

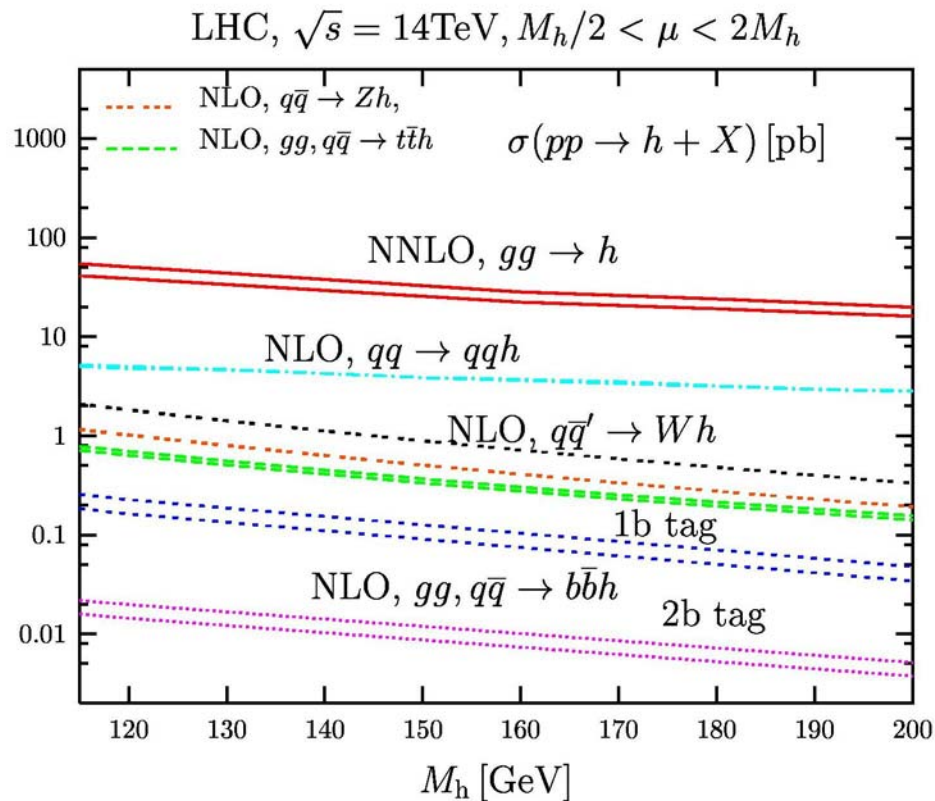
What Do We Know About EWSB?

Sally Dawson (BNL)

Lecture 2

XIII Mexican School of Particles
and Fields, 2008

Production Mechanisms at LHC



Bands show scale dependence

All important channels calculated to NLO or NNLO

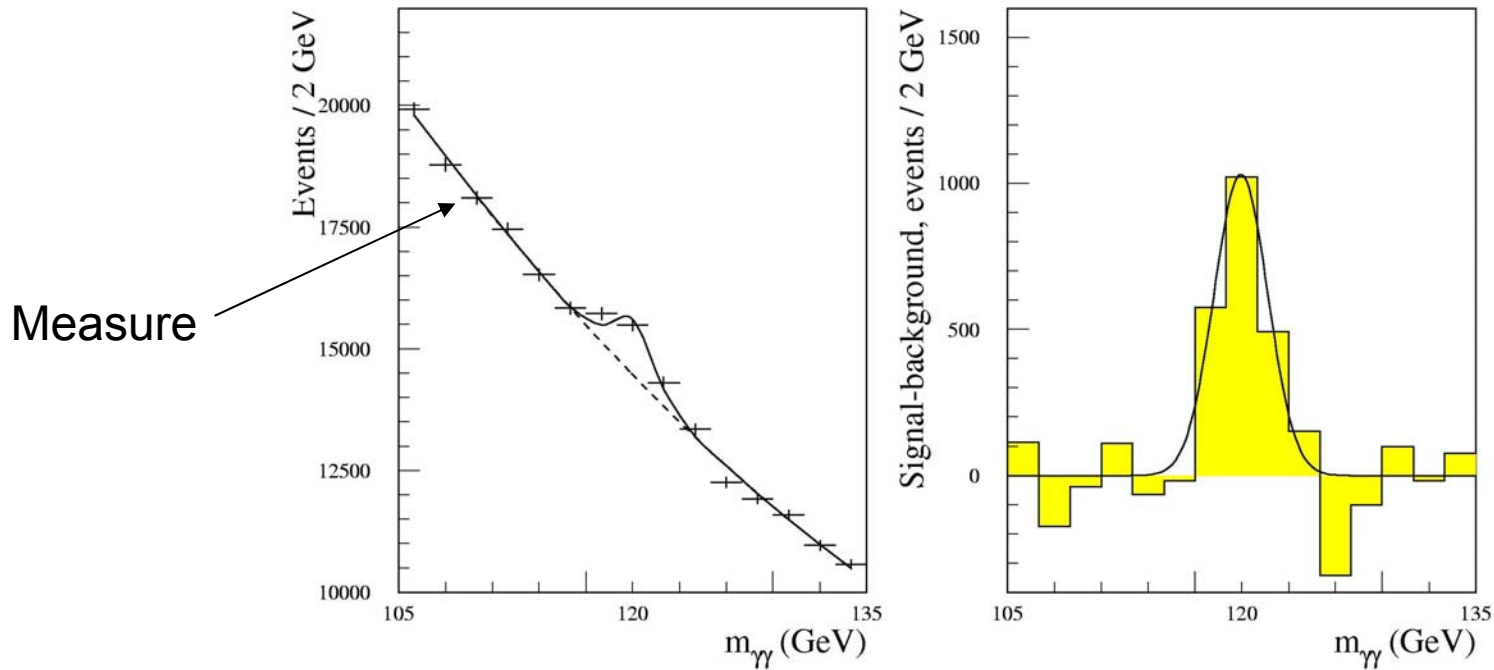
Search Channels at the LHC

$gg \rightarrow H \rightarrow b\bar{b}$ has huge QCD background: Must use rare decay modes of H

- $gg \rightarrow H \rightarrow \gamma\gamma$
 - Small BR ($10^{-3} - 10^{-4}$)
 - Only measurable for $M_H < 140$ GeV
- Largest Background: QCD continuum production of $\gamma\gamma$
- Also from γ -jet production, with jet faking γ , or fragmenting to π^0
- Fit background from data

$$H \rightarrow \gamma\gamma$$

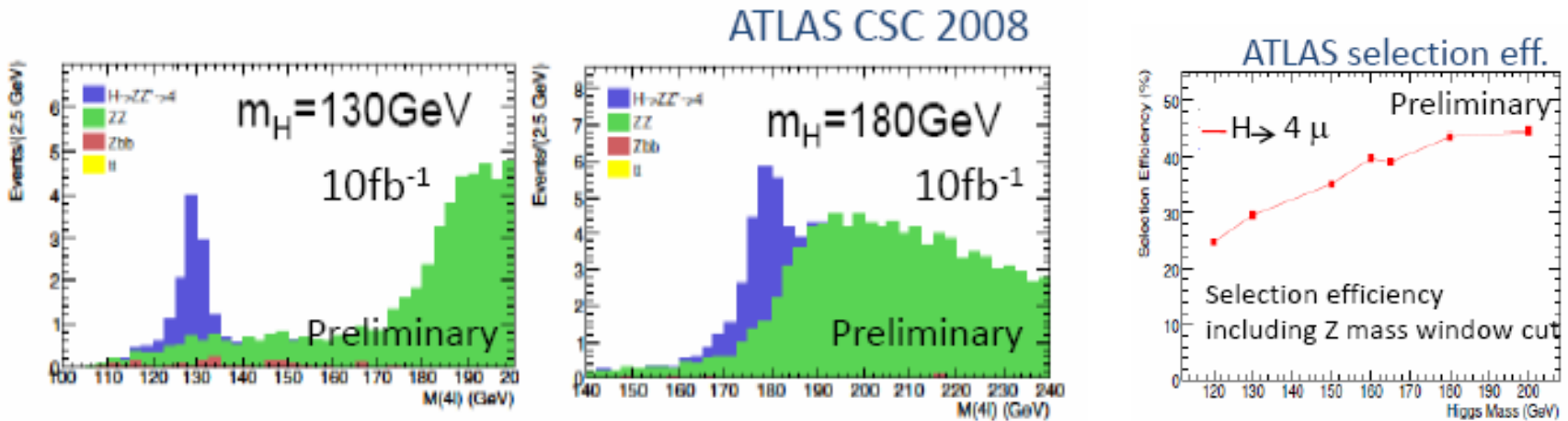
$$M_H = 120 \text{ GeV}; L = 100 \text{ fb}^{-1}$$



$$S/\sqrt{B} = 2.8 \text{ to } 4.3 \sigma$$

Golden Channel: $H \rightarrow ZZ \rightarrow 4$ leptons

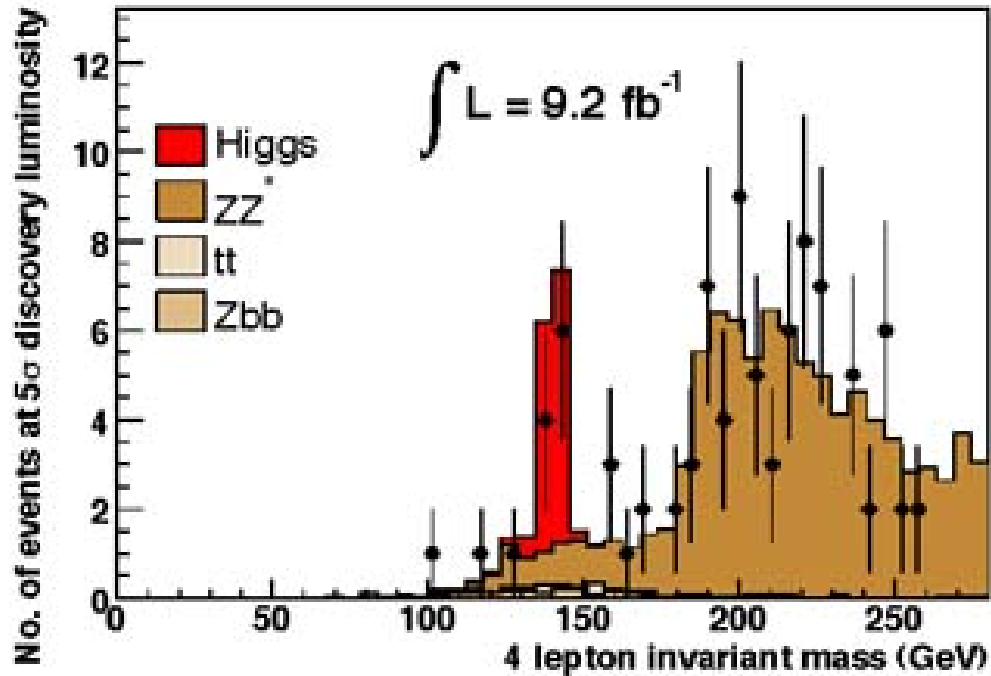
- Need excellent lepton ID



- Below $M_H \sim 130$ GeV, rate is too small for discovery

$H \rightarrow ZZ \rightarrow (4 \text{ leptons})$

CMS:



Possible discovery with $< 10 \text{ fb}^{-1}$

Heavy Higgs in 4-lepton Mode

200 GeV < M_H < 600 GeV:

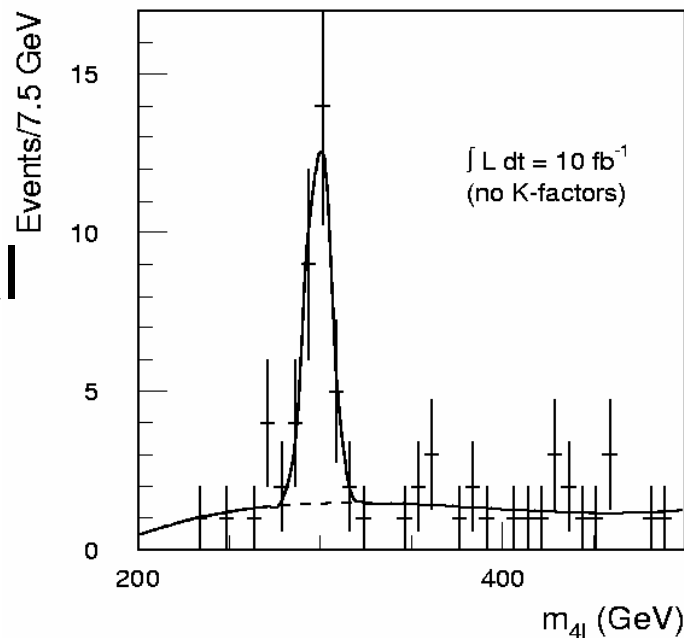
- Discovery in $H \rightarrow ZZ \rightarrow l^+l^-l^+l^-$

• Background smaller than signal

• Higgs width larger than experimental resolution ($M_H > 300$ GeV)

Confirmation in $H \rightarrow ZZ \rightarrow l^+l^-jj$

$H \rightarrow ZZ \rightarrow l^+l^-l^+l^-$



Heavy Higgs

$M_H > 600 \text{ GeV}$:

4 lepton channel statistically limited

Look for:

$$H \rightarrow ZZ \rightarrow l^+l^- \nu\nu$$

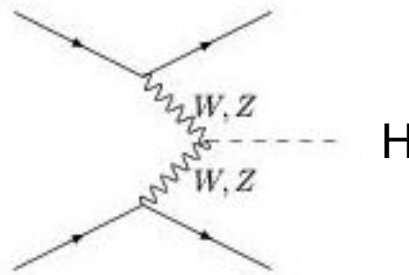
$$H \rightarrow ZZ \rightarrow l^+l^- jj$$

$$H \rightarrow WW \rightarrow l \nu jj$$

-150 times larger BR than 4 lepton channel

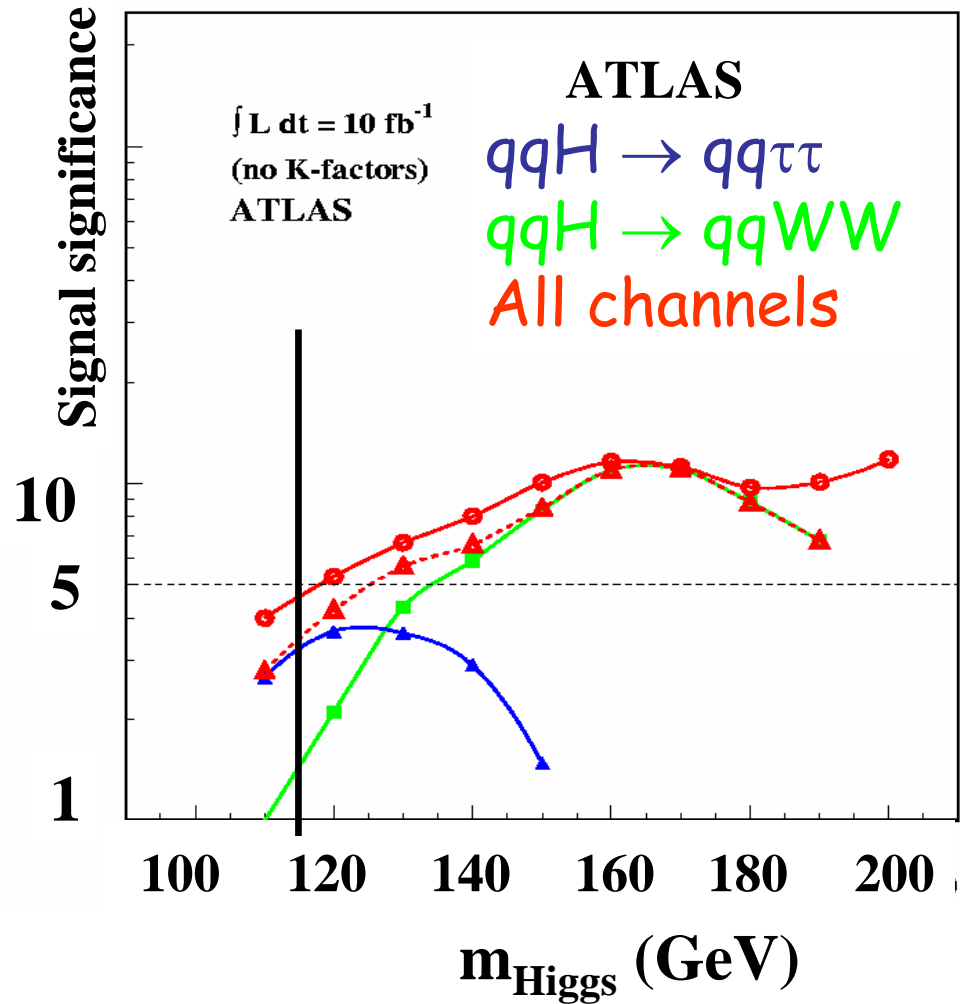
Vector Boson Fusion

- Outgoing jets are mostly forward and can be tagged
 - Little jet activity in central rapidity region
- Idea: Look for different H decay channels in Vector boson fusion
 - Ratio will have smaller errors than total rate

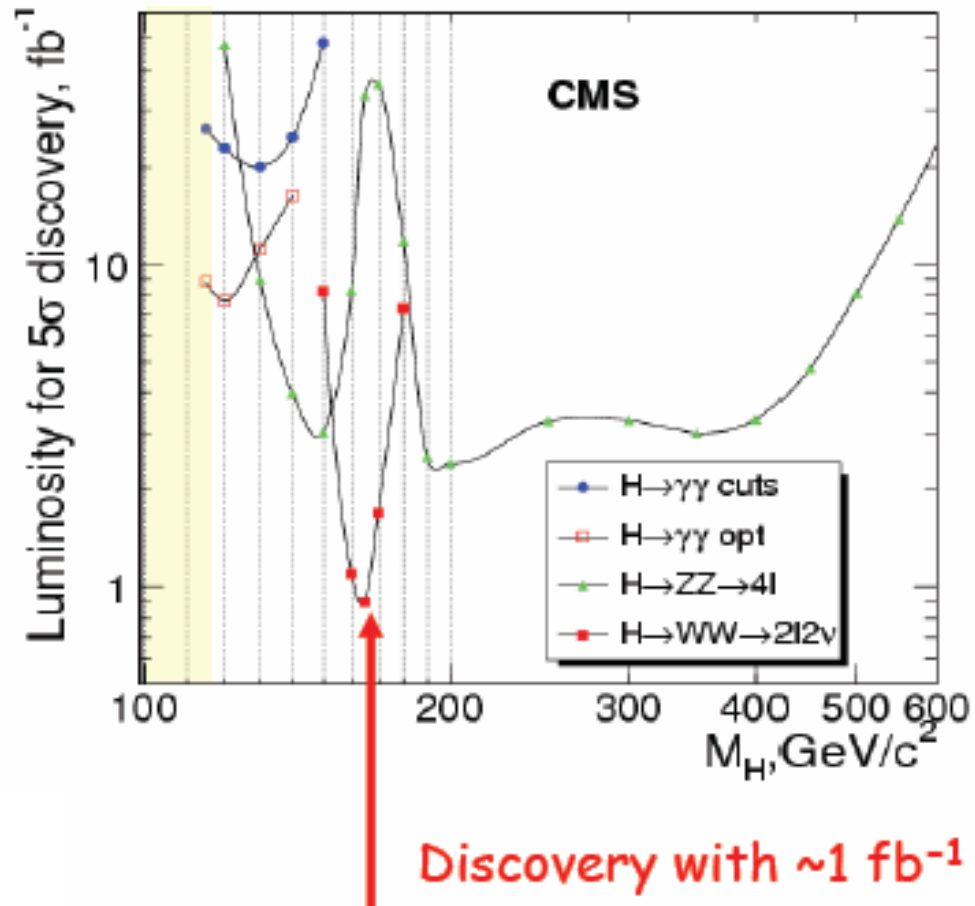


Vector Boson Fusion for light Higgs

Vector boson fusion effective for measuring Higgs couplings

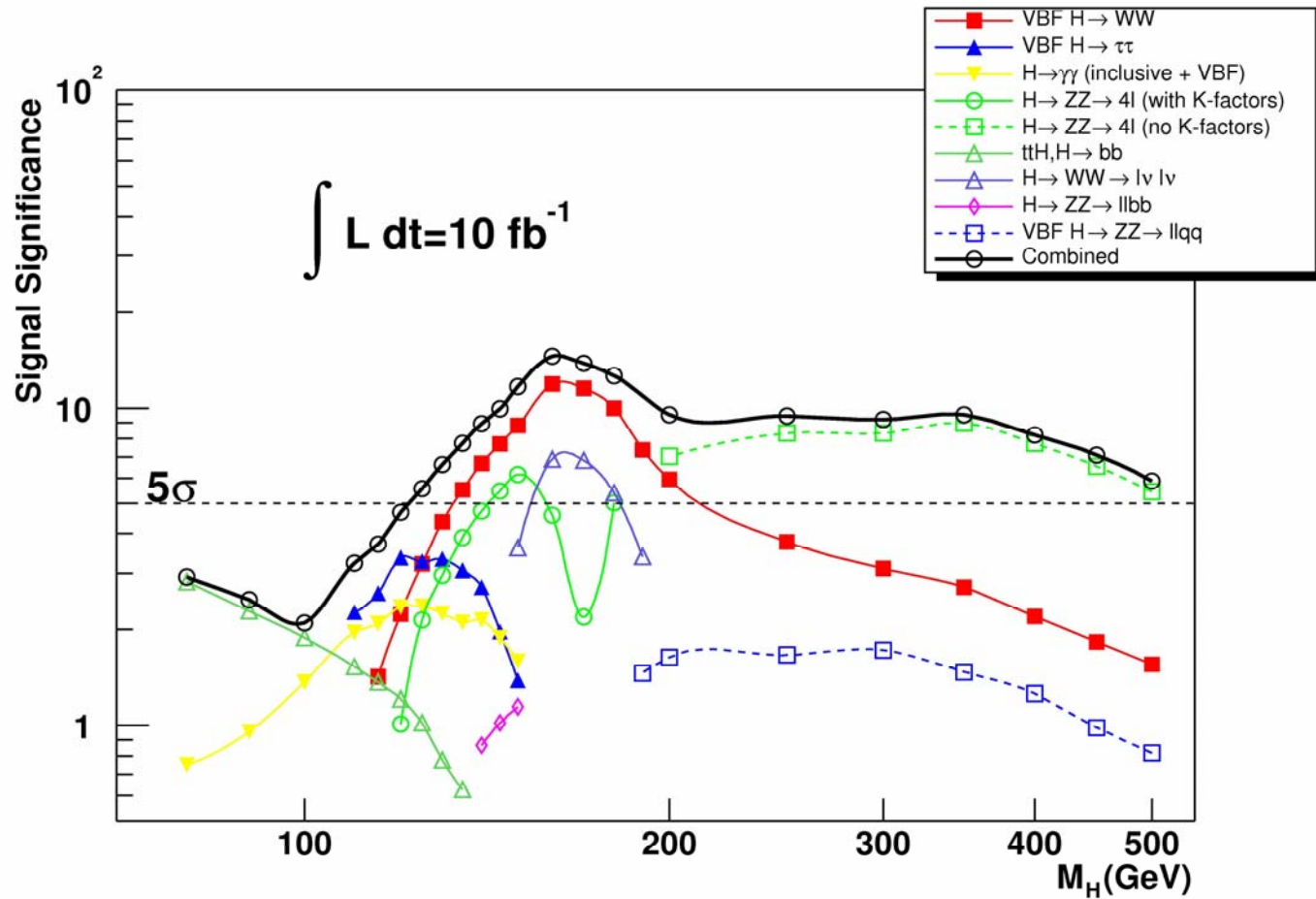


Higgs Discovery Reach at LHC

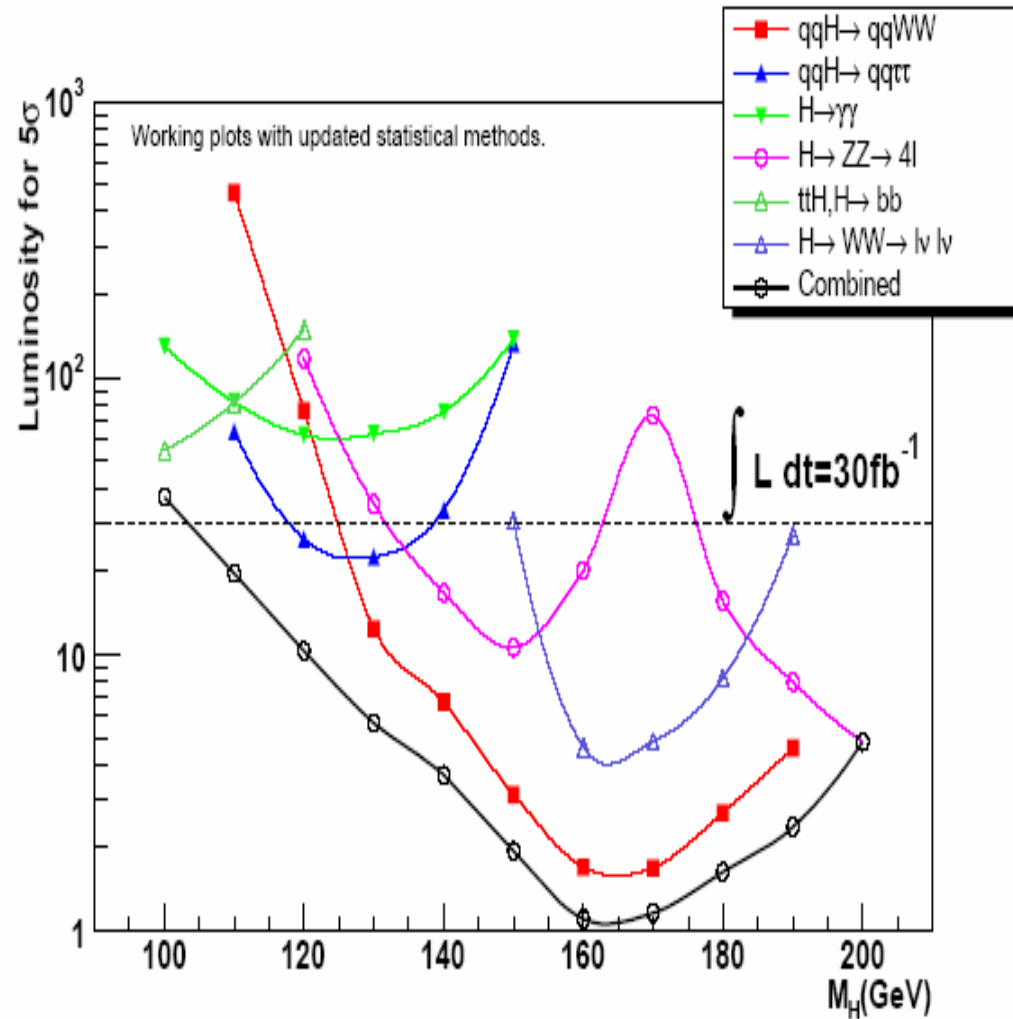


- Improvement in $\gamma\gamma$ channel from earlier studies

Early Higgs Physics



Higgs Discovery at ATLAS



ATLAS

If we find a “Higgs-like” object, what then?

- **We need to:**
 - Measure Higgs couplings to fermions & gauge bosons
 - Measure Higgs spin/parity
 - Reconstruct Higgs potential
 - Is it the SM Higgs?
- **Reminder: Many models have other signatures:**
 - New gauge bosons (little Higgs)
 - Other new resonances (Extra D)
 - Scalar triplets (little Higgs, NMSSM)
 - Colored scalars (MSSM)

Is it a Higgs?

- How do we know what we've found?
- Measure couplings to fermions & gauge bosons

$$\frac{\Gamma(H \rightarrow b\bar{b})}{\Gamma(H \rightarrow \tau^+\tau^-)} \approx 3 \frac{m_b^2}{m_\tau^2}$$

- Measure spin/parity

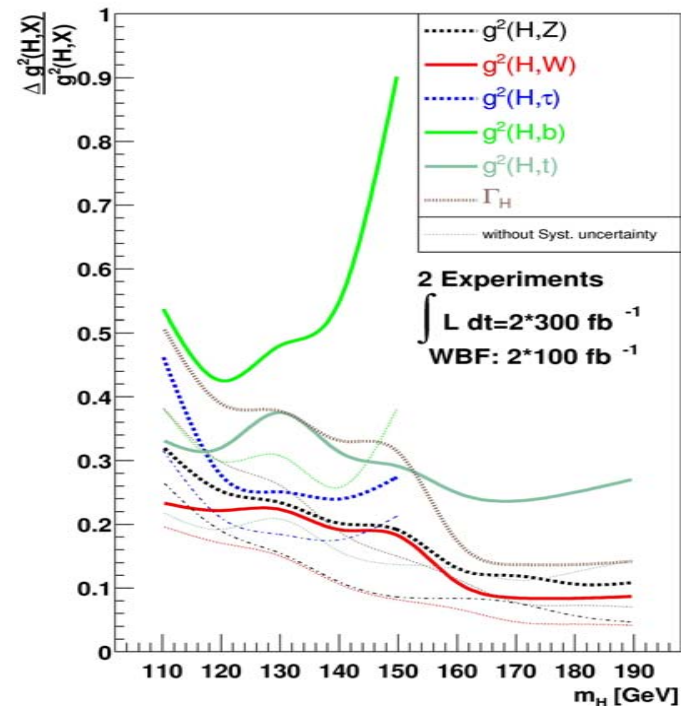
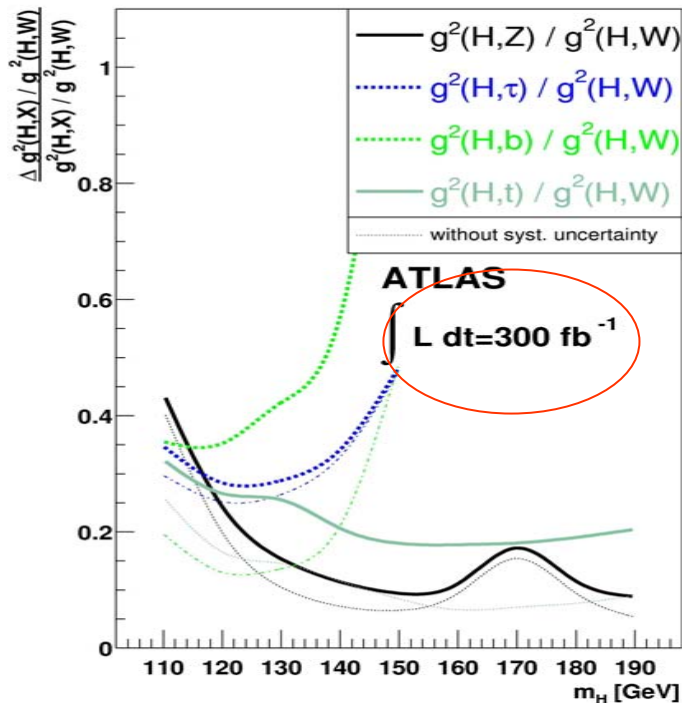
$$J^{PC} = 0^{++}$$

- Measure self interactions

$$V = \frac{M_H^2}{2} H + \frac{M_H^2}{2v} H^3 + \frac{M_H^2}{8v^2} H^4$$

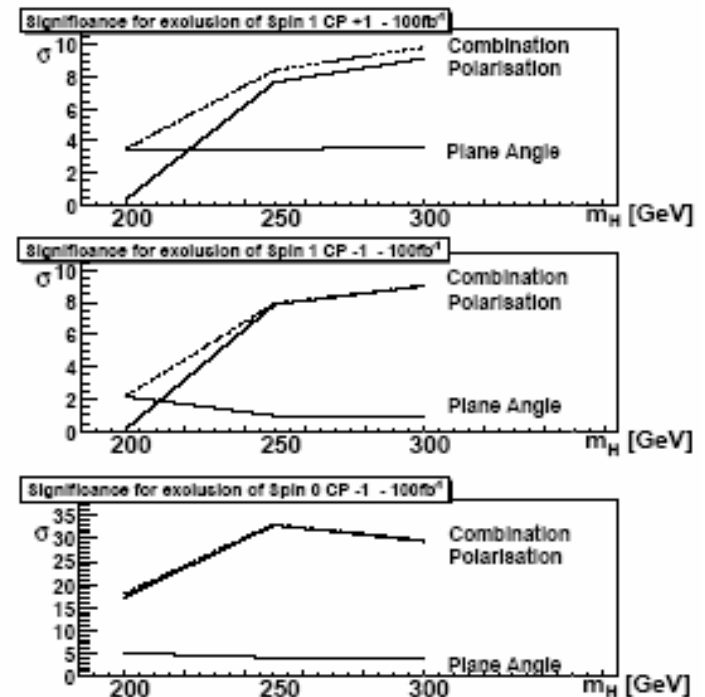
Absolute Measurements of Higgs Couplings

- Ratios of couplings more precisely measured than absolute couplings
- 10-40% measurements of most couplings



What about the Higgs Spin?

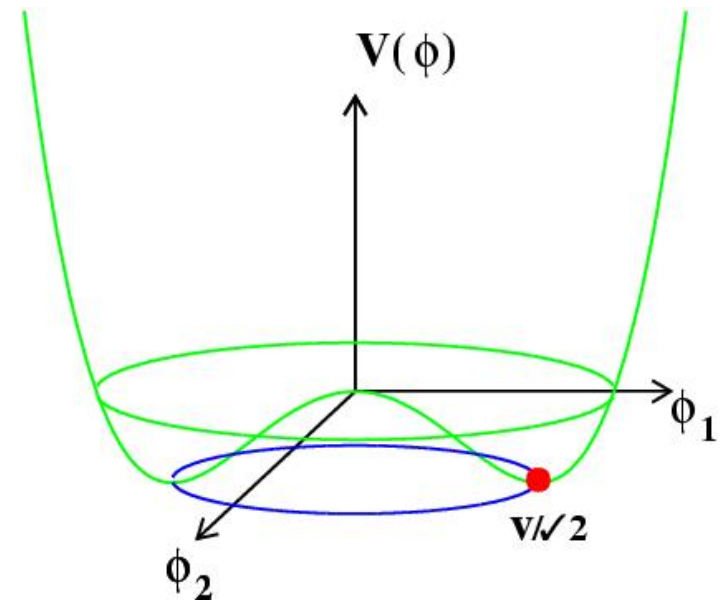
- $H \rightarrow ZZ \rightarrow 4l$ useful above $M_H = 200$ GeV
 - Study polar angle of leptons relative to Z boson
 - Study angle between decay planes of Z's in Higgs rest frame
- For $M_H > 200$ GeV, $> 5\sigma$ discrimination between spin -1,0,1 hypothesis for $L = 100 \text{ fb}^{-1}$
- Spin 1 hypothesis ruled out by observing $H\gamma\gamma$ or Hgg couplings



Can we reconstruct the Higgs potential?

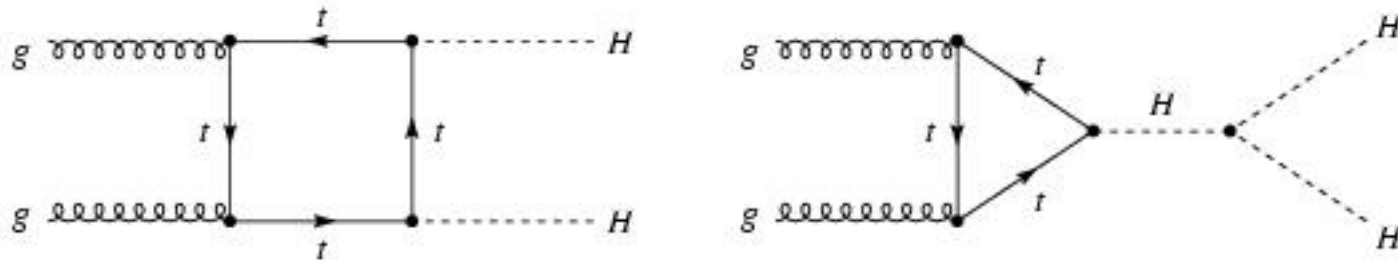
$$V = \frac{M_H^2}{2} H^2 + \lambda_3 v H^3 + \frac{\lambda_4}{4} H^4$$

$$SM : \lambda_3 = \lambda_4 = \frac{M_H^2}{2v^2}$$



- Fundamental test of model!
- We have no idea how to measure λ_4

Reconstructing the Higgs potential



- λ_3 requires 2 Higgs production
- $M_H < 140$ GeV, $H \rightarrow b\bar{b}b\bar{b}$
- Overwhelming QCD background

Can determine whether $\lambda_3 = 0$ at 95% cl
with 300 fb^{-1} for $150 < M_H < 200$ GeV

So we can probably find the Higgs

- Can we learn anything about the Higgs from places other than direct searches?

Basics

- SM is $SU(2) \times U(1)$ theory
 - Two gauge couplings: g and g'
- Higgs potential is $V = -\mu^2\phi^2 + \lambda\phi^4$
 - Two free parameters
- Four free parameters in gauge-Higgs sector

Basics, #2

- Chose parameters in gauge/Higgs sector
 - $\alpha=1/137.0359895(61)$
 - $G_F = 1.16637(1) \times 10^{-5} \text{ GeV}^{-2}$
 - $M_Z = 91.1875 \pm 0.0021 \text{ GeV}$
 - M_H

Express everything else in terms of these parameters

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{\pi\alpha}{2\left(1 - \frac{M_W^2}{M_Z^2}\right)M_W^2}$$

Inadequacy of Tree Level Calculations

- Mixing angle is predicted quantity
 - On-shell definition $\cos^2\theta_W = M_W^2/M_Z^2$
 - Predict M_W

$$M_W^2 = \pi\sqrt{2} \frac{\alpha}{G_F} \left(1 - \sqrt{1 - \frac{4\pi\alpha}{\sqrt{2}G_F M_Z^2}} \right)^{-1}$$

$$s_W^2 c_W^2 = \frac{\pi\alpha}{G_F M_Z^2}$$

- Plug in numbers:
 - M_W predicted = 80.939 GeV
 - $M_W(\text{exp}) = 80.399 \pm 0.025$ GeV
- Need to calculate beyond tree level

Quantum Corrections

- Relate tree level to one-loop corrected masses

$$-i\Pi_{XY}^{\mu\nu} = \text{wavy line} \text{---} \text{circle} \text{---} \text{wavy line}$$

$$\Pi_{XY}^{\mu\nu}(k^2) = g^{\mu\nu}\Pi_{XY}(k^2) + k^\mu k^\nu B_{XY}(k^2)$$

$$M_{V0}^2 = M_V^2 + \Pi_{VV}(M_V^2)$$

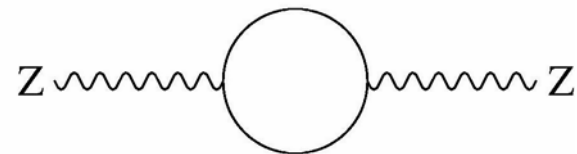
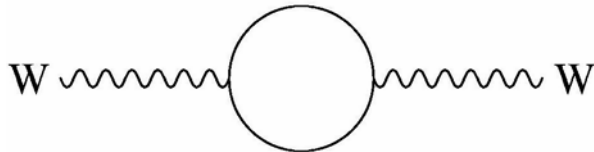
- Majority of corrections at one-loop are from 2-point functions

Note sign conventions for 2-point functions

Example of Quantum Corrections

- Example: $\rho = \frac{M_W^2}{M_Z^2 \cos^2 \theta_W} = 1 + \delta\rho$

$$\delta\rho = \frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2}$$



Top quark contributes to W and Z 2-point functions

Top Quark Corrections to ρ Parameter

- 2-point functions of W, Z

$$\Pi_{ZZ}(0) = \frac{g^2 N_c}{32\pi^2 c_W^2} \left(\frac{4\pi}{m_t^2} \right)^\varepsilon \frac{m_t^2}{\varepsilon} (R_t - L_t)^2$$

$$L_t = 1 - 4s_W^2/3$$

$$R_t = -4s_W^2/3$$

$$\Pi_{WW}(0) = \frac{g^2 N_c}{32\pi^2} \left(\frac{4\pi}{m_t^2} \right)^\varepsilon m_t^2 \left(\frac{1}{\varepsilon} + \frac{1}{2} \right)$$

$$\delta\rho = \frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2} = \frac{G_F}{\sqrt{2}} \frac{N_c}{8\pi^2} \left(\frac{m_t^2}{M_W^2} \right)$$

(Neglecting $\log(m_t)$ pieces and using on-shell definition of $\sin\theta_W$)

Heavy Higgs Contribution to $\delta\rho$



$$\rho = \frac{-3\alpha}{16\pi c_w^2} \log\left(\frac{M_H^2}{M_W^2}\right)$$

- In on-shell scheme, Higgs contributes logarithmically to quantum corrections & top quark contributes quadratically

Modification of tree level relations

$$G_F = \frac{\pi\alpha}{\sqrt{2}M_W^2 \sin^2 \theta_W} \frac{1}{(1 - \Delta r)}$$

- Δr is a physical quantity which incorporates 1-loop corrections
- Contributions to Δr from top quark and Higgs loops

$$\Delta r^t = -\frac{3G_F m_t^2}{8\sqrt{2}\pi^2} \left(\frac{\cos^2 \theta_W}{\sin^2 \theta_W} \right)$$

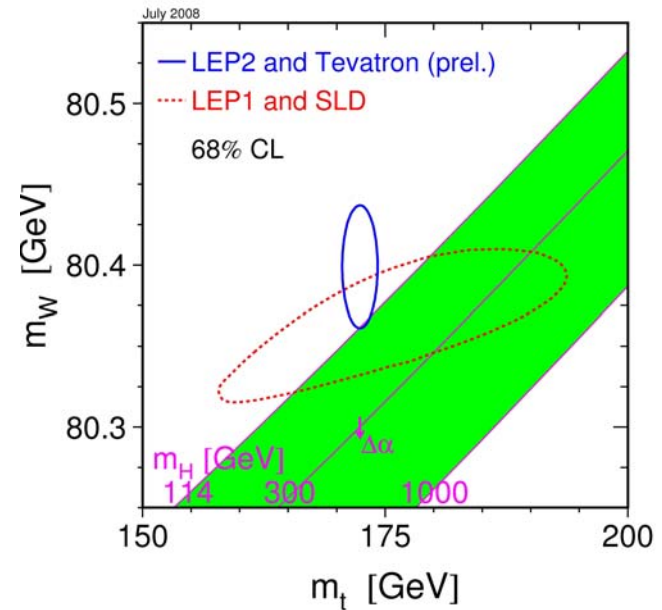
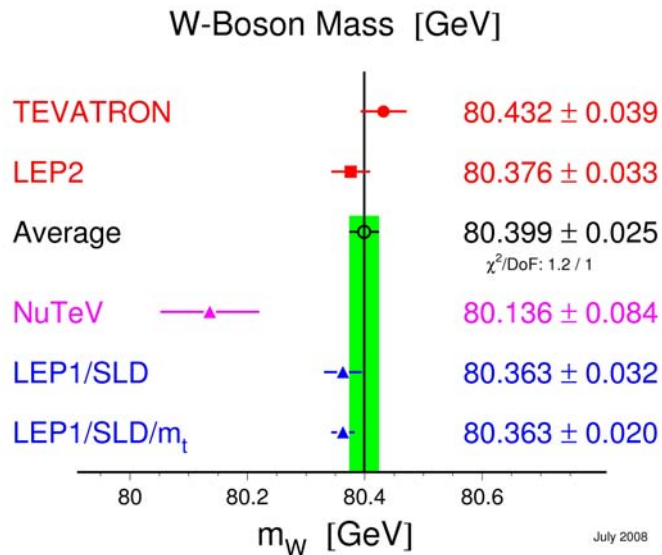
$$\Delta r^H = \frac{11G_F M_W^2}{24\sqrt{2}\pi^2} \left(\ln \frac{M_H^2}{M_W^2} - \frac{5}{6} \right)$$

Extreme sensitivity of precision measurements to m_t

Understanding Higgs Limit

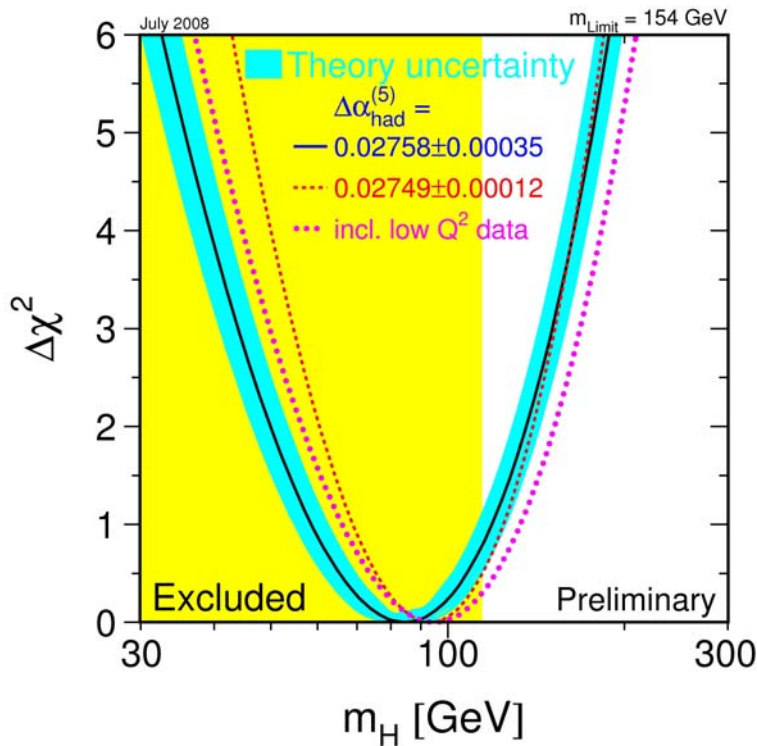
Theory: Input M_Z , G_F , α

→ Predict M_W



Consistency between direct and indirect measurements of M_W and m_t a strong test of theory!

Precision Measurements Limit M_H



- LEP EWWG (July, 2008):

- $m_t = 172.4 \pm 1.2 \text{ GeV}$
- $M_H = 84^{+34}_{-26} \text{ GeV}$
- $M_H < 154 \text{ GeV}$ (one-sided 95% cl)
- $M_H < 185 \text{ GeV}$ (Precision measurements plus direct search limit)

Best fit in region excluded from direct searches

Caveats

- Low Q^2 data not included in fit
 - Doesn't include atomic parity violation in cesium, parity violation in Moller scattering, & neutrino-nucleon scattering (NuTeV)
 - Higgs fit not hugely sensitive to low Q^2 data
- $M_H < 185 \text{ GeV}$
 - Higgs limit moves around with m_t

Higgs limit assumes SM!

Theoretical Limits on M_H

- Unitarity
 - If unitarity is violated, interactions grow with energy (cf longitudinal W's)
- Perturbativity of couplings
 - If perturbativity is violated, loop corrections may be larger than tree
- Limits tell us where minimal SM is valid
 - Unitarity and perturbativity provide strong limits on beyond the SM physics
 - These are model builders tools
- Renormalization of Higgs mass
 - What about naturalness?

Unitarity

- Consider $2 \rightarrow 2$ elastic scattering

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} |A|^2$$

- Partial wave decomposition of amplitude

$$A = 16\pi \sum_{l=0}^{\infty} (2l+1) P_l(\cos\theta) a_l$$

- a_l are the spin / partial waves

Unitarity

- $P_l(\cos\theta)$ are Legendre polynomials:

$$\int_{-1}^1 dx P_l(x) P_{l'}(x) = \frac{2\delta_{l,l'}}{2l+1}$$

$$\begin{aligned}\sigma &= \frac{8\pi}{s} \sum_{l=0}^{\infty} (2l+1) \sum_{l'=0}^{\infty} (2l'+1) a_l a_{l'}^* \int_{-1}^1 d \cos \theta P_l(\cos \theta) P_{l'}(\cos \theta) \\ &= \frac{16\pi}{s} \sum_{l=0}^{\infty} (2l+1) |a_l|^2\end{aligned}$$

- Sum of positive definite terms

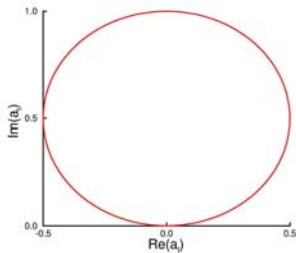
More on Unitarity

- Optical theorem $\sigma = \frac{1}{s} \text{Im}[A(\theta = 0)] = \frac{16\pi}{s} \sum_{l=0}^{\infty} (2l + 1) |a_l|^2$

$$\text{Im}(a_l) = |a_l|^2$$

Optical theorem derived
assuming only conservation
of probability

- Unitarity requirement:



$$|\text{Re}(a_l)| \leq \frac{1}{2}$$

More on Unitarity

- Idea: Use unitarity to limit parameters of theory

Cross sections which grow with energy always violate unitarity at some energy scale

- Remember $W_L(p)$ with $\varepsilon_L \sim p/M_W$

Aside on WW Scattering

$$A(V_L^1 \dots V_L^N \rightarrow V_L^1 \dots V_L^{N'}) = (i)^N (-i)^{N'} A(\omega_1 \dots \omega_N \rightarrow \omega_1 \dots \omega_{N'}) \\ + \mathcal{O}\left(\frac{M_W^2}{E^2}\right)$$

This is a statement about scattering amplitudes, NOT individual Feynman diagrams

ω^\pm , z are Goldstone bosons which are eaten by the Higgs mechanism to give the W & Z bosons their longitudinal components

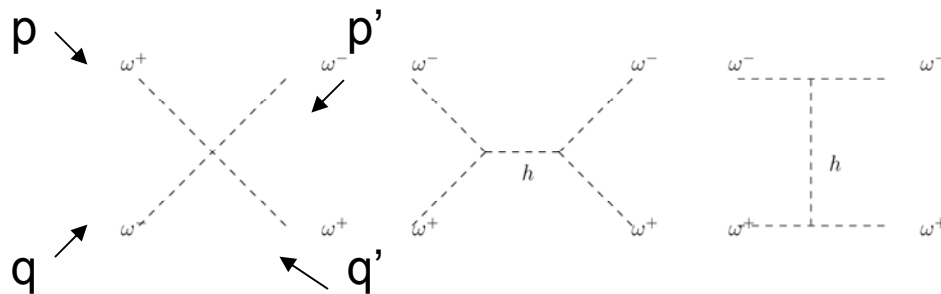
$W^+W^- \rightarrow W^+W^-$

- Recall scalar potential

$$V = \frac{M_H^2}{2} H^2 + \frac{M_H^2}{2v} H(H^2 + z^2 + 2\omega^+\omega^-) + \frac{M_H^2}{8v^2} (H^2 + z^2 + 2\omega^+\omega^-)^2$$

- $\omega^+\omega^- \rightarrow \omega^+\omega$

$$iA(\omega^+\omega \rightarrow \omega^+\omega^-) = -2i\frac{M_H^2}{v^2} + \left(-i\frac{M_H^2}{v}\right)^2 \frac{i}{t - M_H^2} + \left(-i\frac{M_H^2}{v}\right)^2 \frac{i}{s - M_H^2}$$



$$s = (p + q)^2$$

$$t = (p - p')^2$$

$$u = (q - p')^2$$

$$\omega^+ \omega^- \rightarrow \omega^+ \omega^-$$

- Two interesting limits:

$$-s, t \gg M_H^2$$

$$A(\omega^+ \omega^- \rightarrow \omega^+ \omega^-) \rightarrow -2 \frac{M_H^2}{v^2}$$

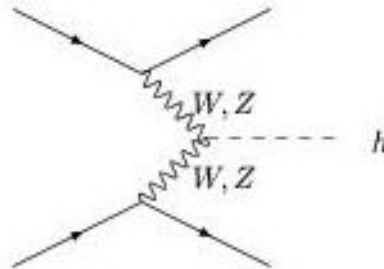
$$-s, t \ll M_H^2$$

$$A(\omega^+ \omega^- \rightarrow \omega^+ \omega^-) \rightarrow -\frac{u}{v^2}$$

$$a_0^0 \rightarrow -\frac{M_H^2}{8\pi v^2}$$

$$a_0^0 \rightarrow -\frac{s}{32\pi v^2}$$

Remember: This is physical process



Use Unitarity to Bound Higgs

$$|\operatorname{Re}(a_l)| \leq \frac{1}{2}$$

- High energy limit:

$$a_0^0 \rightarrow -\frac{M_H^2}{8\pi v^2}$$

$$M_H < 800 \text{ GeV}$$

- Heavy Higgs limit

$$a_0^0 \rightarrow -\frac{s}{32\pi v^2}$$

$$E_c \sim 1.7 \text{ TeV}$$

→ ***New physics at the TeV scale***

Can get more stringent bound from coupled channel analysis

Another Sort of Limit: Landau Pole

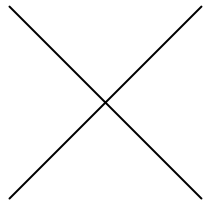
- M_H is a free parameter in the Standard Model
- Can we derive limits from consistency?
- Consider a scalar potential:

$$V = \frac{M_H^2}{2} H^2 + \frac{\lambda}{4} H^4$$

- This is potential at electroweak scale
- Parameters evolve with energy in a calculable way

Consider $HH \rightarrow HH$

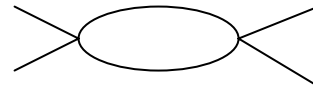
- Real scattering, $s+t+u=4M_H^2$
- Consider momentum space-like and off-shell:
 $s=t=u=Q^2 < 0$
- Tree level: $iA_0 = -6i\lambda$



HH → HH, #2

- One loop:

$$\begin{aligned} iA_s &= (-6i\lambda)^2 \frac{1}{2} \int \frac{d^n k}{(2\pi)^2} \frac{i}{k^2 - M_H^2} \frac{i}{(k + p + q)^2 - M_H^2} \\ &= \frac{9\lambda^2}{8\pi^2} (4\pi\mu^2) \Gamma(\varepsilon) (M_H^2 - Q^2 x(1-x))^{-\varepsilon} \end{aligned}$$



- $A = A_0 + A_s + A_t + A_u$

$$A = -6\lambda \left(1 + \frac{9\lambda}{16\pi^2} (4\pi\mu^2) \Gamma(\varepsilon) (M_H^2 - Q^2 x(1-x))^{-\varepsilon} + \dots \right)$$

HH→HH, #3

- Sum the geometric series to define running coupling

$$A = -6\lambda \left(1 + \frac{9\lambda}{16\pi^2} \log \frac{Q^2}{M_H^2} \right) + \dots$$

$$A = \frac{6\lambda}{1 - \frac{9\lambda}{8\pi^2} \log \left(\frac{Q}{M_H} \right)} \equiv 6\lambda(Q)$$

- $\lambda(Q)$ blows up as $Q \rightarrow \infty$ (called Landau pole)

HH \rightarrow HH, #4

- This is independent of starting point
- BUT.... Without $\lambda\phi^4$ interactions, theory is non-interacting
- Require quartic coupling be finite

$$\frac{1}{\lambda(Q)} > 0$$

HH→HH, #5

- Use $\lambda=M_H^2/(2v^2)$ and approximate $\log(Q/M_H) \rightarrow \log(Q/v)$
- Requirement for $1/\lambda(Q)>0$ gives upper limit on M_H

$$M_H^2 < \frac{32\pi^2 v^2}{9 \log\left(\frac{Q^2}{v^2}\right)}$$

- Assume theory is valid to 10^{16} GeV
 - Gives upper limit on $M_H < 180$ GeV
- Can add fermions, gauge bosons, etc.

We expect Higgs at electroweak scale

High Energy Behavior of λ

- Renormalization group scaling $\frac{1}{\lambda(Q)} = \frac{1}{\lambda(\mu)} + (\dots) \log\left(\frac{Q}{\mu}\right)$

$$16\pi^2 \frac{d\lambda}{dt} = 12\lambda^2 + 12\lambda g_t^2 - 12g_t^4 + (\text{gauge})$$

$$t \equiv \log\left(\frac{Q^2}{\mu^2}\right)$$

$$g_t = \frac{M_t}{v}$$

- *Large λ (Heavy Higgs)*: self coupling causes λ to grow with scale
- *Small λ (Light Higgs)*: coupling to top quark causes λ to become negative

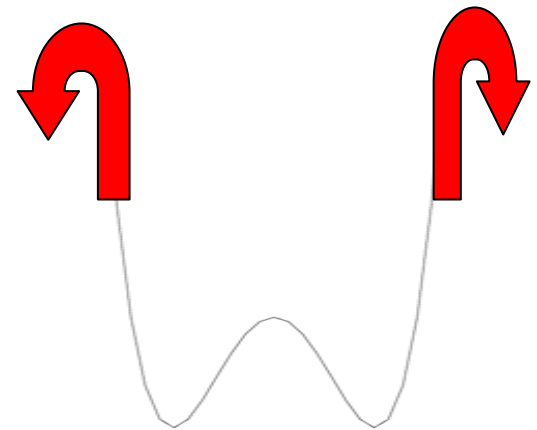
Does Spontaneous Symmetry Breaking Happen?

- SM requires spontaneous symmetry
- This requires $V(v) < V(0)$
- For small λ

$$16\pi^2 \frac{d\lambda}{dt} \approx -16g_t^4$$

- Solve

$$\lambda(\Lambda) \approx \lambda(v) - \frac{3g_t^4}{4\pi^2} \log\left(\frac{\Lambda^2}{v^2}\right)$$



Does Spontaneous Symmetry Breaking Happen? (#2)

- $\lambda(\Lambda) > 0$ gives lower bound on M_H

$$M_H^2 > \frac{3v^2}{2\pi^2} \log\left(\frac{\Lambda^2}{v^2}\right)$$

- If Standard Model valid to 10^{16} GeV

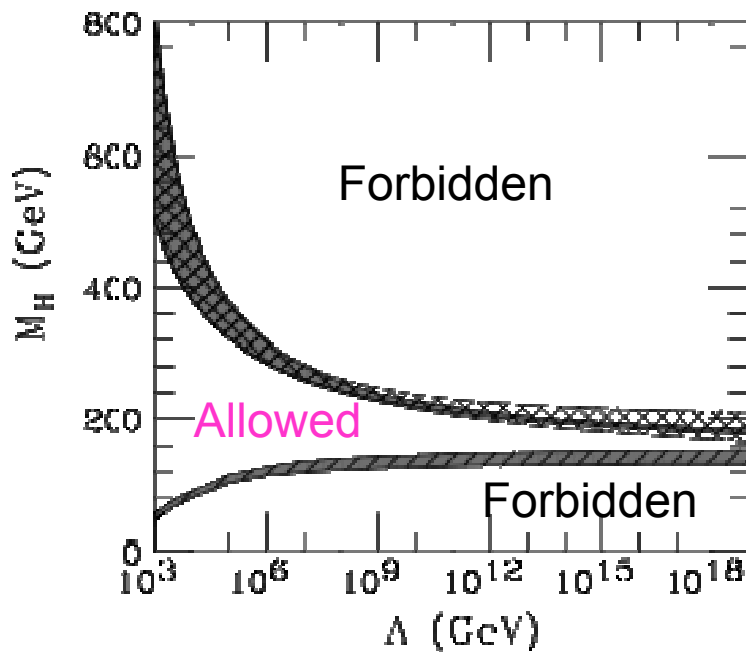
$$M_H > 130 \text{ GeV}$$

- For any given scale, Λ , there is a theoretically consistent range for M_H

Light Higgs Theoretically Attractive

➤ Extrapolate Higgs potential to high scale Λ

$$V = \lambda (\Phi^\dagger \Phi - v^2)^2$$



- Standard Model is only consistent to GUT scale for small range of Higgs masses
- Heavy Higgs implies new physics at some low scale