

# An Introduction to High Energy Nuclear Collisions

QCD under extreme conditions

*Jamal Jalilian-Marian*  
*Baruch College, City University of New York*

*many thanks to the organizers  
and  
colleagues whose slides I have “borrowed”*

## Fundamental particles and their interactions

QUARKS $S=1/2$		LEPTONS $S=1/2$		GAUGE BOSONS $S=1$
$Q = -2/3$	$Q = -1/3$	$Q = -1$	$Q = 0$	quanta
u u u $m=(1-4) 10^{-3}$	d d d $m=(5-8) 10^{-3}$	e $m=5.11 10^{-4}$	$\nu_e$ $m < 3 10^{-9}$	$g_1 \dots g_8$ $m < \text{a few } 10^{-3}$
c c c $m=1.0-1.4$	s s s $m=0.08-0.15$	$\mu$ $m=0.10566$	$\nu_\mu$ $m < 1.9 10^{-4}$	$\gamma$ $m < 2 10^{-25}$
t t t $m=174.3 \pm 5.1$	b b b $m=4.0-4.5$	$\tau$ $m=1.7770$	$\nu_\tau$ $m < 18.2 10^{-3}$	$W^\pm, Z^0$ $m_W = 80.432 \pm 0.39,$ $m_Z = 91.1876 \pm 0.0021$

All masses in GeV units

**QCD** →

**QED** ↗

Interaction	exchanged boson	relative strength	example
Strong	Gluon (g)	1	
Electromagnet.	Photon ( $\gamma$ )	$\frac{1}{137}$	
Weak	$W^\pm, Z^0$	$10^{-14}$	
Gravitation	Graviton (G) ?	$10^{-40}$	

# *An Introduction to High Energy Nuclear Collisions*

## *Lecture I: What is a nucleus?*

Deeply Inelastic Scattering (DIS), Parton Model, QCD, Structure Functions

## *Lecture II: What does a nucleus look like at high energy?*

QCD at small  $x$ , Renormalization Group, Saturation, Color Glass Condensate

## *Lecture III: Particle production in H.E. nuclear collisions*

Classical fields, particle production, Instability, Thermalization?

# WHY?

## ★ *Early Universe*

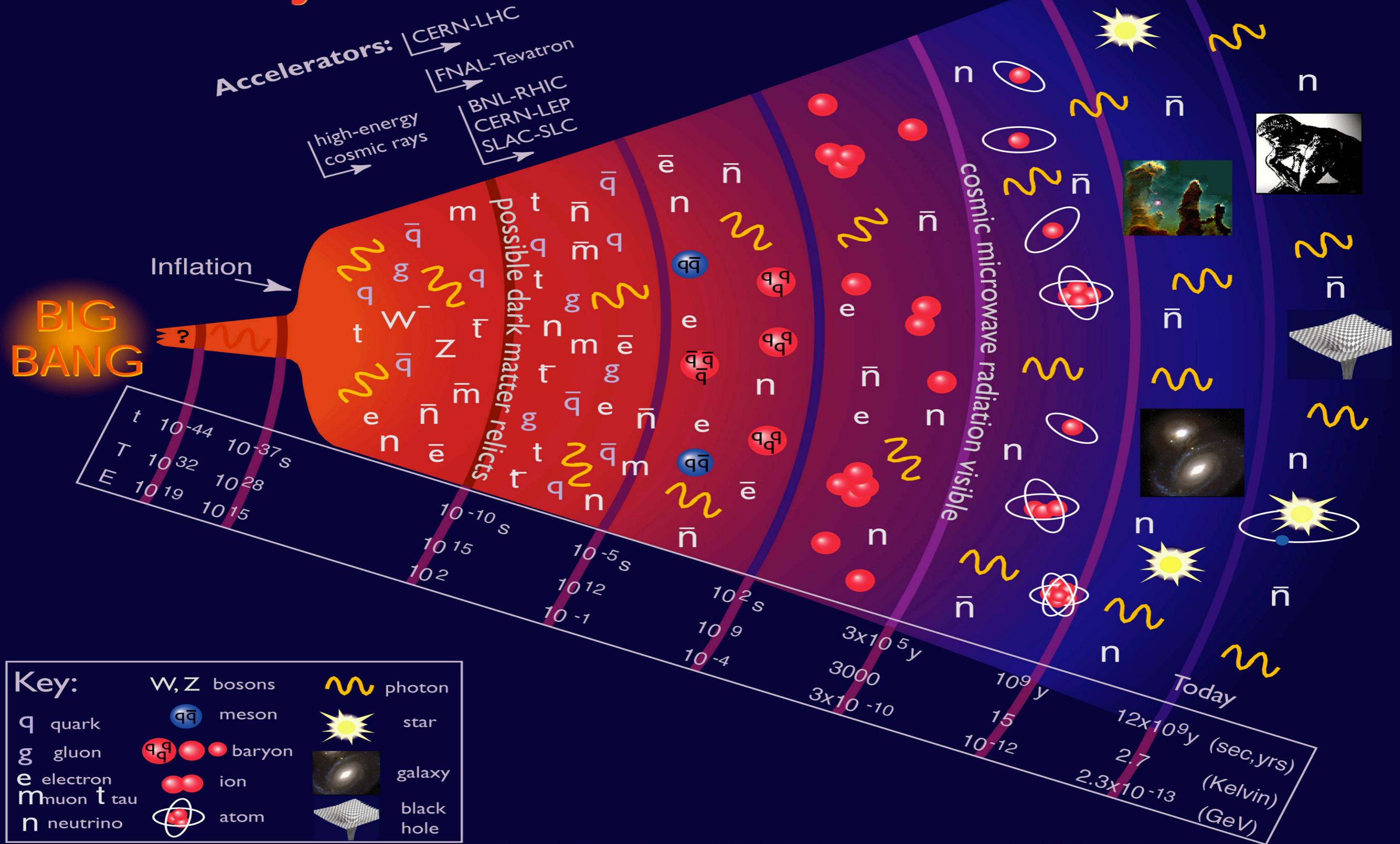
★ *A hot and dense soup (Quark-Gluon Plasma)*

★ *Can we recreate this in lab and study its properties ?*

★ *Strong interactions in the extreme limit*

★ *Can we probe/learn about new aspects of strong interactions ?*

# History of the Universe



Particle Data Group, LBNL, © 2000. Supported by DOE and NSF

Few microseconds after the Big Bang the entire Universe was in a QGP state

# *WHY?*

## ★ *Ultra-high energy cosmic rays*

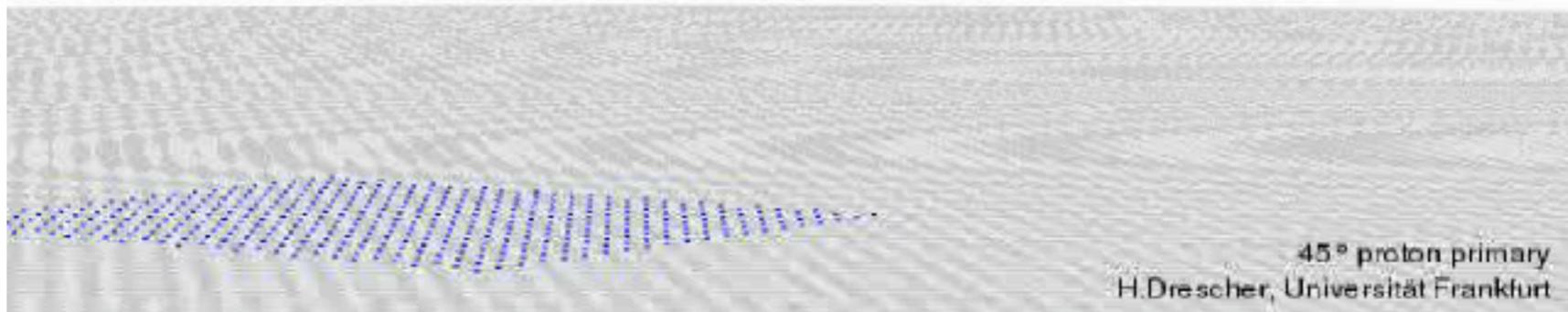
★ *What is their source?*

★ *A window into new physics ?*

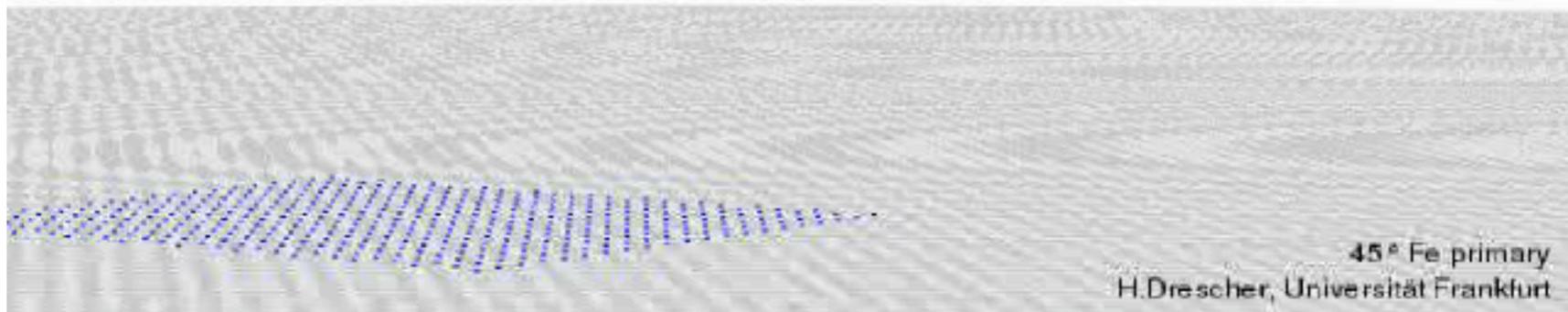
★ *How do they interact?*

★ *Can they shed light on extreme limit of strong interactions ?*

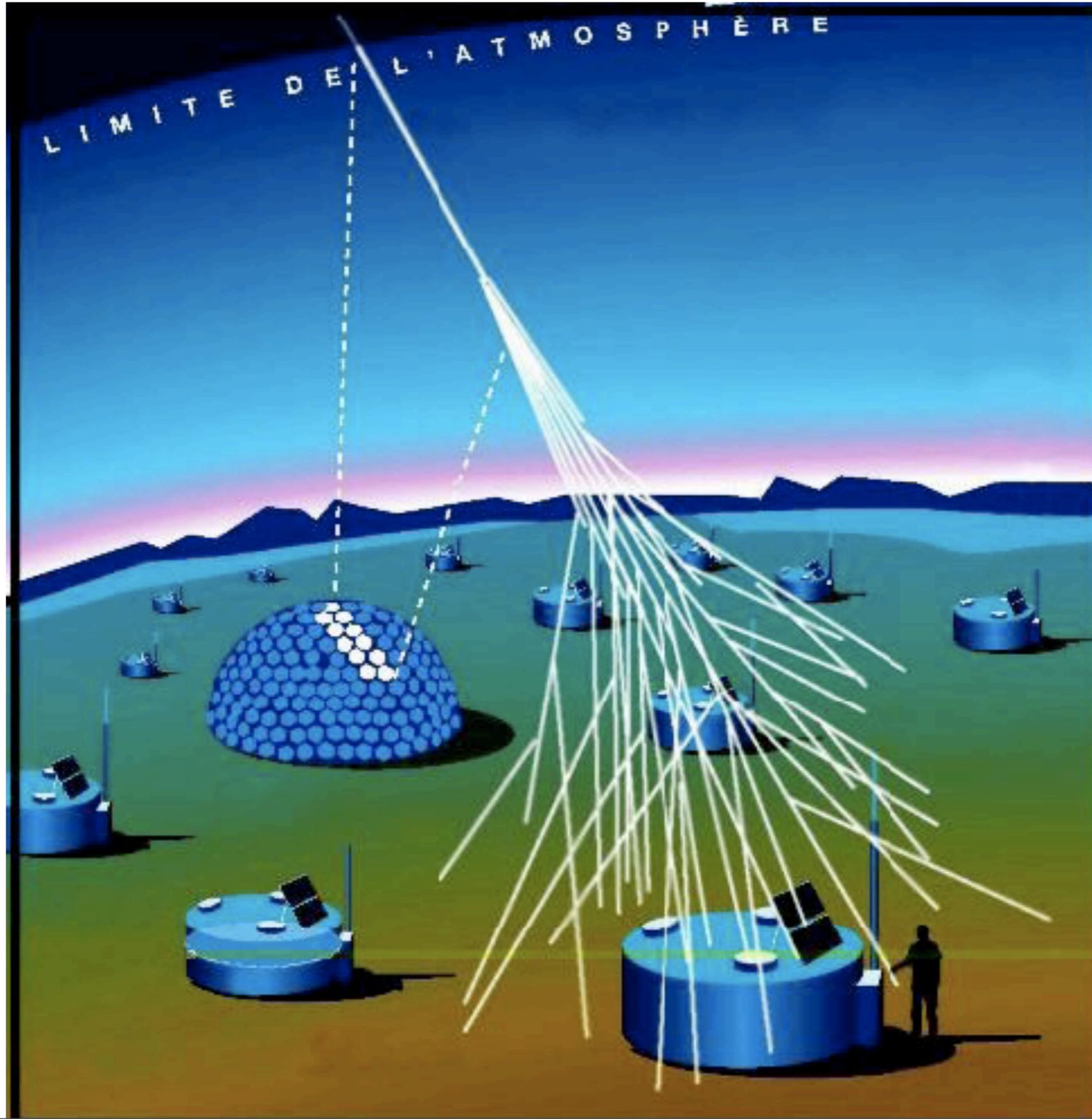
time=-266 $\mu$ s



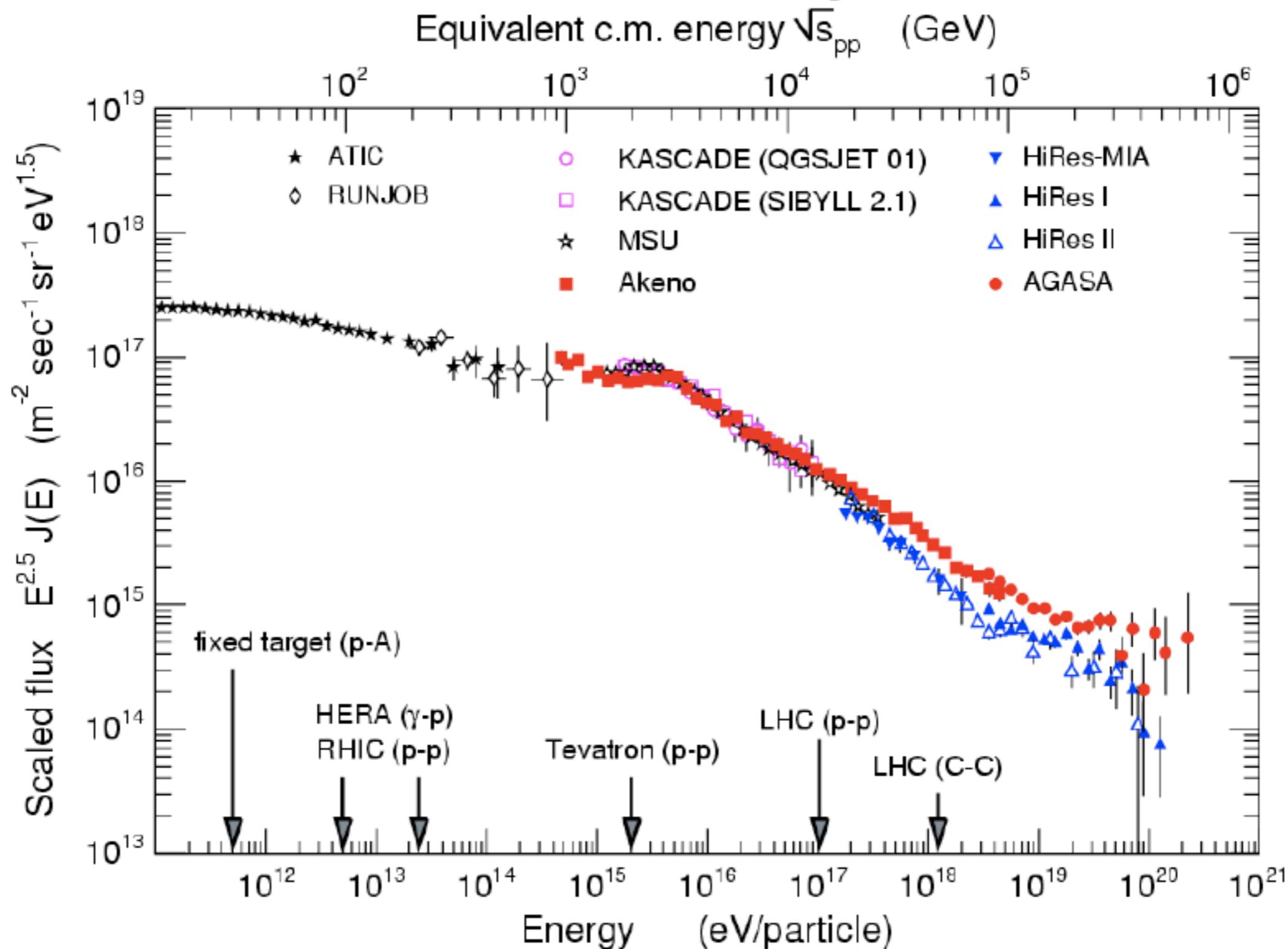
time=-266 $\mu$ s



# *Ultra-High Energy Cosmic Rays*



# The flux of cosmic rays on earth

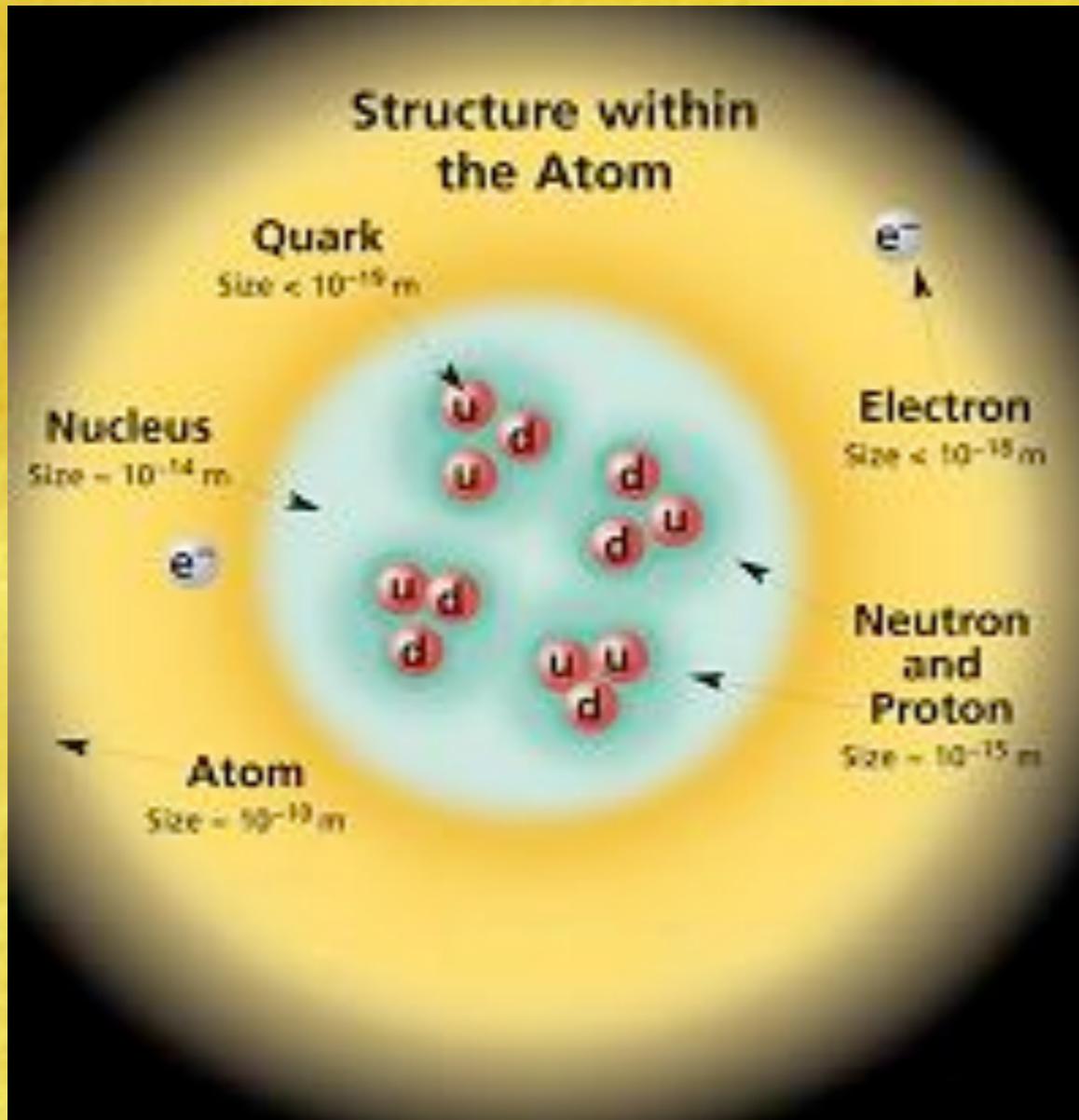


# **Quantum ChromoDynamics (QCD)**

*the theory of strong interactions: hadrons, nuclei*

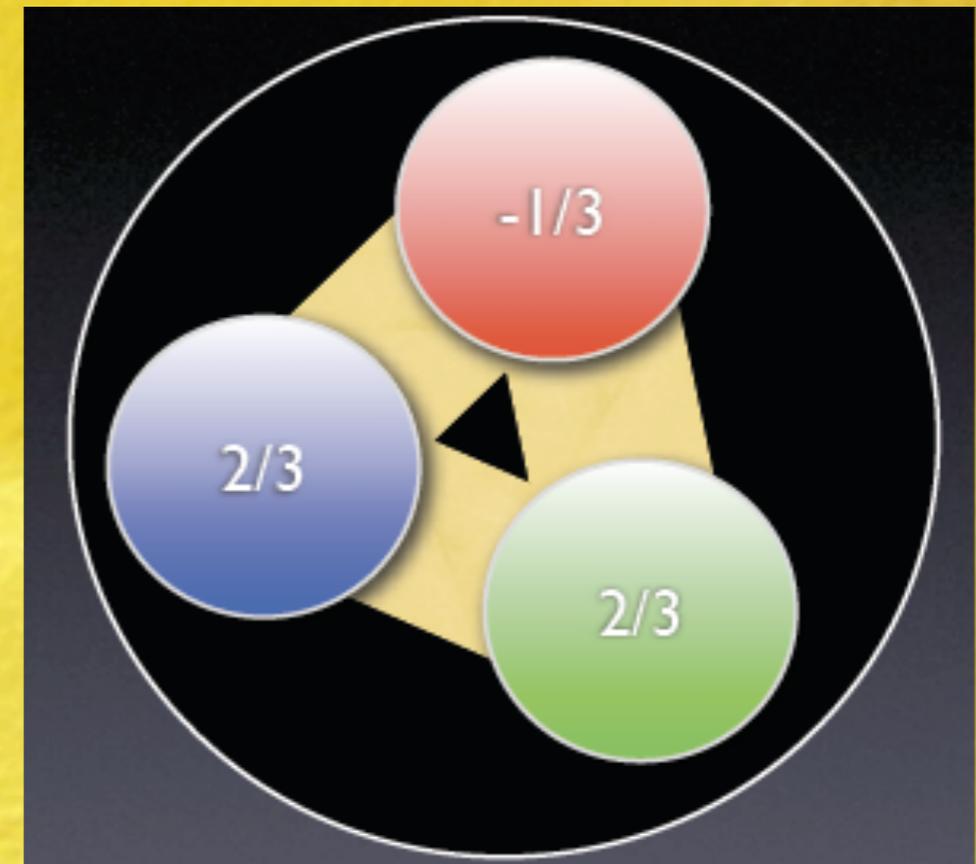
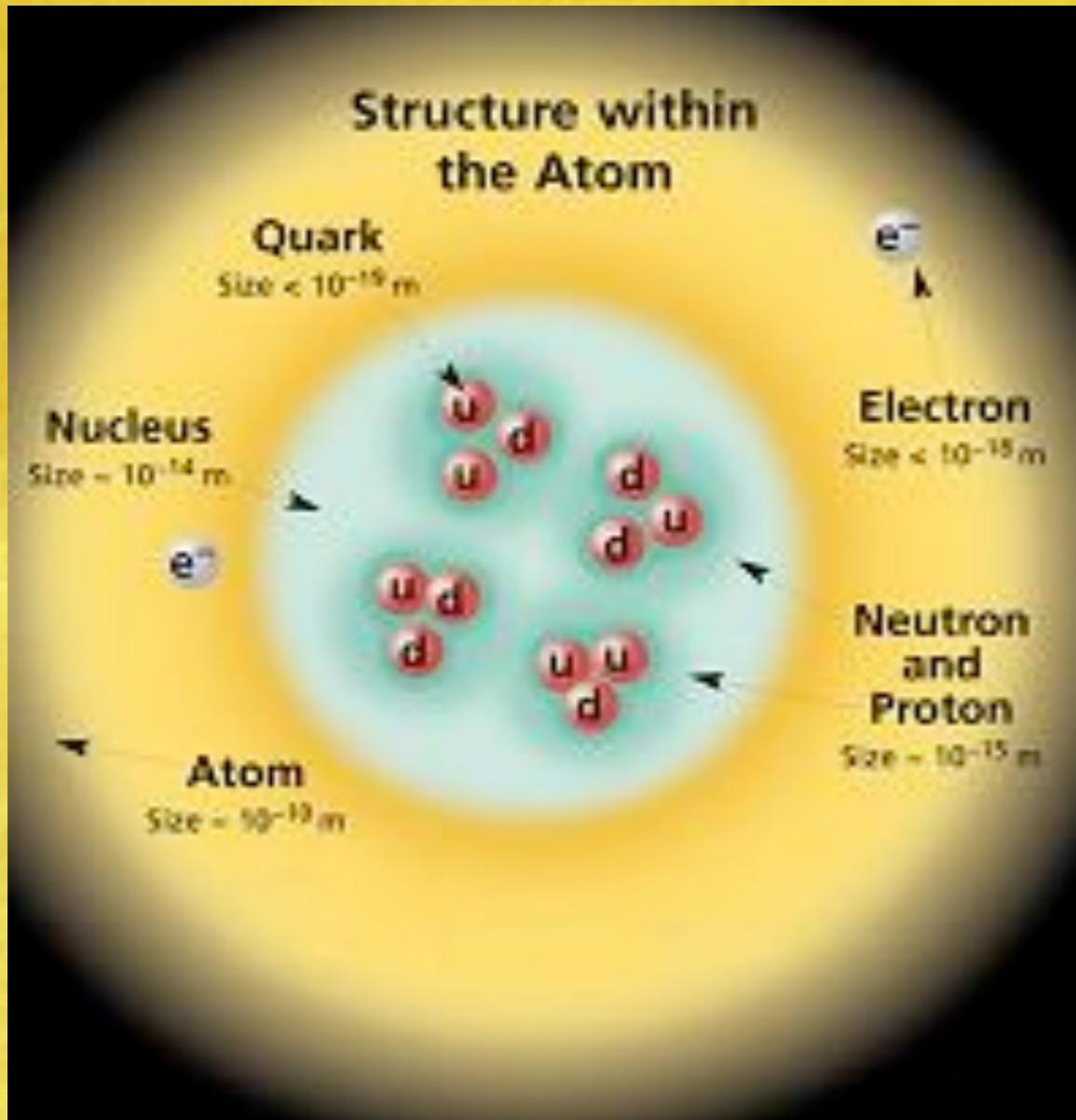
# Quantum ChromoDynamics (QCD)

*the theory of strong interactions: hadrons, nuclei*

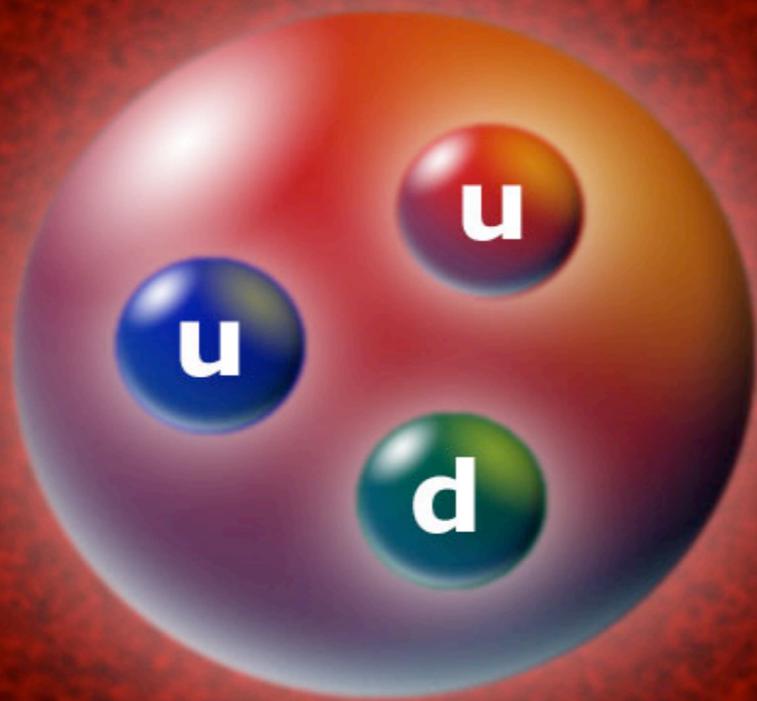


# Quantum ChromoDynamics (QCD)

*the theory of strong interactions: hadrons, nuclei*



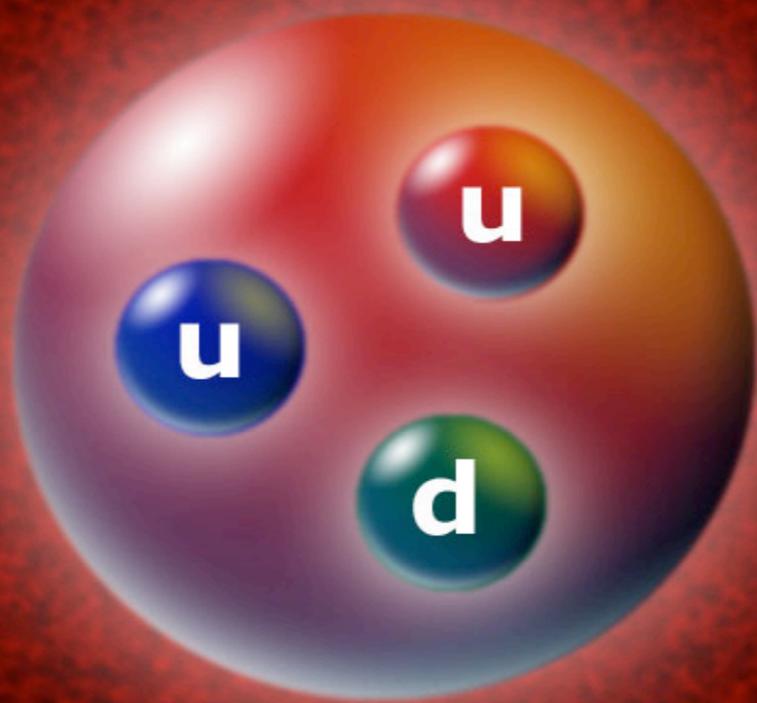
# *pre-QCD: Quarks*



**proton**



# *pre-QCD: Quarks*

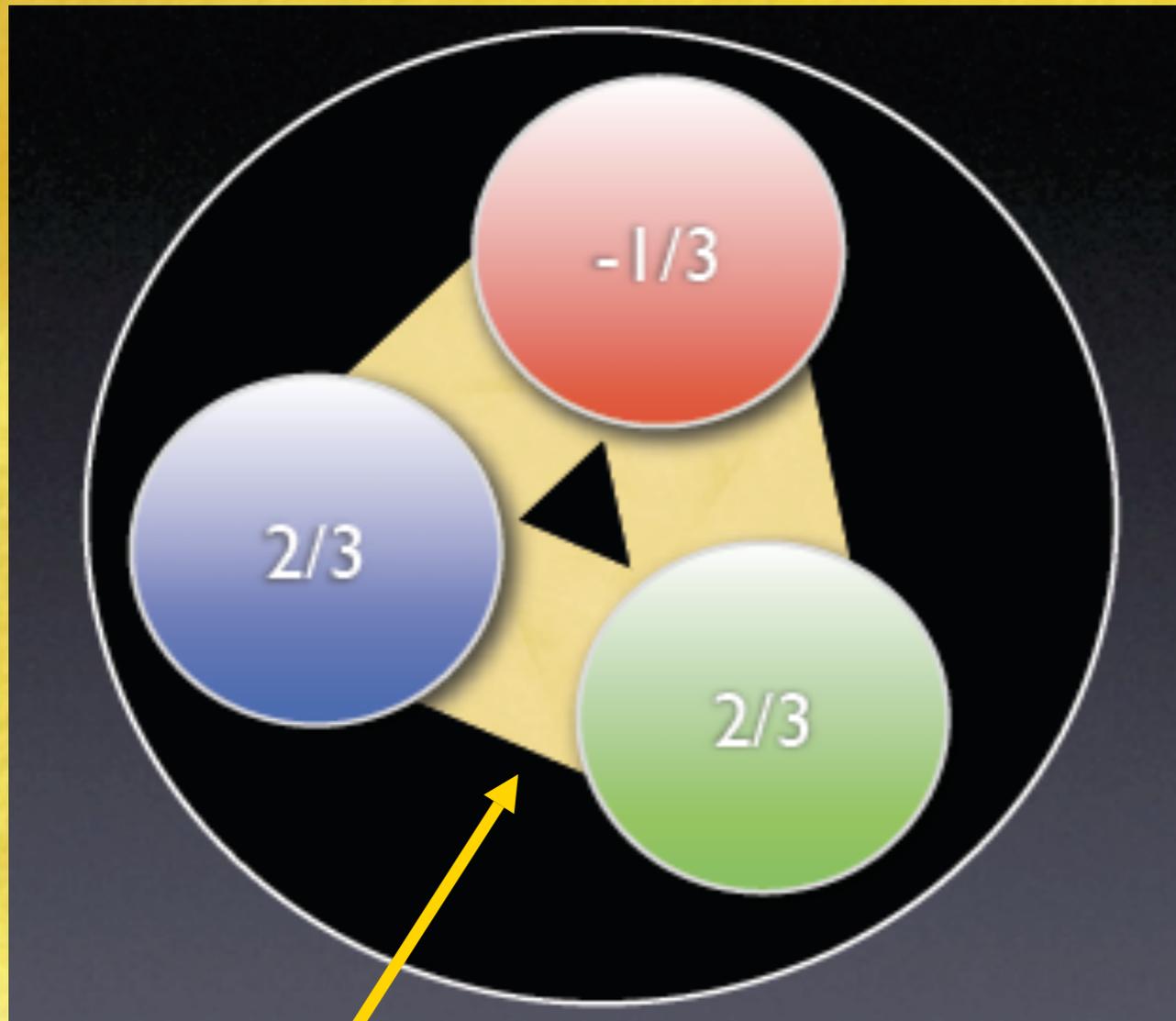


**proton**



**Gell-Mann and Ne'eman**

# *pre-QCD: Yang-Mills*



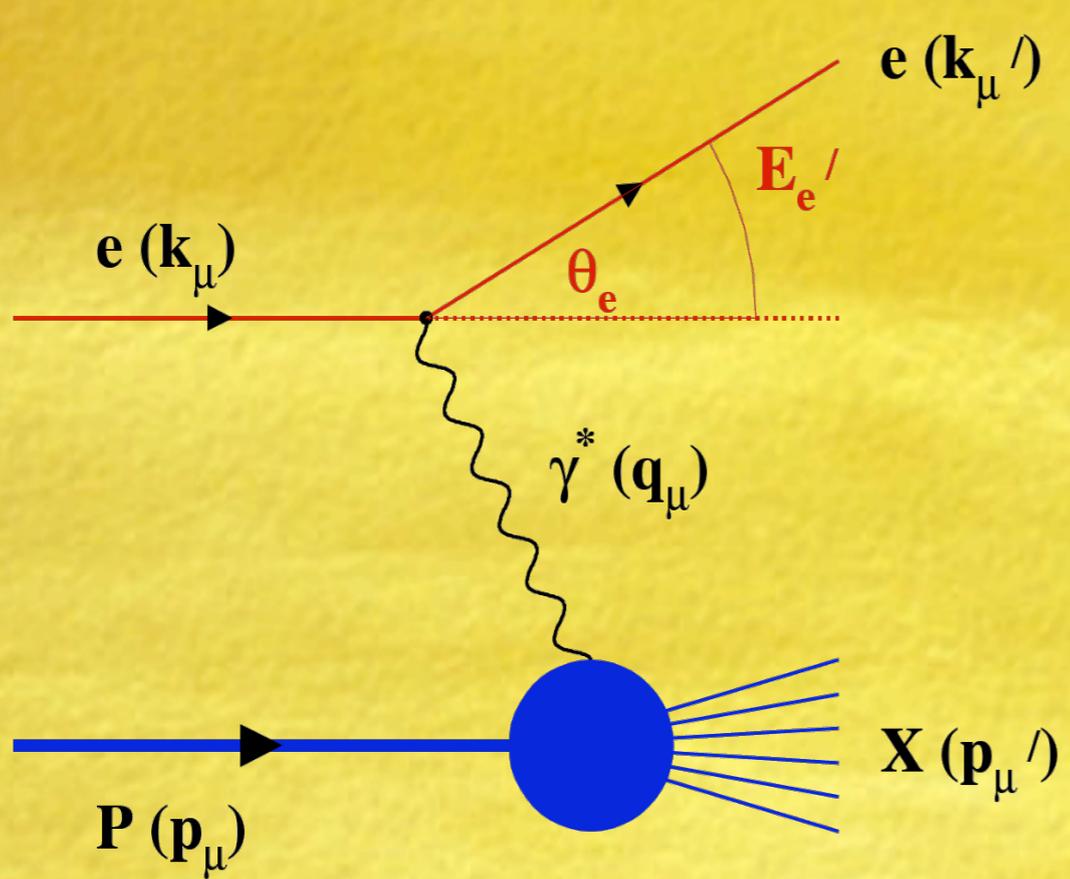
gluons



Yang and Mills

# Deeply Inelastic Scattering (DIS)

probing hadron structure



$$Q^2 = -q^2 = -(k_\mu - k'_\mu)^2$$

$$Q^2 = 4E_e E'_e \sin^2\left(\frac{\theta'_e}{2}\right)$$

$$y = \frac{pq}{pk} = 1 - \frac{E'_e}{E_e} \cos^2\left(\frac{\theta'_e}{2}\right)$$

$$x = \frac{Q^2}{2pq} = \frac{Q^2}{sy}$$

Measure of resolution power

Measure of inelasticity

Measure of momentum fraction of struck quark

# ***DIS***

$$\frac{d\sigma}{dx dQ^2} \propto L_{\mu\nu} W^{\mu\nu}$$

**Hadronic tensor:**

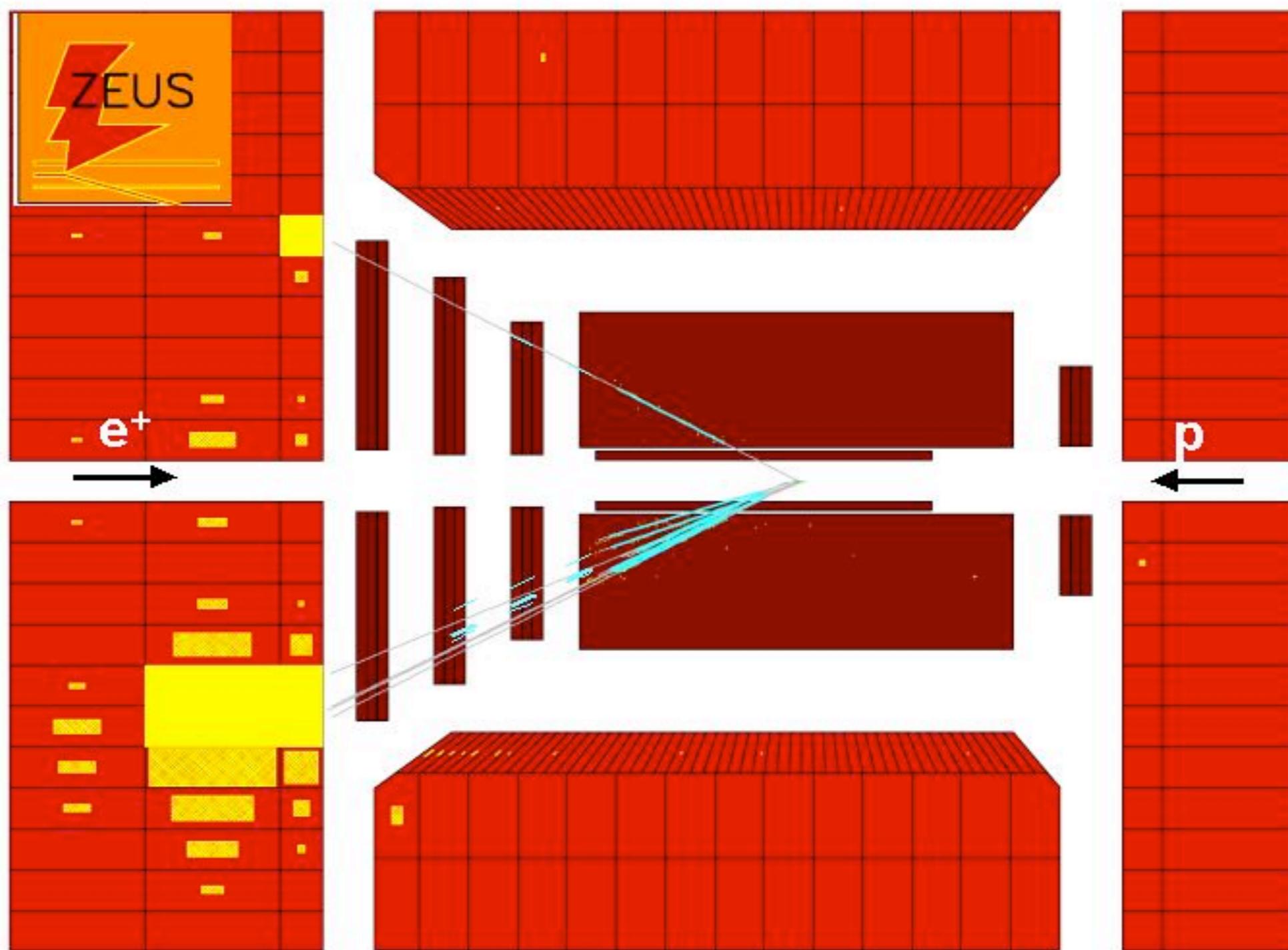
$$W^{\mu\nu} = 2 \text{ Disc. } T^{\mu\nu}(q^2, P \cdot q) = \frac{1}{2\pi} \text{Im.} \int d^4x e^{iq \cdot x} \langle P | T(J^\mu(x) J^\nu(0)) | P \rangle$$

$$J^\mu(x) = \bar{\psi} \gamma^\mu \psi$$

hadronic tensor can be written in terms of two structure functions,  $F_2$ ,  $F_L$

$$\frac{d^2\sigma^{eh \rightarrow eX}}{dx dQ^2} = \frac{4\pi\alpha_{em}^2}{xQ^4} \left[ \left( 1 - y + \frac{y^2}{2} \right) F_2(x, Q^2) - \frac{y^2}{2} F_L(x, Q^2) \right]$$

# A Deep Inelastic electron-proton Scattering event



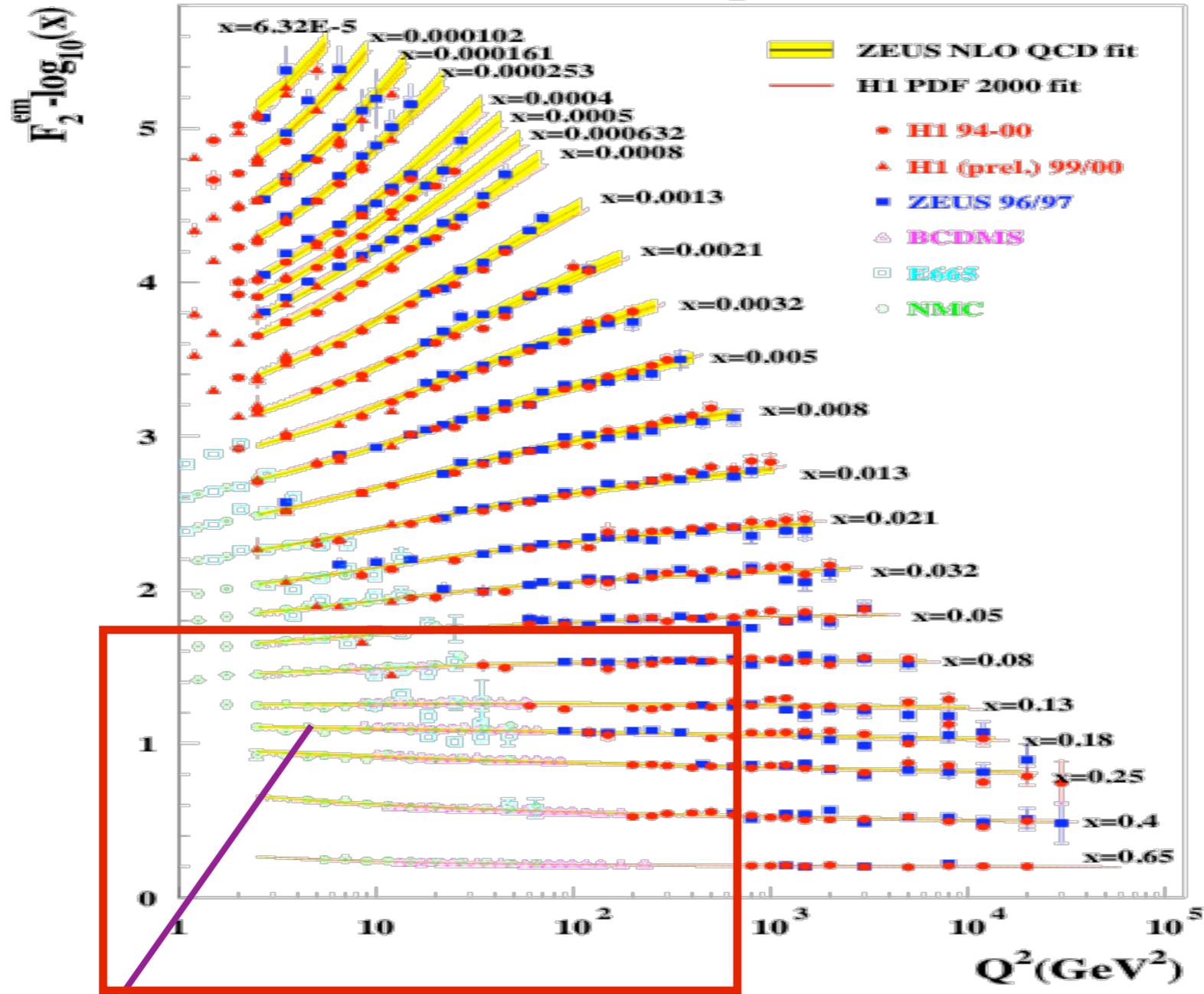
**ZEUS at DESY**



# Nobel to Friedman, Kendall, Taylor



HERA  $F_2$



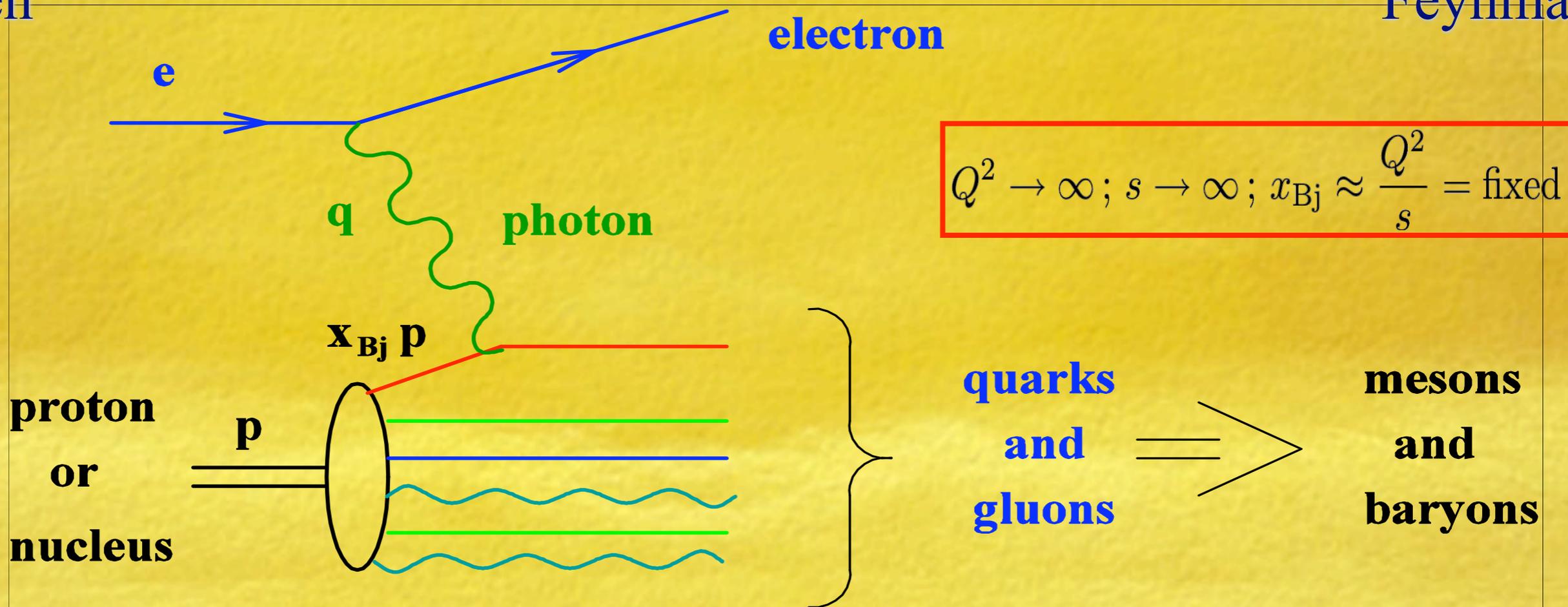
**Bj-scaling: apparent scale invariance of structure functions**



# Parton Model: Bjorken & Feynman

Bjorken

Feynman

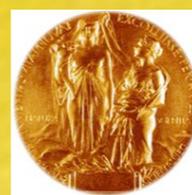


Parton constituents of hadron are "quasi-free" on time scale  $1/q$

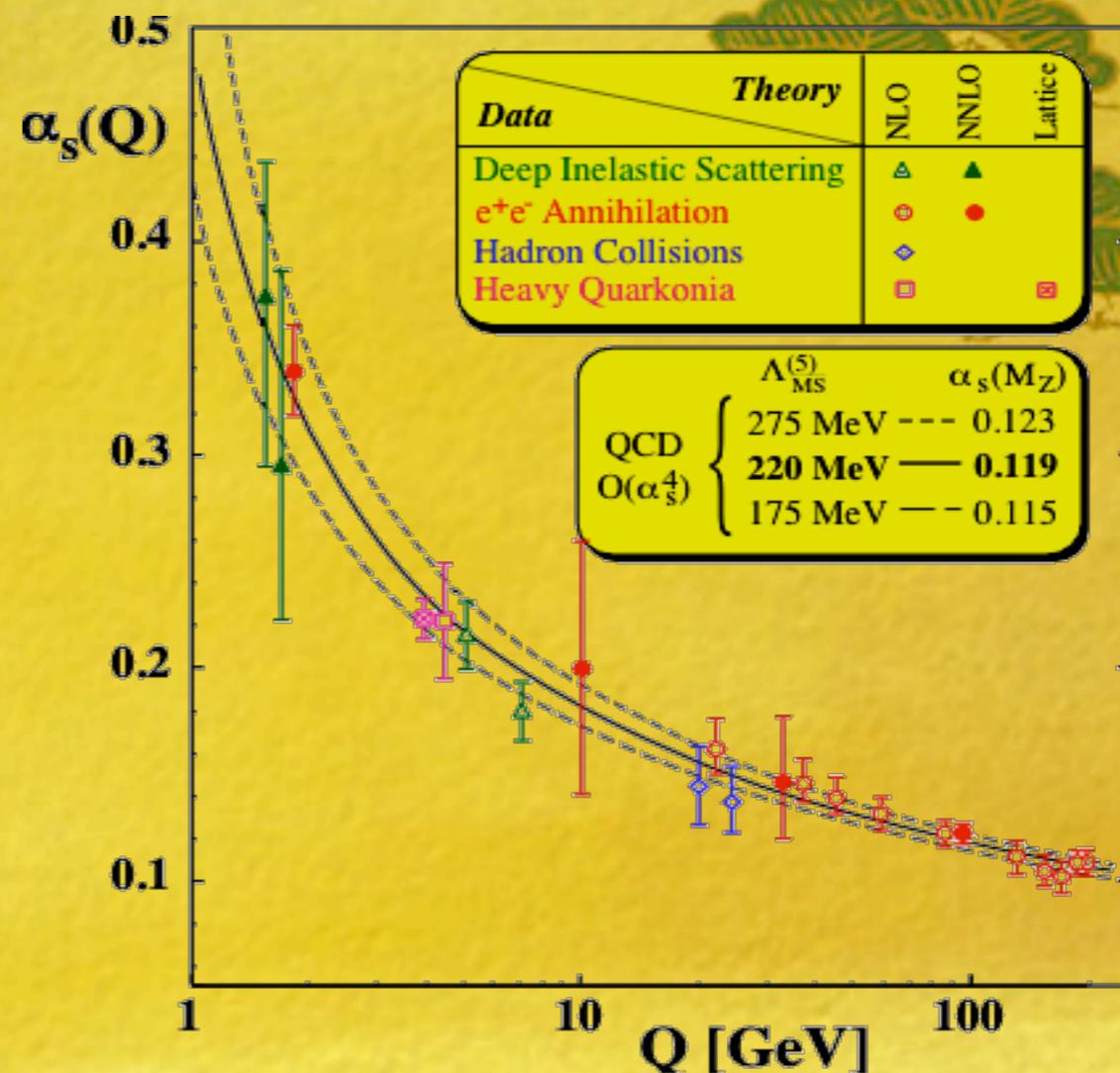
$x_{Bj}$  = Fraction of hadron momentum carried by a parton

$$F_2 \sim e^2 [xq(x) + x\bar{q}(x)]$$

# Asymptotic Freedom



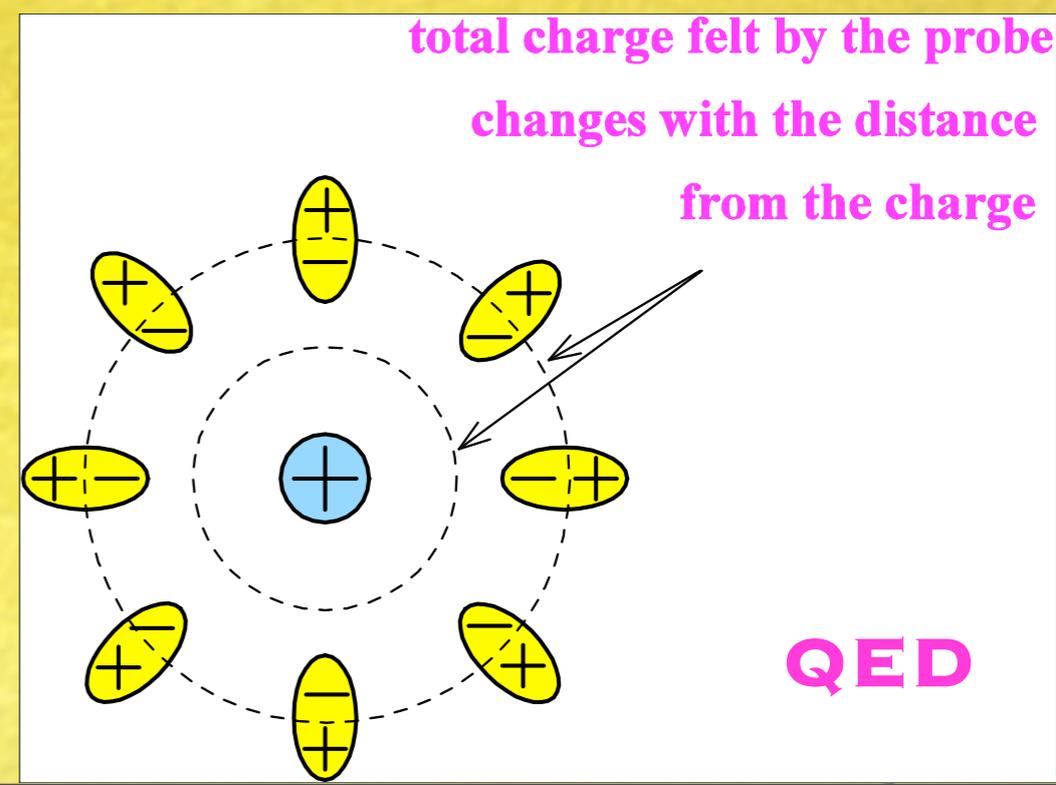
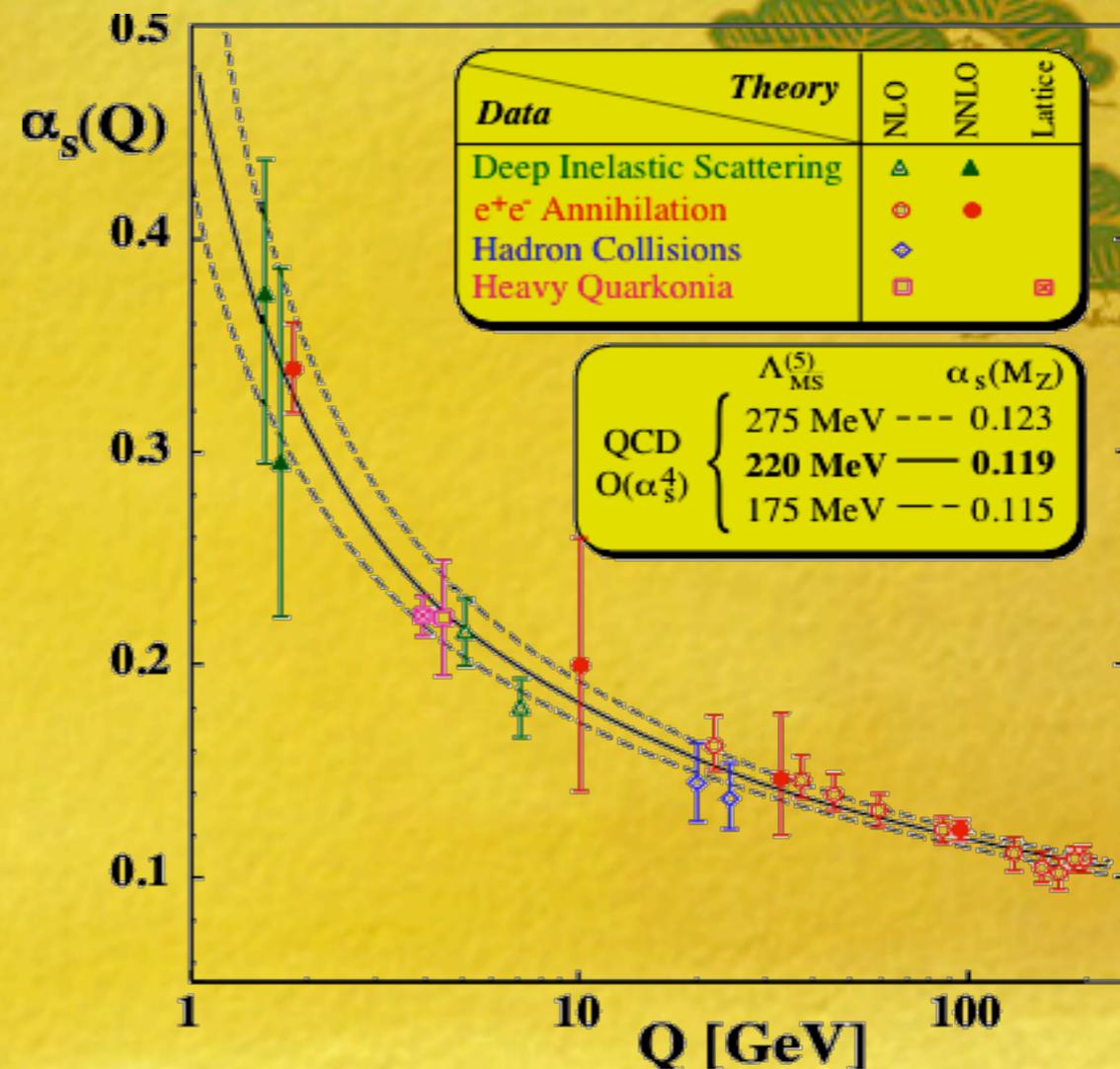
Gross, Wilczek, Politzer



# Asymptotic Freedom



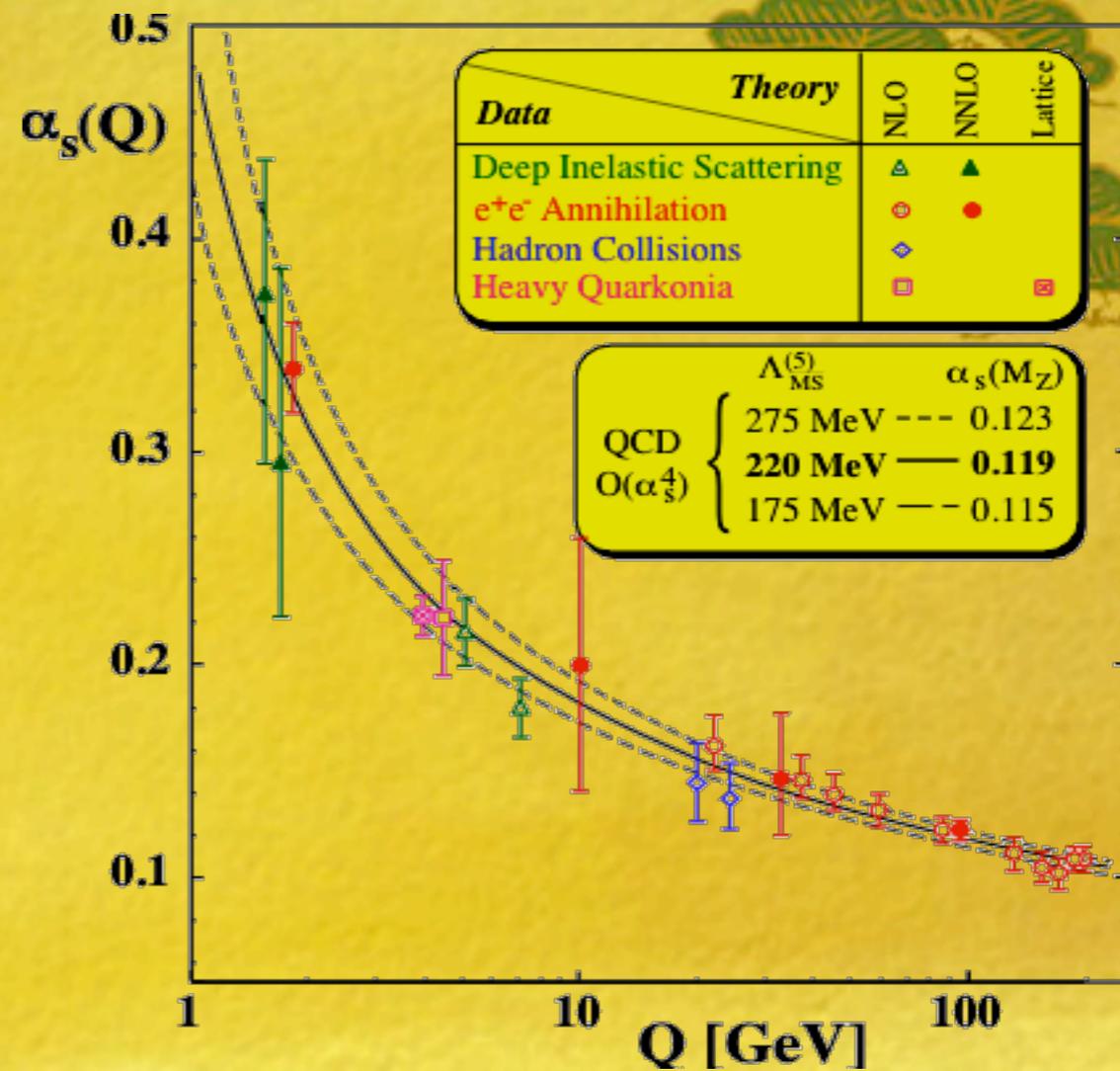
Gross, Wilczek, Politzer



# Asymptotic Freedom



Gross, Wilczek, Politzer



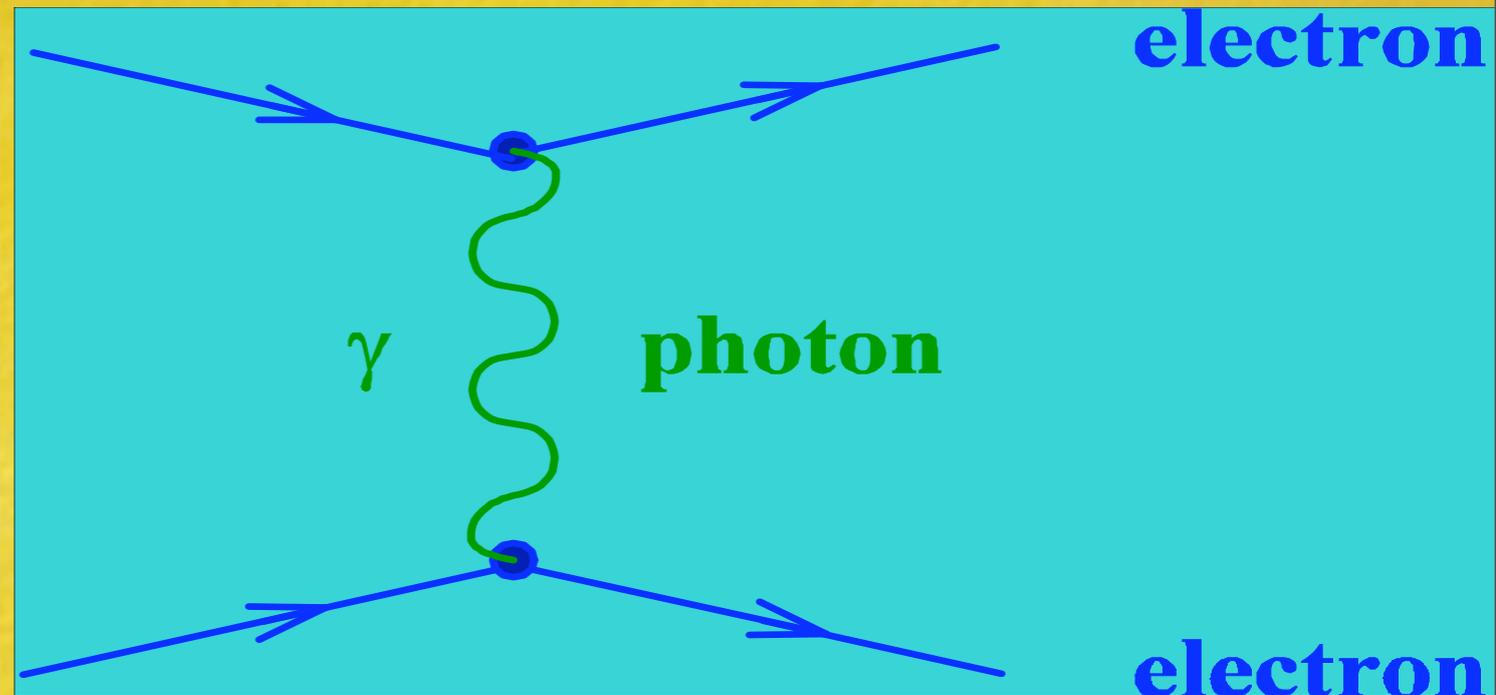
$$\begin{aligned}
 \mathcal{L}_{\text{QCD}} &= \bar{\psi}_i (i\gamma^\mu (D_\mu)_{ij} - m \delta_{ij}) \psi_j - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu} \\
 &= \bar{\psi}_i (i\gamma^\mu \partial_\mu - m) \psi_i - g G_\mu^a \bar{\psi}_i \gamma^\mu T_{ij}^a \psi_j - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu}
 \end{aligned}$$

$$G_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a - g f^{abc} G_\mu^b G_\nu^c$$

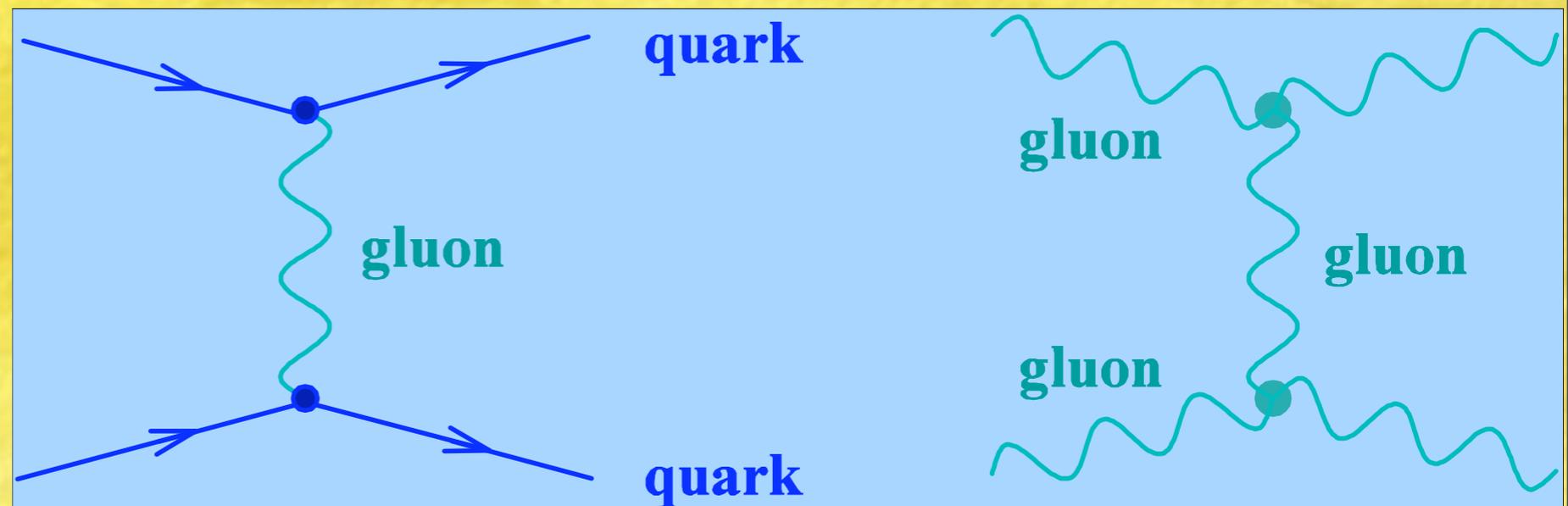
# Perturbative QCD

expansion in  $\alpha_s$

**QED:**  
photons don't interact



**QCD:**  
gluons interact



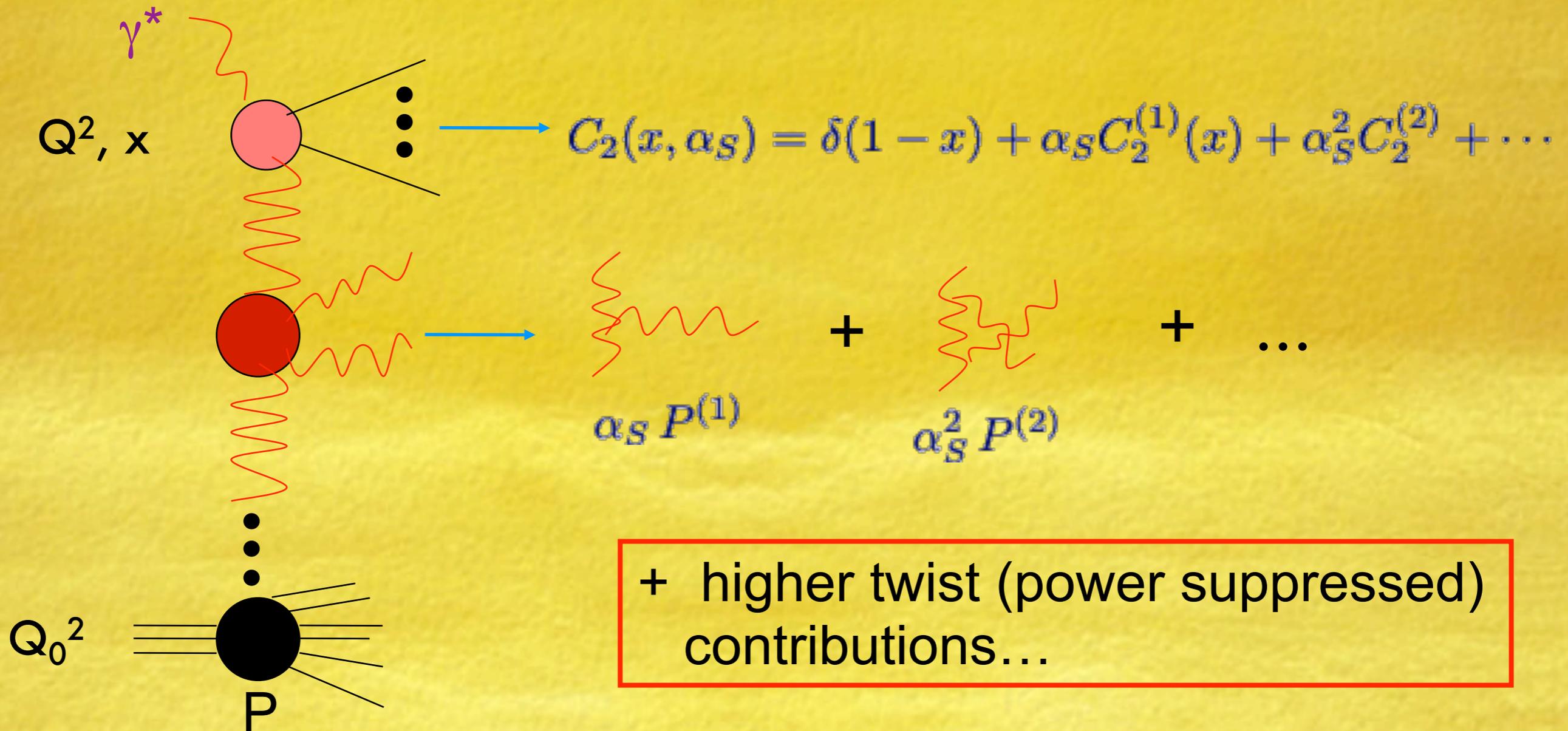
# *The Bjorken limit*



$$Q^2 \rightarrow \infty; s \rightarrow \infty; x_{\text{Bj}} \approx \frac{Q^2}{s} = \text{fixed}$$

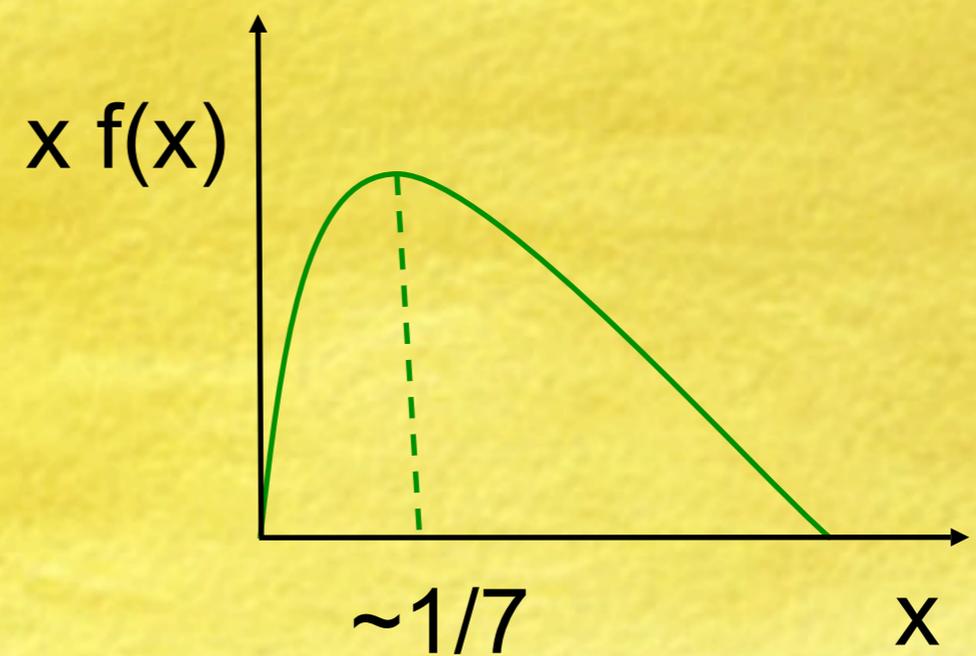
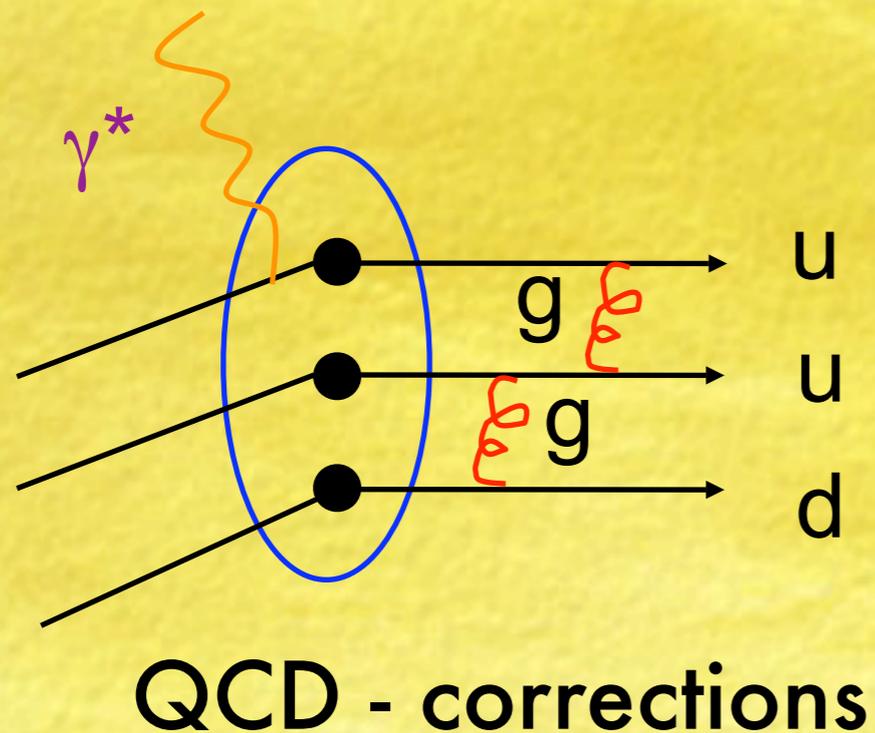
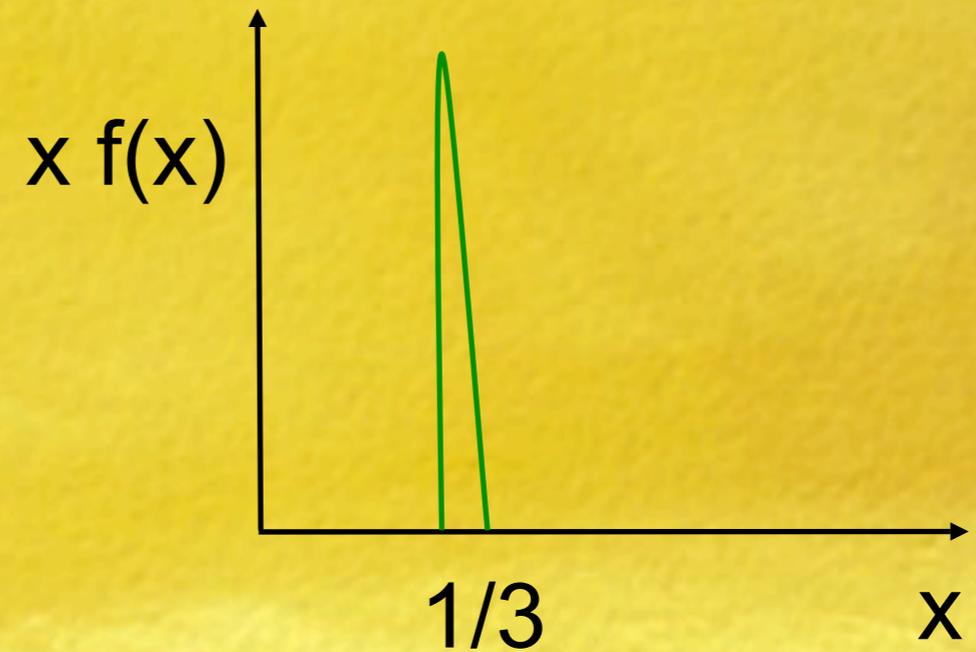
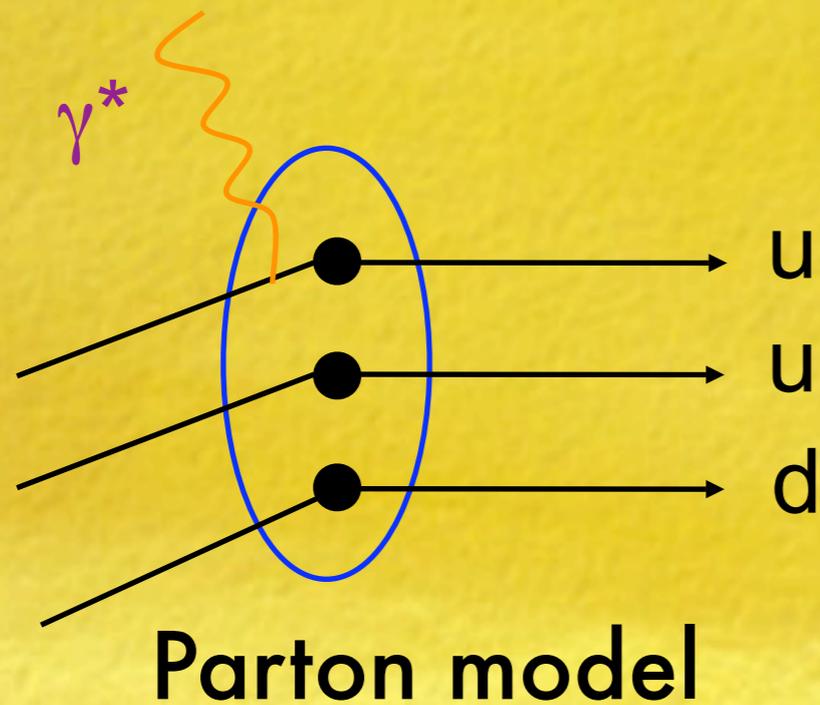
- \* Operator Product Expansion (OPE)  
Factorization theorems  
machinery of precision physics in QCD

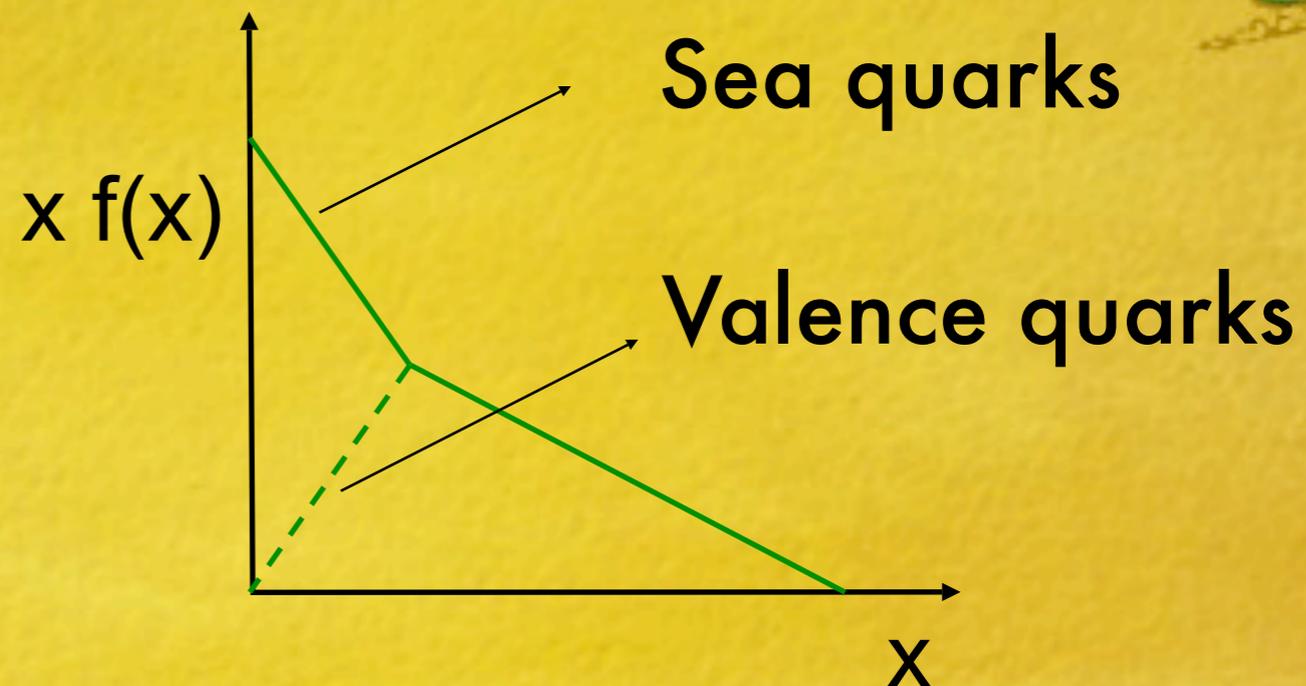
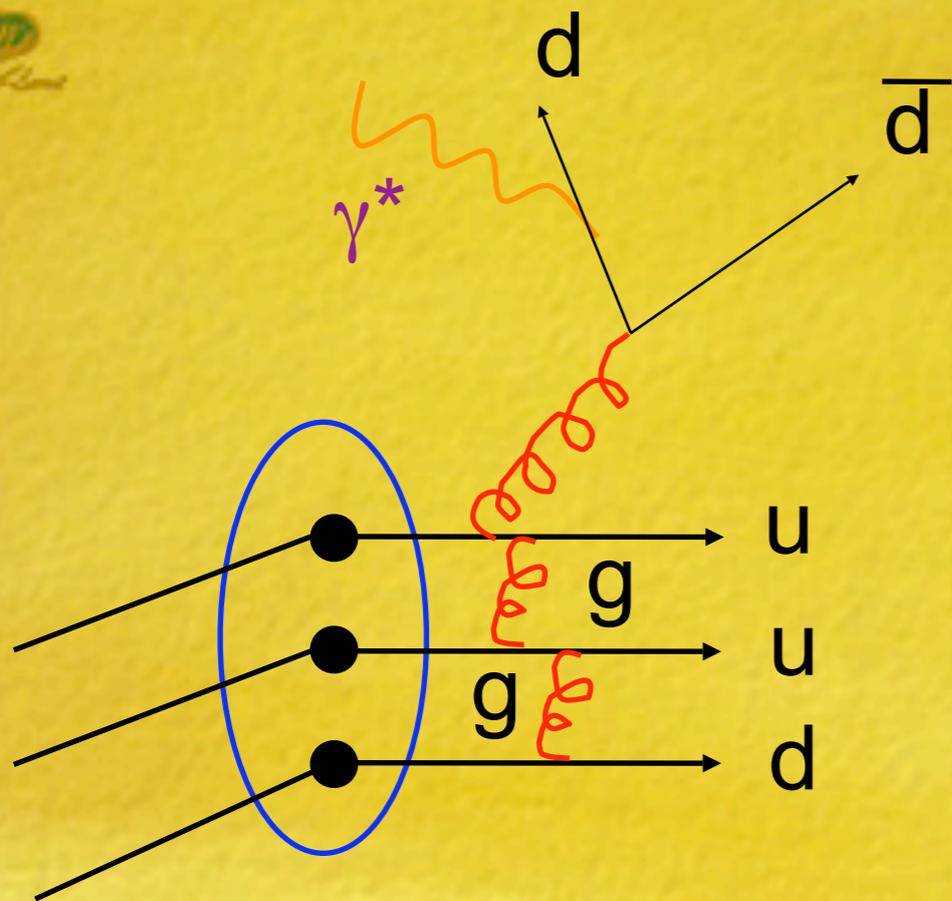
# Structure of higher order contributions in pQCD



- ★ Coefficient functions C - computed to NNLO for many processes
- ★ Splitting functions P - computed to 3-loops

# Proton structure functions





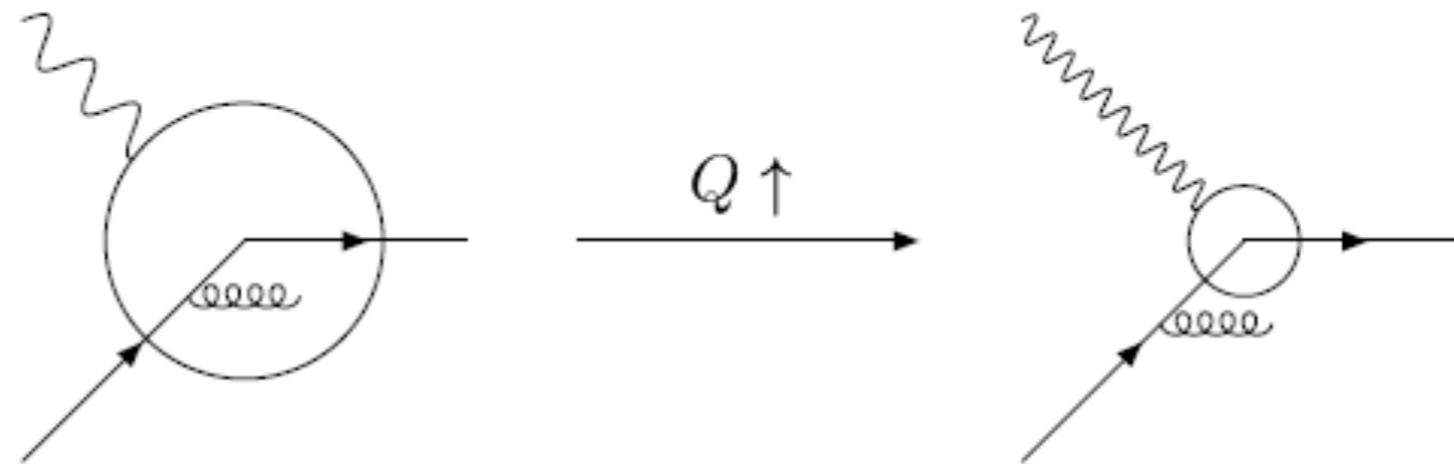
"x-QCD" - small \$x\$ evolution

$$\int_0^1 \frac{dx}{x} (xq(x) - x\bar{q}(x)) = 3 \longrightarrow \# \text{ of valence quarks}$$

$$\int_0^1 \frac{dx}{x} (xq(x) + x\bar{q}(x)) \rightarrow \infty \longrightarrow \# \text{ of quarks}$$

# ***DIS: resolution scale***

- As  $Q$  increases, the photon sees more and more of the inner structure of the nucleus  $A$



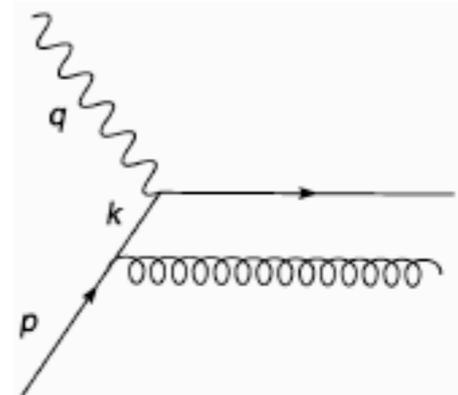
- Thus at higher  $Q$ , more constituents with smaller values of the Feynman  $x$
- A rapid increase of  $G(x, Q^2)$  at small  $x$  is measured

# Structure functions: DGLAP

One gluon emission gives the logarithmic  $Q^2$  dependence

$$\alpha_s \int^{Q^2} \frac{dk_{\perp}^2}{k_{\perp}^2} \sim \alpha_s \ln Q^2$$

$$\rightarrow q(x, Q^2) \sim \frac{\alpha_s}{2\pi} \left( P(x) \ln Q^2 + \dots \right)$$



Change of the resolution scale  $Q^2$

$\rightarrow$  evolution equation for the parton distribution functions

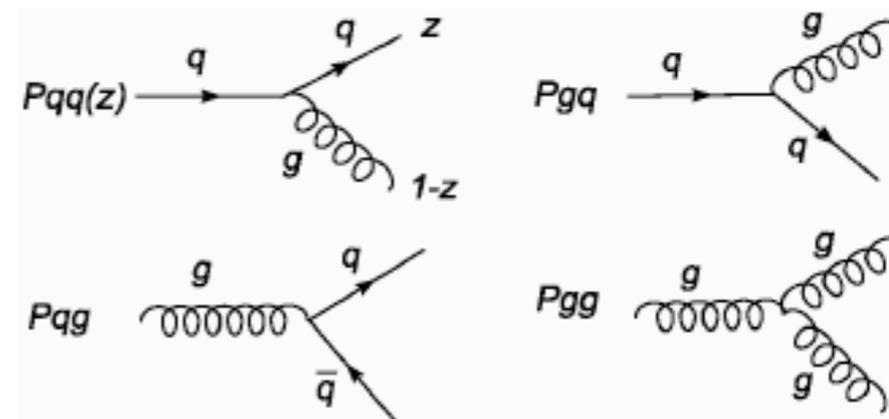
**DGLAP equation** (Dokshitzer-Gribov-Lipatov-Altarelli-Parisi)

$$Q^2 \frac{\partial}{\partial Q^2} \begin{pmatrix} q_i(x, Q^2) \\ g(x, Q^2) \end{pmatrix} = \frac{\alpha_s}{2\pi} \sum_{q_j, \bar{q}_j} \int_x^1 \frac{dz}{z} \begin{pmatrix} P_{q_i q_j}(\frac{x}{z}) & P_{q_i g}(\frac{x}{z}) \\ P_{g q_j}(\frac{x}{z}) & P_{g g}(\frac{x}{z}) \end{pmatrix} \begin{pmatrix} q_j(z, Q^2) \\ g(z, Q^2) \end{pmatrix}$$

“Splitting function”  $P_{ij}(x)$ :

a probability of finding a parton of type “ $i$ ” in a parton of type “ $j$ ”.

The equation for the change of “transverse resolution”



# DGLAP at small $x$

Splitting functions at leading order  $O(\alpha_s^0)$   $(x \neq 1)$

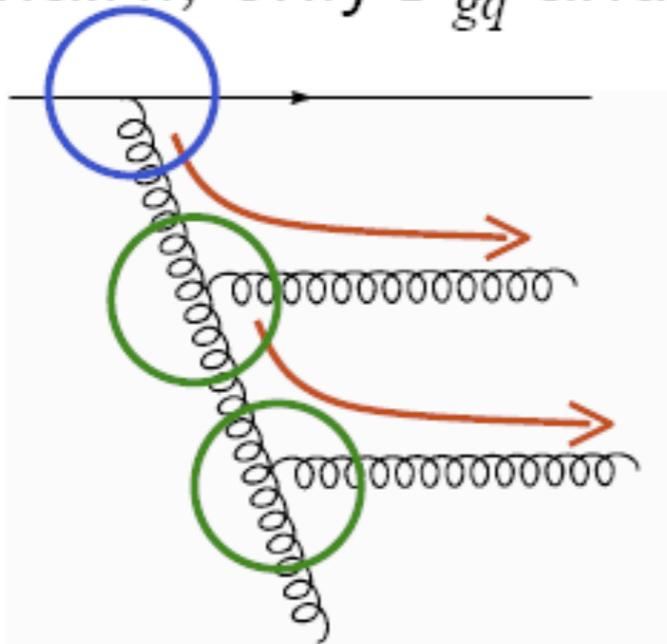
$$P_{qq}^{(0)}(x) = C_F \frac{1+x^2}{1-x}$$

$$P_{qg}^{(0)}(x) = \frac{1}{2} [x^2 + (1-x)^2]$$

$$P_{gq}^{(0)}(x) = C_F \frac{1+(1-x)^2}{x}$$

$$P_{gg}^{(0)}(x) = 2C_A \left[ \frac{x}{1-x} + \frac{1-x}{x} + x(1-x) \right]$$

At small  $x$ , only  $P_{gq}$  and  $P_{gg}$  are relevant.

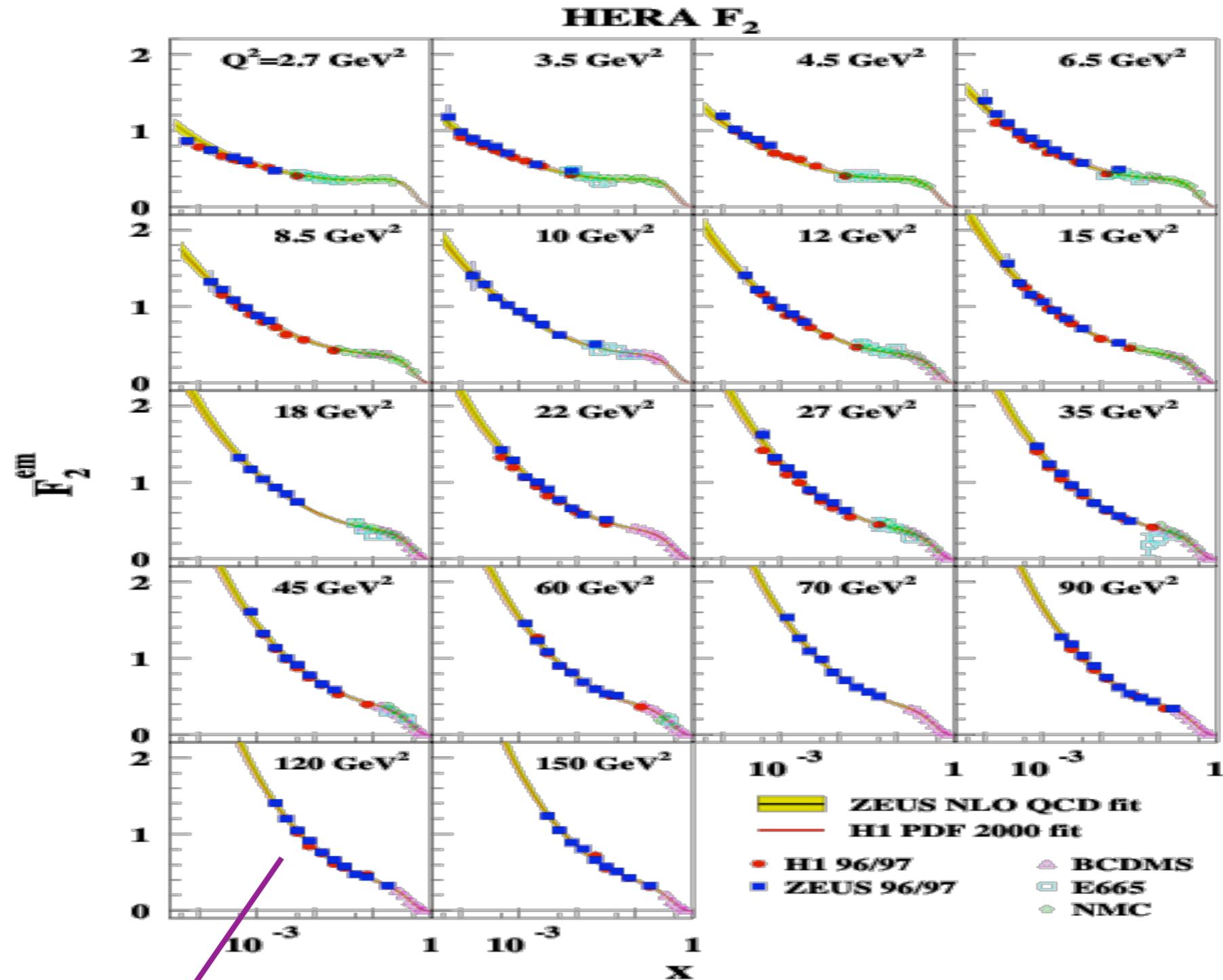


→ **Gluon dominant at small  $x$ !**

The double log approximation (DLA) of DGLAP is easily solved.

-- increase of gluon distribution at small  $x$

$$F_2(x, Q^2) = \sum_{\substack{q=u,c,t \\ d,s,b}} e_q^2 (x q(x, Q^2) + x \bar{q}(x, Q^2))$$



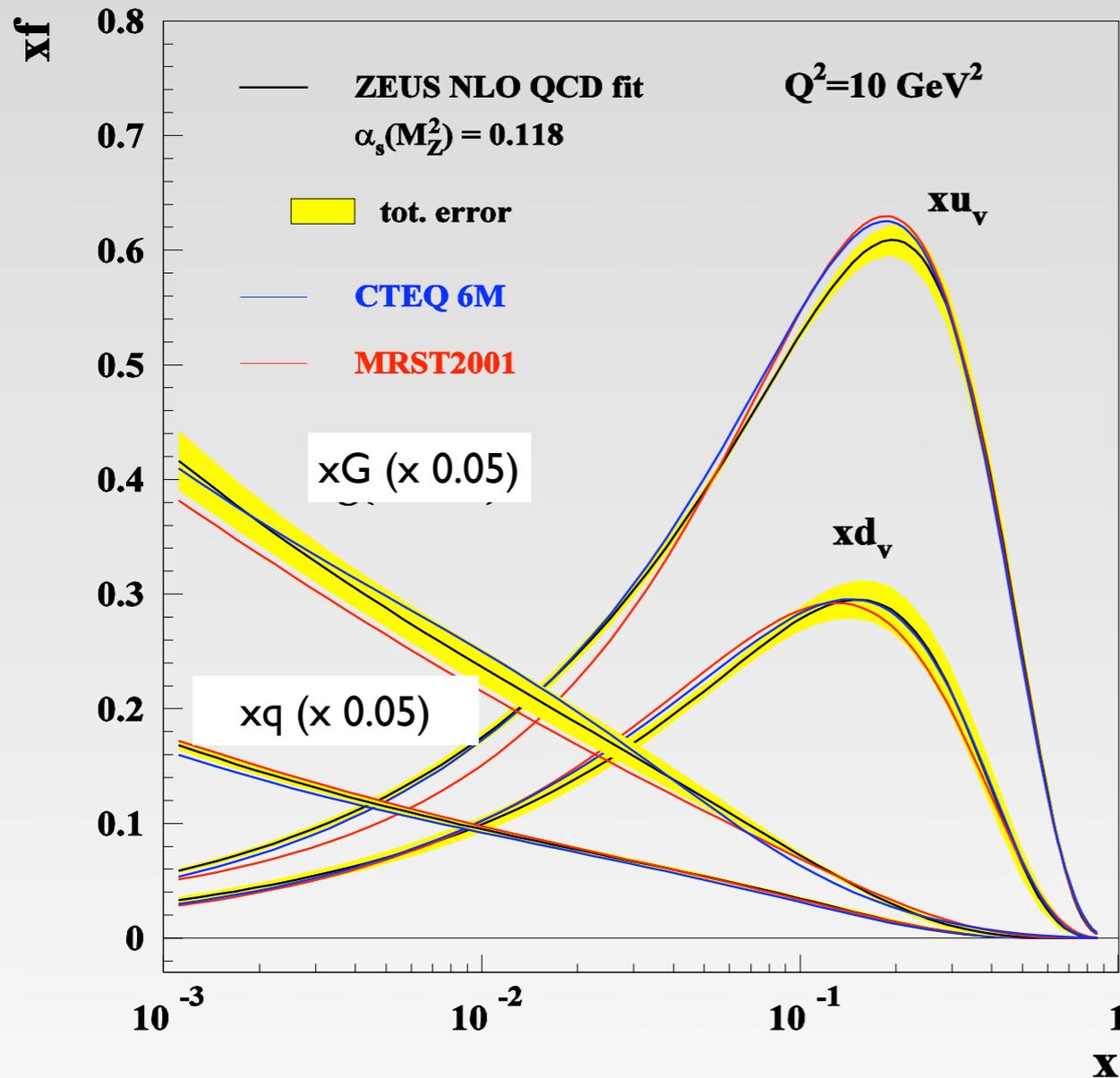
Structure functions grow rapidly at small  $x$

# What have we learned at HERA?

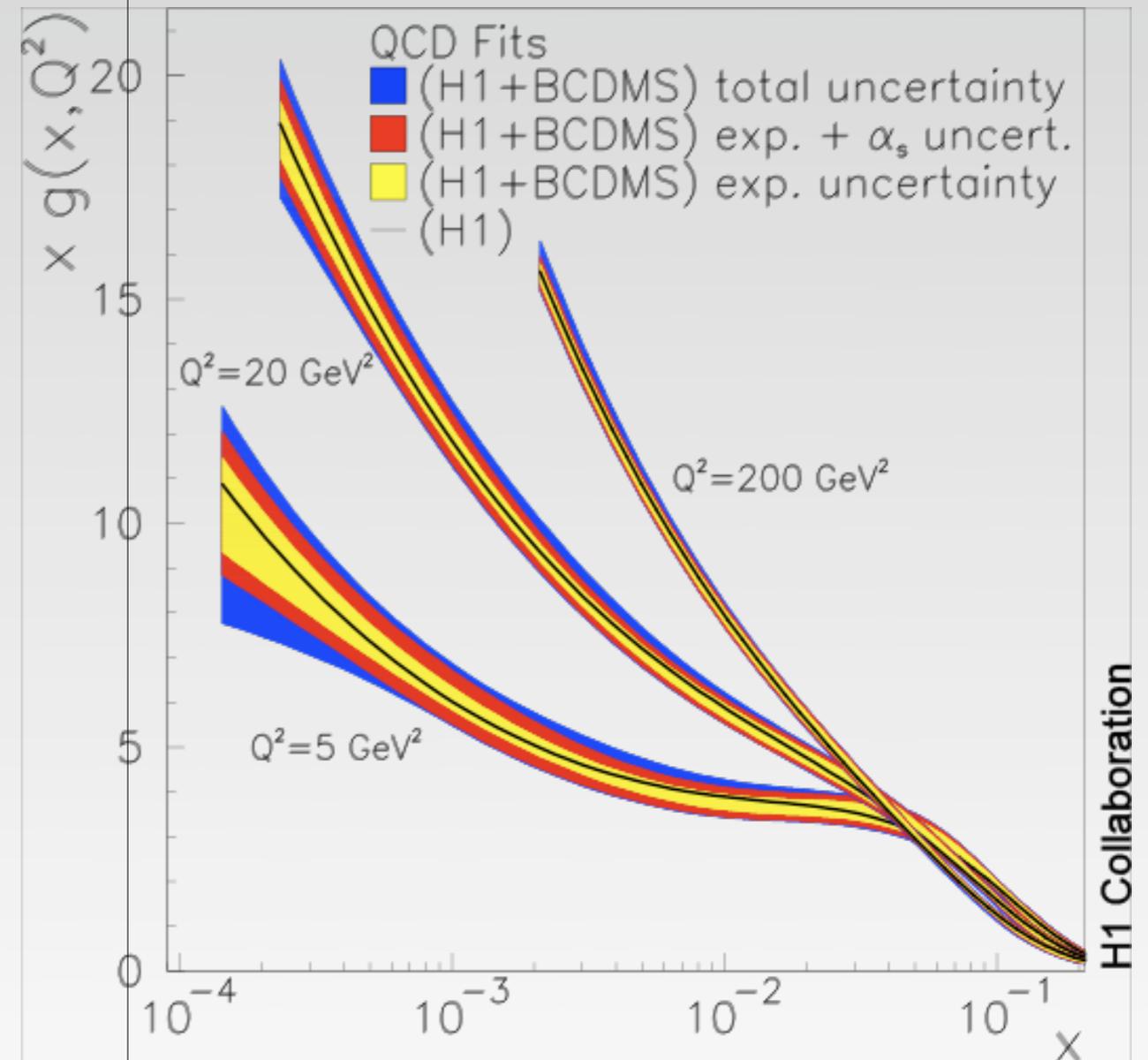
(protons)

Gluons and Quarks

ZEUS



Gluons only

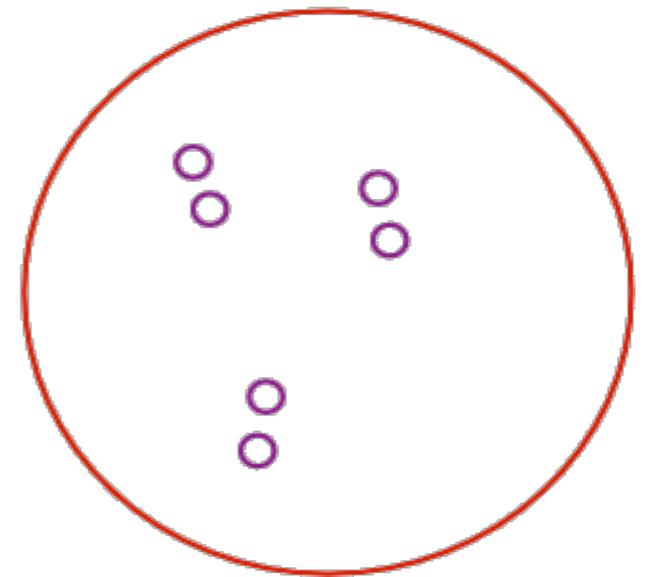
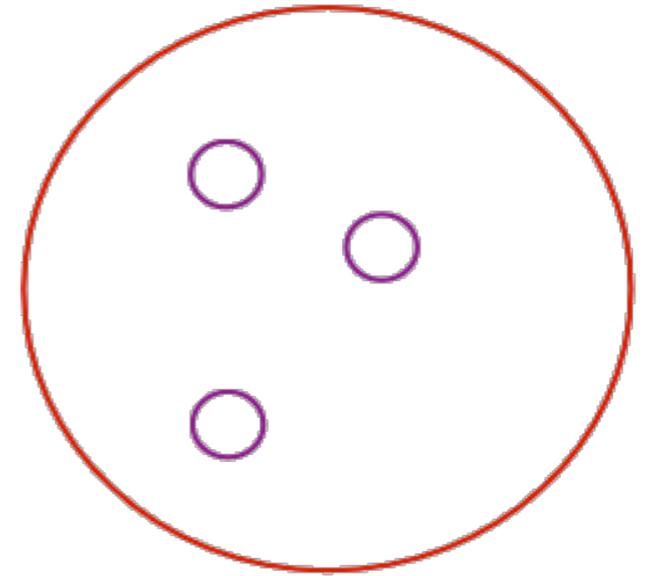


what happens when  $x \rightarrow 0$  ?

# *Resolving the hadron*

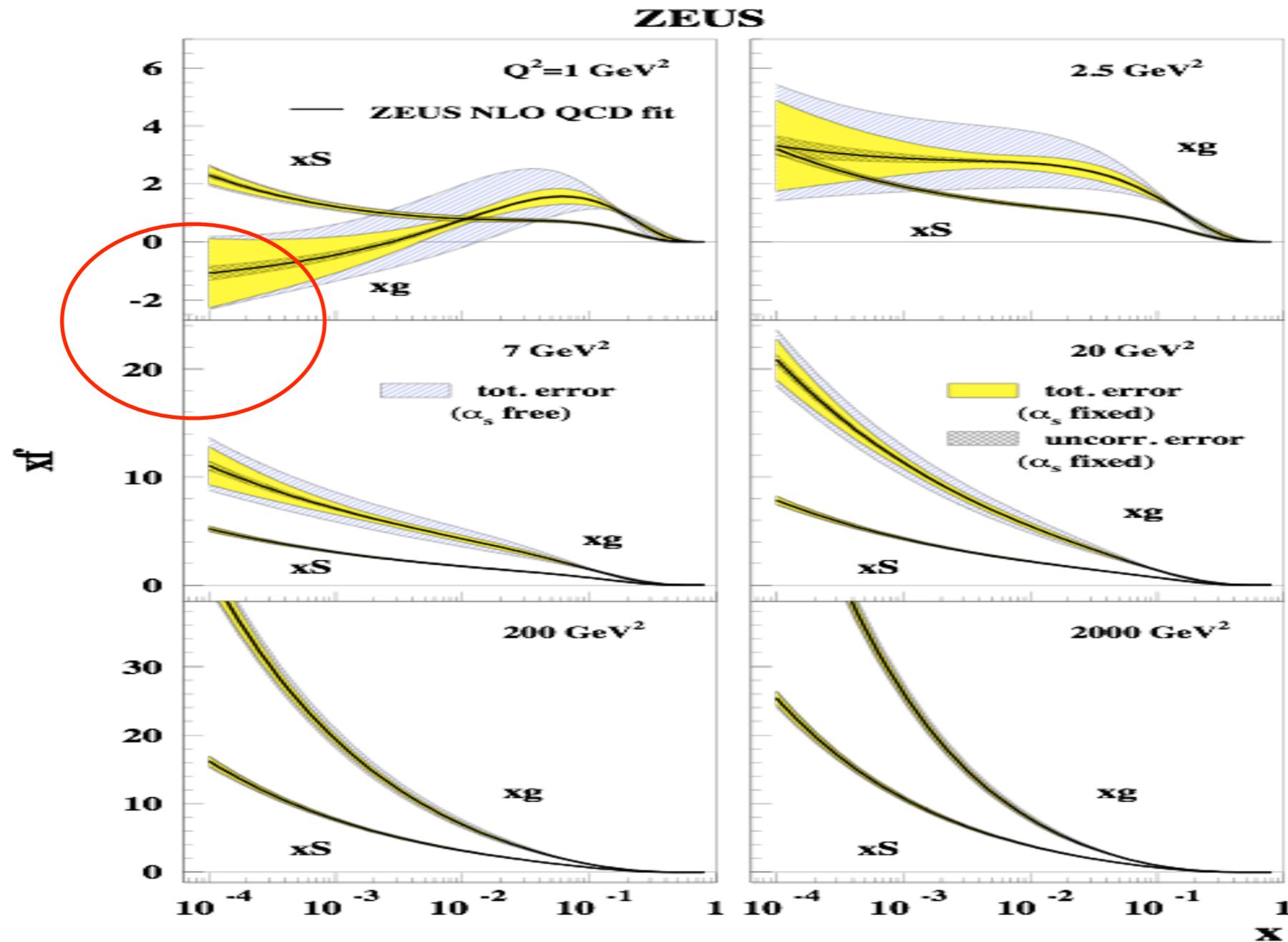
Ren. Group-DGLAP evolution  
(sums large logs in  $Q^2$ )

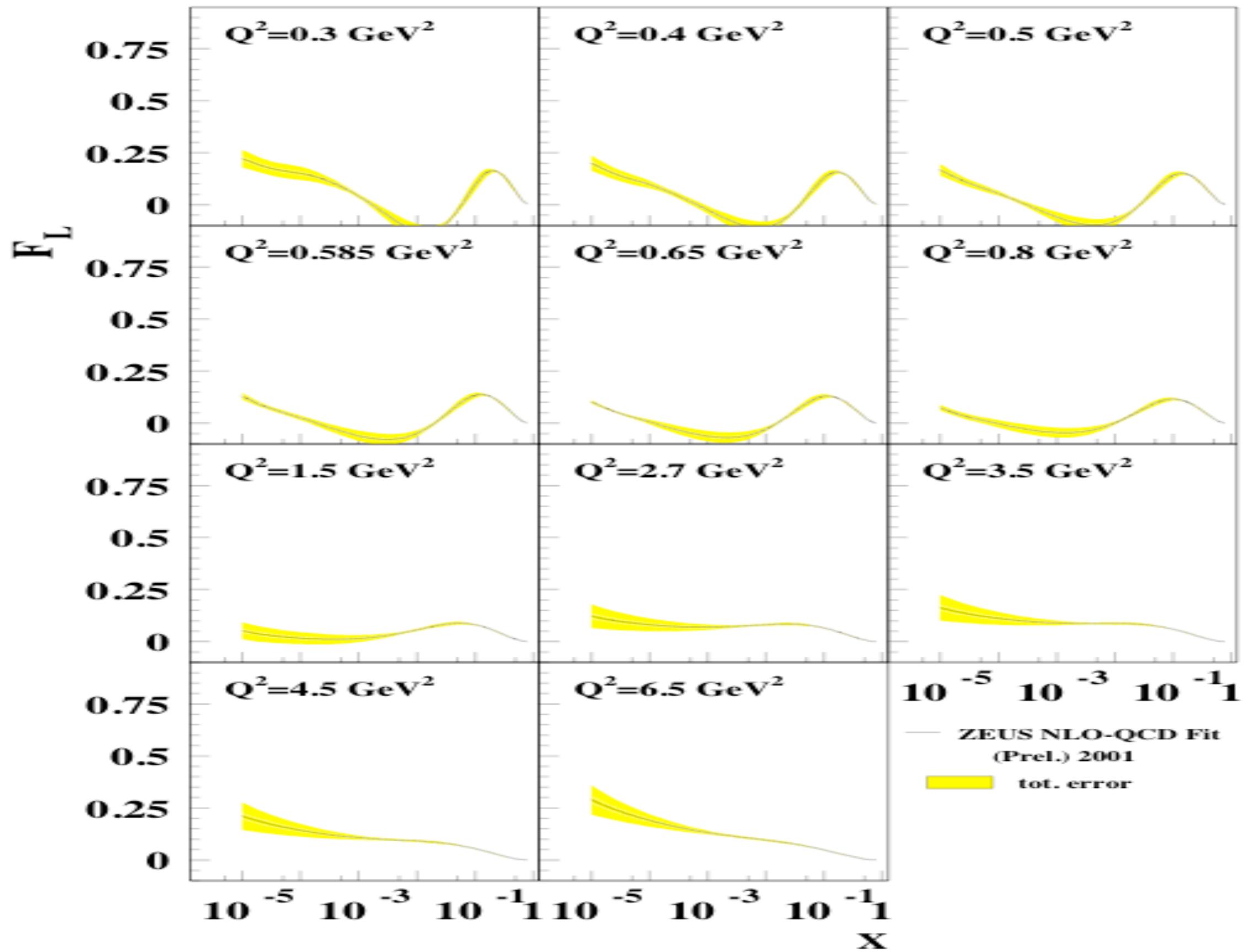
increasing  $Q^2$



Phase space density decreases  
(# partons / area /  $Q^2$  )  
proton becomes more dilute...

# Where may the standard “DGLAP” picture fail?

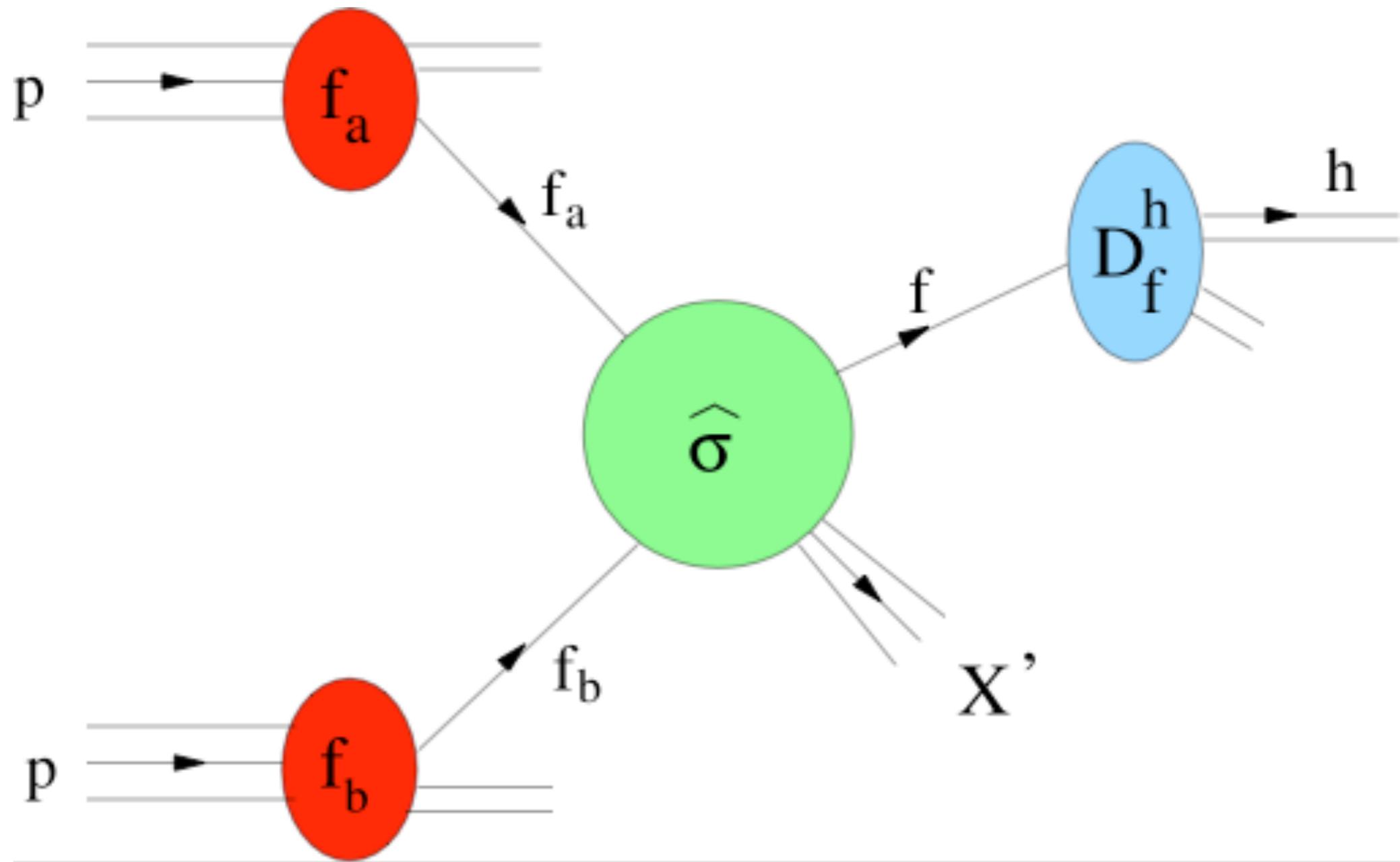




$F_L (\propto \alpha_S x G(x, Q^2))$  is a positive definite quantity

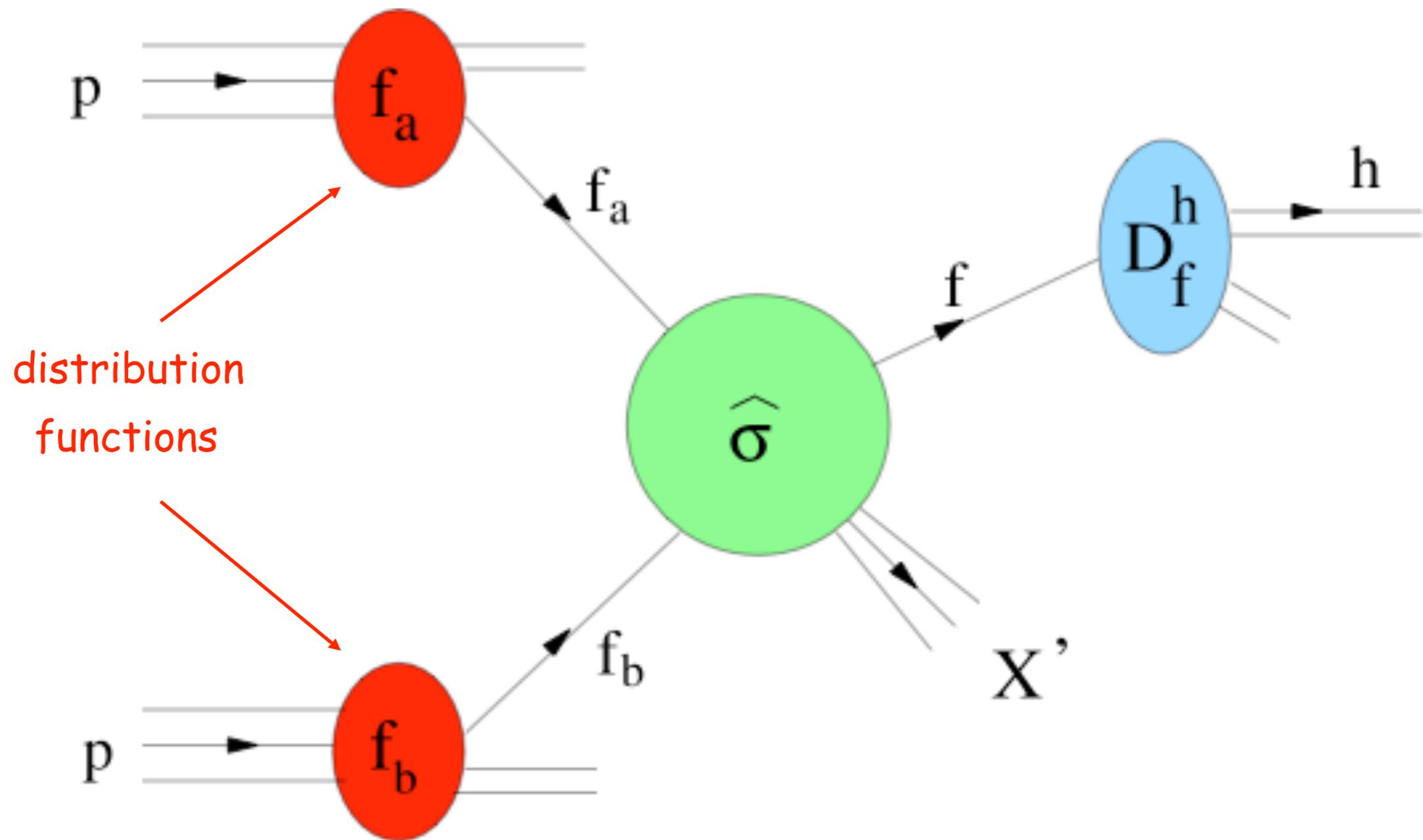
# ***pQCD in proton-proton Collisions***

factorization: separation of long and short distances



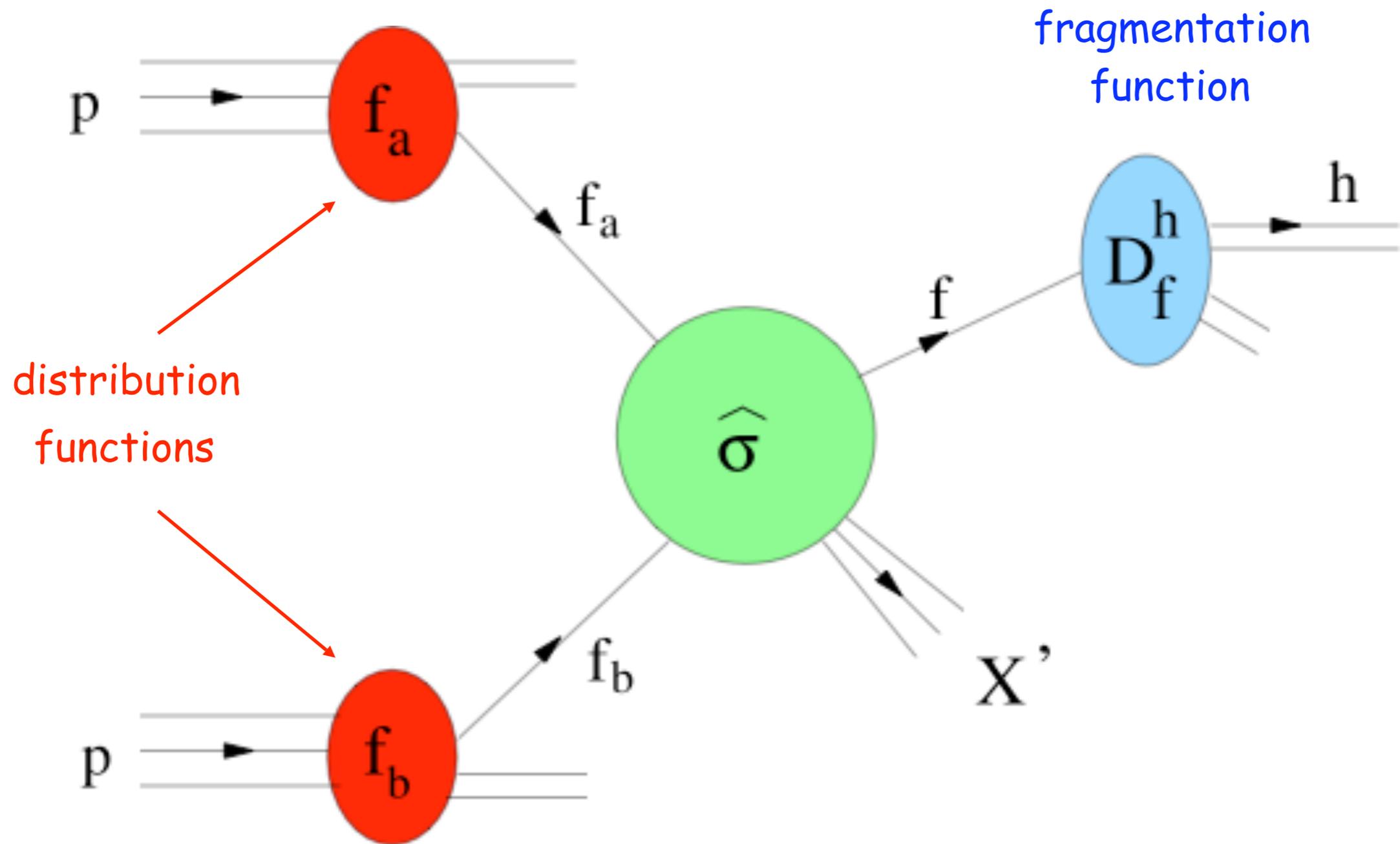
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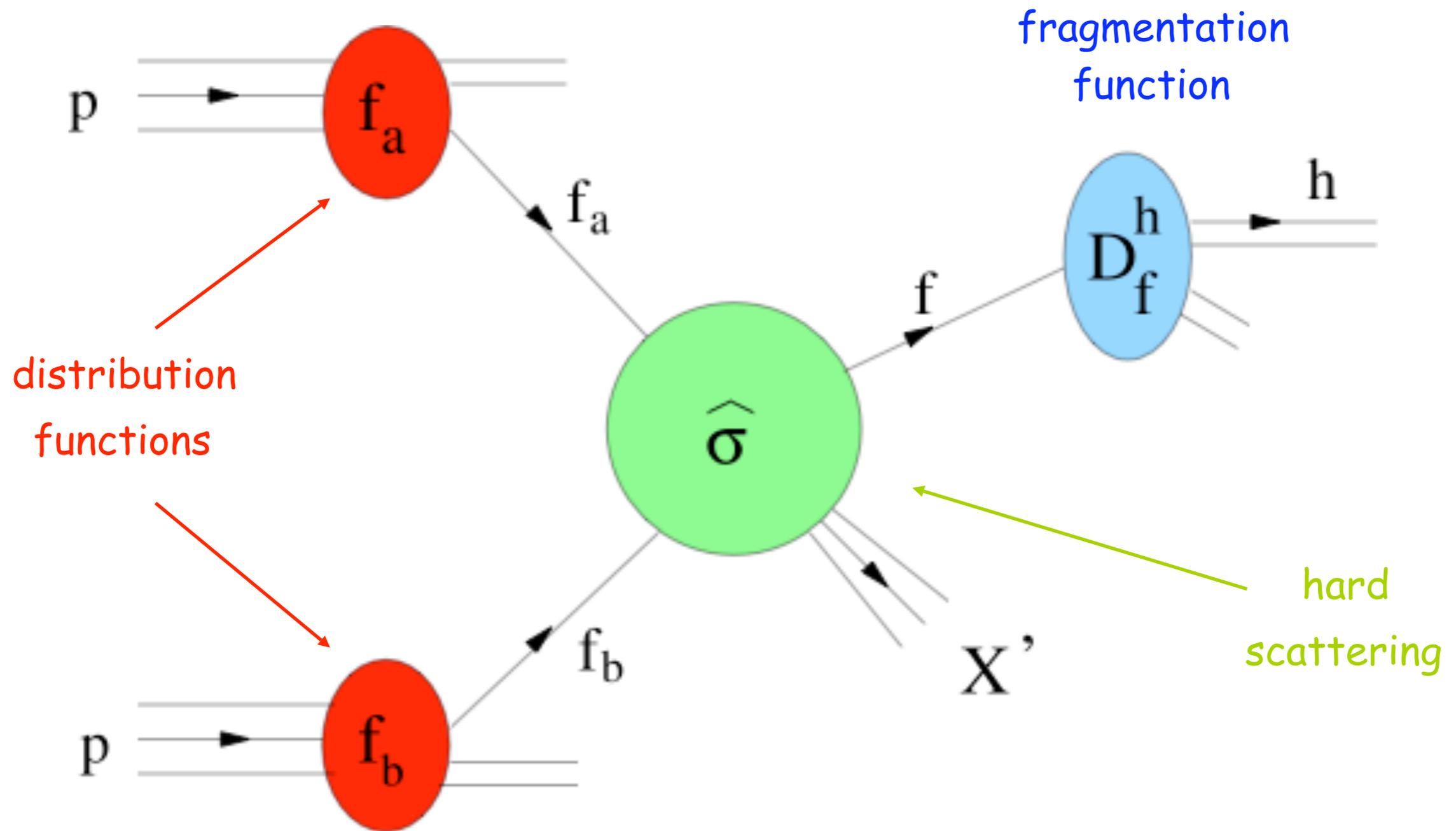
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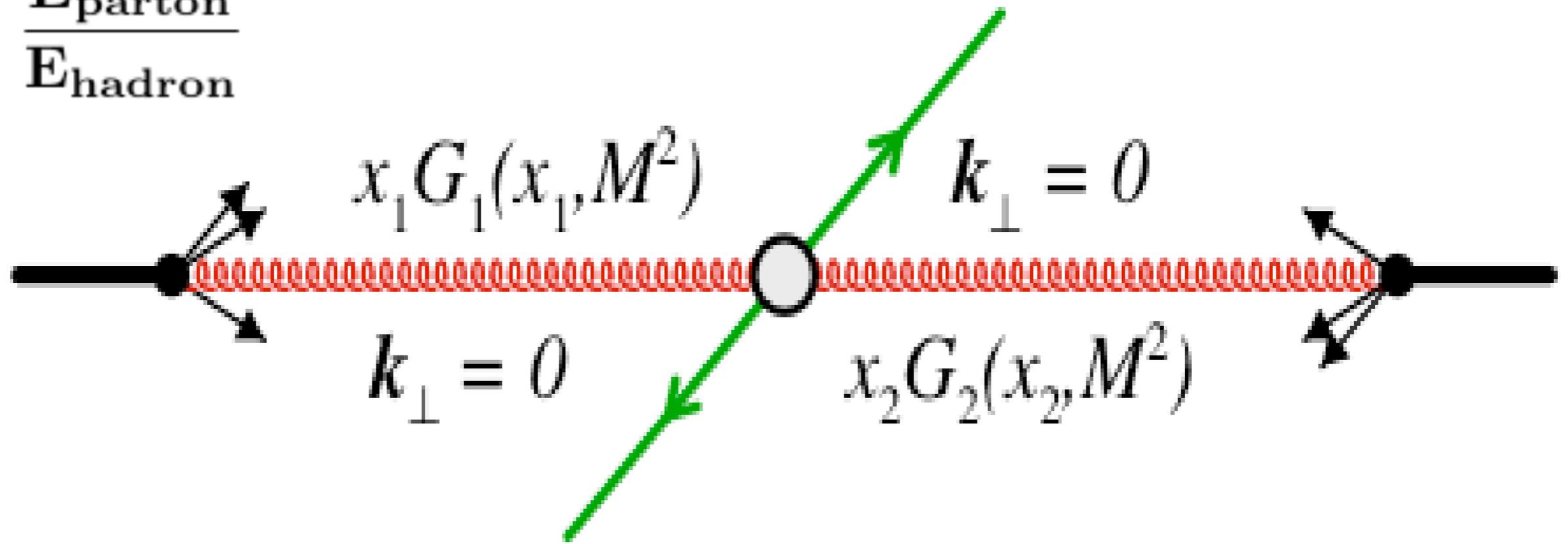


# *collinear factorization*

Incoherence: independent probabilities

$$M \sim \sqrt{S} \gg \Lambda_{\text{QCD}}$$

$$X \equiv \frac{E_{\text{parton}}}{E_{\text{hadron}}}$$

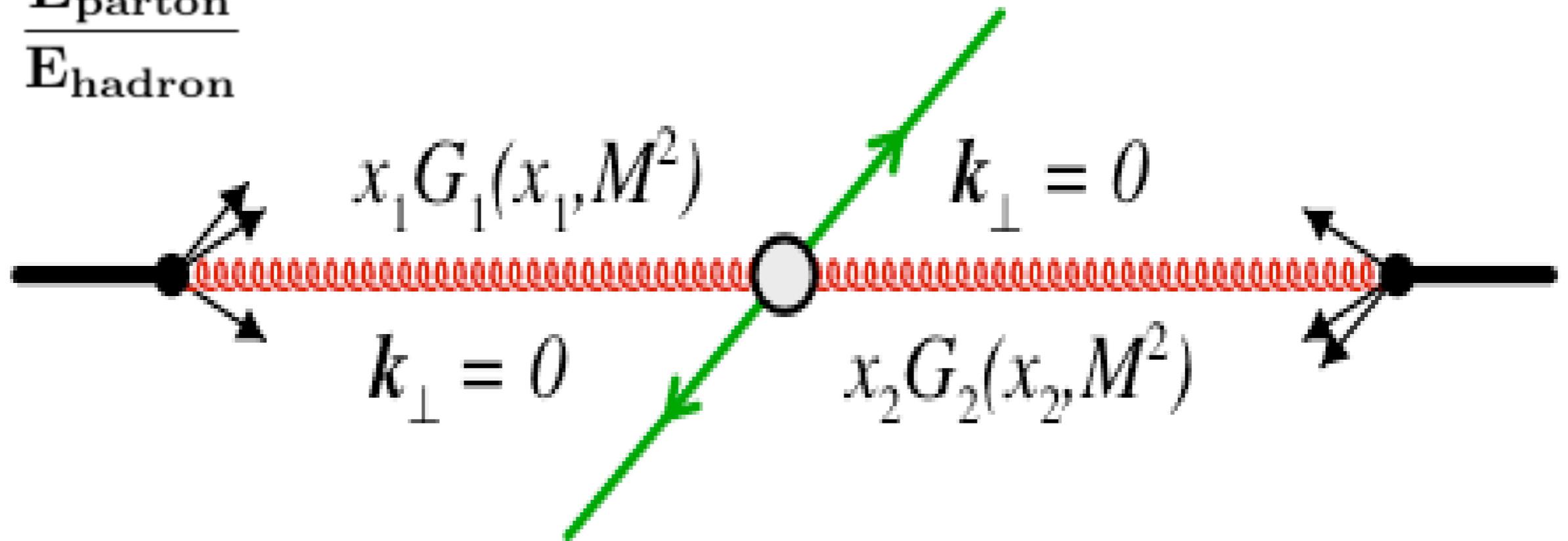


# *collinear factorization*

Incoherence: independent probabilities

$$M \sim \sqrt{S} \gg \Lambda_{\text{QCD}}$$

$$X \equiv \frac{E_{\text{parton}}}{E_{\text{hadron}}}$$



Incoming partons have  $k_{\perp}=0$

Quark and gluon distributions are universal, evaluated at hard scale

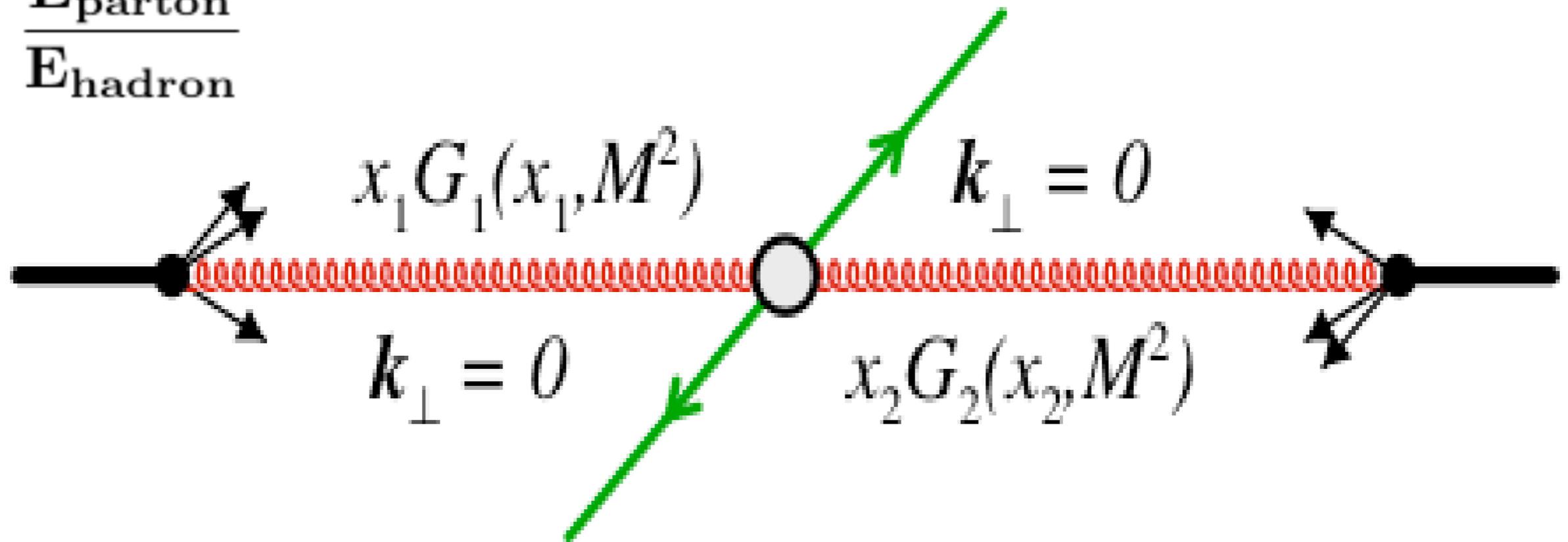
Factorization theorems are proven to all order in  $\alpha_s$

# *collinear factorization*

Incoherence: independent probabilities

$$M \sim \sqrt{S} \gg \Lambda_{\text{QCD}}$$

$$X \equiv \frac{E_{\text{parton}}}{E_{\text{hadron}}}$$



$$d\sigma = \int dx_1 dx_2 dz f_a^{H1}(x_1, M^2) f_b^{H2}(x_2, M^2) D_c^h(z, M^2) \otimes d\hat{\sigma}_{ab}^c(x_1 P_{H1}, x_2 P_{H2}, P_h/z, M^2)$$

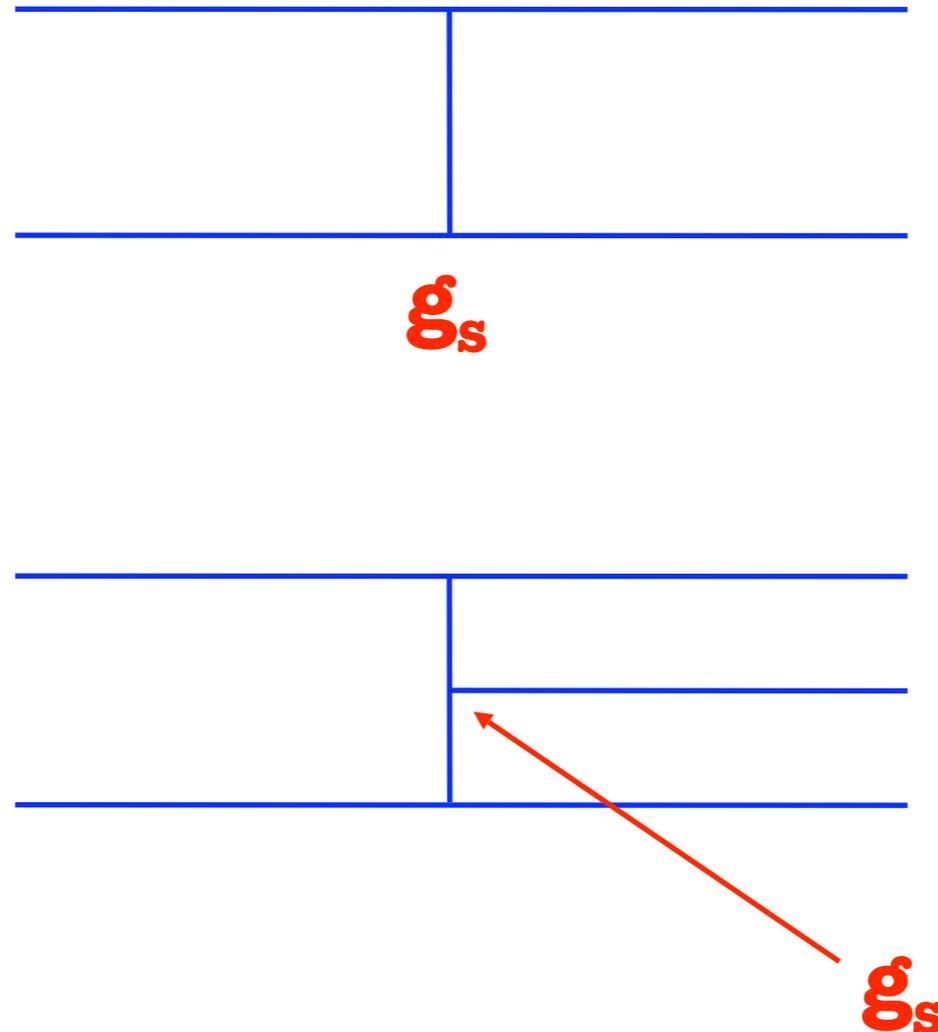
# partonic cross sections

calculable in pQCD

$\alpha_s^2$

+

$\alpha_s^3$

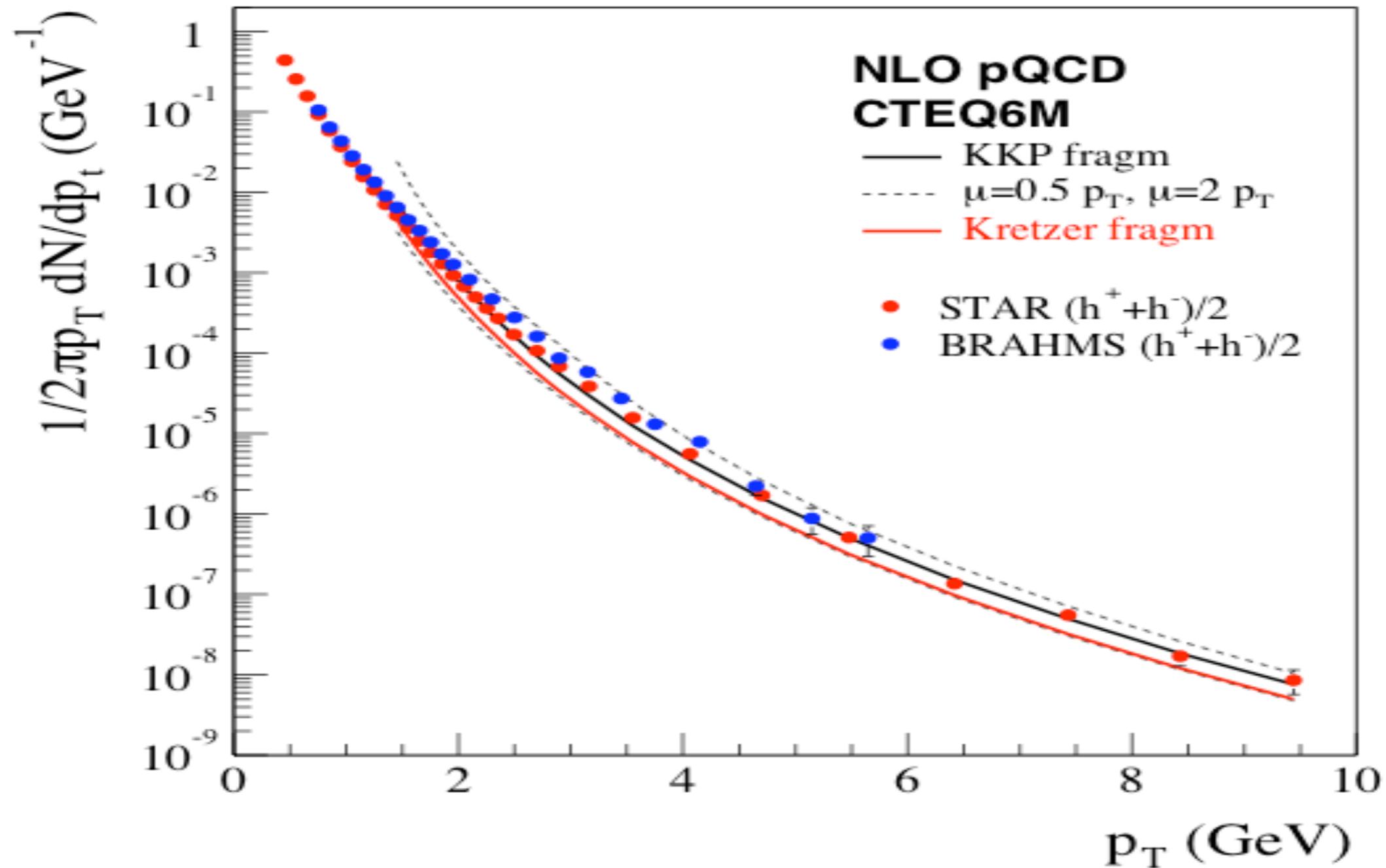


systematic expansion  
in the coupling constant

process  
dependent

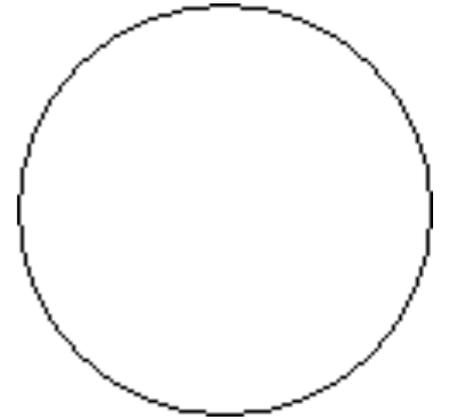
# ***pQCD in pp Collisions at RHIC***

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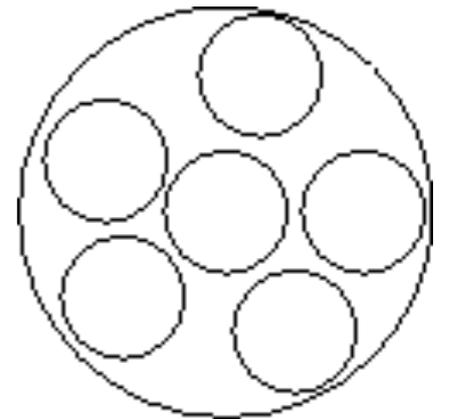


# *How about a nucleus ?*

A point particle:  $\lambda_T \gg 10 \text{ fm}$

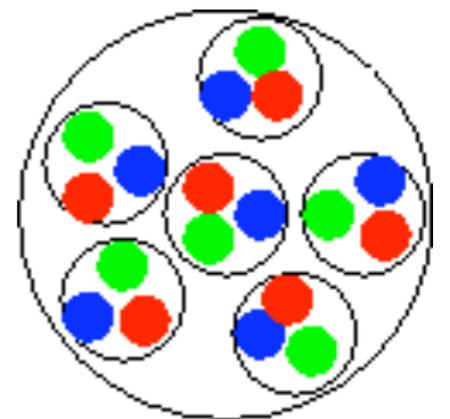


A collection of nucleons:  $\lambda_T \sim 1 \text{ fm}$



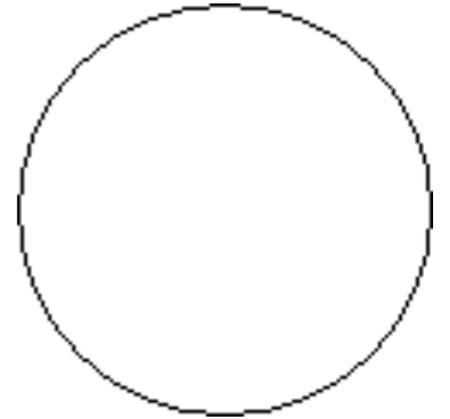
A system of quarks and gluons:

$\lambda_T \ll 1 \text{ fm}$

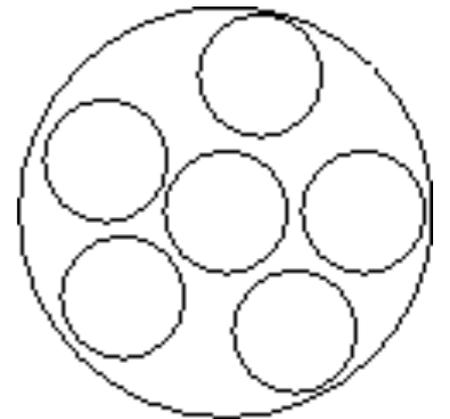


# *How about a nucleus ?*

A point particle:  $\lambda_T \gg 10 \text{ fm}$

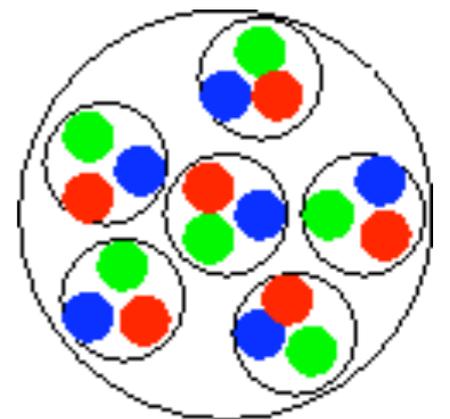


A collection of nucleons:  $\lambda_T \sim 1 \text{ fm}$



A system of quarks and gluons:

$\lambda_T \ll 1 \text{ fm}$



**depends on the resolution<sub>T</sub> scale!**

# *How about scattering of nuclei?*

*RHIC, LHC*

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**I) modification of initial state: "nuclear shadowing"**

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**II) modification of hard scattering: multiple scattering**

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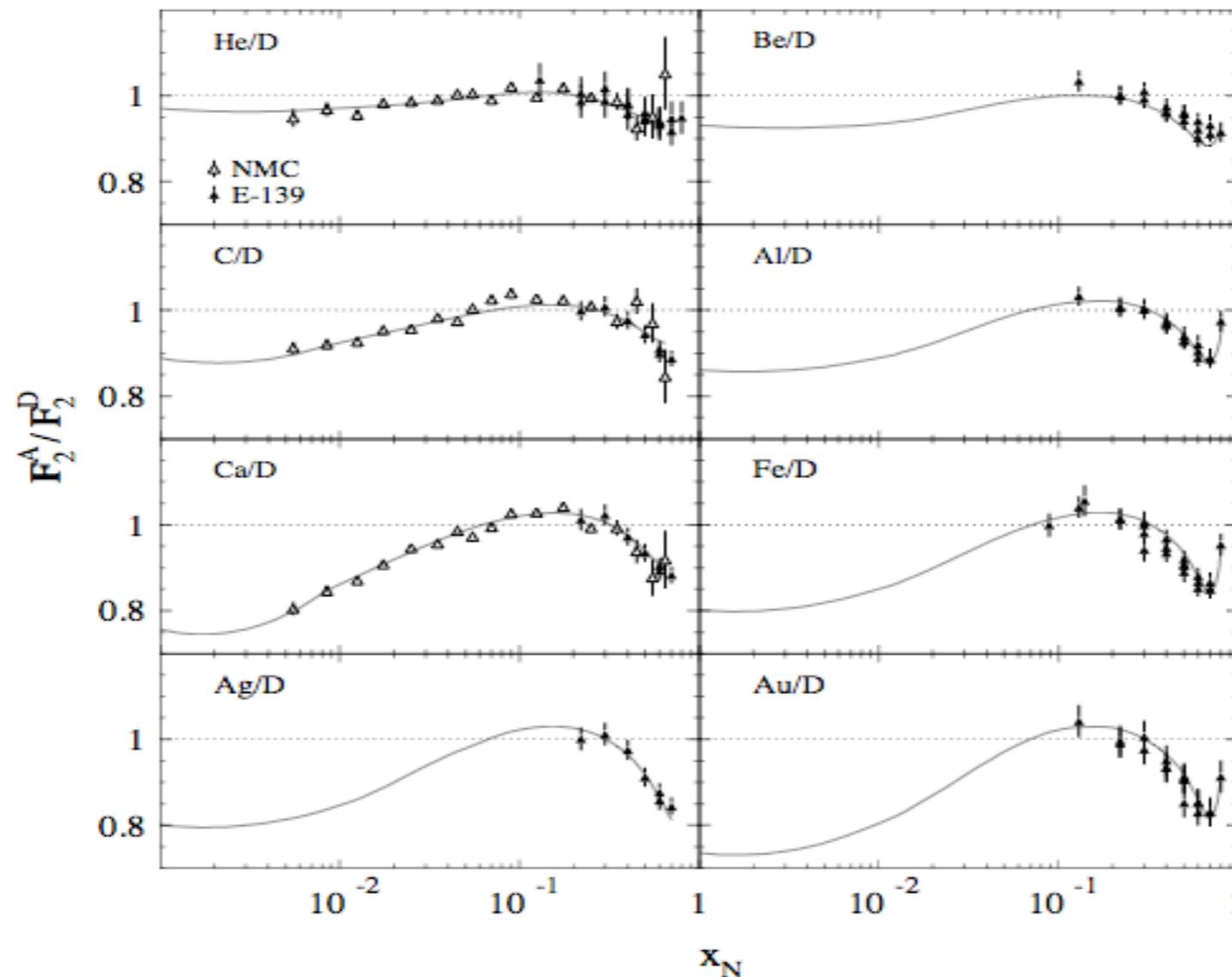
*RHIC, LHC*

**I) modification of initial state: "nuclear shadowing"**

**II) modification of hard scattering: multiple scattering**

**III) modification of fragmentation functions?**

# *Modification of the nuclear structure functions (shadowing)*



*can one understand shadowing from pQCD?*

# *Beyond pQCD in Bj limit*

- ★ Works great for inclusive, high  $Q^2$  processes
- ★ Higher twists important when  $Q^2$  is low
- ★ Problematic for diffractive/exclusive processes
- ★ Formalism not designed to treat shadowing, multiple scattering, diffraction, energy loss, impact parameter dependence, thermalization...

# *The Regge-Gribov limit*



$$x_{Bj} \rightarrow 0; s \rightarrow \infty; Q^2 (\gg \Lambda_{\text{QCD}}^2) = \text{fixed}$$

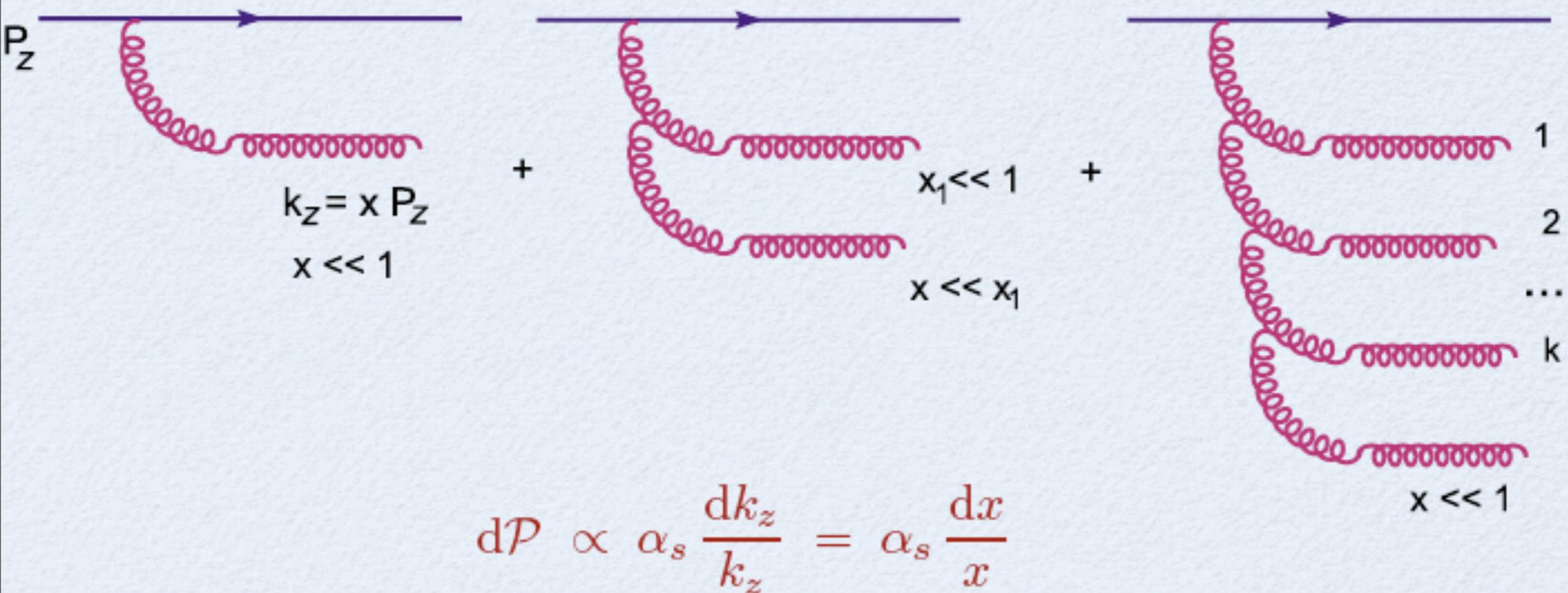
**Physics of strong fields in QCD**

**Multi-particle production**

**Novel universal properties of QCD**

# BFKL evolution

- The infrared sensitivity of bremsstrahlung favors the emission of 'soft' (= small- $x$ ) gluons



- The 'price' of an additional gluon:

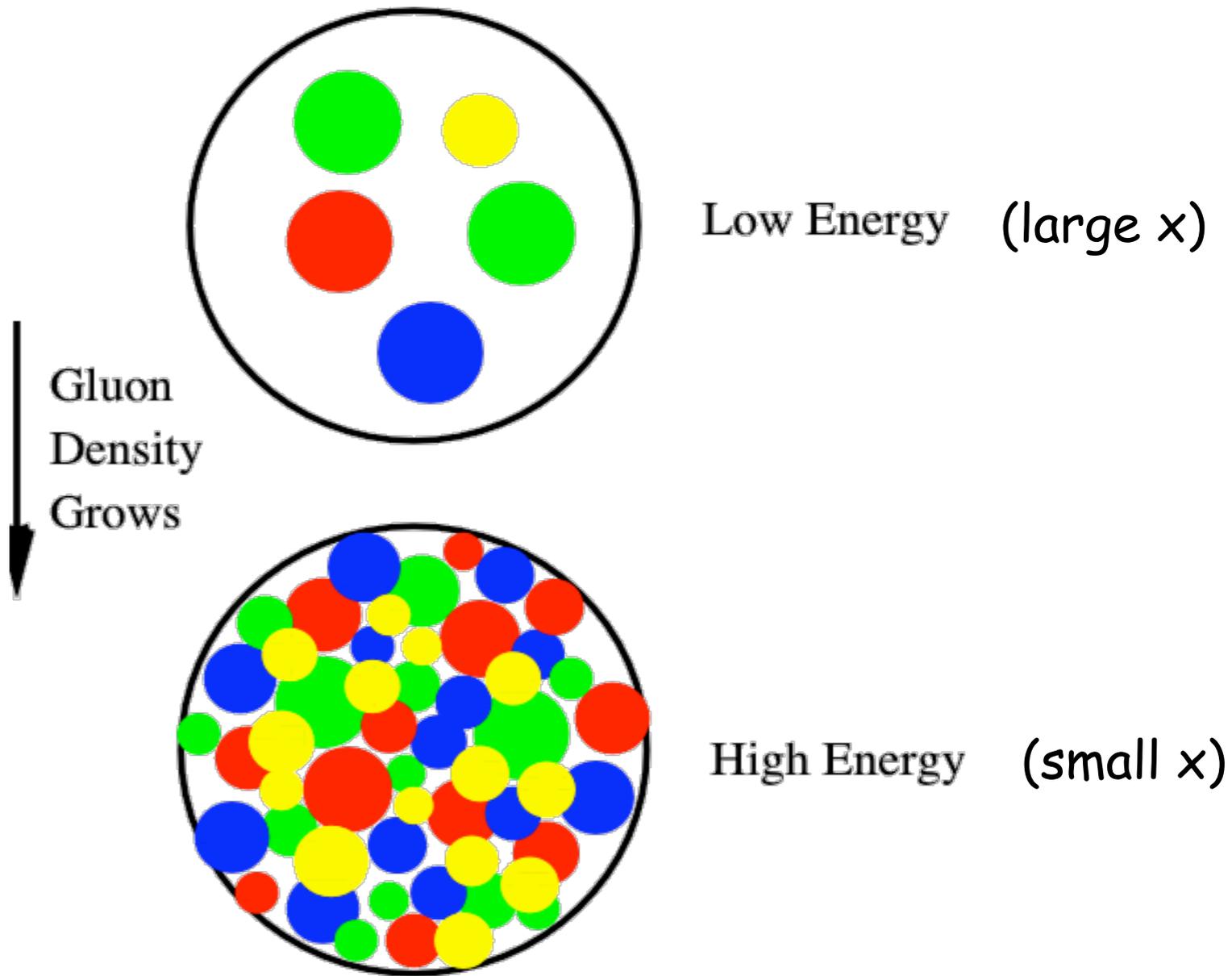
$$\mathcal{P}(1) \propto \alpha_s \int_x^1 \frac{dx_1}{x_1} = \alpha_s \ln \frac{1}{x}$$

number of gluons grows fast

$$n \sim e^{\alpha_s \ln 1/x}$$

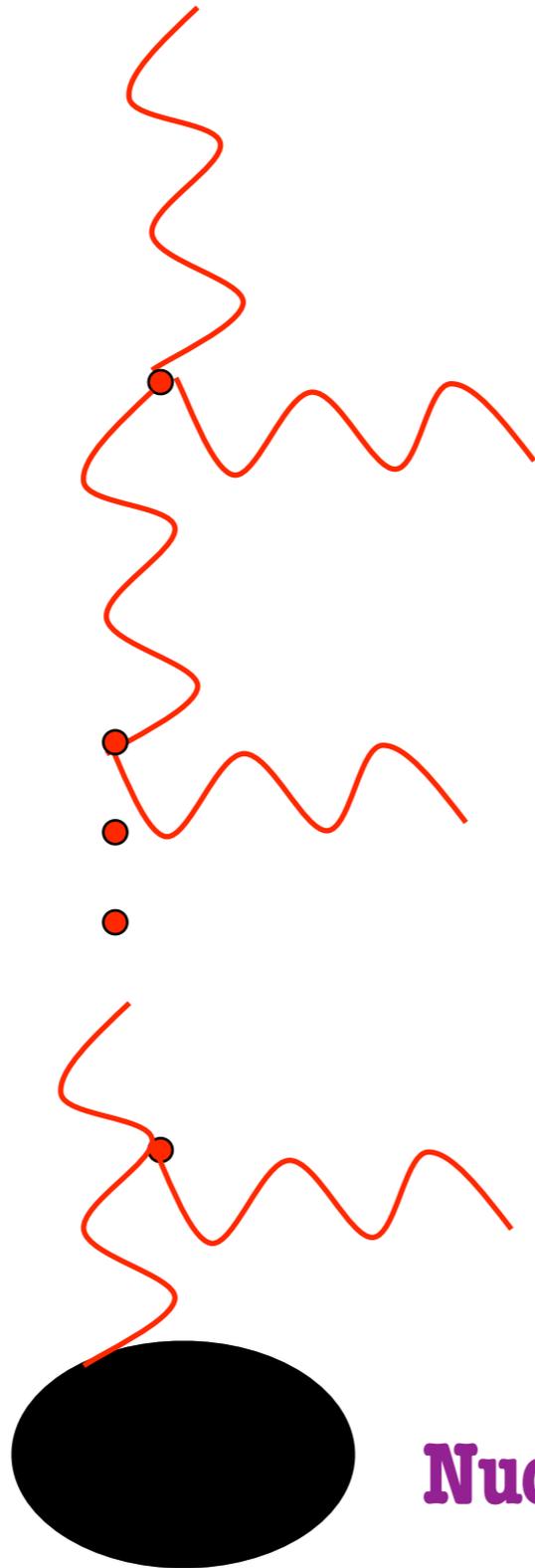
# *Resolving the hadron*

Ren. Group-BFKL evolution  
(sums large logs in  $x$ )



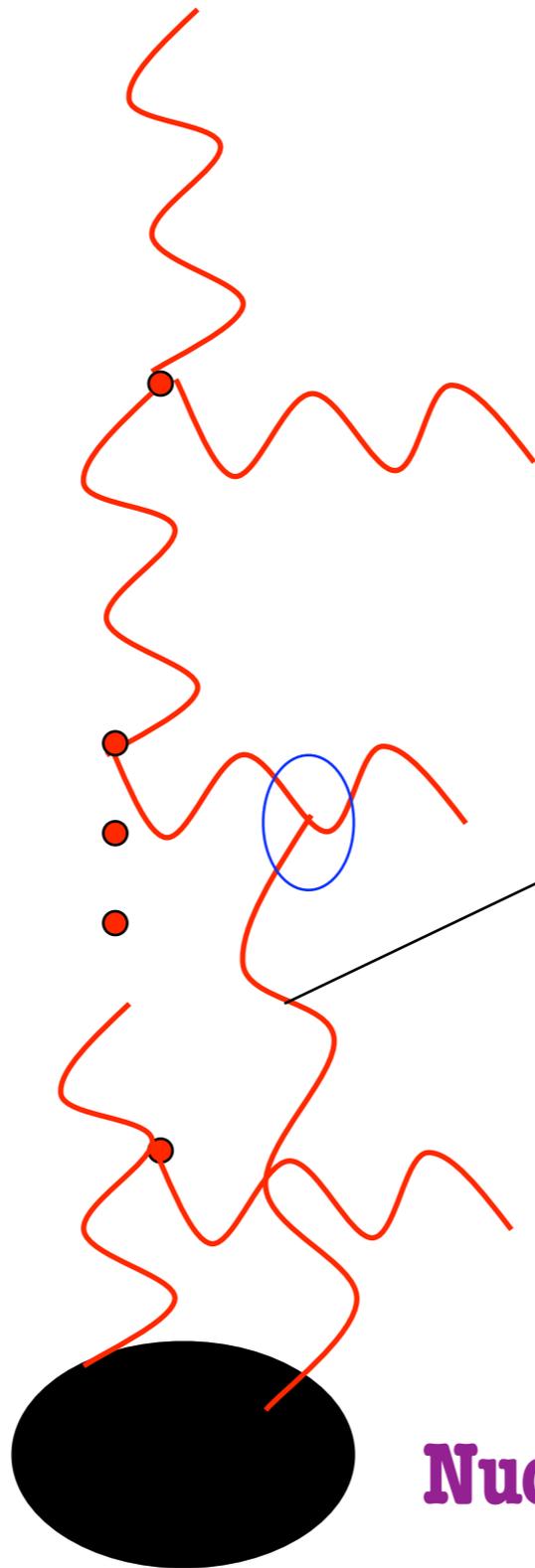
Gluon density saturates at  $f = 1 / \alpha_s$   
- strongest E&M fields in nature...

**QCD Bremsstrahlung**



**Nucleus**

**QCD Bremsstrahlung**



**Nucleus**

**Non-linear evolution:  
Gluon recombination**  
Gribov, Levin, Ryskin

# *Parton Saturation*

- ★ Competition between attractive bremsstrahlung and repulsive recombination effects

Maximum occupation number ( $f = 1/\alpha_S$ ) =>

$$\frac{1}{2(N_c^2 - 1)} \frac{x G(x, Q^2)}{\pi R^2 Q^2} = \frac{1}{\alpha_S(Q^2)}$$

This relation is saturated for

$$Q = Q_s(x) \gg \Lambda_{\text{QCD}} \approx 0.2 \text{ GeV}$$



*Need a new organizing principle to  
explore this novel physics regime of  
high energy QCD*

