{21}|^2 = |V_{23}|^2 - |V_{32}|^2 = |V_{31}|^2 - |V_{13}|^2 = \frac{1}{D(m)} \left\{ \sum_k (m_k (M^2)_{kk} - m_k^2 M_{kk}) \right\}, \quad (3)$$

where

$$D(m) \equiv \begin{vmatrix} 1 & 1 & 1 \\ m_1 & m_2 & m_3 \\ m_1^2 & m_2^2 & m_3^2 \end{vmatrix} = (m_2 - m_1)(m_3 - m_1)(m_3 - m_2) \quad (4)$$

and

$$J(V) \equiv Im(V_{11} V_{12}^* V_{21}^* V_{22}) = \frac{Im(M_{12} M_{23} M_{13}^*)}{D(m)}. \quad (5)$$

Also, in terms of M and its eigenvalues,

$$|V_{k\alpha}|^2 = (N_\alpha)_{kk}, \quad (6)$$

where

$$N_\alpha = \frac{(m_\beta - M)(m_\gamma - M)}{(m_\beta - m_\alpha)(m_\gamma - m_\alpha)}, \qquad \alpha \neq \beta \neq \gamma, \quad (7)$$

with α, β, γ taking values from 1 to 3. Through this equation each $|V_{k\alpha}|$ can be calculated in terms of the eigenvalues (assuming non-degenerate eigenvalues which is true for the quarks) and matrix elements of M. Then, $|V_{k\alpha}|$ so calculated will automatically satisfy the unitarity relations $\sum_k |V_{k\alpha}|^2 = 1 = \sum_\alpha |V_{\alpha k}|^2$. Thus, the calculated $\Delta(V)$ will be unique.

The generalizations of Eq. (5) for the case of two mass matrices M_u and M_d for up and down quarks is [9]

$$Det([M_u, M_d]) = 2i D_u(m) D_d(m) J(V), \quad (8)$$

where $M_u = V_u \hat{M} V_u^\dagger$, $M_d = V_d \hat{M} V_d^\dagger$ and $V = V_u^\dagger U_d$, $D_u(m)$ and $D_d(m)$ correspond to $D(m)$ for up and down quarks. This reduces to our Eq. (5) in the basis in which M_d is diagonal (viz. $V_d = I$) with $V = V_u^\dagger$. Alternatively, when M_u is diagonal, one obtains the equivalent of Eq. (5) for M_d with $V = V_d$.

Equations (3) and (5) provide a simple criterion for selecting suitable mass matrices. In particular, the latter is remarkable in that it shows that if $M_{12}M_{23}M_{13}^*$ is real for a given M, then the Jarlskog invariant for the matrix V which diagonalizes it vanishes.

3 Choice of the mass matrices

The mixing matrix V would be completely determined if one knew M_u and M_d. In practice, however, the mass matrices are guessed at while experiment can only determine the moduli of the CKM matrix elements. The choice of textures (zero matrix elements in the mass matrices) can have an important effect on the properties of V. This problems has been extensively studied for a single mass matrix, which is hermitian or complex symmetric [11, 12].

As an example consider hermitian M with only one off-diagonal element zero (1 texture). In this case, since $M = V\hat{M}V^\dagger$, one has (i $\neq j$)

$$M_{ij} = m_1 V_{i1} V_{j1}^* + m_2 V_{i2} V_{j2}^* + m_3 V_{i3} V_{j3}^* = 0. \tag{9}$$

Multiplying successively by $V_{i3}^* V_{j3}$, $V_{i2}^* V_{j2}$, $V_{i1}^* V_{j1}$ and taking imaginary parts, one obtains $(m_1 - m_2)J(V) = 0$, $(m_1 - m_3)J(V) = 0$ and $(m_2 - m_3)J(V) = 0$. Thus, either M is trivial or $J(V) = 0$! This result is a particular case of Eq(5) above.

The case of 3 textures is mathematically interesting. In this case 3 <u>different</u> matrix elements of M are zero implying the vanishing of 3 linear homogeneous equations in m_i ($i = 1, 2, 3$) whose coefficients are given in terms of the matrix elements V_{ij} of V. So, for a non-trivial solution for m_i a 3x3 determinant formed out of the V_{ij} must vanish. Thus, imposing a constraint on the 4 parameters in V.

The case when the 3 diagonal elements vanish, that is $M_{ii} = 0$, $i = 1, 2, 3$ has been considered [11, 12] in the neutrino sector when M is hermitian or complex symmetric. For the hermitian case, the condition is that the 3x3 determinant

$$D_h \equiv det(|V_{ij}|^2) = 0. \tag{10}$$

Using the standard parametrization [13], this reduces to

$$2cos(2\theta_{12})cos(2\theta_{23})cos(2\theta_{13}) = [3cos^2(\theta_{13}) - 2]sin(2\theta_{12})sin(2\theta_{23})sin(\theta_{13})cos(\delta_{13}). \tag{11}$$

So, V has only three independent parameters and remarkably the CP-violating phase is determined in therms of the mixing angles.

For a complex symmetric M, it's diagonal form $\hat{M} = \tilde{V}MV$, where V is unitary. In this case, $M_{ii} = 0$ ($i = 1, 2, 3$) imply the determinant

$$D_c \equiv det(V_{ij}^2) = 0. \tag{12}$$

A little algebra shows that the complex number $D_c = D_h + iJ(V)$! Thus, Eq (12) implies that in addition to $D_h = 0$ one has

$$J(V) = \frac{1}{8} cos(\theta_{13}) sin(2\theta_{12}) sin(2\theta_{23}) sin(2\theta_{13}) sin(\delta) = 0 \tag{13}$$

Simultaneous satisfaction of the constraints in Eqs(12) and Eqs(13) in the neutrino sector is discussed elsewhere [12].

4 Confrontation of data with different types of mass matrices

Recently, constraints on the possible structure of quark mass matrices were obtained [14, 15] using, as restrictions the experimentally determined values of the six quark masses and the magnitudes of quark mixing matrix elements (V_{ud}, V_{us}, V_{cd}, V_{cs}) and $J(V)$.

We considered three types of mass matrices. In each case, one expressed the parameters (matrix elements) in the mass matrices in terms of the eigenvalues (the quark masses) using the characteristic equations. For $M_q = V_q^\dagger \hat{M}_q V_q$ $(q = u, d)$, the eigenvalues are denoted by (λ_u, λ_c, λ_t) and (λ_d, λ_s, λ_b) for the up and down quark mass matrices respectively. Note that these eigenvalues are real but not necessarily positive. In terms of projectors, one has

$$M_u = \sum_{\alpha=u,c,t} \lambda_\alpha N_\alpha \qquad and \qquad M_d = \sum_{j=d,s,b} \lambda_j N_j. \tag{14}$$

Since $V = V_u V_d^\dagger$, it follows [16],

$$|V_{\alpha j}|^2 = Tr[N_\alpha N_j], \tag{15}$$

where projectors N_α and N_j are

$$N_\alpha = \frac{(\lambda_\beta - M_u)(\lambda_\gamma - M_u)}{(\lambda_\beta - \lambda_\alpha)(\lambda_\gamma - \lambda_\alpha)}, \tag{16}$$

and

$$N_j = \frac{(\lambda_k - M_d)(\lambda_l - M_d)}{(\lambda_k - \lambda_j)(\lambda_l - \lambda_j)}, \tag{17}$$

with (α, β, γ) and (j, k, l) any permutation of (u, c, t) and (d, s, b).

For a given choice of the mass matrices, we first determine the elements of the quark mass matrices in terms of the eigenvalues and then form a χ^2-function which contains eleven summands. The first five compare the theoretical expressions as functions of the elements of the quark mass matrices of the four best measured moduli ($|V_{ud}|$, $|V_{us}|$, $|V_{cd}|$, $|V_{cs}|$) and $J(V)$ with their experimental values [13]. The last six summands constrain the eigenvalues of quark mass matrices to the experimentally deduced quark masses at a specified energy scale. The masses we used are given in Table I for easy reference [13, 17].

A) Fritzsch type mass matrices [11] are given by the hermitian matrices (with 3 textures)

$$M_u = \begin{pmatrix} 0 & A & 0 \\ A^* & 0 & B \\ 0 & B^* & C \end{pmatrix}, \qquad M_d = \begin{pmatrix} 0 & A' & 0 \\ A'^* & 0 & B' \\ 0 & B'^* & C' \end{pmatrix}. \tag{18}$$

Without lack of generality we can take C and C' to be positive and A and B to be real and positive. Then, M_u and M_d have eight real parameters A, B, C, C', $|A'|$, $|B'|$ and the phases $\phi_{A'}$ and $\phi_{B'}$. For CP-violation, one needs only one phase to be non-zero.

This type of of mass matrices, can only be used in the physical basis because an off-diagonal matrix element is zero. This is a shortcoming in my view.

B) Stech type of mass matrices are given by

$$M_u = \begin{pmatrix} \lambda_u & 0 & 0 \\ 0 & \lambda_c & 0 \\ 0 & 0 & \lambda_t \end{pmatrix}, \quad M_d = pM_u + i \begin{pmatrix} 0 & a & d \\ -a & 0 & b \\ -d & -b & 0 \end{pmatrix}. \tag{19}$$

The Stech model has only seven real parameters, however this also can used only in the up-quark diagonal basis. This is a shortcoming in my view.

C) CGS-type hermitian mass matrix was considered first in the references [14] and [15] last year. The CGS-type mass matrix is given by

$$M = \begin{pmatrix} 0 & a & d \\ a^* & 0 & b \\ d^* & b^* & c \end{pmatrix}. \tag{20}$$

For $d = 0$ this reduces to the Fritzsch type. This mass matrix has the virtue that it can be used in all the three bases. To reduce the number of parameters to four we can take a, b, and c to be real and with d as pure imaginary. Thus, in the up (down) quark diagonal basis $M = M_d$ (M_u) there are seven parameters, 4 in M and 3 masses in the diagonal mass matrix. There are eight constraints the other 3 masses and five quantities from the mixing matrix viz, $|V_{ud}|$ etc.

D) CGS - type mass matrix in the physical basis. We take,

$$M_u = \begin{pmatrix} 0 & a & 0 \\ a & 0 & b \\ 0 & b & c \end{pmatrix}, \qquad M_d = \begin{pmatrix} 0 & a' & i|d'| \\ a' & 0 & b' \\ -i|d'| & b' & c' \end{pmatrix}. \tag{21}$$

All the matrix elements are taken to be real and positive, so we have 7 real parameters. This is a small (but important) variation of Eq(18) above. Here M_u is Fritzsch-type while M_d is CGS-type. The fit for the mixed case, namely M_u CGS-type and M_d Fritzsch-type (with 7 real parameters) gives the same χ^2/dof as for the fit for M_u and M_d in Eq(21)! The reason is that there exists an unitary matrix Y such that it can rotate M_u and M_d in Eq(21) to mass matrices $M'_u = Y^\dagger M_u Y$ and $M'_d = Y^\dagger M_d Y$ which are CGS-type and Fritzsch-type respectively. Explicitly, the unitary matrix

$$Y = \frac{1}{b}\begin{pmatrix} \beta & \delta & 0 \\ \delta & \beta & 0 \\ 0 & 0 & b \end{pmatrix}, \tag{22}$$

with $b^2 = \beta^2 + |\delta|^2$ and $\delta = i\eta_\delta|\delta|$ ($\eta_\delta = \pm 1$) is pure imaginary. This gives

$$M'_u = \begin{pmatrix} 0 & a & \delta^* \\ a & 0 & \beta \\ \delta & \beta & c \end{pmatrix}, \qquad M'_d = \begin{pmatrix} 0 & a' & \delta' \\ a' & 0 & \beta' \\ -\delta' & \beta' & c' \end{pmatrix}. \tag{23}$$

The conditions which make Y a unitary matrix ensures M'_u and M_u have the same eigenvalues. Further, in M'_d, $b\beta' = d'\delta^* + \beta b'$ and $b\delta' = \beta d' + b'\delta^*$. Since d' and δ are pure imaginary so is δ' while β' is real. Thus, both M'_u and M'_d are of the CGS-type. By requiring, $\delta' = 0$ we can make M'_d to be F-type. Thus, it is sufficient to consider the case M_u F-type and M_d CGS-type (see Eq(21)) for these fits.

5 Results and conclusions

The results for the varions fits for the varions cases are summarized in Tables I, II, III. For details references CGS-1 and CGS-2 may be consulted.

As can be seen, the fits for Fritzsch (8 parameters) and Stech (7 parameters) types are comparable. However, fixing $\phi_{A'} = -\pi/2$ and $\phi_{B'} = 0$ (as suggested by the free fits) in the Fritzsch case gives a better smaller χ^2/dof. However, the best fits are obtained using a CGS-type matrix (as in §4D).

The stability of this type of analysis with respect to evolution of the quark masses is important. As can be seen the results for χ^2/dof for varions cases at 2 GeV scale (Table II) are very similar to results at M_Z scale in Table III.

A simple way to understand this is to note that if all quark masses are scaled by a common factor then the algebraic expressions for the dimensionless numbers $J(V)$ and the moduli $|V_{\alpha j}|$ ($\alpha = u$, c, $j = d$, s) will be unaffected.

As can be seen from Table I, the ratio $m_q(2GeV)/m_q(M_z) = 1.71 - 1.74$ for $q = u$, d, s, c, b while it is 1.85 for $q = t$.

Table IV gives the results for quark masses at different scales given in row 3 of Table I. Again, the results are similar and choice of mass matrices in §4D is favored Note that masses of the heavier quarks (notable m_t) are very different. This would suggest that the role of the small masses (since they evolve slowly) is possibly more important.

In conclusion, we advocate the use of mass matrices which can be used in all the three bases.

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Energy scale \ Quark flavor	u	d	s	c	b	t
2 GeV	$2.2^{+0.8}_{-0.7}$	5.0±2.0	95 ±25	1.07 ±0.12	$5.04^{+0.16}_{-0.15}$	$318.9^{+13.1}_{-12.3}$
$M_Z = 91.1876 GeV$	$1.28^{+0.50}_{-0.43}$	$2.91^{+1.24}_{-1.20}$	55^{+16}_{-15}	0.624 ±0.083	2.89 ±0.09	172.5 ±3.0
Various	2.25 ±0.75	5.0 ±2.0	95 ±25	1.25 ±0.09	4.20 ±0.07	174.2 ±3.3

Table I: Quark masses at various energy scales. Those in rows 1 and 2 are the evolved masses taken from reference [17]. Row 3 gives masses given in [13]. The m_u, m_d and m_s are at 2 GeV, m_c at m_c, m_b at m_b and m_t by direct observation. Note that u. d and s masses are in MeV while those of c, b and t are in GeV.

Type of mass matrix	Basis	Number of parameters	$\chi^2/(dof)$
Fritzsch	Physical ($\phi_{A'}$ and $\phi_{B'}$ free)	8	$4.23/3 = 1.41$
	Physical ($\phi_{A'} = -\pi/2$ and $\phi_{B'} = 0$)	6	$4.84/5 = 0.97$
Stech	M_u diagonal	7	$9.10/4 = 2.28$
CGS	M_d diagonal	7	$5.92/4 = 1.48$
	M_u diagonal	7	$15.50/4 = 3.88$
M_u Fritzsch-type and M_d CGS-type	Physical	7	$1.89/4 = 0.47$

Table II: Fits for all quark masses at energy scale M_Z. See CGS-1 for details.

Type of mass matrix	Basis	Number of parameters	$\chi^2/(dof)$
Fritzsch	Physical ($\phi_{A'}$ and $\phi_{B'}$ free)	8	$4.80/3 = 1.60$
	Physical ($\phi_{A'} = -\pi/2$ and $\phi_{B'} = 0$)	6	$5.49/5 = 1.10$
Stech	M_u diagonal	7	$11.00/4 = 2.75$
CGS	M_d diagonal	7	$5.39/4 = 1.35$
	M_u diagonal	7	$17.99/4 = 4.50$
M_u Fritzsch-type and M_d CGS-type	Physical	7	$2.47/4 = 0.62$

Table III: Fits for all quark masses at energy scale 2 GeV. See CGS-1 for details.

Type of mass matrix	Basis	Number of parameters	$\chi^2/(dof)$
Fritzsch	Physical ($\phi_{A'}$ and $\phi_{B'}$ free)	8	$3.32/3 = 1.11$
	Physical ($\phi_{A'} = -\pi/2$ and $\phi_{B'} = 0$)	6	$4.27/5 = 0.85$
Stech	M_u diagonal	7	$6.65/4 = 1.66$
CGS	M_d diagonal	7	$53.3/4 = 13.3$
	M_u diagonal	7	$17.19/4 = 4.30$
M_u Fritzsch-type and M_d CGS-type	Physical	7	$0.80/4 = 0.20$

Table IV: Fits for all quark masses given at various energy scales given in row 3 of Table I. See CGS-2 for details.