Neutrino Mass Seesaw Version 3: Recent Developments

Ernest Ma

Physics and Astronomy Department, University of California, Riverside, California 92521, USA

Abstract. The origin of neutrino mass is usually attributed to a seesaw mechanism, either through a heavy Majorana fermion singlet (version 1) or a heavy scalar triplet (version 2). Recently, the idea of using a heavy Majorana fermion triplet (version 3) has gained some attention. This is a review of the basic idea involved, its U(1) gauge extension, and some recent developments.

Keywords: Neutrino Mass, Type III Seesaw, U(1) Gauge Extension
PACS: 14.60.Pq, 14.60.St, 12.60.Cn

INTRODUCTION

In the minimal standard model (SM) of quarks and leptons, the neutrinos $\nu_{e,\mu,\tau}$ are very different from other fermions because they need only exist as the neutral components of the electroweak doublets $L_\alpha = (\nu_\alpha, l_\alpha)$. As such, they are massless two-component spinors and may become massive only if there is new physics beyond the SM. Assuming only the low-energy particle content of the SM, it was pointed out long ago [1] that small Majorana neutrino masses are given by the unique dimension-five operator

$$\mathscr{L}_5 = \frac{f_{\alpha\beta}}{2\Lambda}(\nu_\alpha \phi^0 - l_\alpha \phi^+)(\nu_\beta \phi^0 - l_\beta \phi^+), \tag{1}$$

where $\Phi = (\phi^+, \phi^0)$ is the one Higgs scalar doublet of the SM. The neutrino mass matrix is thus necessarily seesaw in form, i.e. $f_{\alpha\beta} v^2/\Lambda$, where v is the vacuum expectation value of ϕ^0 which breaks the electroweak $SU(2) \times U(1)$ gauge symmetry. It was also pointed out some years ago [2] that there are three (and only three) tree-level realizations of this operator (Fig. 1), as well as three generic one-loop realizations. The most

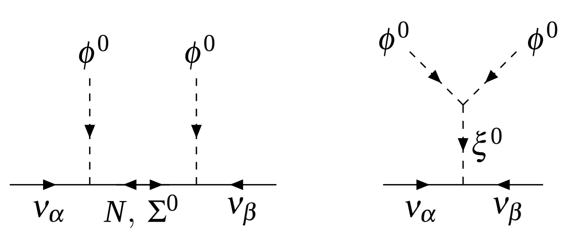

FIGURE 1. Three tree-level realizations of seesaw Majorana neutrino mass.

common thinking regarding the seesaw origin of neutrino mass is to assume a heavy Majorana fermion singlet N (version 1), the next most common is to use a heavy scalar triplet (ξ^{++}, ξ^+, ξ^0) (version 2), whereas the third option, i.e. that of a heavy Majorana fermion triplet $(\Sigma^+, \Sigma^0, \Sigma^-)$ [3] (version 3), has not received as much attention. However, it may be relevant to a host of other issues in physics beyond the SM and is now being studied extensively. I will review in this talk a number of such topics, including gauge-coupling unification in the SM, new U(1) gauge symmetry, and dark matter.

GAUGE-COUPLING UNIFICATION

It is well-known that gauge-coupling unification occurs for the minimal supersymmetric standard model (MSSM) but not the SM. The difference can be traced to the addition of gauginos and higgsinos, transforming under $SU(3)_C \times SU(2)_L \times U(1)_Y$ as (8,1,0), (1,3,0), $(1,2,\pm 1/2)$, and a second Higgs scalar doublet. In particular, the contribution of the $SU(2)_L$ gaugino triplet is crucial in allowing the $SU(2)_L$ and $U(1)_Y$ gauge couplings to meet at high enough an energy scale to be acceptable for suppressing proton decay. Since Σ is exactly such a fermion triplet, it is not surprising that gauge-coupling unification in the SM may be achieved using it [4, 5, 6, 7] together with some other fields.

To understand how this works, consider the one-loop renormalization-group equations governing the evolution of the three gauge couplings with mass scale:

$$\frac{1}{\alpha_i(M_1)} - \frac{1}{\alpha_i(M_2)} = \frac{b_i}{2\pi} \ln \frac{M_2}{M_1}, \tag{2}$$

where $\alpha_i = g_i^2/4\pi$, and the numbers b_i are determined by the particle content of the model between M_1 and M_2. Since

$$\alpha_C(M_U) = \alpha_L(M_U) = (5/3)\alpha_Y(M_U) = \alpha_U \tag{3}$$

is required for unification, but not the actual numerical value of α_U, only $b_Y - b_L$ and $b_L - b_C$ are important for this purpose. These numbers are listed below for the SM, MSSM, and some other models. Focus only on those new particles which transform

TABLE 1. Gauge-coupling unification in the MSSM and other models.

Model	$b_Y - b_L$	$b_L - b_C$	new fermions	new scalars
SM	7.27	3.83	none	none
MSSM	5.60	4.00	(1,3,0), (8,1,0), (1,2,$\pm 1/2$)	(1,2,1/2)
Ref. [4]	5.27	3.83	(1,3,0)	(1,3,0) $\times$ 2, (8,1,0) $\times$ 4
Ref. [5, 6]	5.60	3.00	(1,3,0), (8,1,0)	(1,3,0), (8,1,0)
Ref. [7]	5.87	4.33	(1,3,0)	(1,2,1/2), (8,1,0) $\times$ 2

nontrivially under $SU(2)_L \times U(1)_Y$. Let them be at the electroweak scale, then

$$\ln \frac{M_U}{M_Z} \simeq \frac{\sqrt{2}\pi^2}{(b_Y - b_L) G_F M_W^2} \left(\frac{3}{5 \tan^2 \theta_W} - 1 \right). \tag{4}$$

Hence M_U greater than about 10^{16} GeV implies $b_Y - b_L$ less than about 5.7. In Refs. [5, 6], an intermediate scale of about 10^8 GeV is needed for the color octets.

PHENOMENOLOGY OF $(\Sigma^+, \Sigma^0, \Sigma^-)$

If Σ exists at or below the TeV scale, then it has a rich phenomenology [3, 8, 9, 10] and may be probed at the Large Hadron Collider (LHC). Unless there is a Higgs scalar triplet (s^+, s^0, s^-) [4], the mass splitting between Σ^0 and $\Sigma^\pm$ is radiative and comes from electroweak gauge interactions. It is positive and for large m_Σ, it approaches [11] $G_F M_W^3 (1 - \cos\theta_W)/\sqrt{2}\pi \simeq 168$ MeV, thus allowing the decay of $\Sigma^\pm$ to $\Sigma^0 \pi^\pm$ and $\Sigma^0 l^\pm \nu$. Since Σ also has Yukawa couplings to (ν_α, l_α) and (ϕ^+, ϕ^0), the decays $\Sigma^\pm \to l^\pm h$, $\Sigma^0 \to \nu h$ are possible, as well as $\Sigma^\pm \to l^\pm Z$, $\nu W^\pm$ and $\Sigma^0 \to \nu Z$, $l^\pm W^\mp$ through the mixing of Σ^0 with ν, and $\Sigma^\pm$ with $l^\pm$, unless they are forbidden by a symmetry, in which case Σ^0 is a dark-matter (DM) candidate [4, 11, 12].

The production of Σ is by pairs from quark fusion through the electroweak gauge bosons with a cross section of the order 1 fb for m_Σ of about 1 TeV, and rising to more than 10^2 fb if m_Σ is 300 GeV. Each decay mode of Σ has a huge SM background to contend with. The best chance of digging out the signal is to look for charged-lepton final states. Copying Ref. [10], the prognosis at the LHC for the 5σ discovery of the particles responsible for the three versions of the seesaw mechanism is shown below. A dash means no such state. A cross means no such signal.

TABLE 2. Discovery potential at the LHC for seesaw 1,2,3.

final state	$m_N = 100$ GeV	$m_\xi = 300$ GeV	$m_\Sigma = 300$ GeV
6 leptons	–	–	$\times$
5 leptons	–	–	28 fb^{-1}
$l^\pm l^\pm l^\pm l^\mp$	–	–	15 fb^{-1}
$l^+ l^+ l^- l^-$	–	19 fb^{-1}	7 fb^{-1}
$l^\pm l^\pm l^\pm$	–	–	30 fb^{-1}
$l^\pm l^\pm l^\mp$	<180 fb^{-1}	3.6 fb^{-1}	2.5 fb^{-1}
$l^\pm l^\pm$	<180 fb^{-1}	17.4 fb^{-1}	1.7 fb^{-1}
$l^+ l^-$	$\times$	15 fb^{-1}	80 fb^{-1}
$l^\pm$	$\times$	$\times$	$\times$

LEPTOGENESIS INVOLVING $(\Sigma^+, \Sigma^0, \Sigma^-)$

Jsut as there are three seesaw mechanisms, the decays of the corresponding heavy particles N [13], (ξ^{++}, ξ^+, ξ^0) [14], and $(\Sigma^+, \Sigma^0, \Sigma^-)$ [12] are natural for generating a lepton asymmetry of the Universe, which gets converted [15] into the present observed baryon asymmetry through sphalerons. Just as N may decay into leptons and antileptons because it is a Majorana fermion, the same is true for Σ. Assuming three such triplets, successful leptogenesis requires [12] the lightest to be heavier than about 10^{10} GeV, similar to that for the lightest N. However, since Σ has electroweak gauge interactions, the initial conditions for the Boltzmann equations are determined here through thermal equilibrium, which may not be as simple for N.

There is another interesting correlation. The addition of three $(1,3,0)$ fermion triplets to the SM instead of just one will not lead to gauge-coupling unification unless all three are also roughly at the 10^{10} GeV scale [12]. Whereas other fields are still needed, such as those transforming under $(8,1,0)$, this is another argument for preferring Σ over N.

NEW U(1) GAUGE SYMMETRY

Consider an extension of the SM to include a fermion triplet $(\Sigma^+,\Sigma^0,\Sigma^-)$ per family as well as a new $U(1)_X$ gauge symmetry as listed below. Remarkably [16, 17, 18], $U(1)_X$

TABLE 3. Fermion content of proposed model.

Fermion	$SU(3)_C \times SU(2)_L \times U(1)_Y$	$U(1)_X$
$(u,d)_L$	$(3,2,1/6)$	n_1
u_R	$(3,1,2/3)$	$n_2 = (7n_1 - 3n_4)/4$
d_R	$(3,1,-1/3)$	$n_3 = (n_1 + 3n_4)/4$
$(\nu,e)_L$	$(1,2,-1/2)$	$n_4 \neq -3n_1$
e_R	$(1,1,-1)$	$n_5 = (-9n_1 + 5n_4)/4$
$(\Sigma^+,\Sigma^0,\Sigma^-)_R$	$(1,3,0)$	$n_6 = (3n_1 + n_4)/4$

is free of all anomalies. For example, one can easily check that

$$6n_1^3 - 3n_2^3 - 3n_3^3 + 2n_4^3 - n_5^3 = 3(3n_1+n_4)^3/64 = 3n_6^3. \tag{5}$$

Furthermore, it has been shown [17] that if a fermion multiplet $(1,2p+1,0;n_6)$ per family is added to the SM, the only anomaly-free solutions for $U(1)_X$ are $p=0$ (N) for which the well-known $U(1)_{B-L}$ is obtained, and $p=1$ (Σ) as given above.

The new gauge boson X may be accessible at the LHC. In that case, its decay into quarks and leptons will determine the parameter $r = n_4/n_1$. In particular, the ratios

$$\frac{\Gamma(X \to t\bar{t})}{\Gamma(X \to \mu\bar{\mu})} = \frac{3(65-42r+9r^2)}{81-90r+41r^2}, \quad \frac{\Gamma(X \to b\bar{b})}{\Gamma(X \to \mu\bar{\mu})} = \frac{3(17+6r+9r^2)}{81-90r+41r^2}, \tag{6}$$

are especially good discriminators [19], as shown in Fig. 2 [20].

The scalar sector of this $U(1)_X$ model consists of two Higgs doublets $\Phi_1 = (\phi_1^+,\phi_1^0)$ with charge $(9n_1-n_4)/4$ which couples to charged leptons, and $\Phi_2 = (\phi_2^+,\phi_2^0)$ with charge $(3n_1-3n_4)/4$ which couples to *up* and *down* quarks as well as to Σ. To break the $U(1)_X$ gauge symmetry spontaneously, a singlet χ with charge $-2n_6$ is added, which also allows the Σ's to acquire Majorana masses at the $U(1)_X$ breaking scale. This specific two-Higgs doublet model is different from conventional studies where one doublet couples to *up* quarks and the other to *down* quarks and charged leptons. The resulting detailed differences are verifiable at the LHC.

In general, there is $Z-X$ mixing in their mass matrix, but it must be very small to satisfy present precision electroweak measurements. The condition for zero $Z-X$ mass mixing is $v_1^2/v_2^2 = 3(n_4-n_1)/(9n_1-n_4)$, which requires $1 < n_4/n_1 < 9$. Low-energy precision measurements of SM physics also constrain the contributions of this $U(1)_X$. Let $n_1^2+n_4^2$ be normalized to one, and $\tan\phi = n_4/n_1$, then the 95% confidence-level

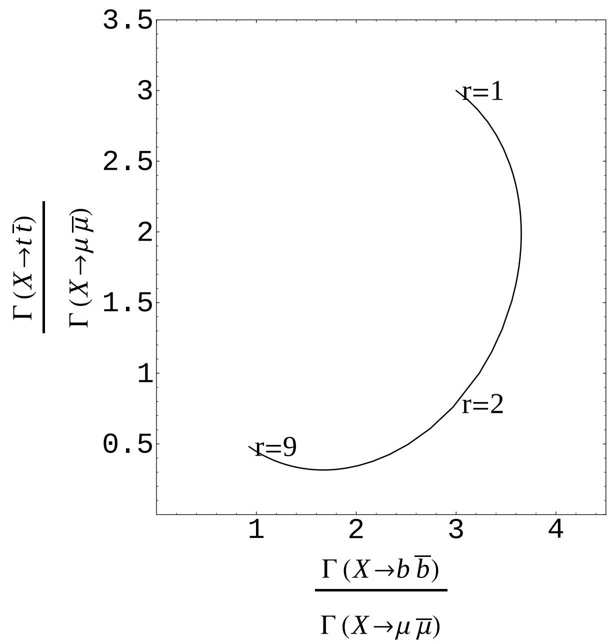

FIGURE 2. Plot of $\Gamma(X \to t\bar{t})/\Gamma(X \to \mu\bar{\mu})$ versus $\Gamma(X \to b\bar{b})/\Gamma(X \to \mu\bar{\mu})$.

lower bound on M_X/g_X is shown in Fig. 3 [20], assuming zero $Z - X$ mixing so that there is no constraint coming from measurements at the Z resonance. Thus only the range $1 < r < 9$, i.e. $\pi/4 < \phi < 1.46$ is actually allowed.

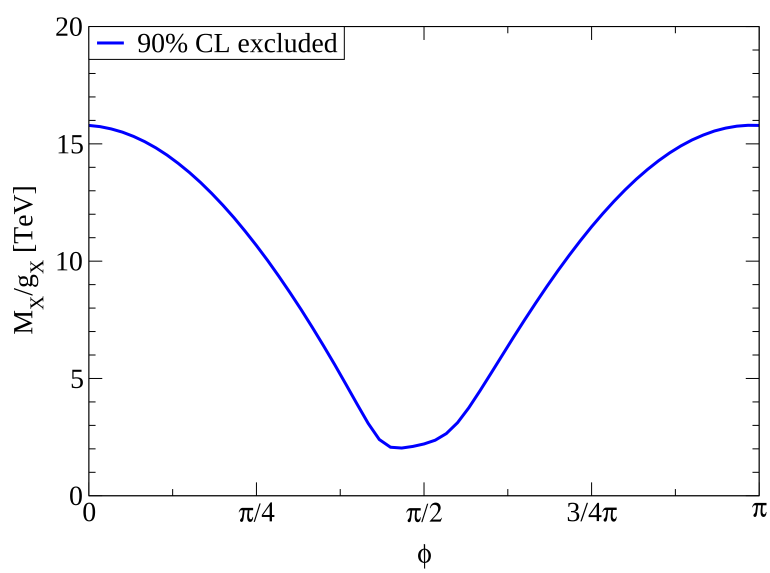

FIGURE 3. Lower bound on M_X/g_X versus ϕ.

SCOTOGENIC RADIATIVE NEUTRINO MASS

There are also three generic one-loop radiative mechanisms [2] for neutrino mass. An intriguing possibility is that the particles in the loop are distinguished from those of the SM by a Z_2 discrete symmetry. The simplest realization [21] is to add a second scalar doublet (η^+, η^0) [22] as well as three fermion singlets N, and let them be odd under Z_2 with all SM particles even. Clearly, Σ may be chosen [7] instead of N and a radiative seesaw neutrino mass is generated as shown in Fig. 4. The allowed quartic scalar term

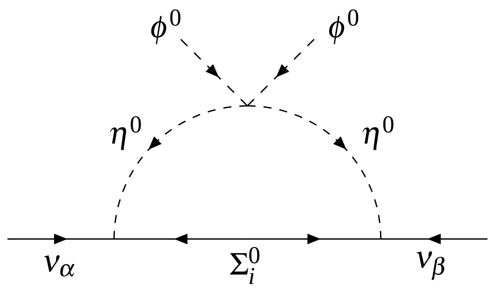

FIGURE 4. One-loop generation of seesaw neutrino mass.

$(\lambda_5/2)(\Phi^\dagger\eta)^2 + H.c.$ is necessary for this mechanism to work. It also splits the complex scalar field η^0 into two mass eigenstates: Re(η^0) and Im(η^0), resulting in

$$(\mathscr{M}_\nu)_{\alpha\beta} = \sum_i \frac{h_{\alpha i} h_{\beta i} M_i}{16\pi^2} \left[\frac{m_R^2}{m_R^2 - M_i^2} \ln \frac{m_R^2}{M_i^2} - \frac{m_I^2}{m_I^2 - M_i^2} \ln \frac{m_I^2}{M_i^2} \right], \tag{7}$$

where $m_R^2 - m_I^2 = 2\lambda_5 v^2$ and M_i are the Σ masses. The lighter one of Re(η^0) and Im(η^0) is then a good candidate [23, 24, 25, 26] for dark matter (DM). Neutrino mass may then be called scotogenic, i.e. being caused by darkness [27].

Σ^0 AS DARK MATTER

In Ref. [21], the lightest N may also be a DM candidate [28, 29], but then its only interaction is with $(\nu_\alpha \eta^0 - l_\alpha \eta^+)$ and these couplings have to be rather large to obtain the requisite DM relic abundance. In that case, flavor-changing radiative decays such as $\mu \to e\gamma$ are generically too big and require delicate fine tuning among the masses and couplings of N to be consistent with data.

If Σ^0 is selected as dark matter, then it can annihilate with itself and coannihilate with the slightly heavier $\Sigma^\pm$ through electroweak gauge interactions to account for the correct relic abundance. Its Yukawa couplings may then be appropriately small, not to upset the constraints from $\mu \to e\gamma$, etc. Using the method developed in Ref. [30] to take coannihilation into account, and the various cross sections times the absolute value of the relative velocity of the DM particles, namely

$$\sigma(\Sigma^0\Sigma^0)|v| \simeq \frac{2\pi\alpha_L^2}{m_\Sigma^2}, \quad \sigma(\Sigma^\pm\Sigma^\pm)|v| \simeq \frac{\pi\alpha_L^2}{m_\Sigma^2}, \tag{8}$$

$$\sigma(\Sigma^+\Sigma^-)|v| \simeq \frac{37\pi\alpha_L^2}{m_\Sigma^2}, \quad \sigma(\Sigma^0\Sigma^\pm)|v| \simeq \frac{29\pi\alpha_L^2}{m_\Sigma^2}, \tag{9}$$

m_Σ is estimated [7] to be in the range 2.28 to 2.42 TeV to reproduce the observed data $\Omega h^2 = 0.11 \pm 0.006$ [31] for its relic abundance. Note that the presence of $\Sigma^\pm$ is important for having a large enough effective annihilation cross section for this to work and that the only free parameter here is m_Σ. The validity of Σ^0 as dark matter depends only on Z_2 and not on whether it is the source of radiative neutrino mass.

Σ AS LEPTON AND N AS BARYON

Assuming neutrino masses come from Σ, an intriguing possibility exists that the heavy fermion singlet N may in fact be a baryon [32, 33, 34, 35, 36]. The crucial ingredient for this unconventional identification is the existence of a scalar diquark $\tilde{h} \sim (3,1,-1/3)$ with baryon number $B = -2/3$ so that the Yukawa couplings $ud\tilde{h}$, $u^c d^c \tilde{h}^*$, and $d^c N \tilde{h}$ are allowed, thereby making N a baryon $(B = 1)$. Since N is a gauge singlet, it is also allowed a large Majorana mass. Hence additive B breaks to multiplicative $(-)^{3B}$ and the decays of the lightest N to udd and $\bar{u}\bar{d}\bar{d}$ through $\tilde{h}$ would produce a baryon asymmetry in the early Universe. Below the mass scale of m_N, baryon number is again additively conserved, allowing this pure B asymmetry to be converted into a conserved $B - L$ asymmetry through the electroweak sphalerons, in analogy to the well-known scenario of leptogenesis [37].

CONCLUSION

Using the fermion triplet $(\Sigma^+, \Sigma^0, \Sigma^0)$ as the seesaw anchor for neutrino masses (version 3), many new and interesting possibilities of physics beyond the SM exist. It may be the missing link for gauge-coupling unification in the SM without going to the MSSM. As a result, the phenomenological landscape at the TeV scale may change significantly and be verifiable at the LHC, where Σ itself is much easier to detect than its singlet counterpart N. There may also be an associated neutral gauge boson, corresponding to an anomaly-free $U(1)_X$, whose decays into quarks and leptons are predicted as a function of a single parameter $r = n_4/n_1$. Furthermore, Σ may be the source of scotogenic radiative neutrino masses and be a dark-matter candidate itself, with a mass around 2.35 TeV. Other recent discussions of fermion triplets are found in Refs. [38, 39, 40, 41, 42].

ACKNOWLEDGEMENTS

I thank Abdel Perez-Lorenzana and the other organizers of the XIII Mexican School of Particles and Fields for their great hospitality and a stimulating meeting in San Carlos. This work was supported in part by the U. S. Department of Energy under Grant No. DE-FG03-94ER40837.

REFERENCES

1. S. Weinberg, Phys. Rev. Lett. **43**, 1566 (1979).
2. E. Ma, Phys. Rev. Lett. **81**, 1171 (1998).
3. R. Foot, H. Lew, X.-G. He, and G. C. Joshi, Z. Phys. **C44**, 441 (1989).
4. E. Ma, Phys. Lett. **B625**, 76 (2005).
5. B. Bajc and G. Senjanovic, JHEP **0708**, 014 (2007).
6. I. Dorsner and P. Fileviez Peez, JHEP **0706**, 029 (2007).
7. E. Ma and D. Suematsu, arXiv:0809.0942 [hep-ph].
8. B. Bajc, M. Nemevsek, and G. Senjanovic, Phys. Rev. **D76**, 055011 (2007).
9. R. Franceschini, T. Hambye, and A. Strumia, Phys. Rev. **D78**, 033002 (2008).
10. F. del Aguila and J. A. Aguilar-Saavedra, arXiv:0808.2468 [hep-ph].
11. M. Cirelli, N. Fornengo, and A. Strumia, Nucl. Phys. **B753**, 178 (2006).
12. W. Fischler and R. Flauger, JHEP **09**, 020 (2008).
13. M. Fukugita and T. Yanagida, Phys. Lett. **B174**, 45 (1986).
14. E. Ma and U. Sarkar, Phys. Rev. Lett. **80**, 5716 (1998).
15. V. A. Kuzmin, V. A. Rubakov, and M. E. Shaposhnikov, Phys. Lett. **B155**, 36 (1985).
16. E. Ma, Mod. Phys. Lett. **A17**, 535 (2002).
17. E. Ma and D. P. Roy, Nucl. Phys. **B644**, 290 (2002).
18. S. M. Barr and I. Dorsner, Phys. Rev. **D72**, 015011; see also S. M. Barr, B. Bednarz, and C. Benesh, Phys. Rev. **D34**, 235 (1986).
19. S. Godfrey and T. A. W. Martin, Phys. Rev. Lett. **101**, 151803 (2008).
20. R. Adhikari, J. Erler, and E. Ma, arXiv:0810.5547 [hep-ph].
21. E. Ma, Phys. Rev. **D73**, 077301 (2006).
22. N. G. Deshpane and E. Ma, Phys. Rev. **D18**, 2574 (1978).
23. R. Barbieri, L. J. Hall, and V. S. Rychkov, Phys. Rev. **D74**, 015007 (2006).
24. L. Lopez Honorez, E. Nezri, J. F. Oliver, and M. H. G. Tytgat, ICAP **02**, 028 (2007).
25. M, Gustafsson, E. Lundstrom, L. Bergstrom, and J. Edsjo, Phys. Rev. Lett. **99**, 041301 (2007).
26. Q.-H. Cao, E. Ma, and G. Rajasekaran, Phys. Rev. **D76**, 095011 (2007).
27. For a brief review of recent developments, see E. Ma, Mod. Phys. Lett. **A23**, 647 (2008).
28. J. Kubo, E. Ma, and D. Suematsu, Phys. Lett. **B642**, 18 (2006).
29. D. Aristizabal Sierra, J. Kubo, D. Restrepo, D. Suematsu, and O. Zapata, arXiv:0808.3340 [hep-ph].
30. K. Griest and D. Seckel, Phys. Rev. **D43**, 3191 (1991).
31. WMAP Collaboration, D. N. Spergel *et al.*, Astrophys. J. Suppl. **148**, 175 (2003); SDSS Collaboration, M. Tegmark *et al.*, Phys. Rev. **D69**, 103501 (2004).
32. E. Ma, Phys. Rev. Lett. **60**, 1363 (1988).
33. E. Ma and M. Raidal, J. Phys. **G28**, 95 (2002).
34. E. Ma, Phys. Lett. **B661**, 273 (2008).
35. E. Ma, Phys. Rev. **D78**, 047701 (2008).
36. E. Ma, arXiv:0806.4386 [hep-ph].
37. For a review, see for example S. Davidson, E. Nardi, and Y. Nir, Phys. Rept. **466**, 105 (2008).
38. P. Fileviez Perez, Phys. Rev. **D76**, 071701 (2007).
39. A. Abada, C. Biggio, F. Bonnet, M. B. Gavela, and T. Hambye, Phys. Rev. **D78**, 033007 (2008).
40. E. Ma, Phys. Rev. **D78**, 017701 (2008).
41. R. N. Mohapatra, N. Okada, and H.-B. Yu, Phys. Rev. **D78**, 075011 (2008).
42. P. Fileviez Perez, arXiv:0809.1202 [hep-ph].