QCD for LHC Physics - 2

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The parton distribution functions give us a very simple model for the cross section for hard-scattering events in proton-proton collisions. Take a parton from each proton, and fold the distributions with the cross sections computed in perturbative QCD:

$$\sigma(pp \to ab + X) = \int d\xi_1 d\xi_2 \sum_{f1,f2} f_{f1}(\xi_1) f_{f2}(\xi_2) \int d\cos\theta_* \frac{d\sigma}{d\cos\theta_*}(f_1 f_2 \to ab)$$

(or, invariantly, integrate over $t = -\frac{1}{2}s(1-\cos\theta_*)$)

An important relation is: $s = \xi_1\xi_2\, s(pp)$ or $s = x_1 x_2\, s(pp)$

The simplest parton-parton cross sections are those for lepton pair production from a quark and an antiquark.

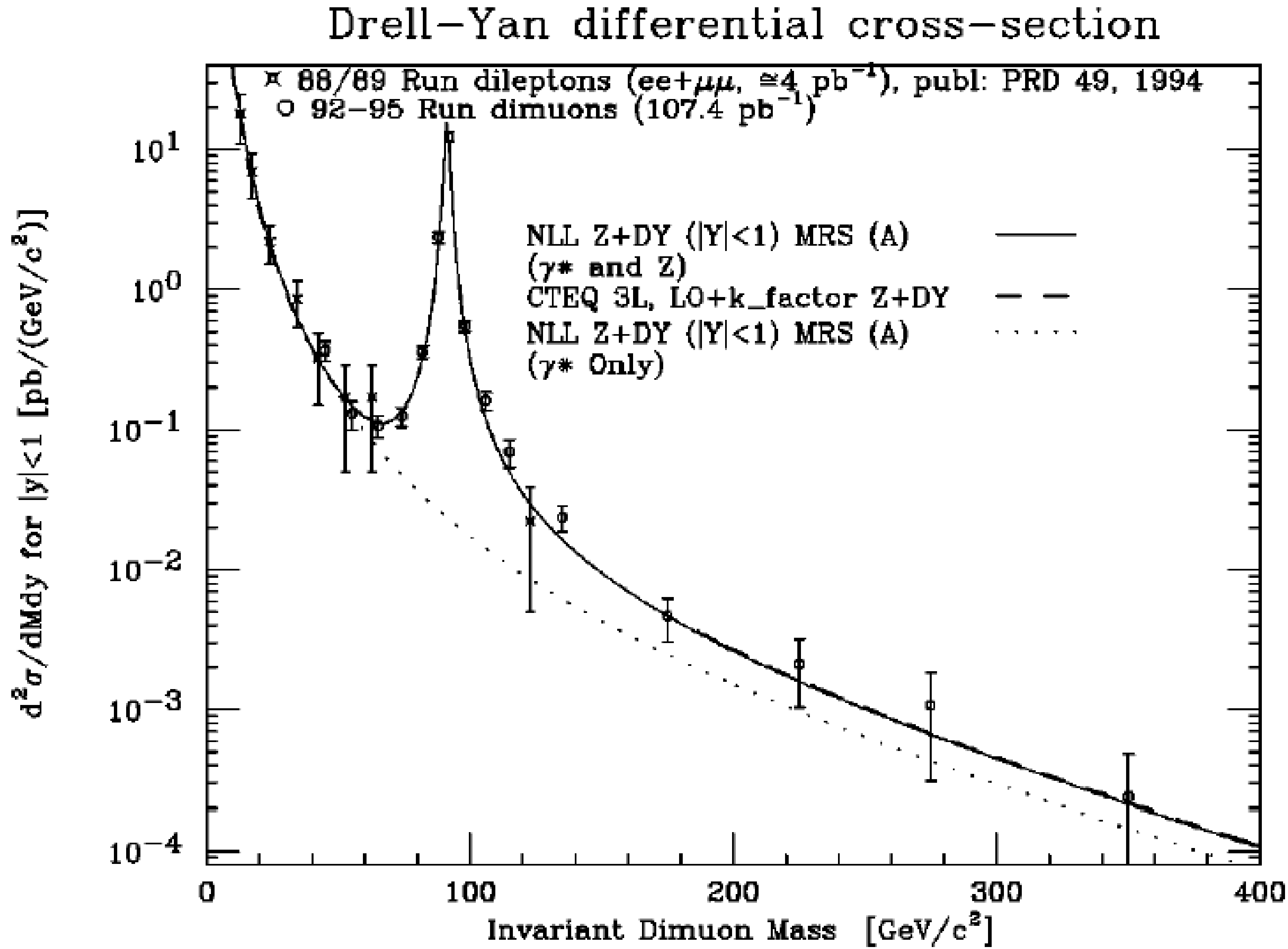
Drell-Yan process:

$$\sigma(q_f \overline{q}_f \to \mu^+ \mu^-) = \frac{4\pi\alpha^2}{3s} \cdot \frac{1}{3} \mathcal{Q}_f^2$$

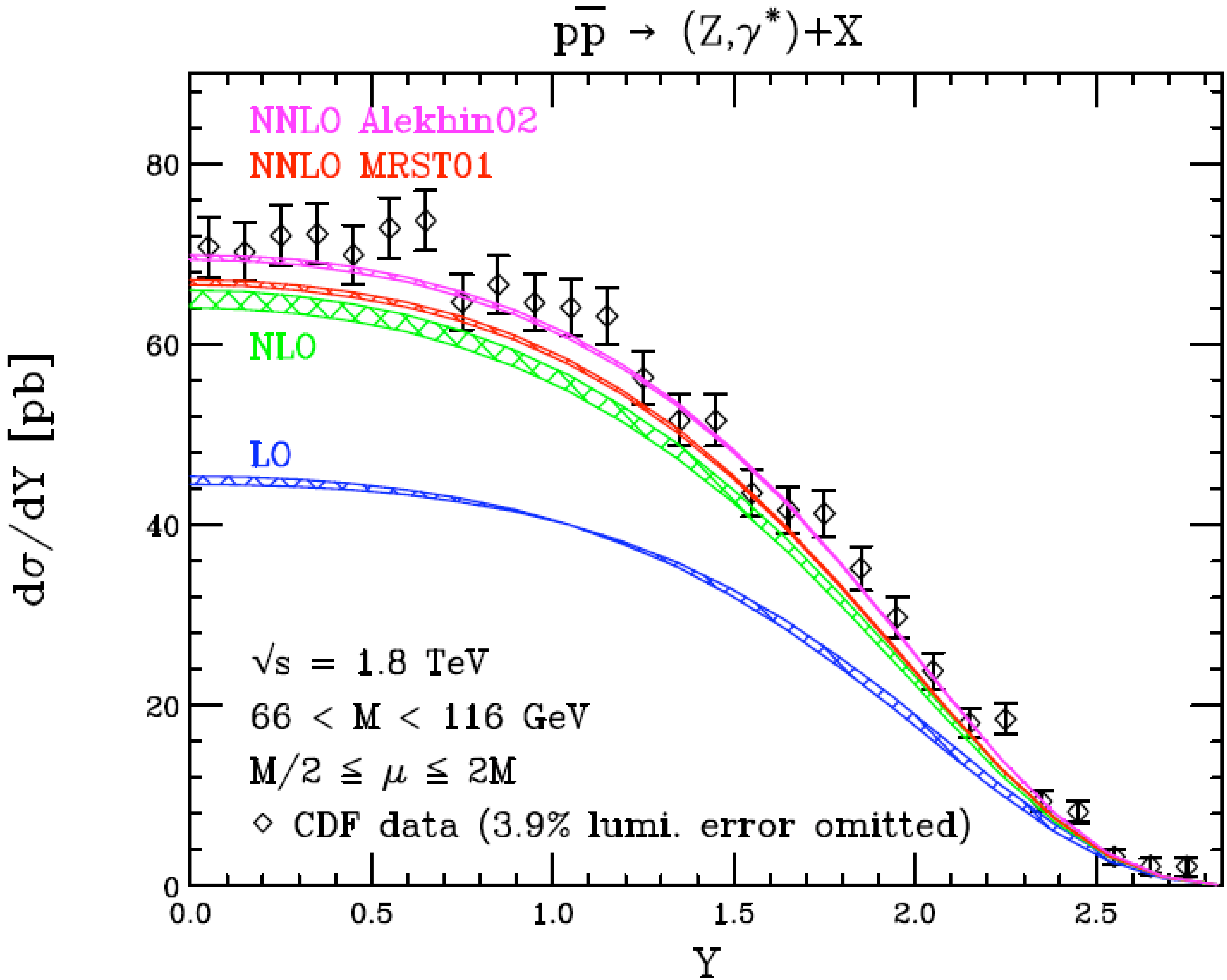
At high energy, the weak interaction resonances are important. Here are the resonant contributions:

$$\sigma(u\overline{d} \to Z^0) = \frac{2\pi^2\alpha_w}{3c_w^2}[(\frac{1}{2} - |\mathcal{Q}_f|s_w^2)^2 + (\mathcal{Q}_f s_w^2)^2]\delta(s - m_Z^2)$$

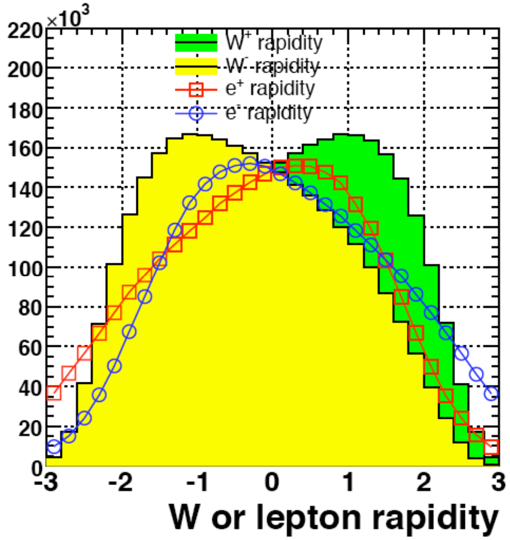
$$\sigma(u\overline{d} \to W^+) = \frac{\pi^2\alpha_w}{3}\delta(s - m_W^2)$$

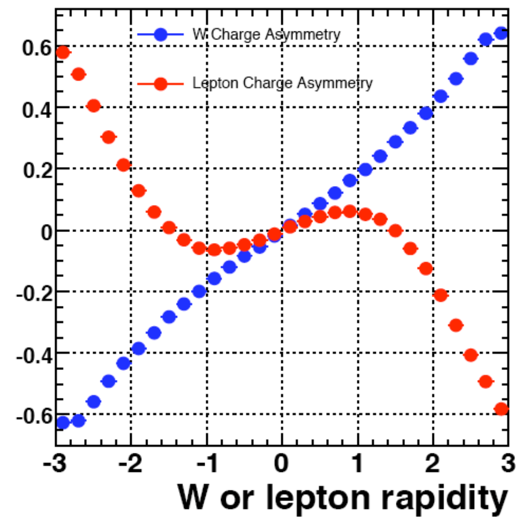

Drell−Yan differential cross−section
88/89 Run dileptons (ee+μμ, ≅4 pb⁻¹), publ: PRD 49, 1994
92−95 Run dimuons (107.4 pb⁻¹)
NLL Z+DY (|Y|<1) MRS (A)
(γ* and Z)
CTEQ 3L, LO+k_factor Z+DY
NLL Z+DY (|Y|<1) MRS (A)
(γ* Only)
d²σ/dMdy for |y|<1 [pb/(GeV/c²)]
Invariant Dimuon Mass [GeV/c²]
10¹
10⁰
10⁻¹
10⁻²
10⁻³
10⁻⁴
0
100
200
300
400

CDF

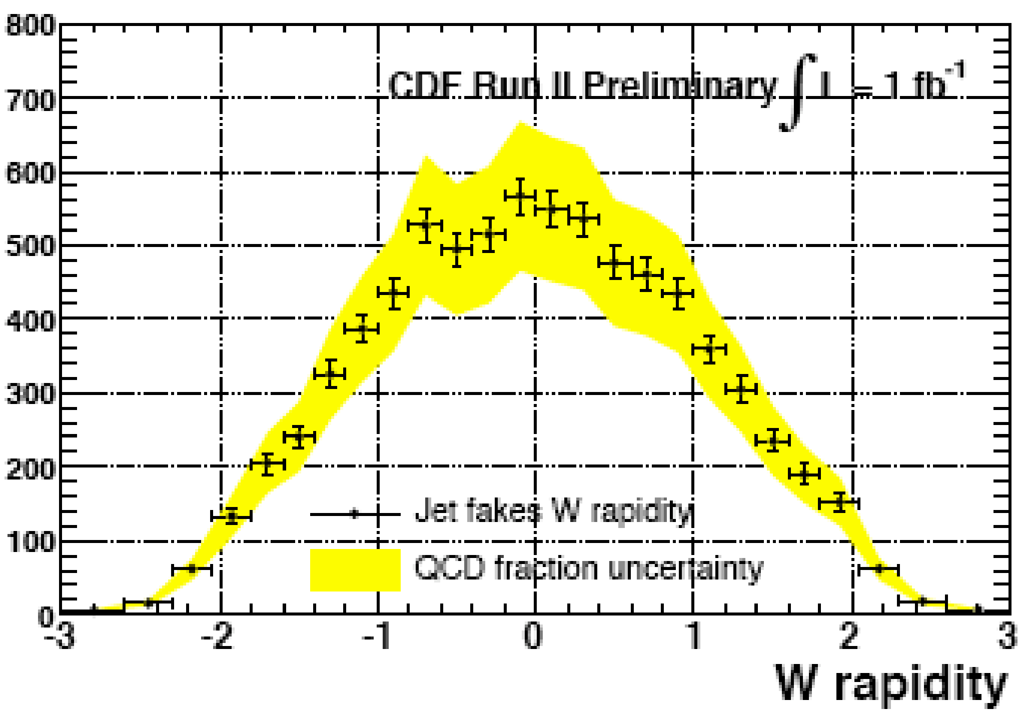


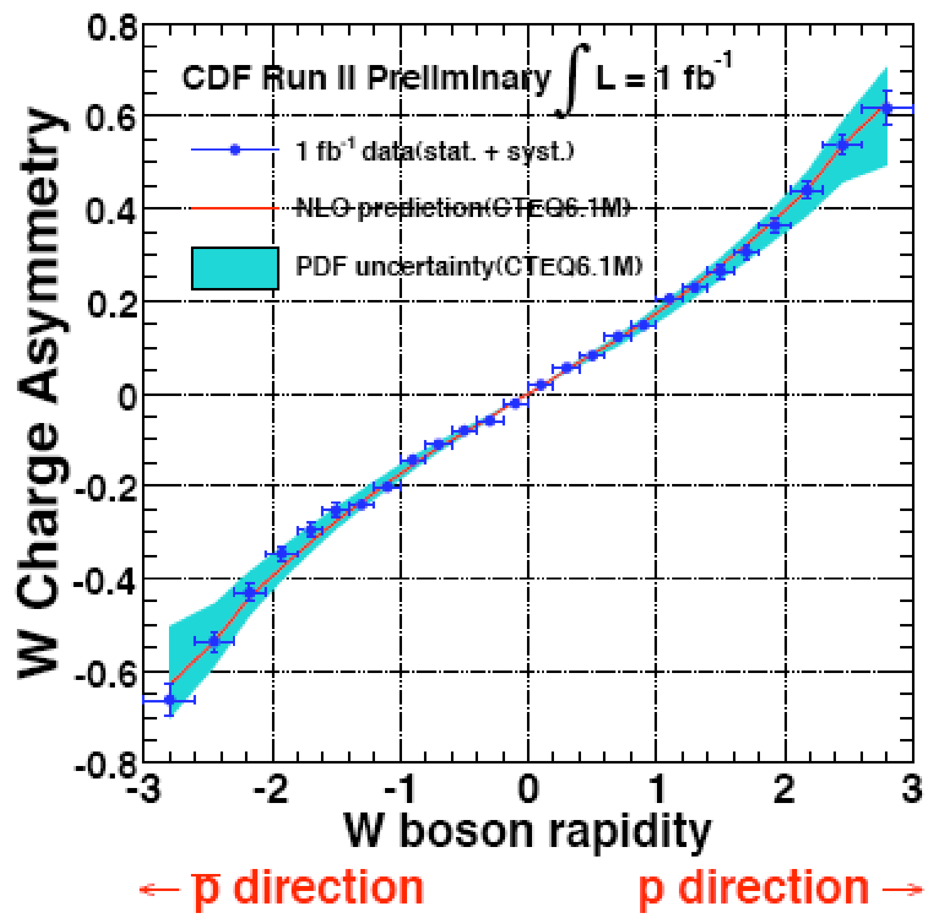
Anastasiou, Dixon, Melnikov, Petriello


W+ rapidity
W- rapidity
e+ rapidity
e- rapidity
W or lepton rapidity


W Charge Asymmetry
Lepton Charge Asymmetry
W or lepton rapidity

CDF


CDF Run II Preliminary ∫ L = 1 fb-1
Jet fakes W rapidity
QCD fraction uncertainty
W rapidity


CDF Run II Preliminary ∫ L = 1 fb-1
1 fb-1 data(stat. + syst.)
NLO prediction(CTEQ6.1M)
PDF uncertainty(CTEQ6.1M)
W Charge Asymmetry
W boson rapidity
← p̄ direction
p direction →

We can apply the same logic to parton-parton hard scattering reactions.

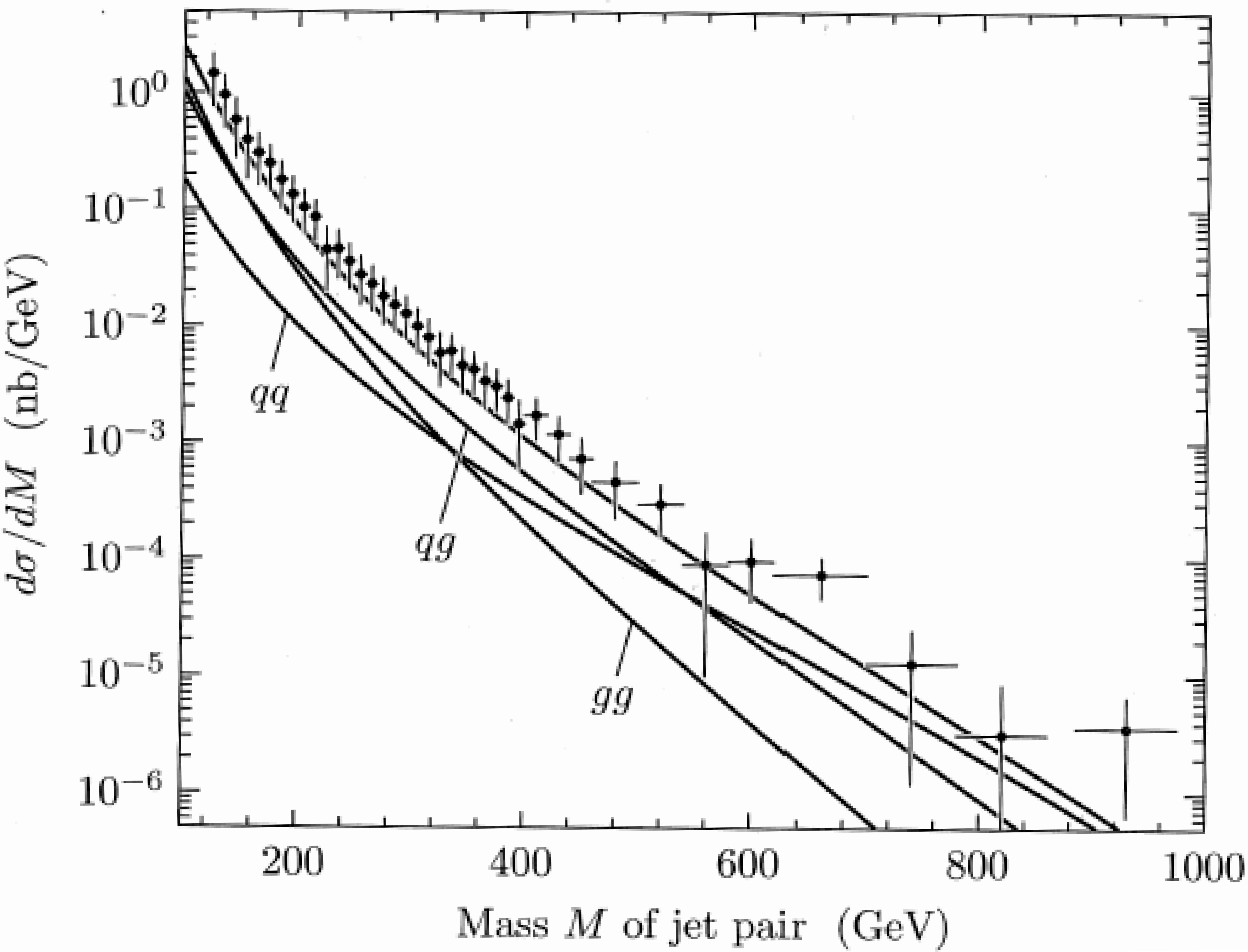
For quark-quark and -antiquark scattering, the cross sections are very similar to the cross sections for electron-quark scattering discussed yesterday.

$$
\begin{aligned}
\frac{d\sigma}{d\cos\theta_*}(u\overline{u} \to d\overline{d}) &= \frac{2}{9}\frac{\pi\alpha_s^2}{s}[\frac{u^2+t^2}{s^2}] \\
\frac{d\sigma}{d\cos\theta_*}(u\overline{u} \to u\overline{u}) &= \frac{2}{9}\frac{\pi\alpha_s^2}{s}[\frac{u^2+t^2}{s^2} + \frac{u^2+s^2}{t^2} - \frac{2}{3}\frac{u^2}{st}] \\
\frac{d\sigma}{d\cos\theta_*}(ud \to u) &= \frac{2}{9}\frac{\pi\alpha_s^2}{s}[\frac{u^2+s^2}{t^2}] \\
\frac{d\sigma}{d\cos\theta_*}(uu \to uu) &= \frac{2}{9}\frac{\pi\alpha_s^2}{s}[\frac{u^2+s^2}{t^2} + \frac{s^2+t^2}{u^2} - \frac{2}{3}\frac{s^2}{ut}]
\end{aligned}
$$

Quark-gluon and gluon-gluon reactions bring in new cross sections. I will derive some of these tomorrow.

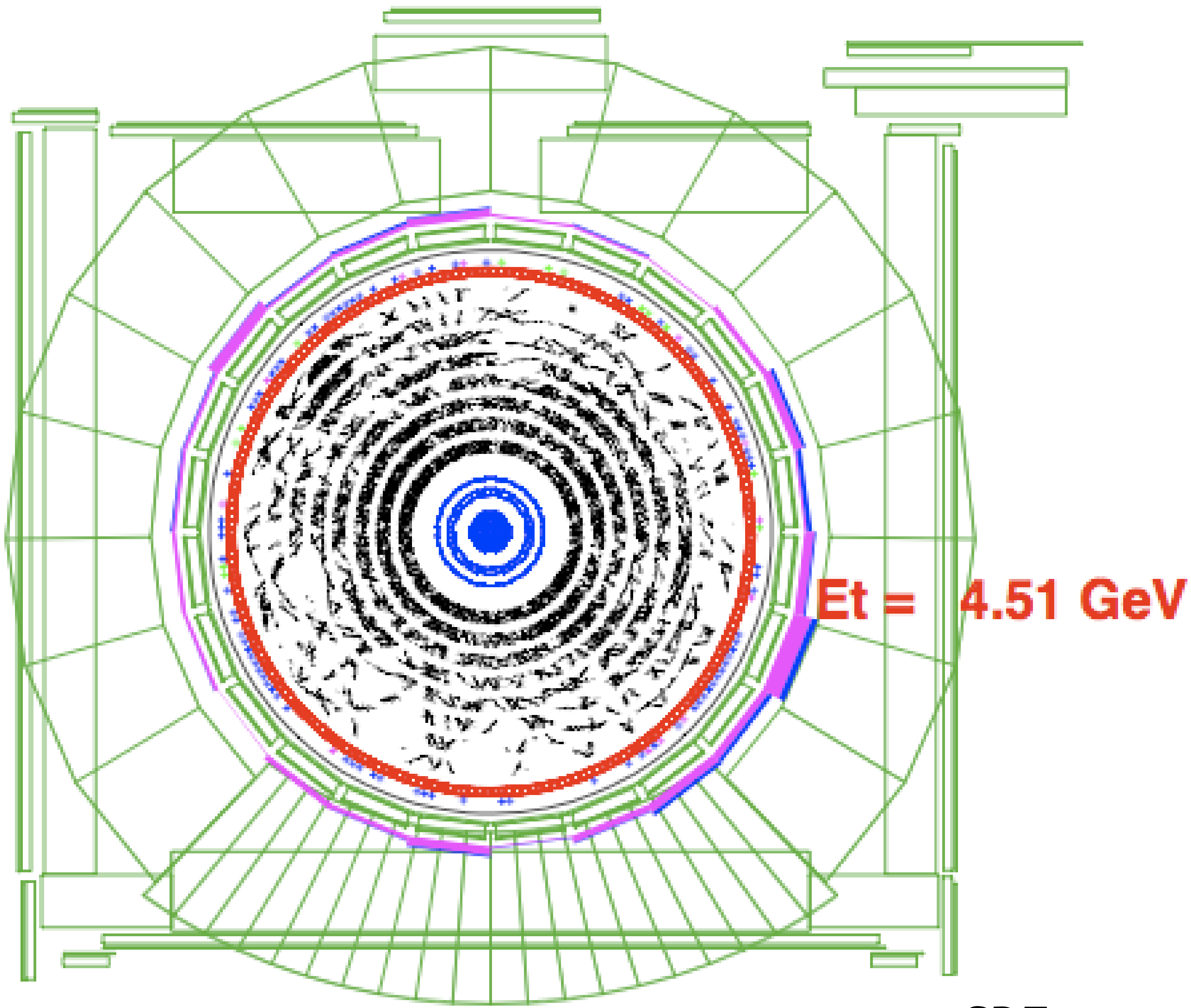
$$\begin{aligned}
\frac{d\sigma}{d\cos\theta_*}(u\overline{u} \to gg) &= \frac{16}{27}\frac{\pi\alpha_s^2}{s}[\frac{u}{t} + \frac{t}{u} - \frac{9}{4}\frac{u^2+t^2}{s^2}] \\
\frac{d\sigma}{d\cos\theta_*}(gg \to u\overline{u}) &= \frac{1}{12}\frac{\pi\alpha_s^2}{s}[\frac{u}{t} + \frac{t}{u} - \frac{9}{4}\frac{u^2+t^2}{s^2}] \\
\frac{d\sigma}{d\cos\theta_*}(ug \to ug) &= \frac{2}{9}\frac{\pi\alpha_s^2}{s}[-\frac{u}{s} - \frac{s}{u} + \frac{9}{4}\frac{u^2+s^2}{t^2}] \\
\frac{d\sigma}{d\cos\theta_*}(gg \to gg) &= \frac{9}{4}\frac{\pi\alpha_s^2}{s}[3 - \frac{st}{u^2} - \frac{su}{t^2} - \frac{ut}{s^2}]
\end{aligned}$$

(For identical particles in the final state, integrate over only half of 4π .)



$d\sigma/dM$ (nb/GeV)
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
qq
qg
gg
200
400
600
800
1000
Mass M of jet pair (GeV)

CDF - 1.8 TeV



CDF event

A much clearer visualization of the event is given by using variables that emphasize the momentum structure transverse to the initial beam directions.

$$(\theta, \phi, p) \to (\eta, \phi, p_T)$$

rapidity:

transverse-moving particle: $(m_T, \vec{p}_T, 0) \quad m_T = (m^2 + p_T^2)^{1/2}$

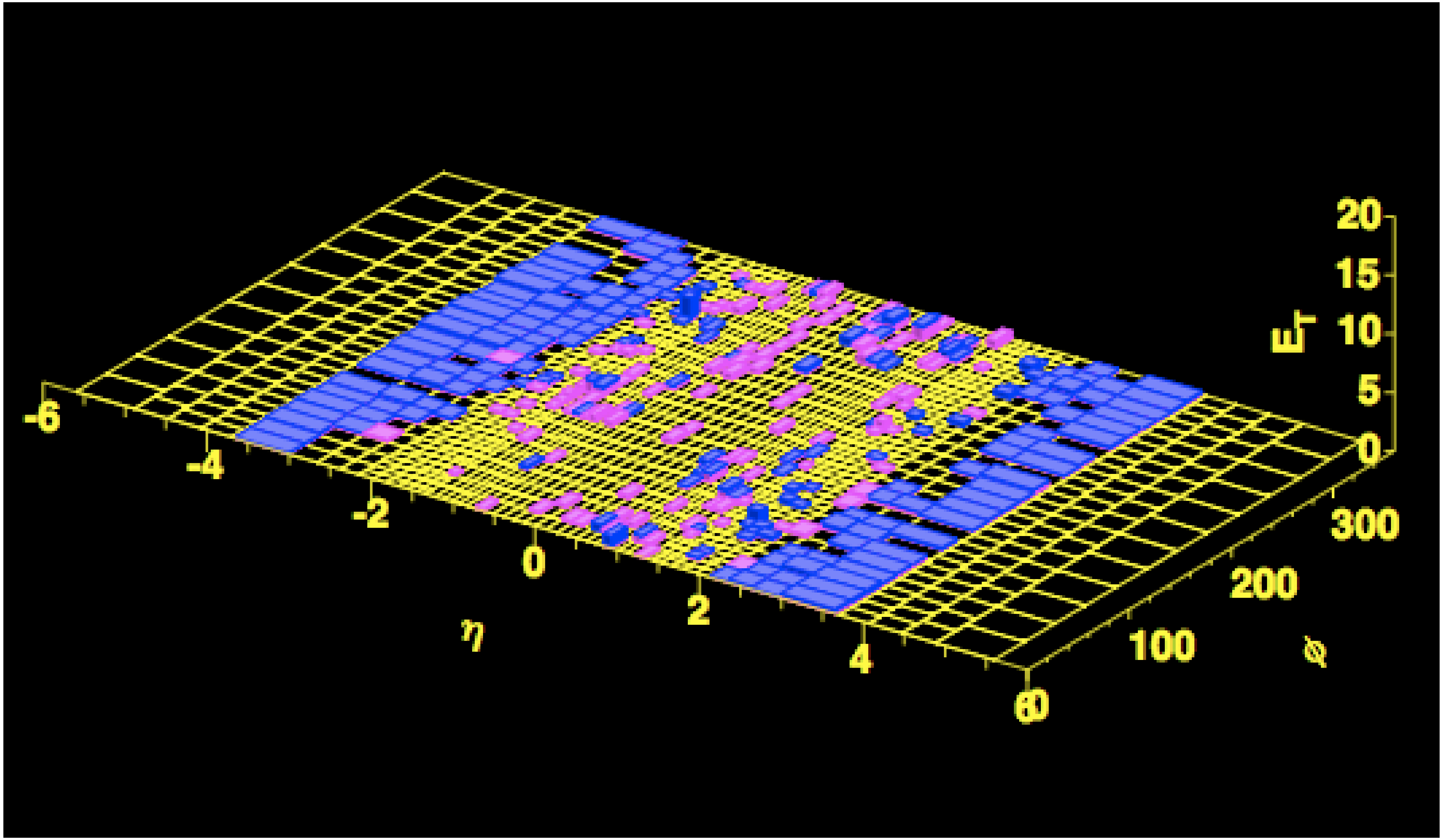
general particle: $(m_T \cosh y, \vec{p}_T, m_T \sinh y)$

pseudo-rapidity:

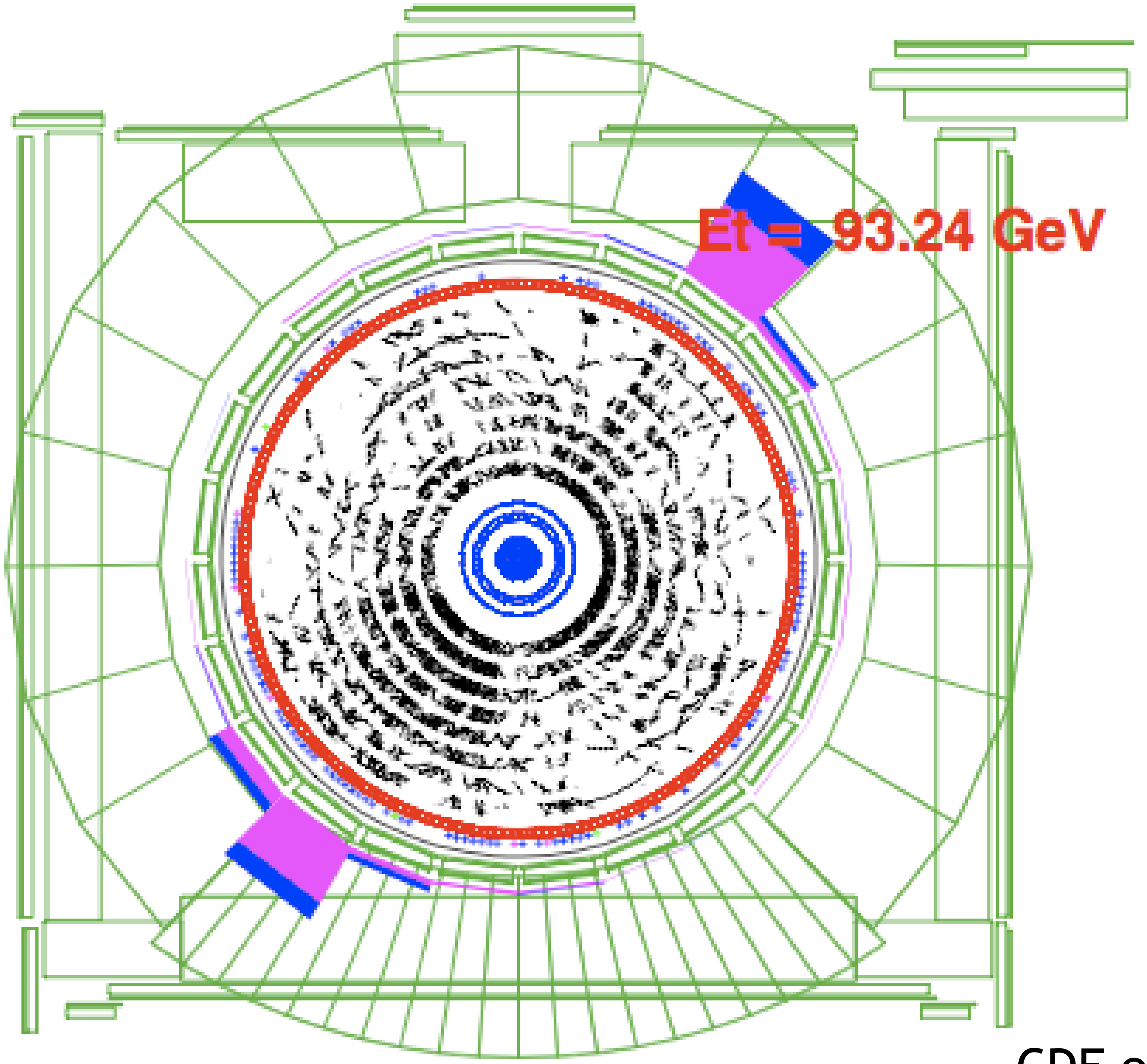
Use the same formulae with the assumption of zero mass. Then there is a direct relation between the rapidity variable and polar angle.

$$\cos\theta = \tanh\eta \qquad \eta = \frac{1}{2}\log\frac{1+\cos\theta}{1-\cos\theta}$$

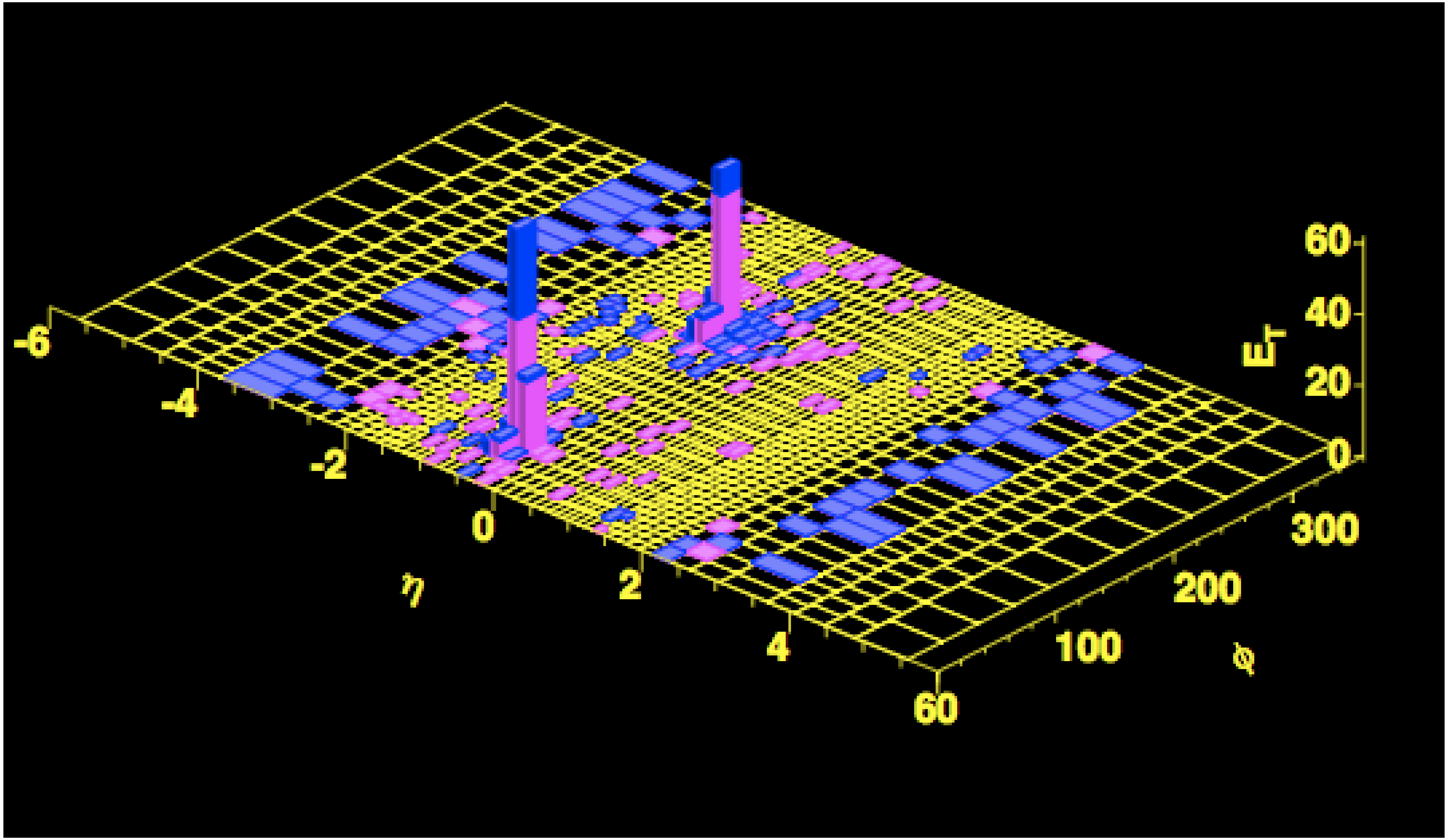
The plot of $|p_T|$ over the plane of (η, ϕ) is called the 'Lego plot'.



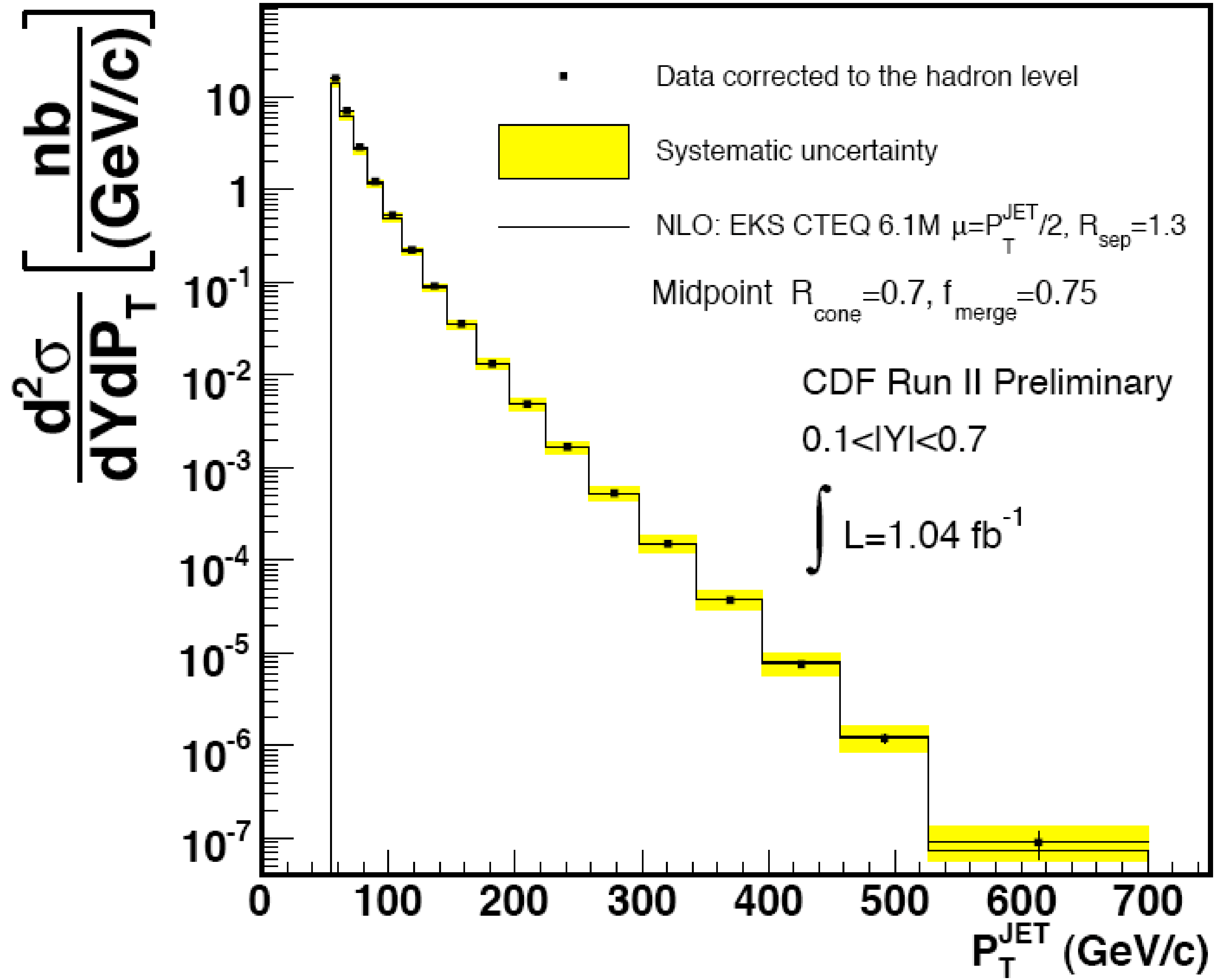
CDF event



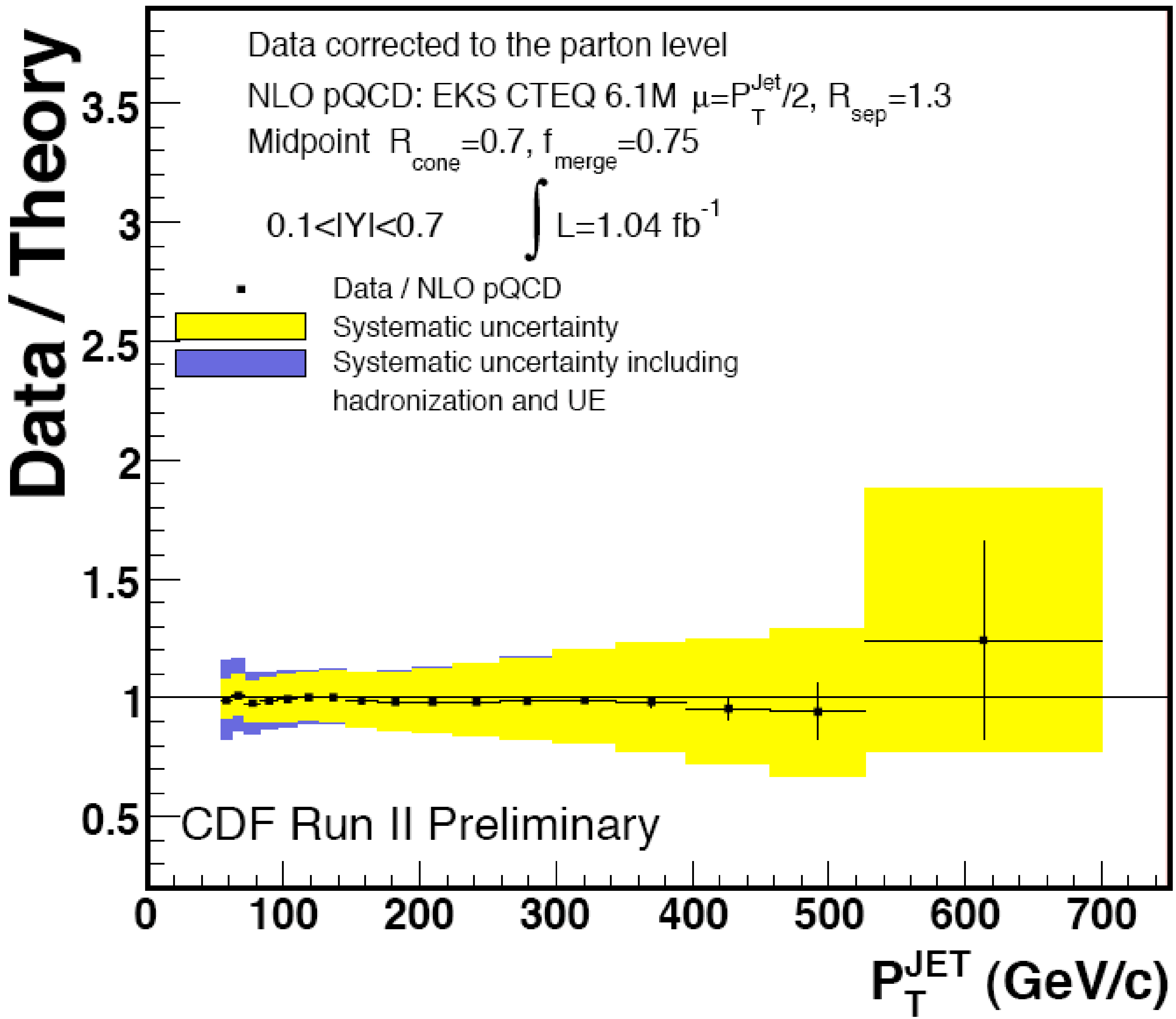
CDF event



CDF event



$\frac{d^2\sigma}{dYdP_T}\left[\frac{nb}{(GeV/c)}\right]$
Data corrected to the hadron level
Systematic uncertainty
NLO: EKS CTEQ 6.1M $\mu=P_T^{JET}/2$, $R_{sep}=1.3$
Midpoint $R_{cone}=0.7$, $f_{merge}=0.75$
CDF Run II Preliminary
0.1<|Y|<0.7
$\int L=1.04\ fb^{-1}$
10
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0
100
200
300
400
500
600
700
P_T^{JET} (GeV/c)


Data / Theory
Data corrected to the parton level
NLO pQCD: EKS CTEQ 6.1M μ=P_T^Jet/2, R_sep=1.3
Midpoint R_cone=0.7, f_merge=0.75
0.1<|Y|<0.7
∫L=1.04 fb^-1
Data / NLO pQCD
Systematic uncertainty
Systematic uncertainty including
hadronization and UE
CDF Run II Preliminary
P_T^JET (GeV/c)
0
100
200
300
400
500
600
700
0.5
1
1.5
2
2.5
3
3.5

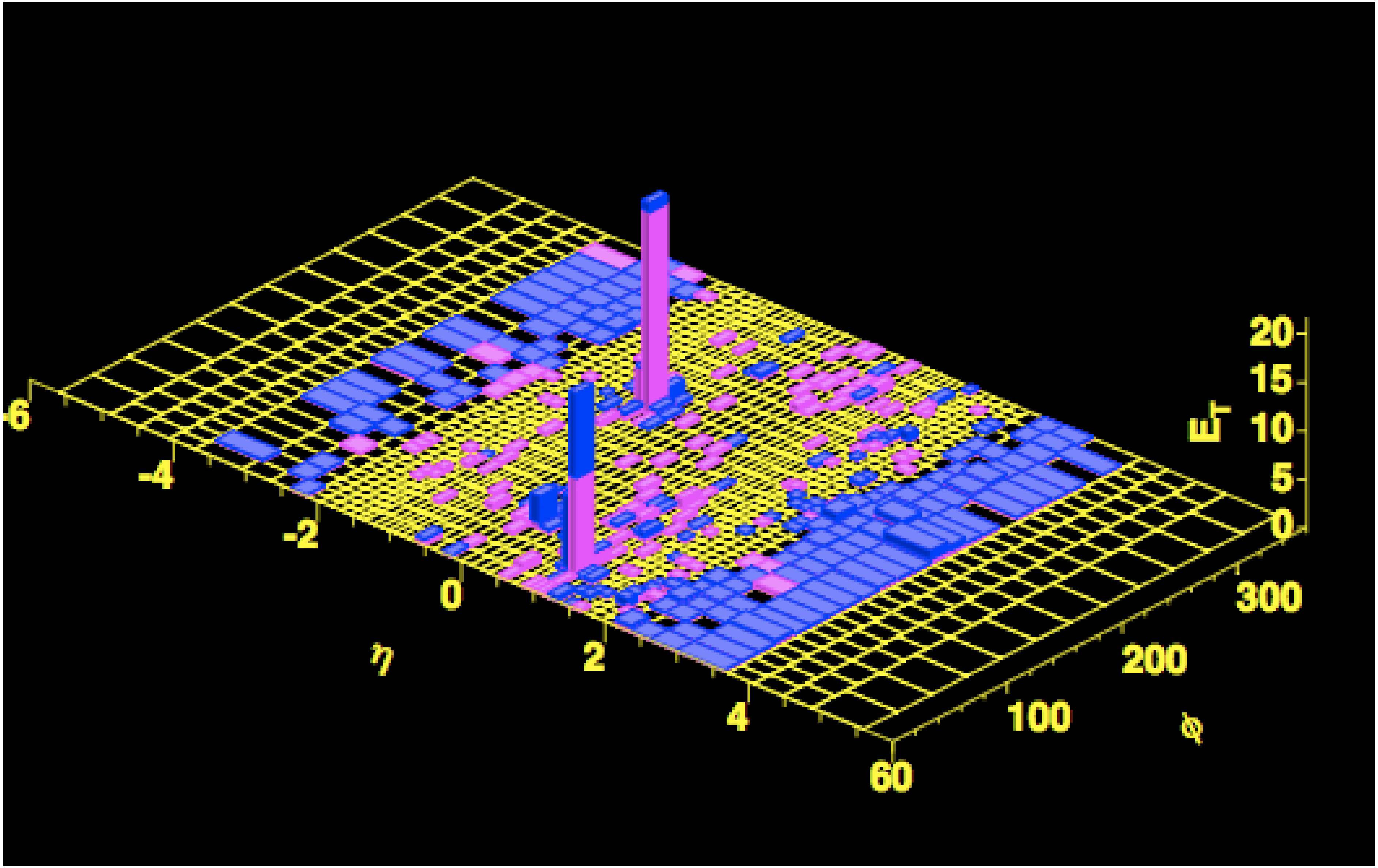
Look more closely at the event pictures in the Lego plot.

How do we define a jet ?   In particular,

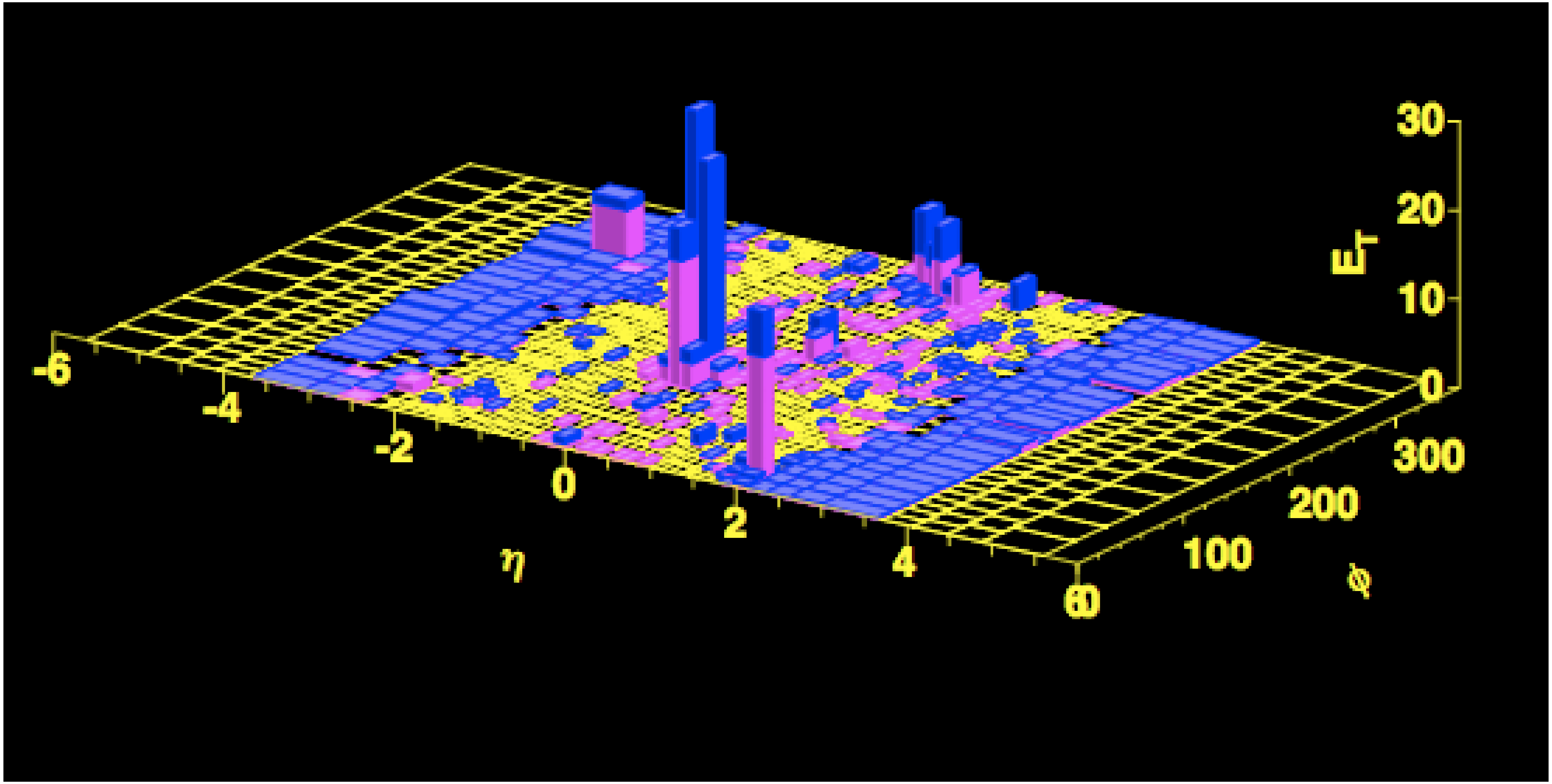
- Which pieces of the transverse momentum distribution are assigned to each jet ?

- What is the internal structure of a jet ?

- How many jets are there in a given event ?



CDF event



CDF event

The simplest version of this question appears in e+e- annihilation. We saw that the probability of gluon emission goes to 1 as we consider more collinear or softer gluons.

At what point do we cross over from a '2-jet event' to a '3-jet event' ?

We can approach this problem in two ways, using a 'jet observable' or a 'jet algorithm'.

A 'jet observable' gives a quantitative answer to the question, to what extent is the momentum flow of the event collimated along a given axis ?

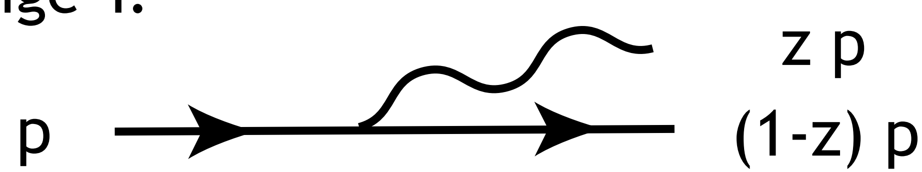
Sphericity:
$$S = \frac{3}{2} \min_{\hat{n}} \frac{\sum_a p_{aT}^2}{\sum_a p_a^2}$$

so that S = 1 for a spherical event, S = 0 for a fully collinear event.

Thrust:
$$T = \max_{\hat{n}} \frac{\sum_a |\vec{p}_a \cdot \hat{n}|}{\sum_a |\vec{p}_a|}$$

so that T = 1/3 for a spherical event, T = 1 for a fully collinear event.

T has the advantage over S in being 'infrared-safe'. The splitting of a quark to a collinear quark-gluon pair changes S but does not change T.

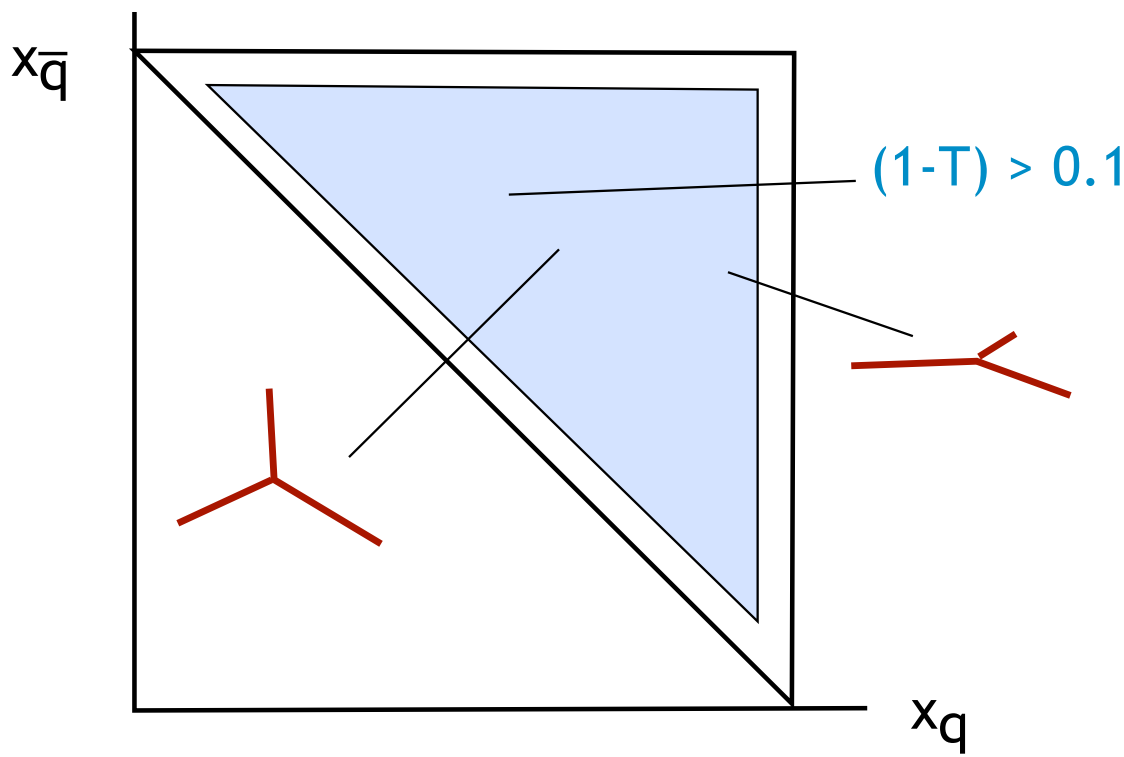


Compute T to $\mathcal{O}(\alpha_s)$

$e^+e^- \to q\bar{q}$ gives T = 1

$e^+e^- \to q\bar{q}g$ gives T = $\max(x_q, x_{\bar{q}}, x_g)$

T= 2/3 for a symmetric (planar) configuration.

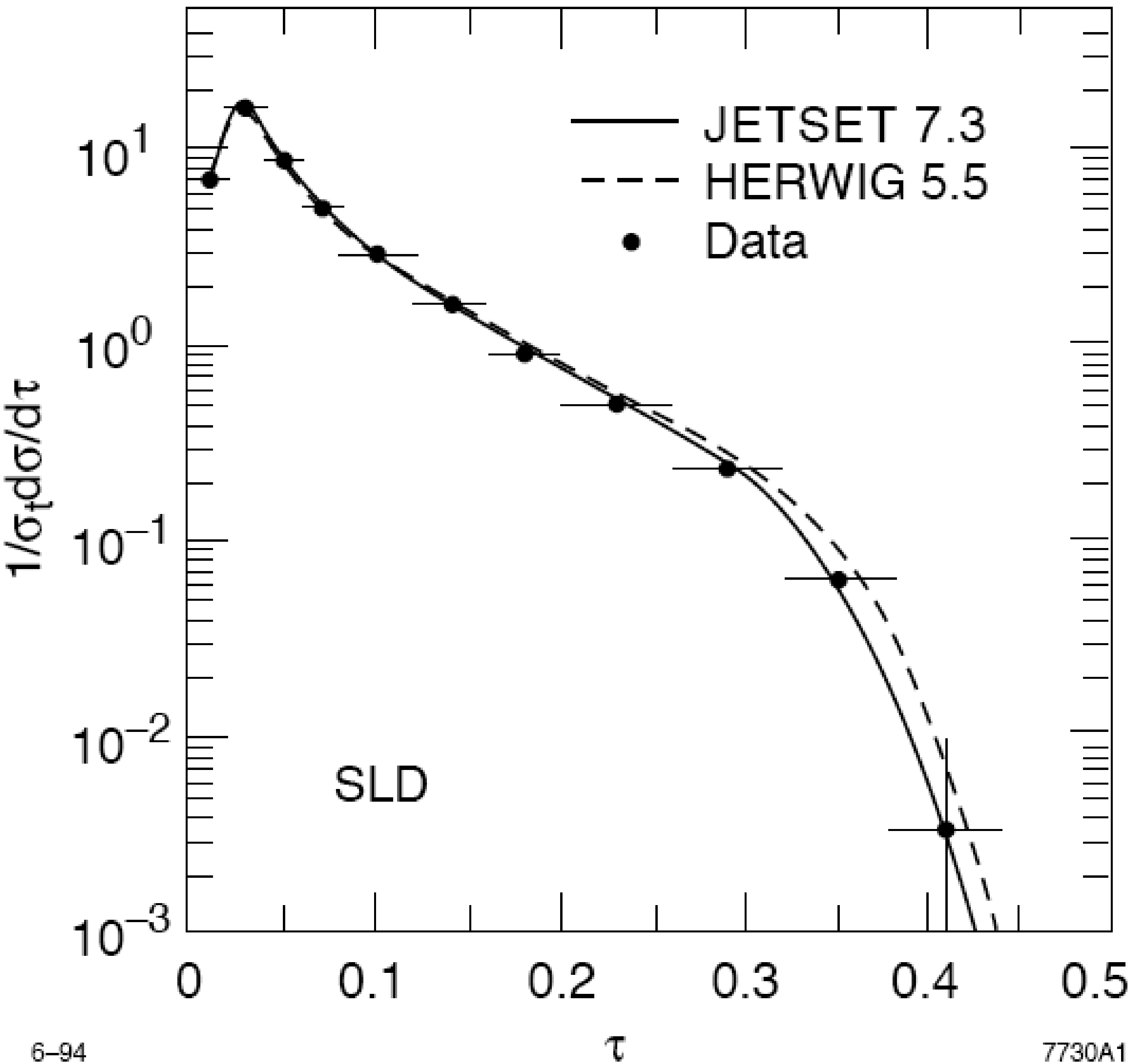


Here is the explicit result. It diverges as $T \to 1$, and it cuts off as $T \to 2/3$.

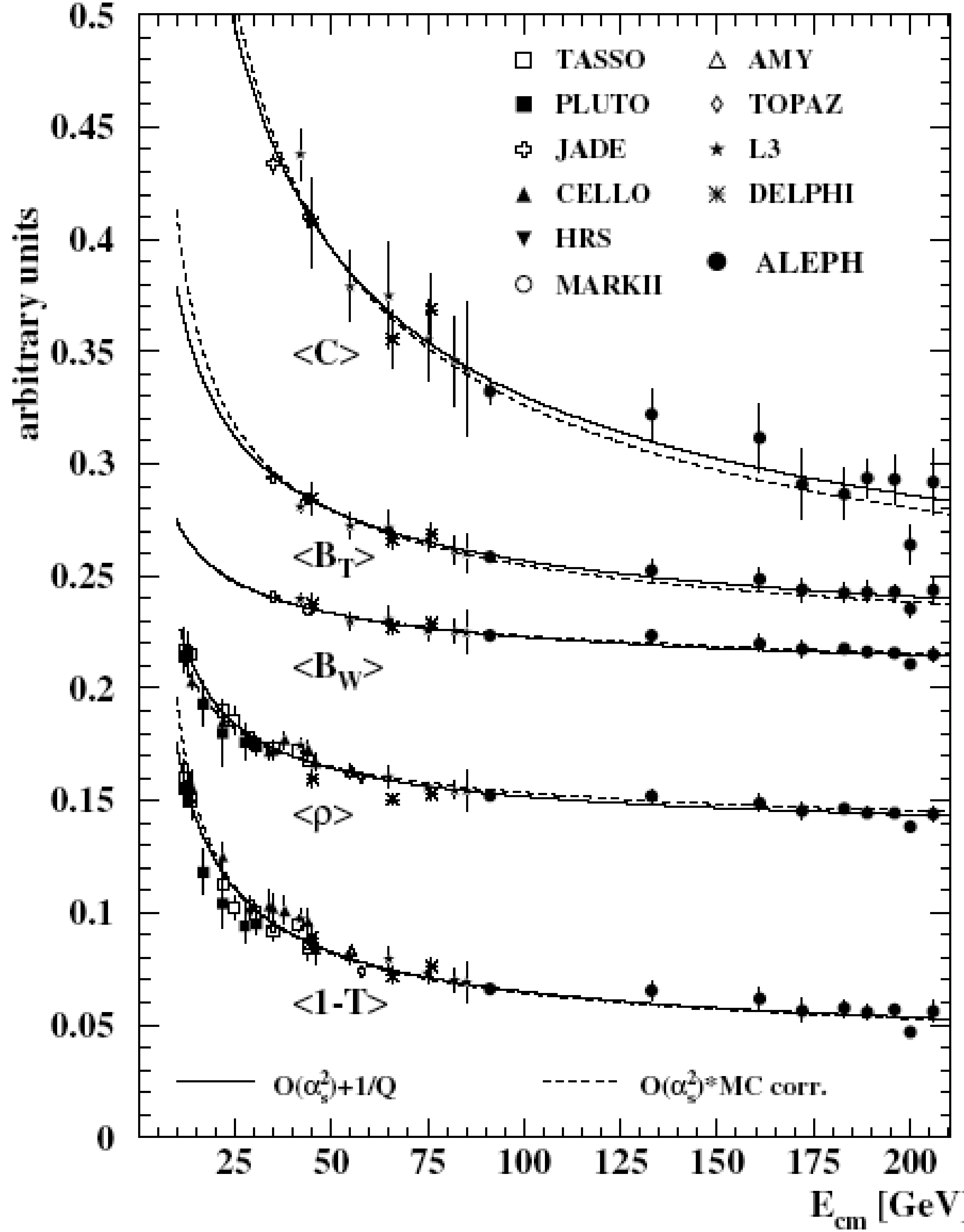
$$\frac{d\sigma}{dT} = \sigma_0 \cdot \frac{2\alpha_s}{3\pi} \cdot \int_{2(1-x_q)}^{x_q} dx_{\overline{q}} \, \frac{x_q^2 + x_{\overline{q}}^2}{(1-x_q)(1-x_{\overline{q}})} \Big|_{x_q = T}$$

$$= \sigma_0 \cdot \frac{2\alpha_s}{3\pi} \cdot \left\{ \frac{2(3T^2 - 3T + 2)}{T(1-T)} \log \frac{2T-1}{1-T} - \frac{3(3T-2)(2-T)}{(1-T)} \right\}$$

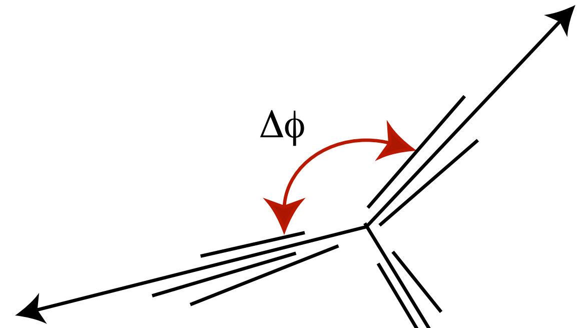
At higher orders, we can go beyond T = 2/3. Nonperturbative corrections are also needed to fit the thrust data as a function of CM energy.



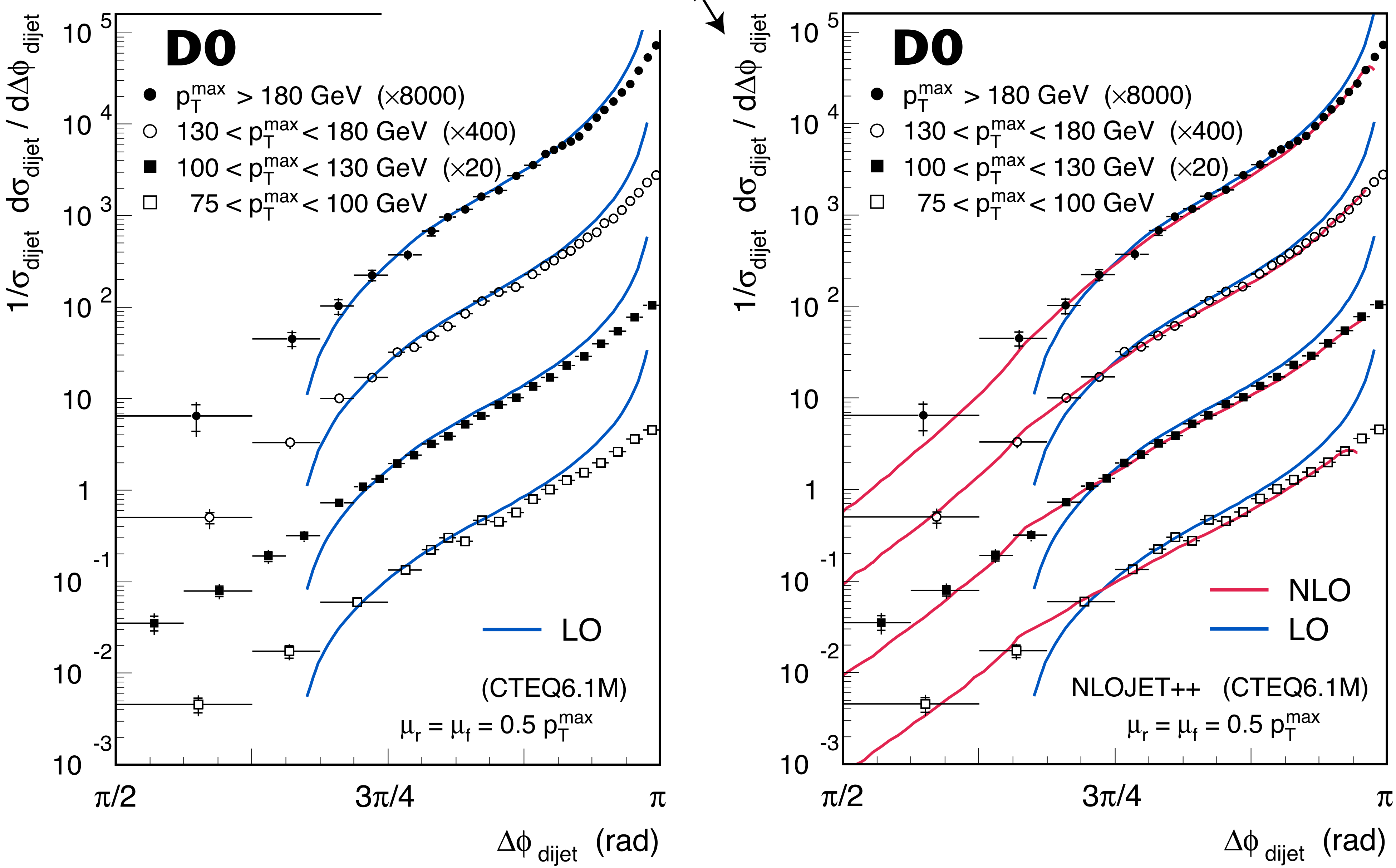
$\tau = 1 - T$



0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
arbitrary units
TASSO
PLUTO
JADE
CELLO
HRS
MARKII
AMY
TOPAZ
L3
DELPHI
ALEPH
<C>
<B$_T$>
<B$_W$>
<ρ>
<1-T>
O(α_s^2)+1/Q
O(α_s^2)*MC corr.
25
50
75
100
125
150
175
200
E$_{cm}$ [GeV]



NLO QCD calculation by Zoltan Nagy



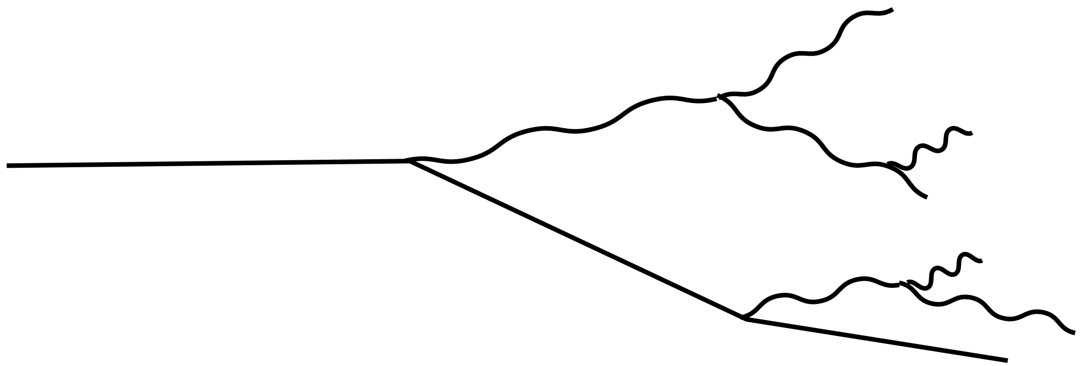
An alternative way to study the jettiness of events is to use a 'jet algorithm'. In this approach, we start from the particles observed in the event and attempt to combine them into clusters. At some point, we will stop, and the clusters at this stage are defined to be the jets.

In a theory calculation, we combine partons. In a real experiment, we might combine energies associated with signals in a detector, e.g. energies in calorimeter towers.

A simple algorithm is the JADE algorithm: Compute the invariant mass of each pair of particles. Find the two particles with the smallest invariant mass, and combine these to a single particle. Continue until the next step gives an invariant mass

$$m_{ij}^2 > y_{cut} s$$

Altarelli-Parisi evolution generates structure for each jet.



Collinear quarks and gluons are radiated at every mass scale. The radiations are distributed on a log scale in Q or in

So a jet is a fractal. For fixed α_s its structure is predicted to be scale-invariant. The running of α_s predicts more structure at smaller mass scales or larger angles.

To predict the structure of a jet, we need to simulate the quark and gluon emissions:

Begin with a simple process with 2 particles in the final state, e.g., $e^+e^- \to q\overline{q}$.

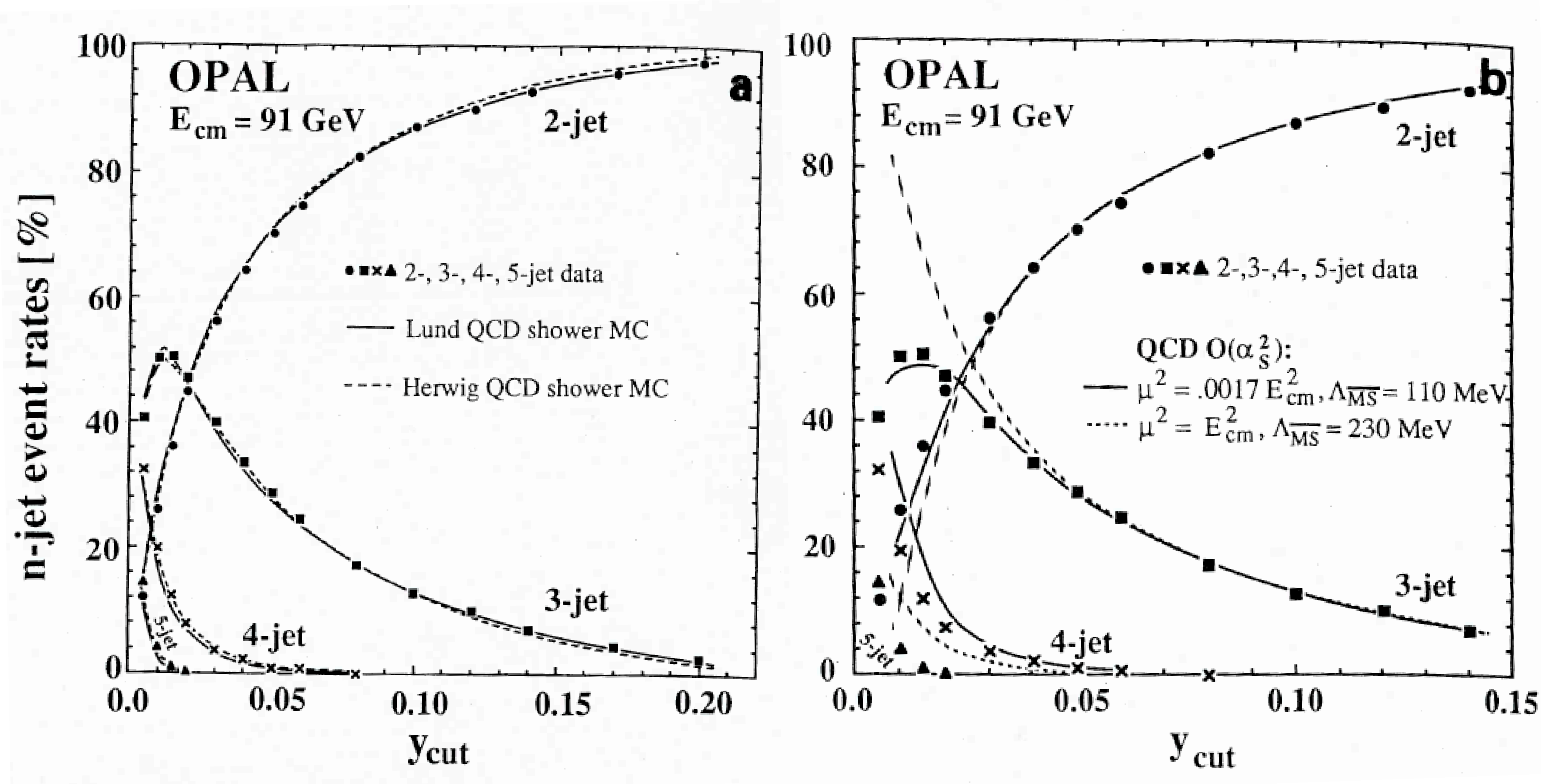
For each outgoing parton, step from $Q = \sqrt{s}$ to $Q \sim \mathrm{GeV}$. At each step, emit a quark or gluon with the probability given by the Altarelli-Parisi equation. (This is specific for collinear emissions; generalize this, somehow, for wide-angle emissions.)

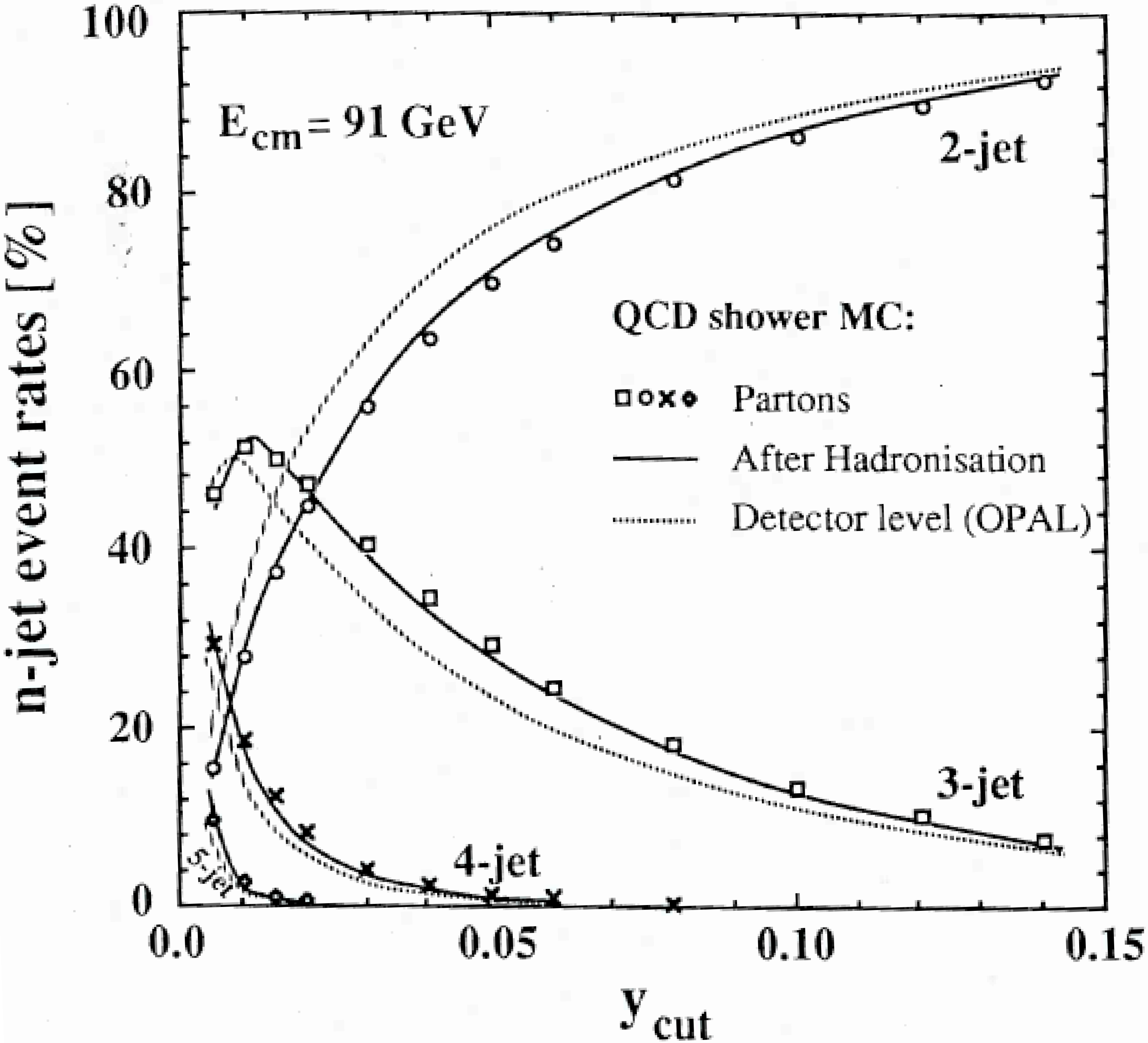
If there are hadrons in the initial state, the initial partons also undergo radiation from parton evolution.

At the end of the process, use a phenomenological model to turn the quarks and gluons into hadrons.

The event simulation programs PYTHIA, HERWIG, SHERPA carry out this program, using somewhat different implementations.

Here is a comparison of the Monte Carlo simulations to data, showing the dependence on the hadronization algorithm and on the QCD parameters used.





100
80
60
40
20
0
0.0
0.05
0.10
0.15
n-jet event rates [%]
y_{cut}
E_{cm} = 91 GeV
2-jet
3-jet
4-jet
5-jet
QCD shower MC:
Partons
After Hadronisation
Detector level (OPAL)

In proton-proton collisions, it is not so easy to define an effective variable to cluster particles into jets.

Detectors cover only a limited angular region, excluding the very forward directions.

The disruption of the two protons creates a large number of particles at low p_T, distributed across the range of rapidity. This particles from this ‘underlying event’ should not be included in the momentum of the jet.

Two commonly used approaches to clustering are

cone jets:

define a jet to be the particles or energies inside a cone of fixed angular size R, where

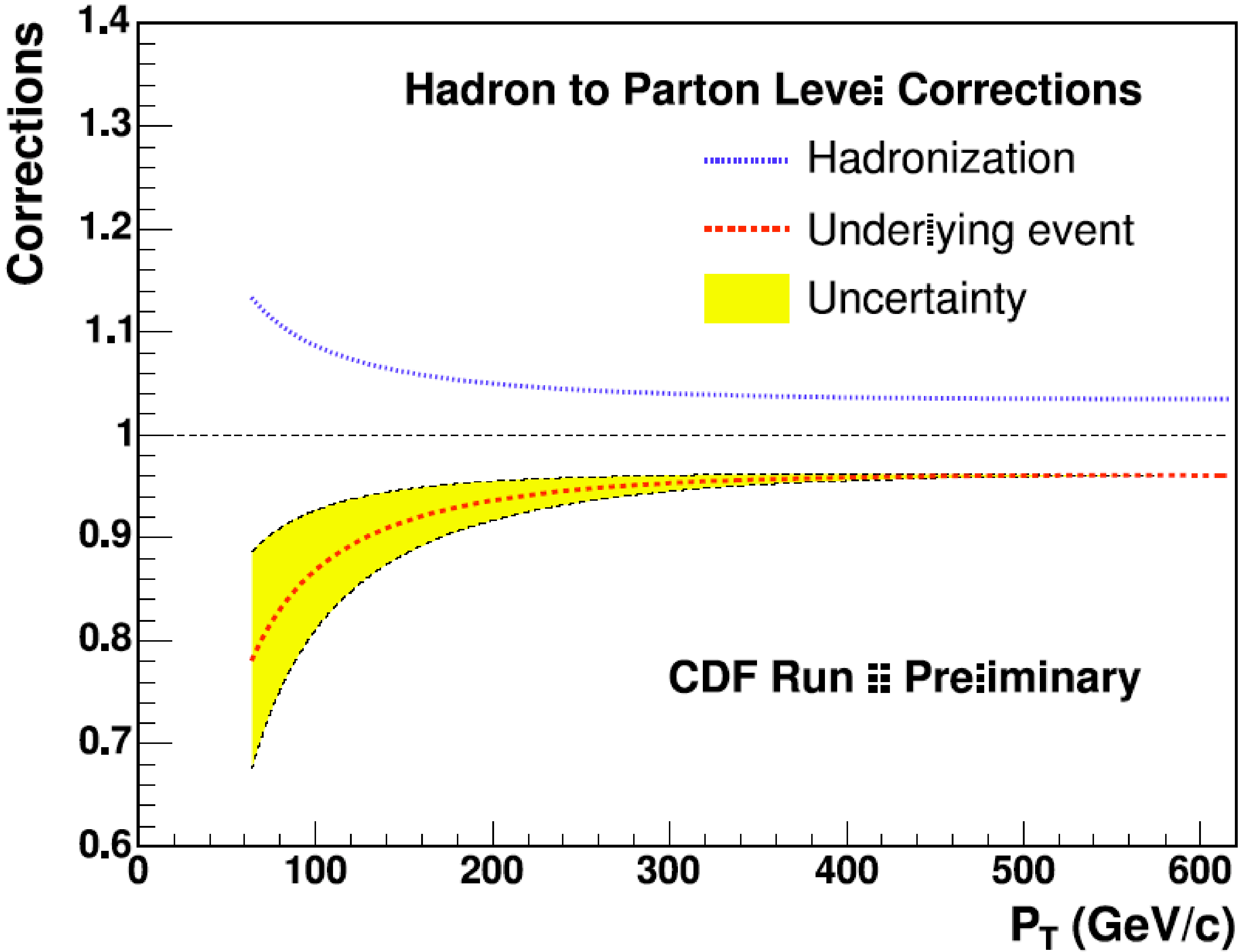
$$R = \sqrt{(\Delta\eta)^2 + (\Delta\phi)^2}$$

A common choice is R = 0.7. Propose cone locations, and move these in (η, ϕ) to maximize the tranverse momentum contained in the cones.

k_T jets:

Combine pairs of particles or energies with the minimum value of

$$k_{T\,ab}^2 = \min_{a,b}(p_{aT}^2, p_{bT}^2) \cdot \Delta R_{ab}^2$$



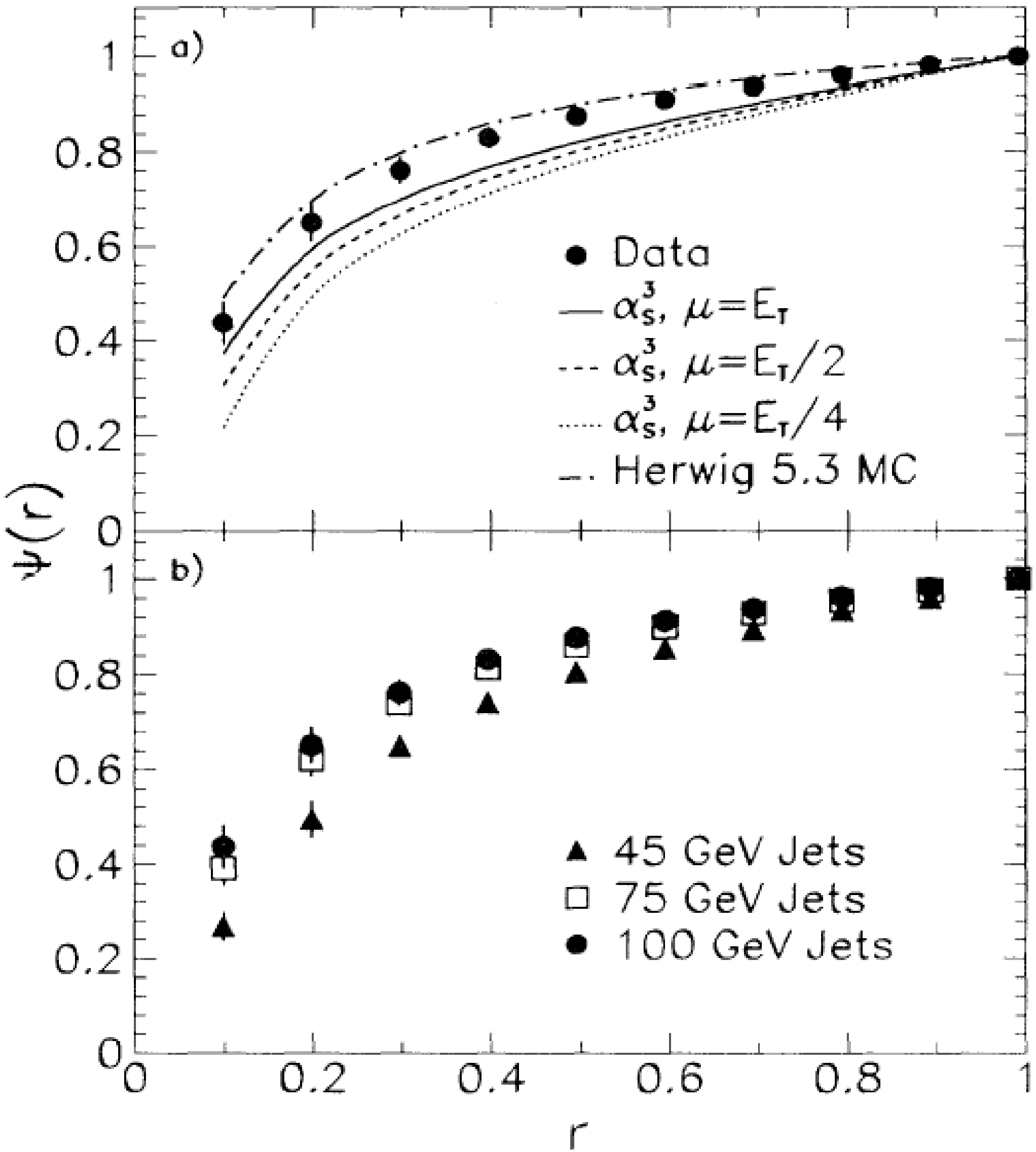
from Campbell-Houston-Stirling for R = 0.7

Peter Loch will discuss these methods in much more detail.

I would just like here to give some evidence that QCD does successfully describe the distribution of energy inside jets.

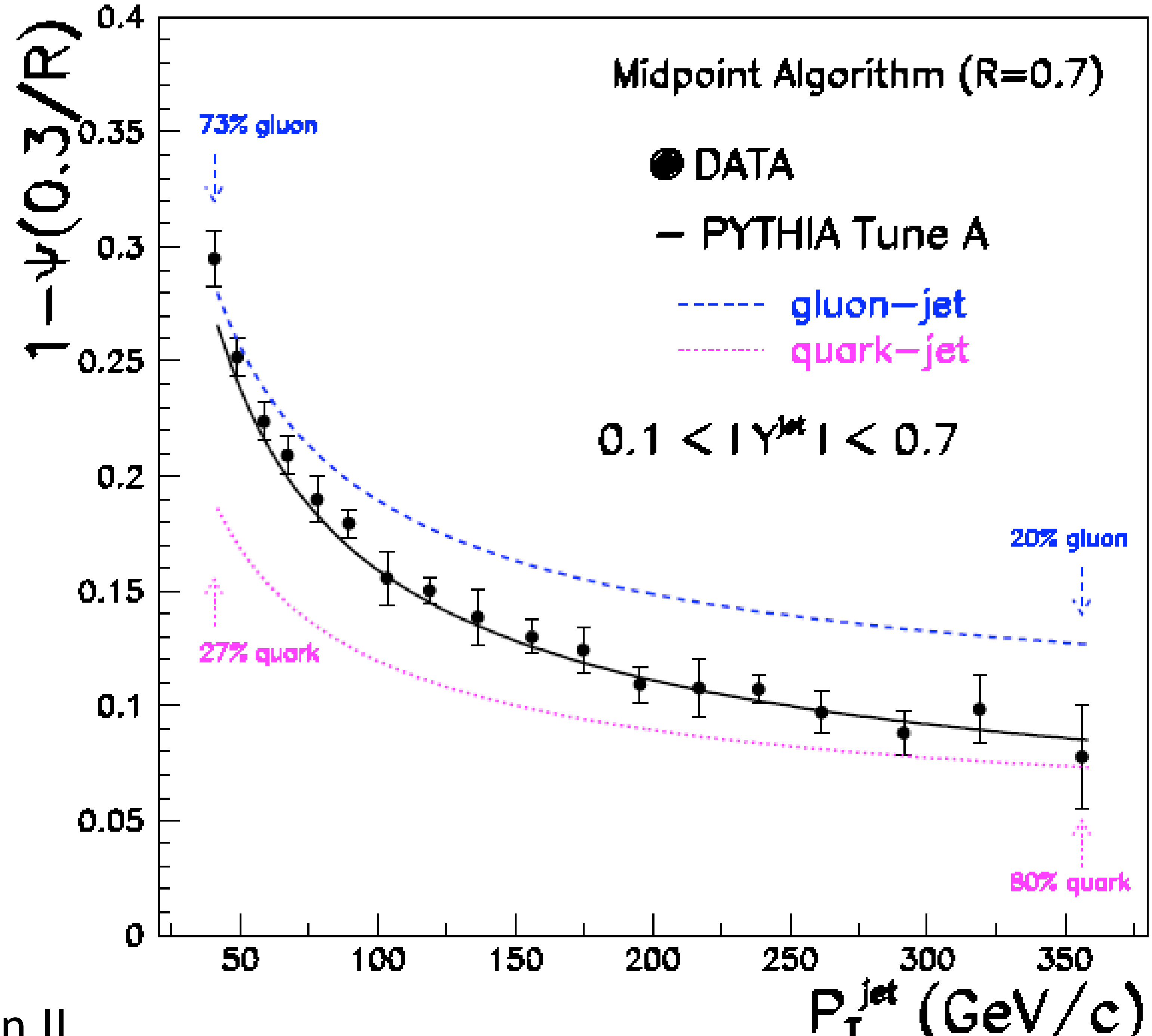
The variable used in the next plots is (0 < r < 1)

$$\psi(r) = \frac{\sum p_T(\text{ cone size rR})}{\sum p_T(\text{ cone size R})}$$



data: CDF

theory:
S. Ellis, Kunszt, and Soper



Midpoint Algorithm (R=0.7)
DATA
PYTHIA Tune A
gluon−jet
quark−jet
0.1 < |Y^jet| < 0.7
73% gluon
27% quark
20% gluon
80% quark
1−ψ(0.3/R)
P_T^jet (GeV/c)

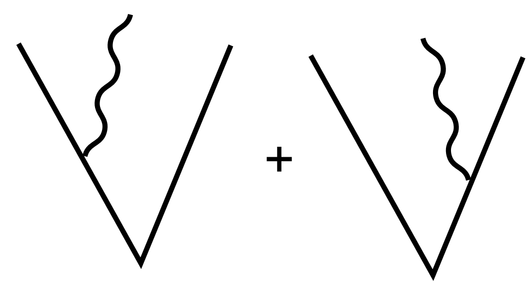
CDF, run II

There are two final issues that I would like to discuss.

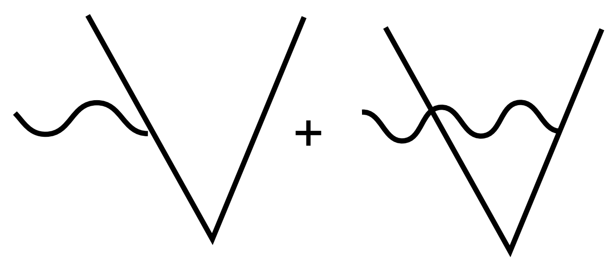
First, it is not quite correct that we can model parton evolution as an independent process for each parton.

For radiation from a color-singlet $q\overline{q}$ dipole, depending on the orientation, we may have

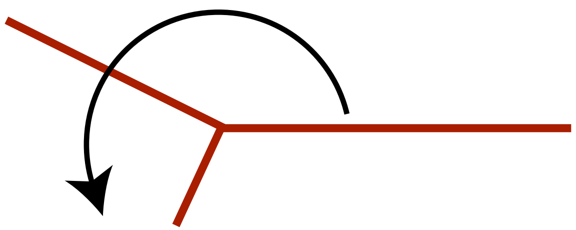
constructive interference:

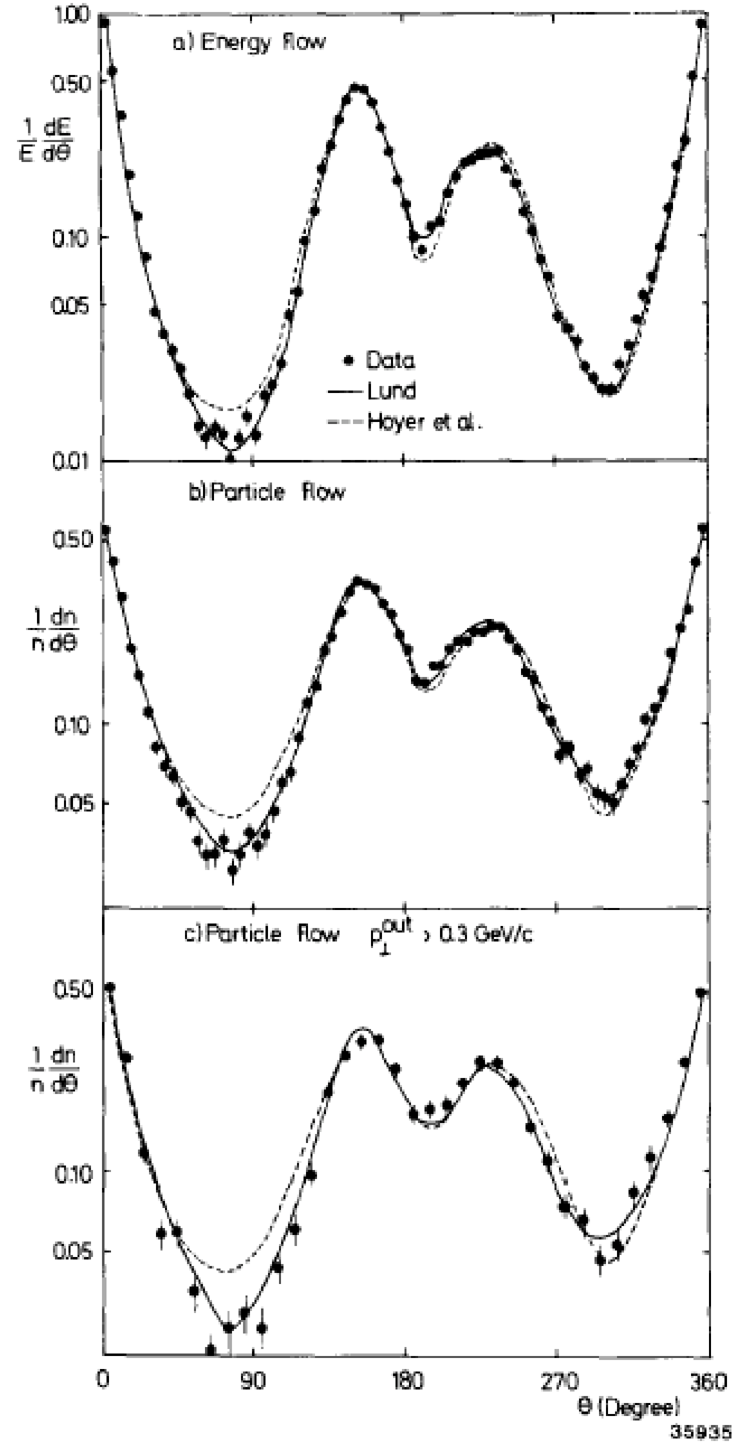


destructive interference:



This effect actually shows up in the data as a modulation of particle production.





JADE

29-36 GeV

This color coherence is implemented in PYTHIA and HERWIG by enforcing angular ordering of the successive emissions in the parton shower.

Second, for many purposes, a simple parton shower is not a sufficiently accurate representation of the final state.

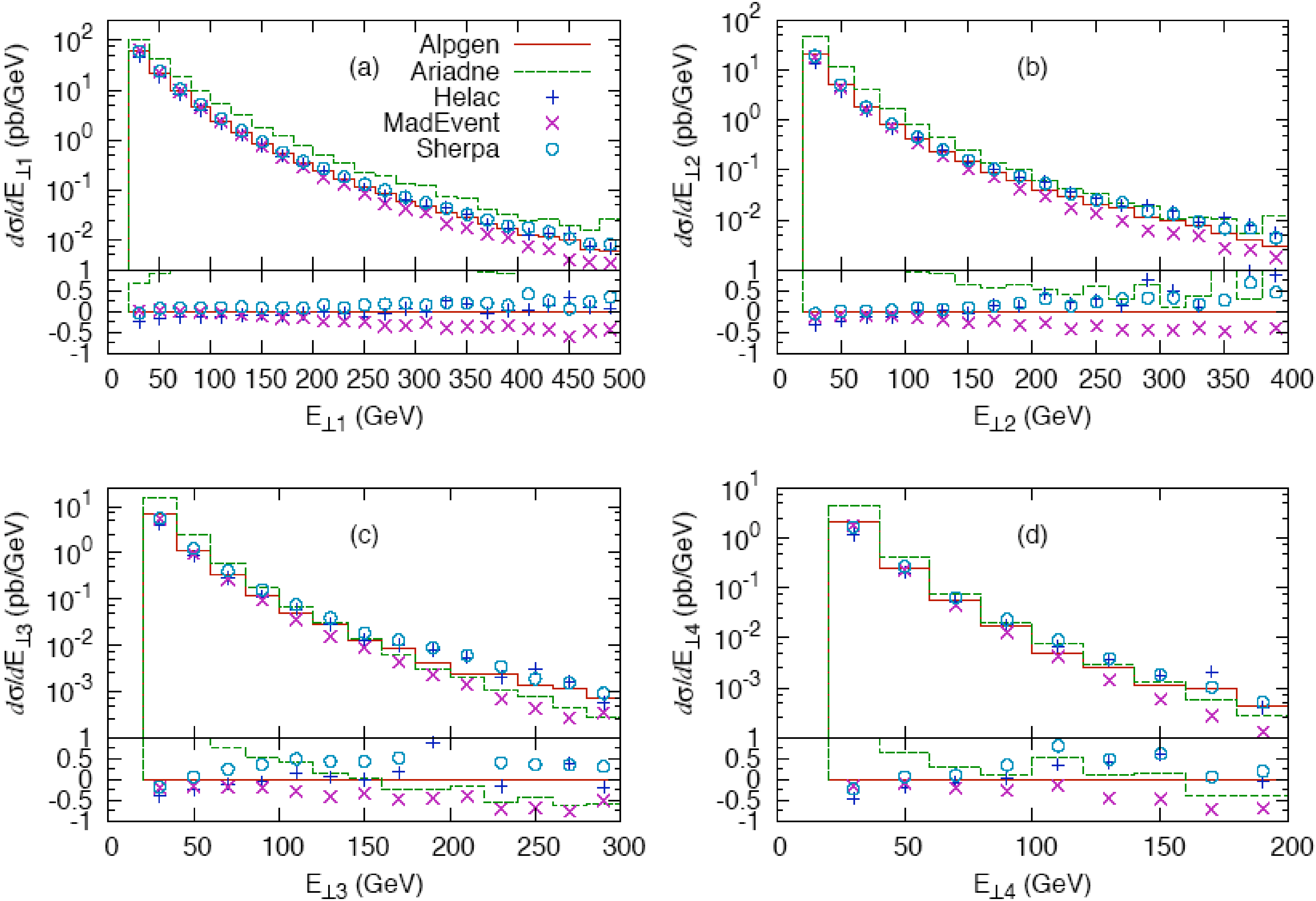
When we search for new particles at the LHC, we will be interested in events with leptons or missing energy + 4 jets, all at large angles to one another. We will want to know the Standard Model rates for such events.

PYTHIA and HERWIG are not designed to estimate the rates for such events correctly. To do this, we need full QCD matrix elements for the multijet processes.

I will discuss next time how to calculate these matrix elements. There is also the problem of integrating the results into event generation without double-counting of parton emissions.

Several codes now compute such matrix elements and generate events, matching the matrix element generation to parton showers.

predictions of the ET spectrum in W+ jets at the LHC for the 4 hardest jets, from J. Alwall et al. arXiv:0706.2569



Some very interesting ideas arise in the computation of multi-parton matrix elements. I will discuss these in the next lecture.