Cosmological simulations: the role of scalar fields

M.A. Rodríguez-Meza

Depto. de Física, Instituto Nacional de Investigaciones Nucleares, Col. Escandón, Apdo. Postal 18-1027, 11801 México D.F.
marioalberto.rodriguez@inin.gob.mx; http://www.astro.inin.mx/mar

Abstract. We present numerical *N*-body simulation studies of large-scale structure formation. The main purpose of these studies is to analyze the several models of dark matter and the role they played in the process of large-scale structure formation. We analyze the standard and more successful case, i.e., the cold dark matter with cosmological constant (ΛCDM). We compare the results of this model with the corresponding results of other alternative models, in particular, the models that can be built from the Newtonian limit of alternative theories of gravity like scalar-tensor theories. An specific model is the one that considers that the scalar field is non-minimally coupled to the Ricci scalar in the Einstein-Hilbert Lagrangian that gives, in the Newtonian limit an effective gravitational force that is given by two contributions: the standard Newtonian potential plus a Yukawa potential that comes from a massive scalar field. Comparisons of the models are done by analyzing the snapshots of the *N*-body system at z=0 for several values of the SF parameters.

INTRODUCTION

In this work we present some results about the role scalar fields (SF) play in cosmological simulations, in particular on the process of large scale structure formation. The main goal of this work is to study the large scale structure formation where the usual approach is that the evolution of the initial primordial fluctuation energy density fields evolve following Newtonian mechanics in an expanding background[1]. The force between particles are the standard Newtonian gravitational force. We will see that we can introduce SF by adding a term in this force. This force will be of Yukawa type with two parameters (α, λ)[2]. We have been studying, in the past years, the effects of this kind of force on some astrophysical phenomena[2, 3, 4, 5]. The Yukawa force comes as a Newtonian limit of a scalar-tensor theory (STT) with the SF non-minimally coupled to gravitation[2]. It is our purpose to find the role these scalar fields play on the large scale structure formation processes. We start by discussing the standard ΛCDM model and the general approach in *N*-body simulations (See Bertschinger[6] for details). Then, we present the modifications we need to do to consider the effects of a static SF and we show the results of this theory for the cosmological concordance model of a ΛCDM universe[7]. To perform the simulations we have modified a standard serial treecode the author has developed [8] and the Gadget 1 [9] (see also `http://www.astro.inin.mx/mar`) in order to take into account the contribution of the Yukawa potential.

EVOLUTION EQUATIONS FOR A ΛCMD UNIVERSE

General Scalar-tensor theory and its Newtonian limit

Let us consider the Einstein field equations of a typical STT[10]

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{1}{\phi}\left[8\pi T_{\mu\nu} + \frac{1}{2}Vg_{\mu\nu} + \frac{\omega}{\phi}\partial_\mu\phi\partial_\nu\phi\right.$$
$$\left. -\frac{1}{2}\frac{\omega}{\phi}(\partial\phi)^2 g_{\mu\nu} + \phi_{;\mu\nu} - g_{\mu\nu}\Box\phi\right], \tag{1}$$

for the metric $g_{\mu\nu}$ and for the massive SF ϕ we have

$$\Box\phi = \frac{1}{3+2\omega}\left[8\pi T - \omega'(\partial\phi)^2 + \phi V' - 2V\right], \tag{2}$$

where $()' \equiv \frac{\partial}{\partial\phi}$. Here $T_{\mu\nu}$ is the energy-momentum tensor with trace T, $\omega(\phi)$ and $V(\phi)$ are in general arbitrary functions that gobern kinetic and potential contribution of the SF. The potential contribution, $V(\phi)$, provides mass to the SF, denoted here by m_{SF}.

The study of large-scale formation in the universe is greatly simplified by the fact that a limiting approximation of general relativity, Newtonian mechanics, applies in a region small compared to the Hubble length cH^{-1} ($cH_0^{-1} \approx 3000h^{-1}$ Mpc, where c is the speed of light, $H_0 = 100h$ km/s/Mpc, is Hubble's constant and $h \approx (0.5 - 1)$), and large compared to the Schwarzschild radii of any collapsed objects. The rest of the universe affect the region only through a tidal field. The length scale cH_0^{-1} is of the order of the largest scales currently accessible in cosmological observations and $H_0^{-1} \approx 10^{10}h^{-1}$ yr characterizes the evolutionary time scale of the universe.

Therefore, in the present study, we need to consider the influence of SF in the limit of a static STT, and then we need to describe the theory in its Newtonian approximation, that is, where gravity and the SF are weak (and time independent) and velocities of dark matter particles are non-relativistic. We expect to have small deviations of the SF around the background field, defined here as $\langle\phi\rangle$ and can be understood as the scalar field beyond all matter. If one defines the perturbations $\phi = \langle\phi\rangle + \bar{\phi}$ and $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, where $\eta_{\mu\nu}$ is the Minkowski metric, the Newtonian approximation gives [2]

$$R_{00} = \frac{1}{2}\nabla^2 h_{00} = \frac{G_N}{1+\alpha}4\pi\rho - \frac{1}{2}\nabla^2\bar{\phi}, \tag{3}$$

$$\nabla^2\bar{\phi} - m_{SF}^2\bar{\phi} = -8\pi\alpha\rho, \tag{4}$$

we have set $\langle\phi\rangle = (1+\alpha)/G_N$ and $\alpha \equiv 1/(3+2\omega)$. In the above expansion we have set the cosmological constant term equal to zero, since on galactic scales its influence should be negligible. We only consider the influence of dark matter due to the boson field of mass m_{SF} governed by Eq. (4), that is the modified Helmholtz equation. However, at cosmological scales we do take into account the cosmological constant contribution, see below. Equations (3) and (4) represent the Newtonian limit of the STT with arbitrary potential $V(\phi)$ and function $\omega(\phi)$ that where Taylor expanded around $\langle\phi\rangle$. The resulting

equations are then distinguished by the constants G_N, α, and $\lambda = h_P/m_{SF}c$. Here h_P is Planck's constant.

The next step is to find solutions for this new Newtonian potential given a density profile, that is, to find the so-called potential–density pairs. General solutions to Eqs. (3) and (4) can be found in terms of the corresponding Green functions, and the new Newtonian potential is[2, 4]

$$
\begin{aligned}
\Phi_N \equiv \frac{1}{2}h_{00} &= -\frac{G_N}{1+\alpha} \int d\mathbf{r}_s \frac{\rho(\mathbf{r}_s)}{|\mathbf{r} - \mathbf{r}_s|} \\
&\quad -\alpha \frac{G_N}{1+\alpha} \int d\mathbf{r}_s \frac{\rho(\mathbf{r}_s)\mathrm{e}^{-|\mathbf{r}-\mathbf{r}_s|/\lambda}}{|\mathbf{r} - \mathbf{r}_s|} + \mathrm{B.C.}
\end{aligned}
\tag{5}
$$

The first term of Eq. (5), is the contribution of the usual Newtonian gravitation (without SF), while information about the SF is contained in the second term, that is, arising from the influence function determined by the modified Helmholtz Green function, where the coupling ω (α) enters as part of a source factor.

Cosmological evolution equations using a static STT

To simulate cosmological systems, the expansion of the universe has to be taken into account. Also, to determine the nature of the cosmological model we need to determine the composition of the universe, i. e., we need to give the values of Ω_i for each component i, taking into account in this way all forms of energy densities that exist at present. If a particular kind of energy density is described by an equation of state of the form $p = w\rho$, where p is the pressure and w is a constant, then the equation for energy conservation in an expanding background, $d(\rho a^3) = -p d(a^3)$, can be integrated to give $\rho \propto a^{-3(1+w)}$. Then, the Friedmann equation for the expansion factor $a(t)$ is written as

$$
\frac{\dot{a}^2}{a^2} = H_0^2 \sum_i \Omega_i \left(\frac{a_0}{a}\right)^{3(1+w_i)} - \frac{k}{a^2}
\tag{6}
$$

where w_i characterizes equation of state of specie i. The most familiar forms of energy densities are those due to pressureless matter with $w_i = 0$ (that is, nonrelativistic matter with rest-mass-energy density ρc^2 dominating over the kinetic-energy density $\rho v^2/2$) and radiation with $w_i = 1/3$. The density parameter contributed today by visible, nonrelativistic, baryonic matter in the universe is $\Omega_B \approx (0.01 - 0.2)$ and the density parameter that is due to radiation is $\Omega_R \approx 2 \times 10^{-5}$. In this work we will consider a model with only two energy density contribution. One which is a pressureless and nonbaryonic dark matter with $\Omega_{DM} \approx 0.3$ that does not couple with radiation. Other, that will be a cosmological constant contribution $\Omega_\Lambda \approx 0.7$ with and equation of state $p = -\rho$. The above equation for $a(t)$ becomes

$$
\frac{\dot{a}^2}{a^2} = H_0^2 \left[\Omega_{DM} \left(\frac{a_0}{a}\right)^3 + \Omega_\Lambda\right] - \frac{k}{a^2}
\tag{7}
$$

Here, we employ a cosmological model with a static SF which is consistent with the Newtonian limit given by Eq. (5). Thus, the scale factor, $a(t)$, is given by the following Friedman model,

$$a^3 H^2 = H_0^2 \left[\frac{\Omega_{m0} + \Omega_{\Lambda 0}\, a^3}{1+\alpha} + \left(1 - \frac{\Omega_{m0} + \Omega_{\Lambda 0}}{1+\alpha} \right) a \right] \tag{8}$$

where $H = \dot{a}/a$, Ω_{m0} and $\Omega_{\Lambda 0}$ are the matter and energy density evaluated at present, respectively. We notice that the source of the cosmic evolution is deviated by the term $1 + \alpha$ when compared to the standard Friedman-Lemaitre model. Therefore, it is convenient to define a new density parameter by $\Omega_i^{(\alpha)} \equiv \Omega_i/(1+\alpha)$. This new density parameter is such that $\Omega_m^{(\alpha)} + \Omega_\Lambda^{(\alpha)} = 1$, which implies a flat universe, and this shall be assumed in our following computations, where we consider $(\Omega_m^{(\alpha)}, \Omega_\Lambda^{(\alpha)}) = (0.3, 0.7)$. For positive values of α, a flat cosmological model demands to have a factor $(1+\alpha)$ more energy content (Ω_m and Ω_Λ) than in standard cosmology. On the other hand, for negative values of α one needs a factor $(1+\alpha)$ less Ω_m and Ω_Λ to have a flat universe. To be consistent with the CMB spectrum and structure formation numerical experiments, cosmological constraints must be applied on α in order for it to be within the range $(-1, 1)$ [11, 12, 13, 14].

The N-Body problem for dark matter

The Vlasov-Poisson equation in an expanding universe describes the evolution of the six-dimensional, one-particle distribution function, $f(\mathbf{x}, \mathbf{p})$. The Vlasov equation is,

$$\frac{\partial f}{\partial t} + \frac{\mathbf{p}}{ma^2} \cdot \frac{\partial f}{\partial \mathbf{x}} - m \nabla \Phi_N(\mathbf{x}) \cdot \frac{\partial f}{\partial \mathbf{p}} = 0 \tag{9}$$

where $\mathbf{x}$ is the comoving coordinate, $\mathbf{p} = ma^2 \dot{\mathbf{x}}$, m is the particle mass, and Φ_N is the self-consistent gravitational potential given by the Poisson equation,

$$\nabla^2 \Phi_N(\mathbf{x}) = 4\pi G_N\, a^2 \left[\rho(\mathbf{x}) - \rho_b(t) \right] \tag{10}$$

where ρ_b is the background mass density. Eqs. (9) and (10) form the Vlasov-Poisson equation, constitutes a collisionless, mean-field approximation to the evolution of the full N-body distribution. An N-body code attempts to solve Eqs. (9) and (10) by representing the one-particle distribution function as

$$f(\mathbf{x}, \mathbf{p}) = \sum_{i=1}^{N} \delta(\mathbf{x} - \mathbf{x}_i)\, \delta(\mathbf{p} - \mathbf{p}_i) \tag{11}$$

Substitution of (11) in the Vlasov-Poisson system of equations yields the exact Newton's equations for a system of N gravitating particles. See Ref. [6] for details.

In the Newtonian limit of STT of gravity, the Newtonian motion equation for a particle i is written as[20]

$$\ddot{\mathbf{x}}_i + 2H\mathbf{x}_i = -\frac{1}{a^3}\frac{G_N}{1+\alpha}\sum_{j\neq i}\frac{m_j(\mathbf{x}_i - \mathbf{x}_j)}{|\mathbf{x}_i - \mathbf{x}_j|^3}F_{SF}(|\mathbf{x}_i - \mathbf{x}_j|, \alpha, \lambda) \qquad (12)$$

where the sum includes all periodic images of particle j, and $F_{SF}(r, \alpha, \lambda)$ is

$$F_{SF}(r, \alpha, \lambda) = 1 + \alpha\left(1 + \frac{r}{\lambda}\right)e^{-r/\lambda} \qquad (13)$$

which, for small distances compared to λ, is $F_{SF}(r < \lambda, \alpha, \lambda) \approx 1 + \alpha\left(1 + \frac{r}{\lambda}\right)$ and, for long distances, is $F_{SF}(r > \lambda, \alpha, \lambda) \approx 1$, as in Newtonian physics.

RESULTS

In this section, we present results of cosmological simulations of a ΛCDM universe with and without SF contribution. We use 256^3 particles in box $256h^{-1}$ Mpc size. We have studied in the past a ΛCDM model in a smaller box and with less resolution than the present case[20], the ΛCDM case that comes with Gadget 1.0[19], and the Santa Barbara cluster[21].

The initial linear power spectrum was generated using the fitting formula by Klypin & Holtzman[15] for the transfer function. This formula is a slight variation of the common BBKS fit[16]. It includes effects from baryon suppression but no baryonic oscillations. We use the standard Zel'dovich approximation[17] to provide the initial 256^3 particles displacement off a uniform grid and to assign their initial velocities in a $256h^{-1}$ Mpc box. The starting redshift is $z_{in} = 50$ and we choose the following cosmology: $\Omega_{DM} = 0.314$ (where Ω_{DM} includes cold dark matter and baryons), $\Omega_B = 0.044$, $\Omega_\Lambda = 0.686$, $H_0 = 71$ km/s/Mpc, $\sigma_8 = 0.84$, and $n = 0.99$. Particle masses are in the order of 1.0×10^{10} M$_\odot$. The individual softening length was 20 kpc/h. This choice of softening length is consistent with the mass resolution set by the number of particles. All these values are in concordance with measurements of cosmological parameters by WMAP[18]. The initial condition is in the Cosmic Data Bank web page (`http://t8web.lanl.gov/people/heitmann/test3.html`). See Heitmann et al. 2005[7] for more details.

We now present the results for the ΛCDM model previously described. Because the visible component is the smaller one and given our interest to test the consequences of including a SF contribution to the evolution equations, our model excludes gas particles, but all its mass has been added to the dark matter. We restrict the values of α to the interval $(-1,1)$ [11, 12, 13, 14] and use $\lambda = 1, 5, 10, 20$ Mpc/h, since these values sweep the scale lengths present in the simulations. In Fig. 1 we show x–y snapshots at redshift $z = 0$ of our ΛCDM model. Fig. 1 (a) presents the standard case without SF, i.e., the interaction between bodies is through the standard Newtonian potential. In (b), (c), (d), and (e) we show the case with $\alpha = 1$, and $\lambda = 1, 5, 10, 20$ Mpc/h, respectively. In (f) and (g) $\lambda = 5$ Mpc/h and $\alpha = -1/2$ and $-1/4$, respectively. One notes clearly how the SF modifies the matter structure of the system. The most dramatic cases are (e) and
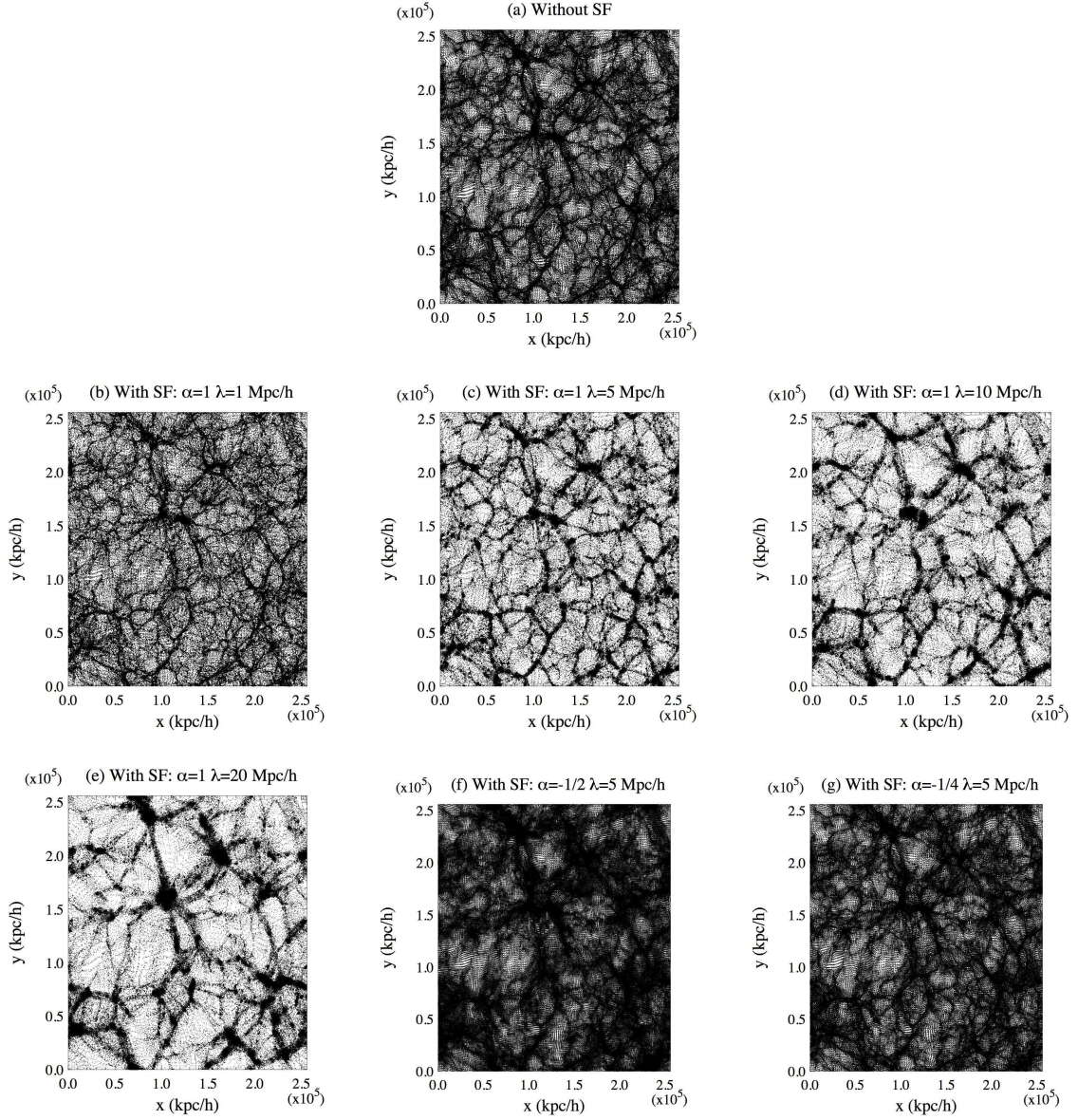
FIGURE 1. *x–y* snapshots at $z = 0$ of a ΛCDM universe without and with SF. See text for details.

(f) where we have used $\alpha = 1$ and $\lambda = 20$ Mpc$/h$, and $\alpha = -1/2$ and $\lambda = 5$ Mpc$/h$, respectively.

We now analyze the general effect that the constant α has on the dynamics. The role of α in our approach is as follows. On one hand, to construct a flat model we have set the condition $\Omega_m^{(\alpha)} + \Omega_\Lambda^{(\alpha)} = 1$, which implies having $(1 + \alpha)$ times the energy content of the standard ΛCDM model. This essentially means that we have an increment by a factor of $(1 + \alpha)$ times the amount of matter, for positive values of α, or a reduction of the same factor for negative values of α. Increasing or reducing this amount of matter affects the matter term on the r.h.s. of the equation of motion (12), but the amount

affected cancels out with the term $(1 + \alpha)$ in the denominator of (12) stemming from the new Newtonian potential. On the other hand, the factor F_{SF} augments (diminishes) for positive (negative) values of α for small distances compared to λ, resulting in more (less) structure formation for positive (negative) values of α compared to the ΛCDM model. For $r \gg \lambda$ the dynamics is essentially Newtonian.

Therefore, for cases in which we use $\lambda = 5$ Mpc$/h$ we have the following. In the case of (c), for $r \ll \lambda$, the effective gravitational pull has been augmented by a factor of 2, in contrast to case (f) where it has diminished by a factor of $1/2$; in model (g) the pull diminishes only by a factor of $3/4$. That is why one observes for $r < \lambda$ more structure formation in (c), less in (f), and lesser in model (g). The effect is then, for a growing positive α, to speed up the growth of perturbations, then of halos and then of clusters, whereas negative α values ($\alpha \to -1$) tend to slow down the growth. Whereas in models (b), (c), (d) and (e) where we keep $\alpha = 1$ and take $\lambda = 1, 5, 10, 20$ Mpc$/h$, we observe less structure in case (b) to more structure in case (e), passing for intermediate structure formation cases. In spite of that the effective gravitational constant has been augmented by a factor of 2 the importance of the Yukawa contribution is very small for distances $r \gg \lambda$. That is way we observe this behavior.

CONCLUSIONS

The theoretical scheme we have used is compatible with local observations because we have defined the background field constant $< \phi > = G_N^{-1}(1 + \alpha)$. A direct consequence of the approach is that the amount of matter (energy) has to be increased for positive values of α and diminished for negative values of α with respect to the standard ΛCDM model in order to have a flat cosmological model. Quantitatively, our model demands to have $\Omega/(1 + \alpha) = 1$ and this changes the amount of dark matter and energy of the model for a flat cosmological model, as assumed. The general gravitational effect is that the interaction including the SF changes by a factor $F_{SF}(r, \alpha, \lambda) \approx 1 + \alpha \left(1 + \frac{r}{\lambda}\right)$ for $r < \lambda$ in comparison with the Newtonian case. Thus, for $\alpha > 0$ the growth of structures speeds up in comparison with the Newtonian case. For the $\alpha < 0$ case the effect is to diminish the formation of structures. For $r > \lambda$ the dynamics is essentially Newtonian. However, this preliminar analysis we have done is insufficient to give us a clear conclusions on the role played by SF in the large-scale structure formation process. We will need to do a systematic study of the evolution of the two-point correlation function which is a mesure of galaxy clustering. We also will need to compute the mass power spectrum and velocity dispersions of the halos. Therefore, we will be able make sistematic comparisons with observations. This work is in process and will be published soon.

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