Hyperon Semileptonic Decays: Some Theoretical Issues

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Background

- Cabibbo[1] proposed a model for weak hadronic currents based on SU(3) symmetry: it led to detailed predictions for hyperon semileptonic decays (HSD).

  The Lorentz structure of the current is $V - A$, and θ_c —the Cabibbo angle— is a parameter to be determined from experimental data.

- Kobayashi and Maskawa[2] generalized Cabibbo universality to three generations of quarks, which could accommodate CP violation.

The matrix V is known as Cabibbo-Kobayashi-Maskawa (CKM) matrix and

$$V_{ud} \approx \cos\theta_c, \qquad V_{us} \approx \sin\theta_c.$$

[1] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963)

[2] M. Kobayashi, T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973)

- Review of Particle Physics, 1996[3]

Analysis of K_{e3} decays yields

$$|V_{us}| = 0.2196 \pm 0.0023$$

... The analysis of hyperon decay data has larger theoretical uncertainties because of first order SU(3) symmetry breaking effects in the axial-vector couplings... but due account of symmetry breaking... gives the corrected value of 0.222 ± 0.003. We average these results to obtain

$$|V_{us}| = 0.2205 \pm 0.0018$$

- Review of Particle Physics, 2008[4]

$|V_{us}|$ may be determined from kaon decays, hyperon decays, and tau decays. Previous determinations have most often used $K\ell 3$ decays... ...with the Leutwyler-Roos calculation of $f_+(0)$ gives

$$|V_{us}| = \lambda = 0.2255 \pm 0.0019. \qquad (11)$$

It should be mentioned that hyperon semileptonic decay fits suggest [5]

$$|V_{us}| = 0.2250(27) \quad \text{Hyperon decays} \qquad (16)$$

modulo SU(3) breaking effects that could shift that value up or down... Similarly, strangeness changing tau decays gives [45]

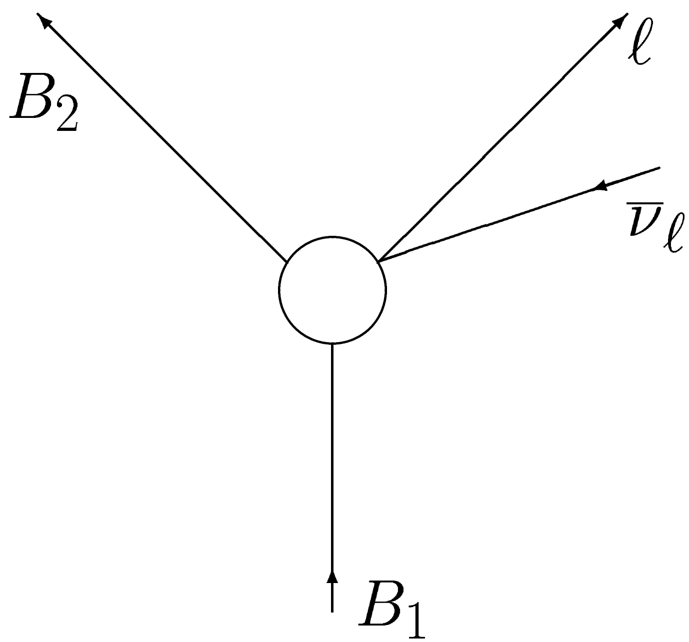
$$|V_{us}| = 0.2208(34) \quad \text{Tau decays} \qquad (17)$$

where the central value depends on the strange quarks mass.

[3] R.M. Barnett *et al.*, Phys. Rev. D **54**, 1 (1996)

[4] C. Amsler, Phys. Lett. **B667**, 1 (2008)

Hyperon Semileptonic Decays



$$B_1 \to B_2 + \ell + \overline{\nu}_\ell$$

	4-momentum	mass
B_1	$p_1 = (E_1, \mathbf{p}_1)$	M_1
B_2	$p_2 = (E_2, \mathbf{p}_2)$	M_2
ℓ	$l = (E, \mathbf{l})$	m
$\overline{\nu}_\ell$	$p_\nu = (E_\nu^0, \mathbf{p}_\nu)$	m_ν

The low-energy weak interaction Hamiltonian for HSD is given by

$$H_W = \frac{G_V}{\sqrt{2}} J_\alpha L^\alpha + \text{H.c.}$$

Here the leptonic current is

$$L^\alpha = \overline{\psi}_e \gamma^\alpha (1 - \gamma_5) \psi_{\nu_e} + \overline{\psi}_\mu \gamma^\alpha (1 - \gamma_5) \psi_{\nu_\mu}$$

and the hadronic current expressed in terms of the vector (V_α) and axial-vector (A_α) currents is

$$\begin{aligned} J_\alpha &= V_\alpha - A_\alpha \,, \\ V_\alpha &= V_{ud} \overline{u} \gamma_\alpha d + V_{us} \overline{u} \gamma_\alpha s \,, \\ A_\alpha &= V_{ud} \overline{u} \gamma_\alpha \gamma_5 d + V_{us} \overline{u} \gamma_\alpha \gamma_5 s \,. \end{aligned}$$

G_V is the weak coupling constant.

MATRIX ELEMENTS OF THE HADRONIC CURRENT

The matrix elements of J_μ between spin-1/2 states can be written as

$$\langle B_2|J_\alpha|B_1\rangle = V_{\rm CKM}\,\overline{u}_{B_2}(p_2)\Bigg[f_1(q^2)\gamma_\alpha + \frac{f_2(q^2)}{M_1}\sigma_{\alpha\beta}q^\beta + \frac{f_3(q^2)}{M_1}q_\alpha + \left(g_1(q^2)\gamma_\alpha + \frac{g_2(q^2)}{M_1}\sigma_{\alpha\beta}q^\beta + \frac{g_3(q^2)}{M_1}q_\alpha\right)\gamma_5\Bigg]u_{B_1}(p_1),$$

$q = p_1 - p_2$ is the momentum transfer, $V_{\rm CKM}$ is either V_{ud} or V_{us}, as the case may be, and

f_1	vector f.f.	g_1	axial-vector f.f.
f_2	weak magnetism f.f.	g_2	weak electricity f.f.
f_3	induced scalar f.f.	g_3	induced pseudoscalar f.f.

In the limit of exact flavor SU(3) symmetry

- V_μ and A_μ belong to SU(3) octets and

$$f_k(q^2) = C_F^{B_2 B_1} F_k(q^2) + C_D^{B_2 B_1} D_k(q^2)\,,$$
$$g_k(q^2) = C_F^{B_2 B_1} F_{k+3}(q^2) + C_D^{B_2 B_1} D_{k+3}(q^2)\,,$$

  $F_i(q^2)$ and $D_i(q^2)$ are reduced form factors.

- The weak vector currents and the em current are members of the same SU(3) octet, which fixes f_1 and f_2:

$$F_1(0) = 1\,, \quad D_1(0) = 0\,, \quad F_2(0) = \kappa_p + \frac{1}{2}\kappa_n\,, \quad D_2(0) = -\frac{3}{2}\kappa_n\,.$$

- $F_3(q^2) = D_3(q^2) = 0$ by conservation of the em current so $f_3(q^2) = 0$.
- g_1 is given in terms of F and D (undetermined parameters).
- $g_2 = 0$ by hermiticity and time reversal invariance for diagonal matrix elements of hermitian currents.

TRANSITION AMPLITUDE FOR HSD

The transition amplitude M_0 for the process

$$B_1 \to B_2 + \ell + \overline{\nu}_\ell$$

is given by

$$\mathsf{M}_0 = \frac{G_V}{\sqrt{2}}[\overline{u}_{B_2}(p_2)W^\mu(p_1,p_2)u_{B_1}(p_1)][\overline{u}_\ell(l)O_\mu v_\nu(p_\nu)],$$

where

$$\begin{aligned} W_\mu &= f_1(q^2)\gamma_\mu + \frac{f_2(q^2)}{M_1}\sigma_{\mu\nu}q^\nu + \frac{f_3(q^2)}{M_1}q_\mu \\ &\quad + \left[g_1(q^2)\gamma_\mu + \frac{g_2(q^2)}{M_1}\sigma_{\mu\nu}q^\nu + \frac{g_3(q^2)}{M_1}q_\mu\right]\gamma_5, \end{aligned}$$

and $O_\mu = \gamma_\mu(1-\gamma_5)$. Hereafter $f_i \equiv f_i(0)$, $g_i \equiv g_i(0)$.

Differential Decay Rate $d\Gamma$

$d\Gamma$ is obtained from M_0. Different choices of the variables in the final states lead to appropriate expressions:

- In the rest frame of B_1 $[B_2]$ when it is polarized along the direction s_1 $[s_2]$, and with ℓ and $\overline{\nu}_\ell$ going into the solid angles $d\Omega_\ell$ and $d\Omega_\nu$.

- In the rest frame of B_1 $[B_2]$ when it is polarized along the direction s_1 $[s_2]$, by leaving E and E_2 as the relevant variables.

  This choice allows detailed studies of the Dalitz plot (DP):

$$d\Gamma(E, E_2) = d\Phi_3 \left[A_0'(E, E_2) - A_0''(E, E_2)\, \hat{\mathbf{s}}_i \cdot \hat{\mathbf{p}}\right],$$

  with $\hat{\mathbf{p}} = \hat{\mathbf{l}}$, $\hat{\mathbf{p}}_{\mathbf{2}}$.

Integrated Observables in HSD

The (uncorrected) decay rate R for HSD (rough approximation) is

$$R^0 = G_V^2 \frac{(\Delta M)^5}{60\pi^3} \left[\left(1 - \frac{3}{2}\beta + \frac{6}{7}\beta^2\right) f_1^2 + \frac{4}{7}\beta^2 f_2^2 + \left(3 - \frac{9}{2}\beta + \frac{12}{7}\beta^2\right) g_1^2 + \frac{12}{7}\beta^2 g_2^2 + \frac{6}{7}\beta^2 f_1 f_2 - (4\beta - 6\beta^2) g_1 g_2 \right]$$

where $\Delta M = M_{B_1} - M_{B_2}$, and $\beta = \Delta M / M_1$.

The q^2-dependence of the form factors can be incorporated as

$$f_1(q^2) = f_1(0) + \frac{q^2}{M_1^2}\lambda_1^f, \qquad g_1(q^2) = g_1(0) + \frac{q^2}{M_1^2}\lambda_1^g,$$

where λ_1^f and λ_1^g are slope parameters of order unity.

R^0 then gets the contribution

$$G_V^2 \frac{(\Delta M)^5}{60\pi^3} \left(\frac{4}{7}\beta^2\right) \left(f_1 \lambda_1^f + 5 g_1 \lambda_1^g\right).$$

The angular spin-asymmetry coefficients are defined as

$$\alpha_A^0 = 2 \frac{N\left(\theta_A < \frac{1}{2}\pi\right) - N\left(\theta_A > \frac{1}{2}\pi\right)}{N\left(\theta_A < \frac{1}{2}\pi\right) + N\left(\theta_A > \frac{1}{2}\pi\right)}$$

$A = B_2, \ell, \nu_\ell$. θ_A is the angle between the A-direction and polarization direction of B_1. Likewise, $\alpha_{e\nu}$ can be defined.

For more precise formulas and when the charged lepton mass is retained, $d\Gamma(E, E2)$ and $\alpha_A(E, E2)$ should be integrated numerically:

$$R^0 = \sum_{i \le j = 1}^{6} a_{ij}^R f_i f_j + \sum_{i \le j = 1}^{6} b_{ij}^R (f_i \lambda_{f_j} + f_j \lambda_{f_i}).$$

Radiative Corrections

Current high-statistics experiments in HSD make Dalitz-plot measurements feasible. The analysis of such measurements requires the application of radiative corrections (RC) so it is necessary that theoretical expressions as general and accurate as possible be available.

In computing RC one faces some difficulties. They depend on:

- ▷ An ultraviolet cutoff.
- ▷ The strong interaction, and on details of the weak interaction other than the effective $V - A$ theory. In a few words, RC have a model-dependent part.
- ▷ The charge assignments of the decaying and emitted baryons.
- ▷ The observed kinematical and angular variables, and on certain experimental conditions.

Advances in RC	Unpolarized	B_1	Order $(\alpha/\pi)(q/M_1)^0$	R [1]; R [2]
			Order $(\alpha/\pi)(q/M_1)$	R [3]; R [4,5]
		B_2	Order $(\alpha/\pi)(q/M_1)^0$	R [1]
			Order $(\alpha/\pi)(q/M_1)$	Work in progress
	Polarized	B_1	Order $(\alpha/\pi)(q/M_1)^0$	$\alpha_{e\nu}, \alpha_e, \alpha_\nu$ [1]; α_B [6]; α_e [8]; α_B [9];
			Order $(\alpha/\pi)(q/M_1)$	$\alpha_\nu, \alpha_e, \alpha_B, \alpha, \beta$ [7]; α_B [10]; α_e [11]; α_B [12]
		B_2	Order $(\alpha/\pi)(q/M_1)^0$	$\hat{\alpha}_e, \hat{\alpha}_\nu, A, B$ [1]; $\hat{\alpha}_e$ [13]
			Order $(\alpha/\pi)(q/M_1)$	Work in progress

[1] A. Garcia *et.al.*, Lecture Notes in Physics 222 (Springer-Verlag, 1985). E and E_ν. Both CDH and NDH.

[2] D.M. Tun, *et. al.*, PRD **40**, 2967 (1989). E and E_2. Both CDH and NDH.

[3] F. Glück, K. Tóth, PRD **41**, 2160 (1990); E and E_2. Numerical results.

[4] D.M. Tun, *et. al.*, PRD **44**, 3589 (1991). E and E_2. CDH only.

[5] A. Martinez, *et. al.*, PRD **47**, 3984 (1993). E and E_2. NDH only.

[6] R.F.M. *et. al.*, PRD **55**, 5702 (1997). E and E_2. Both CDH and NDH.

[7] F. Glück, K. Tóth, PRD **46**, 2090 (1992); E and E_2. Numerical results. Both CDH and NDH

[8] A. Martinez, *et. al.*, PRD **63**, 014025 (2001). E and E_2. Both CDH and NDH. TBR and FBR.

[9] R.F.M. *et. al.*, PRD **65**, 074002 (2002). E and E_2. Both CDH and NDH. TBR and FBR.

[10] J. J. Torres, *et. al.*, PRD **70**, 093012 (2004). E and E_2. Both CDH and NDH.

[11] M. Neri, *et. al.*, PRD **72**, 057503 (2005). E and E_2. Both CDH and NDH.

[12] J. J. Torres, *et. al.*, PRD **74**, 077501 (2006). E and E_2. Both CDH and NDH. TBR and FBR.

[13] M. Neri *et al*, PRD **78** 054018 (2008). E and E_1. Both CDH and NDH. TBR only.

Virtual Radiative Corrections

Sirlin[5] implemented an approach to deal with the RC to the β decay of a physical nucleon. He showed that the virtual RC can be separated out in two parts:

- One is model-independent, finite in the ultraviolet region and contains the infrared divergence.
- The other one contains all the complications due to strong interactions and the presence of the intermediate vector boson and the ultraviolet divergence (model-dependent part).

This separation procedure is gauge invariant.

Later, it was shown[6] that the method could be generalized to other observables and that it remained valid even when q is not negligible.

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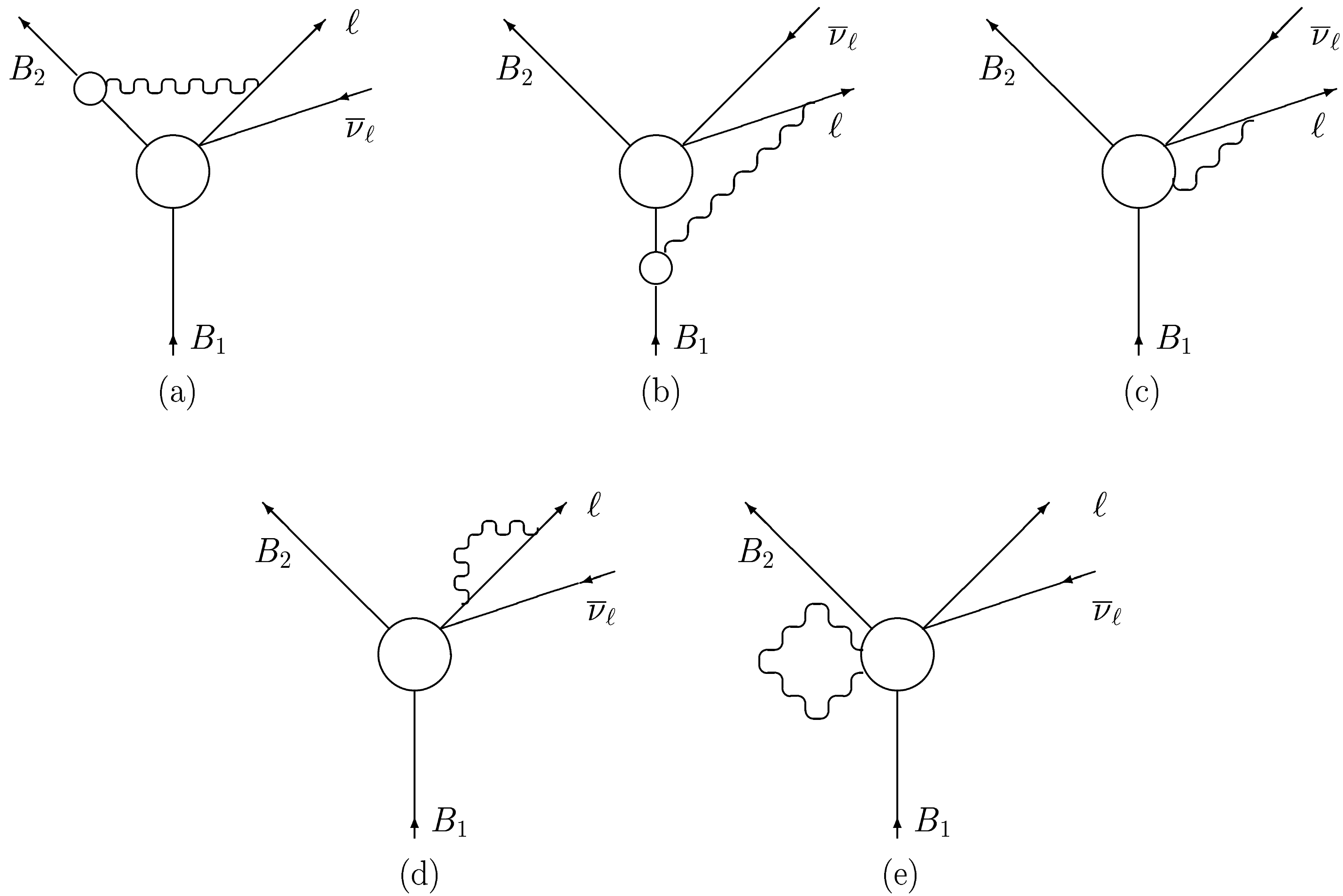
[5] A. Sirlin, Phys. Rev. **164**, 1767 (1967).

[6] A. Garcia and R. Juarez W., Phys. Rev. D **22**, 1132 (1980)

Order $\mathcal{O}(\alpha)$ Virtual Radiative Corrections



DECAY AMPLITUDE WITH VIRTUAL RC

The transition amplitude with virtual RC for the process

$$B_1 \to B_2 + \ell + \overline{\nu}_\ell$$

is[7]

$$\mathsf{M_V} = \mathsf{M_0} + \mathsf{M_v} + \mathsf{S},$$

All the model-dependence is contained in S,

$$\mathsf{S} = \frac{\alpha}{\pi} \bar{u}_{B_2} \left(c\gamma_\mu + d\gamma_\mu\gamma_5 \right) u_{B_1}\, \bar{u}_\ell O^\mu v_\nu,$$

where c and d constants so that

$$f_1'(0) \equiv f_1(0) + \frac{\alpha}{\pi} c, \qquad g_1'(0) \equiv g_1(0) + \frac{\alpha}{\pi} d,$$

and the transition amplitude reduces to

$$\mathsf{M_V} = \mathsf{M_0}' + \mathsf{M_v}.$$

7 A. Sirlin, Phys. Rev. **164**, 1767 (1967); A. Garcia and S.R. Juarez, Phys. Rev. D **22**, 1132 (1980).

Decay Rate with Virtual RC

The polarization of B_1 can be studied with the projection operator

$$\Sigma(s_1) = \frac{1 - \gamma_5 \not{s}_1}{2}, \qquad u_{B_1}(p_1) \to \Sigma(s_1) u_{B_1}(p_1)$$

with the conditions $s_1 \cdot s_1 = -1$ and $s_1 \cdot p_1 = 0$.

In the rest system of B_1, with p_2 along the z axis, the DP can be written as

$$\begin{aligned} d\Gamma_V = d\Phi_3 \Big\{ & A_0' + \frac{\alpha}{\pi}(A_1'\phi + A_1''\phi') \\ & -\hat{\mathbf{s}}_1 \cdot \hat{\mathbf{p}}_2 \Big[A_0'' + \frac{\alpha}{\pi}(A_2'\phi + A_2''\phi') \Big] \Big\} \end{aligned}$$

The infrared divergence is contained in $\phi(E)$.

A_0', A_0'', A_i', and A_i'' depend on the kinematical variables and are quadratic functions of the form factors.

Bremsstrahlung Radiative Corrections

The inner-bremsstrahlung contributions must be added to the virtual RC so we need to consider the four-body process

$$B_1(p_1) \to B_2(p_2) + \ell(l) + \overline{\nu}_\ell(p_\nu) + \gamma(k),$$

where γ represents a photon with momentum $k = (\omega, \mathbf{k})$.

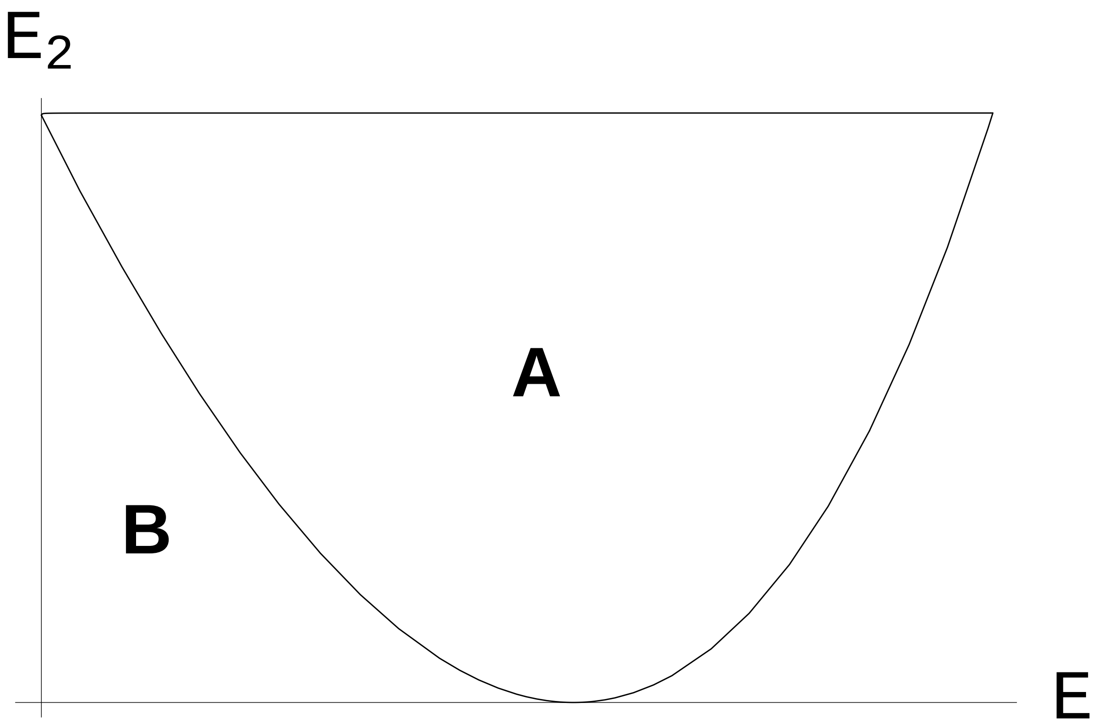
- The bremsstrahlung RC is a four-body decay whose DP covers entirely the DP of the three-body decay.

  We define:

  - ▷ Three-body region (TBR): The DP of the three-body decay.
  - ▷ Four-body region (FBR): The non-overlap of the DP of the four-body decay and the DP of the three-body decay.

- The FBR region is present when real photons cannot be discriminated in an experimental analysis of HSD.

Kinematical Allowed Region for the Four-Body Decay



Region A

$$E_2^{\min} \leq E_2 \leq E_2^{\max}, \qquad m \leq E \leq E_m,$$

$$E_m = (M_1^2 - M_2^2 + m^2)/2M_1$$

Region B

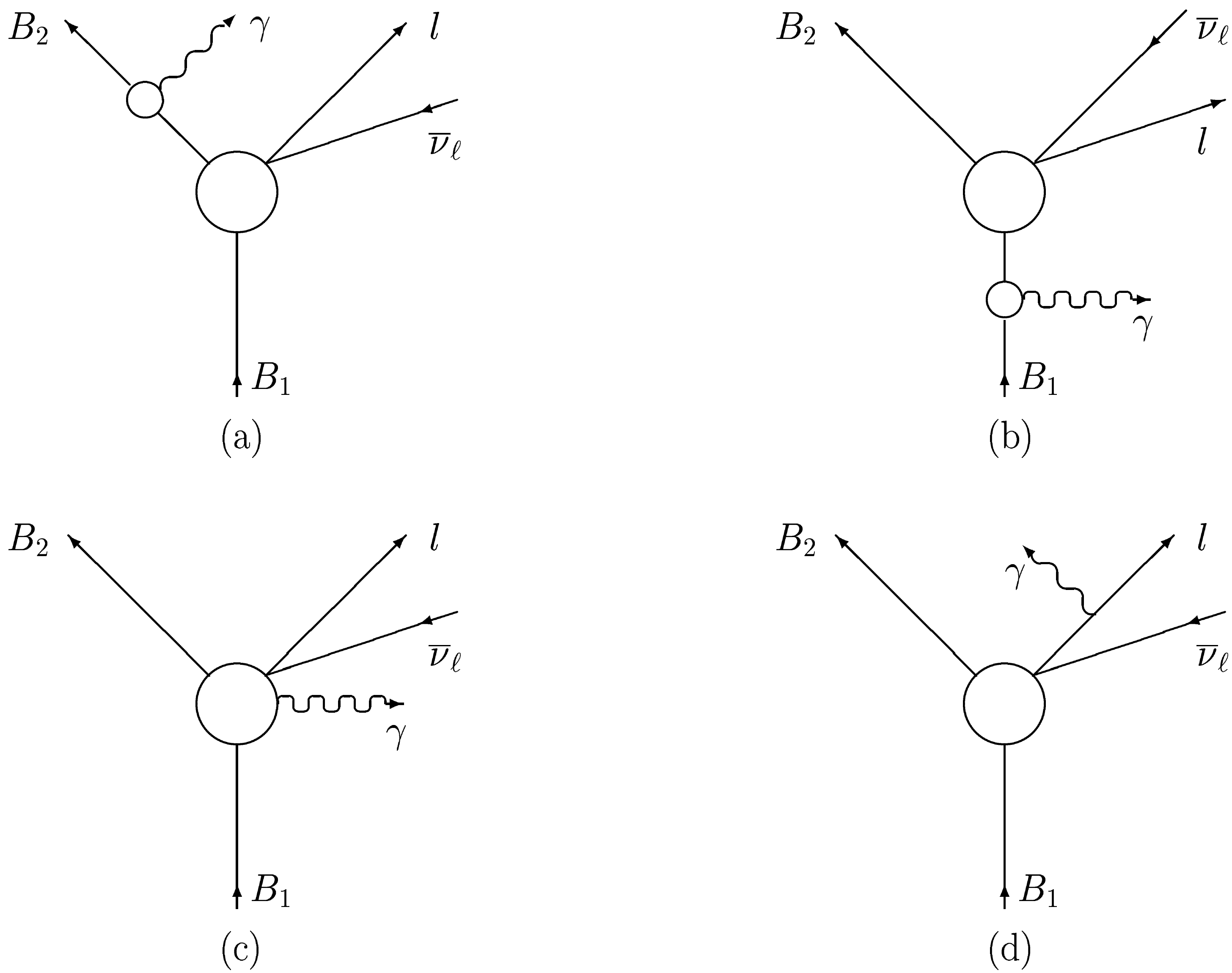
$$M_2 \leq E_2 \leq E_2^{\min}, \qquad m \leq E \leq E_b,$$

$$E_b = [(M_1 - M_2)^2 + m^2]/2(M_1 - M_2)$$

An event in B demands the existence of a fourth particle which carries away finite energy and momentum. In A this may or may not be the case. Thus B corresponds to the FBR whereas A is the TBR.

The analysis of bremsstrahlung RC considers the process $B_1 \to B_2 + \ell + \overline{\nu}_\ell + \gamma$ with both TBR and FBR contributions.

Order $\mathcal{O}(\alpha)$ Bremsstrahlung Radiative Corrections



Decay Amplitude with Bremsstrahlung RC

- To order $\mathcal{O}(\alpha q/\pi M_1)$, the amplitude for the four-body decay can be obtained in a model-independent fashion by virtue of the Low theorem.[8]
- The model-dependence will show up when including terms of order $\mathcal{O}(\alpha q^2/\pi M_1^2)$ or higher.

The amplitude with bremsstrahlung RC can be summarized as[9]

$$\mathsf{M}_\mathsf{B} = \mathsf{M}_1 + \mathsf{M}_2 + \mathsf{M}_3,$$

where M_3 is one order in q higher than M_1 and M_2, and is written in terms of the electromagnetic static parameters of the baryons.

The calculation of $d\Gamma_B$ is performed with standard techniques.

[8]F.E. Low, Phys. Rev. **110**, 974 (1958). H. Chew, Phys. Rev. **123**, 377 (1961)
[9]D.M. Tun, S.R. Juarez, A. Garcia, Phys. Rev. D **44**, 3589 (1991).

DP with Bremsstrahlung RC

The DP with bremsstrahlung RC, $d\Gamma_B$, can be organized as

$$d\Gamma_B = d\Gamma_B^{\rm ir} + d\Gamma_B^{\rm TBR} + d\Gamma_B^{\rm FBR},$$

where

$$d\Gamma_B^{\rm ir} = \frac{\alpha}{\pi} I_0(\lambda)\, d\Omega + d\Gamma_B^0,$$

contains the infrared divergence in the first summand whereas the remaining terms are finite.

$d\Gamma_B^{\rm TBR}$ and $d\Gamma_B^{\rm FBR}$ still must be integrated over the photon variables with the appropriate limits. Two ways to do it:

- Numerical integration
- Analytical integration

DP with RC: Putting Everything Together

In the rest system of B_1, with p_2 along the z axis, the complete radiatively corrected DP to order $\mathcal{O}(\alpha q/\pi M_1)$ can be cast into[10]

$$\begin{aligned} d\Gamma(B_1 \to B_2 \ell \overline{\nu}_\ell) &= d\Gamma_V + d\Gamma_B \\ &= d\Omega \Big\{ A_0' + \frac{\alpha}{\pi}(\Phi_1 + \Phi_{1F}) \\ &\qquad - \hat{\mathbf{s}}_\mathbf{1} \cdot \hat{\mathbf{p}}_\mathbf{2} \Big[B_0'' + \frac{\alpha}{\pi}(\Phi_2 + \Phi_{2F}) \Big] \Big\} \end{aligned}$$

where the $\Phi_i(E, E_2)$ depend quadratically on the form factors.

In the rest system of B_1, with ℓ along the z axis, a similar expression can be obtained. The decay of charged and neutral baryons need to be analyzed separately.

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[10] J.J. Torres *et. al.*, PRD **74**, 077501 (2006).

Features

- ▷ To order $\mathcal{O}(\alpha q/\pi M_1)$, the expression for $d\Gamma$ has no infrared divergences, it does not contain an ultraviolet cutoff, and is model-independent.

- ▷ It can be useful in the analysis of the Dalitz plot of precision experiments involving light and heavy quarks.

- ▷ It is not compromised to fixing the form factors at prescribed values.

Spin-asymmetry coefficient α_B

The DP so organized allows the calculation of α_B:

$$\alpha_B = 2\frac{N^+ - N^-}{N^+ + N^-}$$

N^+ $[N^-]$ denotes the number of emitted hyperons with momenta in the forward [backward] hemisphere with respect to s_1. One gets

$$\alpha_B = -\frac{\Delta_2 + (\alpha/\pi)(\Psi_2 + \Psi_{2F})}{\Delta_1 + (\alpha/\pi)(\Psi_1 + \Psi_{1F})},$$

where

$$\Delta_2 = \int_m^{E_m} \int_{E_2^{\min}}^{E_2^{\max}} B_0'' \, dE_2 dE, \qquad \Delta_1 = \int_m^{E_m} \int_{E_2^{\min}}^{E_2^{\max}} A_0' \, dE_2 dE,$$

$$\Psi_i = \int_m^{E_m} \int_{E_2^{\min}}^{E_2^{\max}} \Phi_i \, dE_2 dE, \qquad \Psi_{iF} = \int_m^{E_b} \int_{M_2}^{E_2^{\min}} \Phi_{iF} \, dE_2 dE,$$

σ	(a)									
0.8067	0.5	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1
0.8043	50.7	0.3	0.1	0.1	0.0	0.0	0.0	0.0	0.1	0.3
0.8020		1.2	0.3	0.2	0.1	0.1	0.1	0.0	0.1	
0.7997		5.4	0.7	0.3	0.2	0.1	0.1	0.1	0.1	
0.7974			1.6	0.6	0.3	0.2	0.1	0.1	0.1	
0.7951			4.4	1.1	0.5	0.3	0.2	0.1	0.1	
0.7928			19.8	2.1	0.9	0.4	0.2	0.1		
0.7904				5.2	1.5	0.7	0.3			
0.7881					3.2	1.1	0.3			
0.7858					9.8	2.2	0.2			
	(b)									
0.8067	0.6	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1
0.8043	51.2	0.3	0.1	0.0	0.0	0.0	0.0	0.0	0.1	0.2
0.8020		1.2	0.3	0.1	0.1	0.0	0.1	0.1	0.1	
0.7997		5.5	0.6	0.2	0.1	0.1	0.1	0.1	0.2	
0.7974			1.4	0.4	0.2	0.1	0.1	0.1	0.2	
0.7951			4.1	0.8	0.3	0.2	0.1	0.1	0.1	
0.7928			18.5	1.7	0.6	0.3	0.2	0.1		
0.7904				4.4	1.1	0.4	0.2	0.1		
0.7881					2.4	0.7	0.2			
0.7858					8.3	1.4	0.1			
δ	0.05	0.15	0.25	0.35	0.45	0.55	0.65	0.75	0.85	0.95

Table 1: Percentage $\delta\alpha_B(E, E_2)$ with RC over the TBR in $\Sigma^- \to n e \overline{\nu}$ decay (a) to order $\mathcal{O}(\alpha/\pi)$ and (b) to order $\mathcal{O}(\alpha q/\pi M_1)$.

σ	(a)									
0.8067	0.6	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1
0.8043	51.2	0.3	0.1	0.0	0.0	0.0	0.0	0.0	0.1	0.2
0.8020		1.2	0.3	0.1	0.1	0.0	0.1	0.1	0.1	
0.7997		5.5	0.6	0.2	0.1	0.1	0.1	0.1	0.2	
0.7974			1.4	0.4	0.2	0.1	0.1	0.1	0.2	
0.7951			4.1	0.8	0.3	0.2	0.1	0.1	0.1	
0.7928			18.5	1.7	0.6	0.3	0.2	0.1		
0.7904				4.4	1.1	0.4	0.2	0.1		
0.7881					2.4	0.7	0.2			
0.7858					8.3	1.4	0.1			
	(b)									
0.8067	0.6	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1
0.8044	50.7	0.3	0.1	0.0	0.0	0.0	0.0	0.0	0.1	0.2
0.8020		1.2	0.2	0.1	0.1	0.0	0.0	0.1	0.1	
0.7997		5.4	0.6	0.2	0.1	0.1	0.1	0.1	0.2	
0.7974			1.4	0.4	0.2	0.1	0.1	0.1	0.2	
0.7951			4.0	0.8	0.3	0.2	0.1	0.1	0.1	
0.7928			18.4	1.7	0.6	0.3	0.2	0.1		
0.7904				4.4	1.0	0.4	0.2	0.1		
0.7881				11.1	2.4	0.7	0.2			
0.7858					8.2	1.4	0.1			
δ	0.05	0.15	0.25	0.35	0.45	0.55	0.65	0.75	0.85	0.95

Table 2: Percentage $\delta\alpha_B(E, E_2)$ with RC over the TBR in $\Sigma^- \to ne\overline{\nu}$ decay (a) to order $\mathcal{O}(\alpha q/\pi M_1)$ and (b) computed by Gluck and Toth [Phys. Rev. D **46**, 2090 (1992)].

σ	(a)									
0.8067	0.5	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1
0.8043	50.7	0.3	0.1	0.1	0.0	0.0	0.0	0.0	0.1	0.3
0.8020	54.3	1.2	0.3	0.2	0.1	0.1	0.1	0.0	0.1	
0.7997	55.0	5.4	0.7	0.3	0.2	0.1	0.1	0.1	0.1	
0.7974	55.0	31.1	1.6	0.6	0.3	0.2	0.1	0.1	0.1	
0.7951	54.5	43.3	4.4	1.1	0.5	0.3	0.2	0.1	0.1	
0.7928	53.1	45.4	19.8	2.1	0.9	0.4	0.2	0.1		
0.7904	50.2	44.5	28.7	5.2	1.5	0.7	0.3			
0.7881	44.1	40.1	31.2	−5.2	3.2	1.1	0.3			
0.7858	29.4	27.3	23.0	14.3	9.8	2.2	0.2			
	(b)									
0.8067	0.6	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1
0.8044	50.7	0.3	0.1	0.0	0.0	0.0	0.0	0.0	0.1	0.2
0.8020	60.7	1.2	0.2	0.1	0.1	0.0	0.0	0.1	0.1	
0.7997	62.4	5.4	0.6	0.2	0.1	0.1	0.1	0.1	0.2	
0.7974	63.7	47.3	1.4	0.4	0.2	0.1	0.1	0.1	0.2	
0.7951	64.5	56.7	4.0	0.8	0.3	0.2	0.1	0.1	0.1	
0.7928	64.5	58.7	18.4	1.7	0.6	0.3	0.2	0.1		
0.7904	62.8	58.1	45.1	4.4	1.0	0.4	0.2	0.1		
0.7881	57.1	53.6	45.7	11.1	2.4	0.7	0.2			
0.7858	39.8	37.8	33.6	25.1	8.2	1.4	0.1			
δ	0.05	0.15	0.25	0.35	0.45	0.55	0.65	0.75	0.85	0.95

Table 3: Percentage $\delta\alpha_B(E,E_2)$ with RC over the TBR and FBR in $\Sigma^- \to ne\overline{\nu}$ decay (a) to order $\mathcal{O}(\alpha/\pi)$ and (b) computed by Gluck and Toth [Phys. Rev. D **46**, 2090 (1992)].

INTEGRATED α_B WITH RC: TBR CONTRIBUTION

Decay	α_B^0	$\delta\alpha_B$		
		$\mathcal{O}(\alpha q/\pi M_1)$	$\mathcal{O}(\alpha/\pi)$	Toth & Gluck '92
$\Lambda \to pe\overline{\nu}$	-58.6	-0.09	-0.2	-0.1
$\Sigma^- \to ne\overline{\nu}$	66.7	0.05	0.1	0.0
$\Sigma^- \to \Lambda e\overline{\nu}$	7.2	0.08		

Table 4: Values of α_B and comparison with other works. We set $f_1 = 1.27$, $g_1 = 0.89$, and $f_2 = 1.20$ for $\Lambda \to pe\overline{\nu}$; $f_1 = 1$, $g_1 = -0.34$, and $f_2 = -0.97$ for $\Sigma^- \to ne\overline{\nu}$; and $f_1 = 0$, $g_1 = 0.60$, and $f_2 = 1.17$ for $\Sigma^- \to \Lambda e\overline{\nu}$.

Weak Form Factors

Fits to the data on HSD used to be made under the assumption of exact SU(3) symmetry in order to extract V_{us}. Currently, the experiments are precise enough to the extent that this assumption no longer provides a reliable fit.

Thus, the determination of V_{us} from HSD requires an understanding of the SU(3) symmetry breaking effects in the form factors.

For the leading form factors:

- f_1 is protected by the Ademollo-Gatto theorem against SU(3) breaking corrections to lowest order in $(m_s - \hat{m})$.
- g_1 gets first-order SU(3) breaking effects so it introduces larger theoretical uncertainties.

The remaining form factors

- The contributions of f_2 to the different observables of HSD in the SU(3) limit are first-order symmetry breaking contributions because of the kinematic factor of q.

- Reasonable shifts[11] from the SU(3) predictions of f_2 do not have any observable effect upon χ^2 or g_1 in a global fit to experimental data. f_2 can be used in its SU(3) symmetric value.

- The data are not accurate enough for an extraction of the small g_2-dependence of the decay amplitudes, so we use the value $g_2 = 0$.

- Contributions of f_3 and g_3 in the different HSD observables are proportional to the square mass electron and can be safely ignored.

[11] RFM, A. García, and G. Sánchez-Colón, Phys. Rev. D **54**, 6855 (1996)

Some Approaches to Determine HSD Form Factors

▷ Quark model (relativistic and non-relativistic)

- F. Schlumpf, PRD **51**, 2262 (1995): f_1, g_1 • J. Donoghue, *et. al.*, PRD **35**, 934 (1987): f_1, g_1

▷ Chiral perturbation theory

- A. Krause, Helv. Phys. Acta **63**, 3 (1990): f_1 • J. Anderson *et. al.*, PRD **47**, 4975 (1993): f_1
- E. Jenkins and A. Manohar, PLB **255**, 558 (1991); **259**, 353 (1991): g_1
- A. Faessler *et al* Phys. Rev. D**77**, 114007 (2008): f_1, g_1

▷ The $1/N_c$ expansion

- J. Dai, E. Jenkins, A. Manohar, PRD **53**, 273 (1996): g_1
- RFM, E. Jenkins, A. Manohar, PRD **58**, 094028 (1998): f_1, g_1
- RFM, PRD **70**, 114036 (2004): f_1, g_1

▷ The combination of HBCHPT and the $1/N_c$ expansion

- RFM, C.P. Hofmann, PRD **74**, 094001 (2006): g_1

▷ Lattice Gauge Theory

- D. Guadagnoli *et al*, Nucl. Phys. **B**761 (2007): f_1, g_1.

Fits to the Experimental Data

The experimentally measured quantities in HSD are

- the total decay rate R,
- the angular correlation coefficients $\alpha_{e\nu}$, and
- the angular spin-asymmetry coefficients α_e, α_ν, α_B, A, and B

An alternative choice are R and the ratio g_1/f_1.

Fits in HSD should include

- the radiative corrections to the different observables
- the momentum-transfer contribution of the form factors

$$f_1(q^2) = f_1(0)\left(1 + 2\frac{q^2}{M_V}\right), \quad g_1(q^2) = g_1(0)\left(1 + 2\frac{q^2}{M_A}\right),$$

where $M_V, M_A \sim 1$ GeV.

FITTING THE DATA [RFM, PRD **70**, 114036 (2004)]

Process	RFM	Anderson & Luty	Donoghue *et al.*	Krause	Schlumpf
$\Lambda \to p$	1.02±0.02	1.024	0.987	0.943	0.976
$\Sigma^- \to n$	1.04±0.02	1.100	0.987	0.987	0.975
$\Xi^- \to \Lambda$	1.04±0.04	1.059	0.987	0.957	0.976
$\Xi^- \to \Sigma^0$	1.07±0.05	1.011	0.987	0.943	0.976
$\Xi^0 \to \Sigma^+$	1.07±0.05				
V_{us}	0.2199±0.0026	0.2177±0.0019	0.2244±0.0019	0.2274±0.0019	0.2256±0.0019

Table 5: SB pattern for f_1. The entries correspond to $f_1/f_1^{\mathrm{SU}(3)}$. The fit to data was performed by using the decay rate and asymmetry coefficients.

- From K_{l3} decays (Particle Data Group, 2008)

$$V_{us} = 0.2255 \pm 0.0019$$

- From hyperon semileptonic decays [Cabibbo, Swallow, and Winston, Annu. Rev. Nucl. Part. Sci. **53**, 39 (2003)]

$$V_{us} = 0.2250 \pm 0.0027$$

with no indication of flavor SU(3)-breaking effects.

- In the context of the $1/N_c$ expansion

$V_{us} = 0.2238 \pm 0.0019$	(No SB effects)
$V_{us} = 0.2230 \pm 0.0019$	(First order SB in g_1)
$V_{us} = 0.2199 \pm 0.0026$	(Second order SB in f_1)

Conclusions

- Analyses in HSD still face some interesting theoretical problems.
- On the one hand, the computation of RC to order $\mathcal{O}(\alpha q/\pi M_1)$ is under reasonable control. Model-independent theoretical expression for the Dalitz plot which cover both the three-body and the four-body regions are now available.
- The results are suitable for high-statistics experiments.
- On the other hand, a deep understanding of the SU(3) symmetry breaking effects in the weak form factors is not settled down yet.
- The $1/N_c$ expansion has been useful in the analysis of SB effects. Fits to data prefer the values $f_1/f_1^{\mathrm{SU(3)}} > 1$ (up to 7 %), opposite to the value < 1 predicted by the quark model (relativistic and non-relativistic).
- The value of V_{us} from HSD is similar in precision to the one derived from K_{l3}.
- More work, theoretical and experimental, will be welcome in the near future.