An Introduction to High Energy Nuclear Collisions

QCD under extreme conditions

Jamal Jalilian-Marian
Baruch College, City University of New York

Lecture III: Applications

DIS, particle production in pp,pA,AA, Instability, Thermalization?

The Regge-Gribov limit

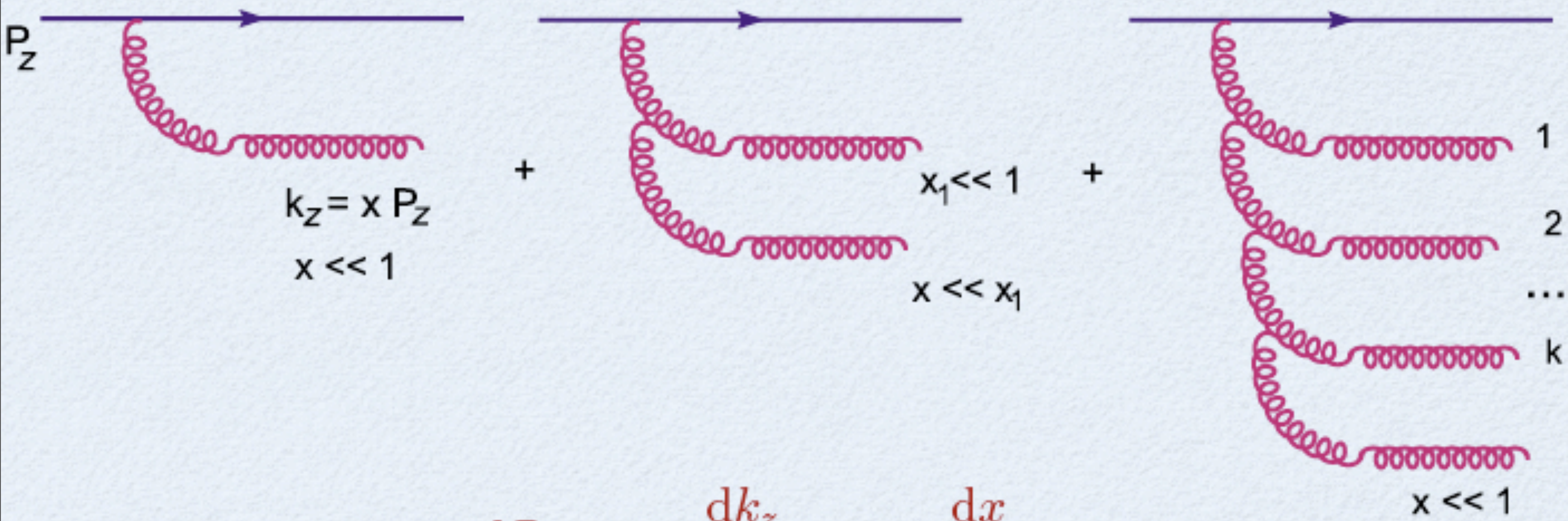
$$x_{\mathrm{Bj}} \rightarrow 0\,;\; s \rightarrow \infty\,;\; Q^2 (>> \Lambda^2_{\mathrm{QCD}}) = \text{fixed}$$

Physics of strong fields in QCD

Multi-particle production

Novel universal properties of QCD

- The infrared sensitivity of bremsstrahlung favors the emission of 'soft' (= small–x) gluons



$$d\mathcal{P} \propto \alpha_s \frac{dk_z}{k_z} = \alpha_s \frac{dx}{x}$$

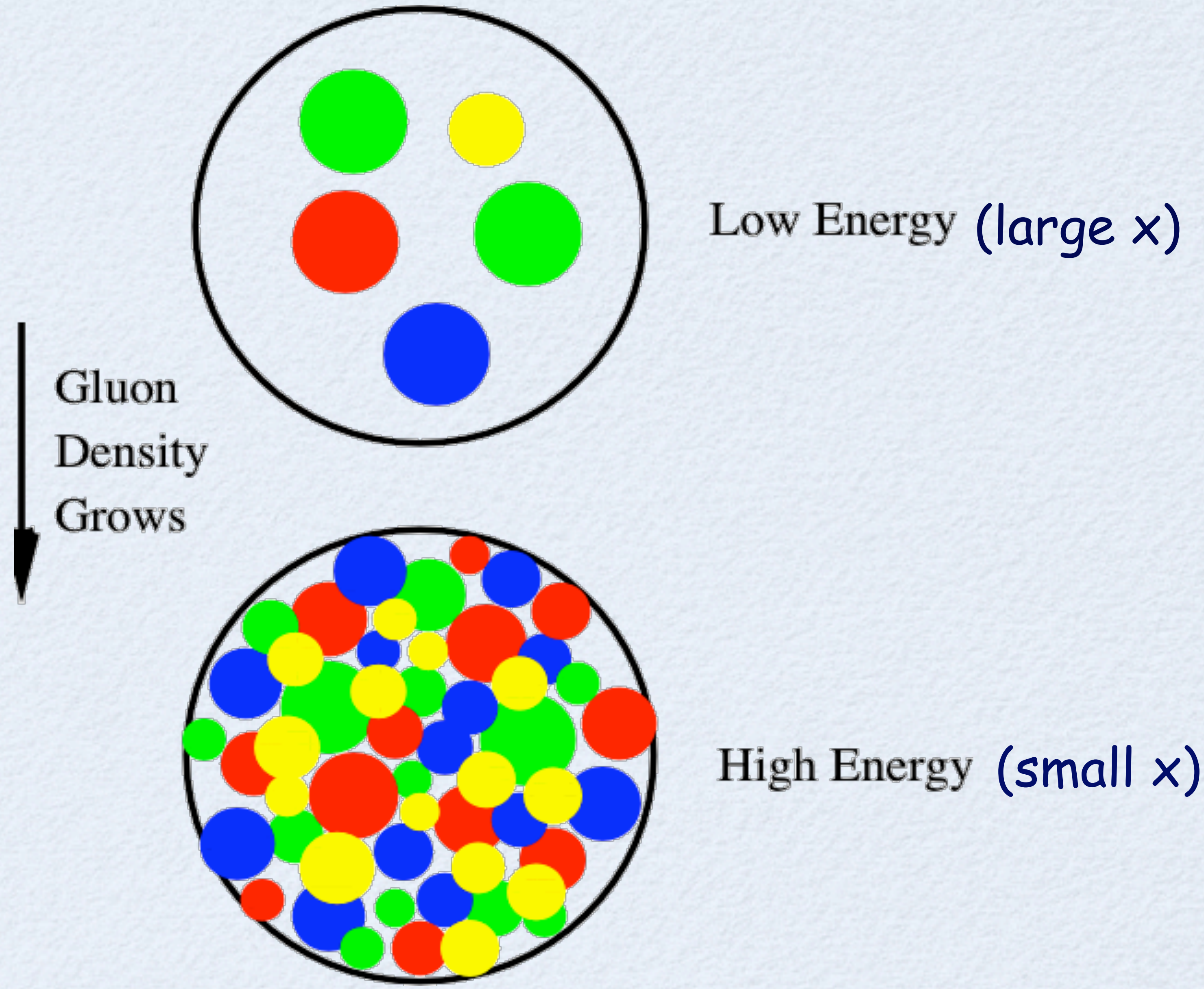
- The 'price' of an additional gluon:

$$\mathcal{P}(1) \propto \alpha_s \int_x^1 \frac{dx_1}{x_1} = \alpha_s \ln \frac{1}{x}$$

number of gluons grows fast $n \sim e^{\alpha_s ln 1/x}$

Resolving the hadron/nucleus at high energy

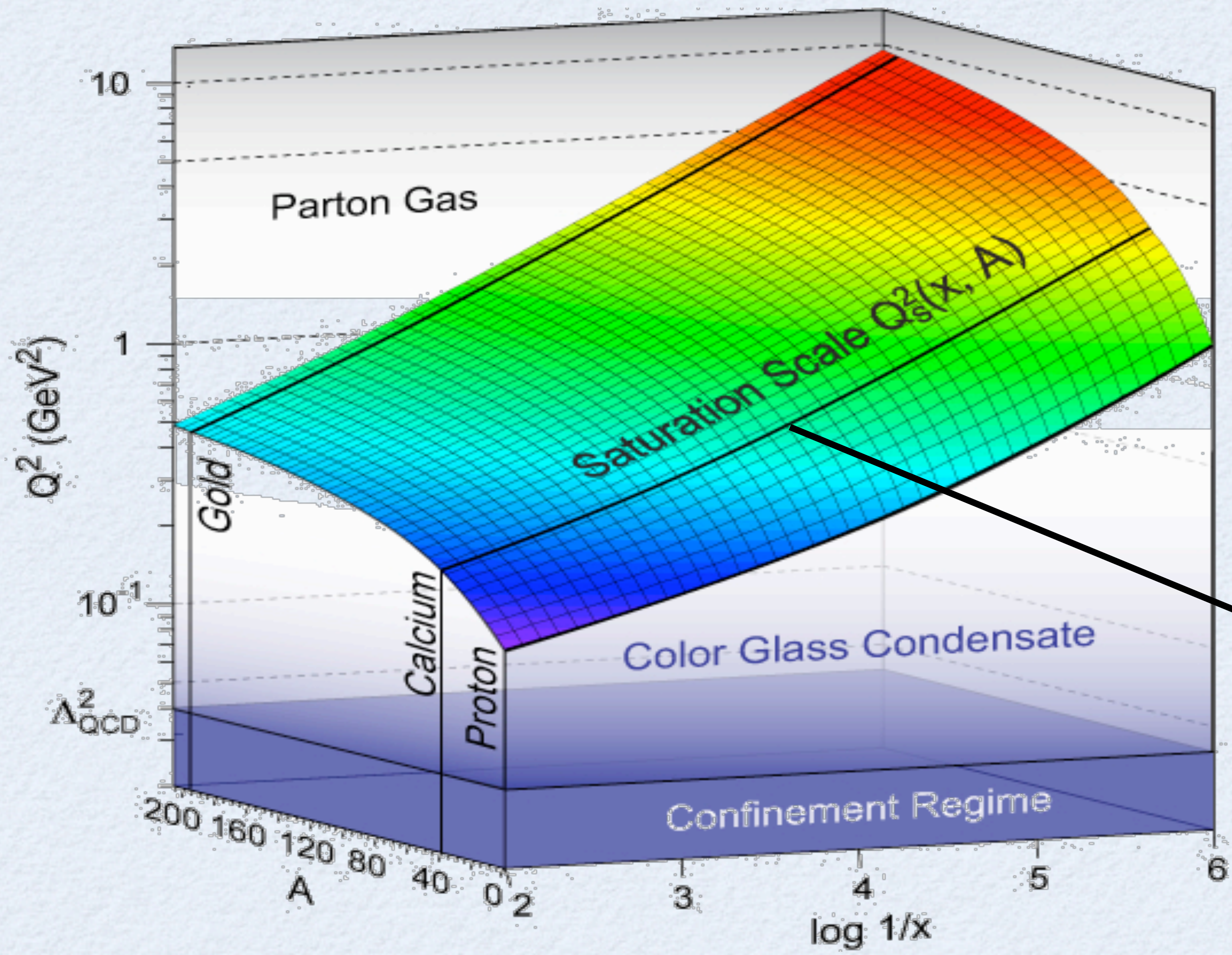
Ren.Group in x (energy)



Gluon density saturates at $f = 1/\alpha_S$

- strongest E&M fields in nature...

The saturation scale $Q_s(x,A,b_t)$



$\times \frac{9}{4}$ **for gluon**

$\alpha_S(Q_S^2) << 1$

QCD in high gluon density regime

Need a new organizing principle to explore this novel regime of high energy QCD

multiple scattering: classical fields

+

energy (x) dependence: ln (1/x)

DIS cross section

$$\frac{d\sigma}{dx dQ^2} \propto L_{\mu\nu} W^{\mu\nu}$$

Hadronic tensor

$$W^{\mu\nu} = 2\,\mathrm{Disc.}\,T^{\mu\nu}(q^2, P\cdot q) = \frac{1}{2\pi}\,\mathrm{Im.}\int d^4x\, e^{iq\cdot x}\langle P|T(J^\mu(x)J^\nu(0))|P\rangle$$

$$J^\mu(x) = \bar{\psi}\gamma^\mu\psi$$

In full generality

$$W^{\mu\nu}(q^2, P{\cdot}q) = \frac{1}{2\pi}\,\frac{P^+}{M}\,\mathrm{Im.}\int d^3X\int d^4x\, e^{iq\cdot x}\,\langle \mathrm{Tr}\,(\gamma^\mu G_A(X+x/2, X-x/2)\gamma^\nu G_A(X-x/2, X+x/2))\rangle$$

Quark propagator in background gauge field A of hadron/nucleus

Can compute $W^{\mu\nu}$ (and F_2, F_L) systematically in expansion about classical field (no OPE!)

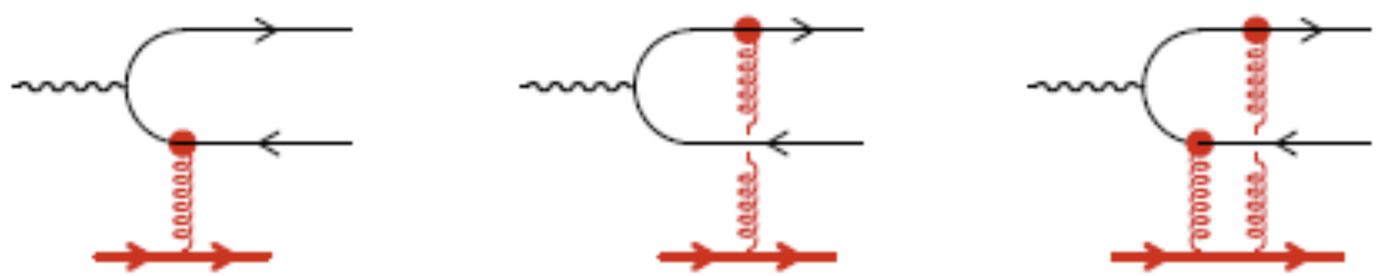
DIS cross section

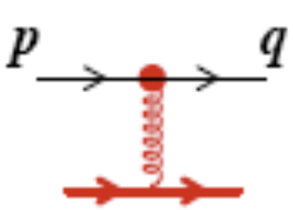
- Differential photon-target cross-section : $\gamma^\star\, p(A) \to q\,\bar{q}\,X$

$$d\sigma_{\gamma^* T} = \frac{d^3\boldsymbol{k}}{(2\pi)^2 2E_{\boldsymbol{k}}}\frac{d^3\boldsymbol{p}}{(2\pi)^3 2E_{\boldsymbol{p}}}\;\frac{1}{2q^-}2\pi\delta(q^- - k^- - p^-)$$
$$\times \left\langle \mathcal{M}^\mu(\boldsymbol{q}|\boldsymbol{k},\boldsymbol{p})\mathcal{M}^{\nu *}(\boldsymbol{q}|\boldsymbol{k},\boldsymbol{p})\right\rangle \epsilon_\mu(Q)\epsilon^*_\nu(Q)\;,$$

  - $\boldsymbol{k},\boldsymbol{p}$: momenta of the quark and antiquark
  - $\boldsymbol{q}$: momentum of the virtual photon

- Scattering amplitude :

$\mathcal{M}^\mu(\boldsymbol{q}|\boldsymbol{k},\boldsymbol{p}) =$ 



$$= 2\pi\,\delta(p^- - q^-)\,\gamma^- \int d^2\vec{\boldsymbol{x}}_\perp\; e^{i(\vec{\boldsymbol{q}}_\perp - \vec{\boldsymbol{p}}_\perp)\cdot\vec{\boldsymbol{x}}_\perp}\;\left[U(\vec{\boldsymbol{x}}_\perp) - 1\right]$$

DIS cross section

- The sum of the three terms simplifies considerably :

$$\mathcal{M}^\mu(k|q,p) = \frac{i}{2}\int \frac{d^2\vec{l}_\perp}{(2\pi)^2}\int d^2\vec{x}_{1\perp}d^2\vec{x}_{2\perp}\,\Big[\overline{u}(\vec{q})\;\Gamma^\mu\;v(\vec{p})\Big]$$
$$\times\, e^{i\vec{l}_\perp\cdot\vec{x}_{1\perp}}e^{i(\vec{p}_\perp+\vec{k}_\perp-\vec{q}_\perp-\vec{l}_\perp)\cdot\vec{x}_{2\perp}}\,\Big[U(\vec{x}_{1\perp})U^\dagger(\vec{x}_{2\perp})-1\Big]$$

with

$$\Gamma^\mu \equiv \frac{\gamma^-(\not{K}-\not{L}+m)\gamma^\mu(\not{K}-\not{Q}-\not{L}+m)\gamma^-}{p^-[(\vec{k}_\perp-\vec{l}_\perp)^2+m^2-2k^-q^+]+k^-[(\vec{k}_\perp-\vec{q}_\perp-\vec{l}_\perp)^2+m^2]}$$

- By inserting this into the DIS cross-section, we see that the differential cross-section (with two resolved quark jets in the final state) depends on the correlator of four Wilson lines

DIS cross section

- If we integrate out the final quark and antiquark, two of the Wilson lines cancel and we get :

$$\sigma_{\gamma^* T} = \int_0^1 dz \int d^2\vec{\boldsymbol{r}}_\perp \, |\psi(\boldsymbol{q}|z, \vec{\boldsymbol{r}}_\perp)|^2 \, \sigma_{\rm dipole}(\vec{\boldsymbol{r}}_\perp)$$

with

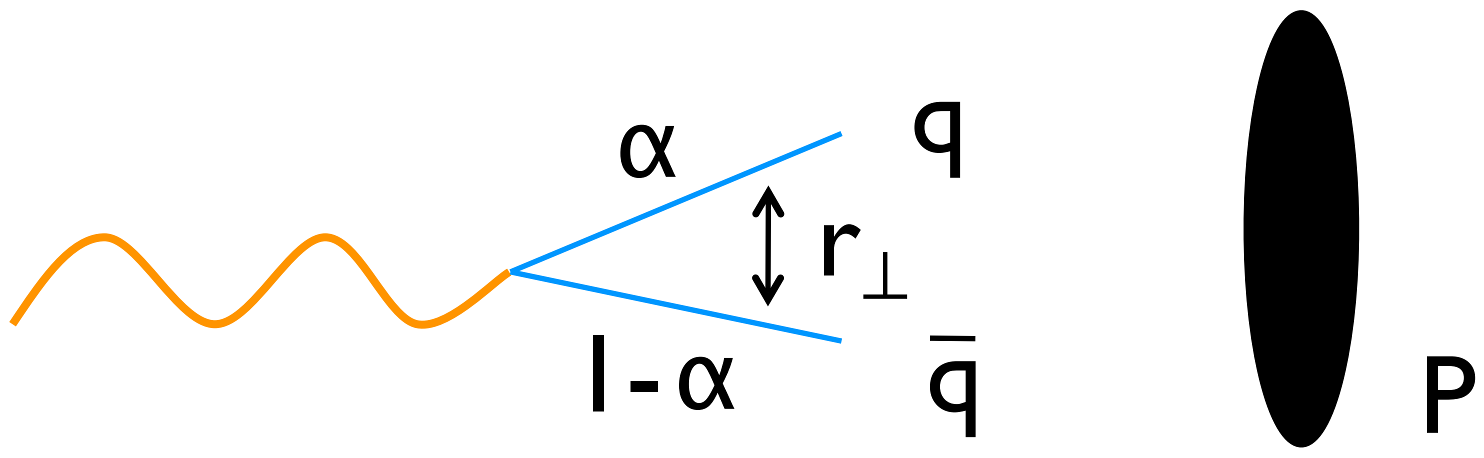
$$\sigma_{\rm dipole}(\vec{\boldsymbol{r}}_\perp) \equiv \frac{2}{N_c} \int d^2\vec{\boldsymbol{X}}_\perp \, {\rm Tr} \left\langle 1 - U(\vec{\boldsymbol{X}}_\perp + \frac{\vec{\boldsymbol{r}}_\perp}{2}) U^\dagger(\vec{\boldsymbol{X}}_\perp - \frac{\vec{\boldsymbol{r}}_\perp}{2}) \right\rangle$$

and

$$|\psi(\boldsymbol{q}|z, \vec{\boldsymbol{r}}_\perp)|^2 \equiv \frac{N_c \, \epsilon_\mu(Q) \epsilon^*_\nu(Q)}{64\pi (q^-)^2 z(1-z)} \int \frac{d^2\vec{\boldsymbol{l}}_\perp}{(2\pi)^2} \frac{d^2\vec{\boldsymbol{l}}'_\perp}{(2\pi)^2} \, e^{i(\vec{\boldsymbol{l}}_\perp - \vec{\boldsymbol{l}}'_\perp)\cdot \vec{\boldsymbol{r}}_\perp} \times {\rm Tr}\left((\not{k} + m) \Gamma^\mu (\not{p} - m) \Gamma^{\nu\prime} \right)$$

$$U(x_t) \equiv \hat{P} \exp[-ig \int dx^- \, \frac{1}{\partial_t^2} \, \rho^a(x^-, x_t) \, T^a]$$

DIS cross section



in coordinate space (target rest frame):

- virtual photon splits into a quark anti-quark pair
- the quark anti-quark pair (dipole) scatters on the target

eikonal propagation of the dipole through nucleus

the dipole scattering probability satisfies the JIMWLK (BK) equation

DIS cross section

$$F_2(x,Q^2) = \frac{Q^2}{4\pi^2\alpha_{em}}(\sigma_T + \sigma_L)$$

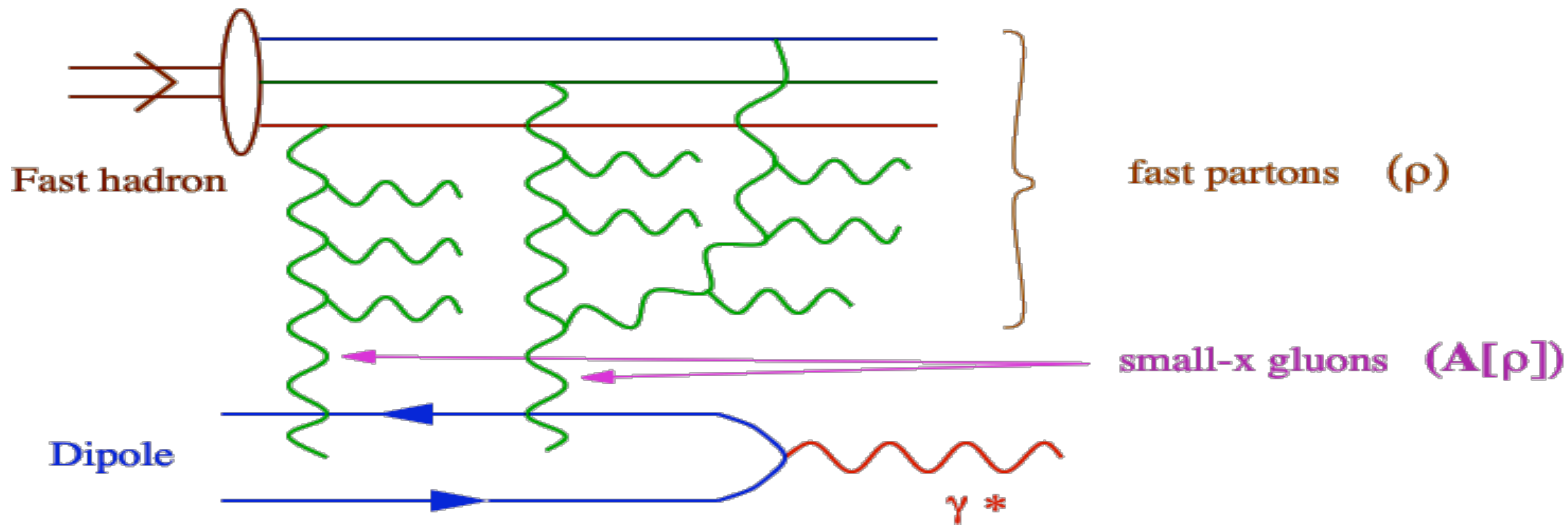
$$\sigma_{T,L} = \int d^2r\, d\alpha \; |\Psi_{T,L}(r,\alpha,Q^2)|^2 \, \sigma_{dip}(x,r)$$

$$|\Psi_L(r,\alpha,Q^2)|^2 = \frac{3\alpha_{em}}{\pi^2} \sum_f e_f^2 \, 4Q^2\alpha^2(1-\alpha)^2 K_0^2(\varepsilon\, r)$$

$$|\Psi_T(r,\alpha,Q^2)|^2 = \frac{3\alpha_{em}}{2\pi^2} \sum_f e_f^2 \left\{[\alpha^2 + (1-\alpha)^2]\, \varepsilon^2 K_1^2(\varepsilon\, r) + m_f^2 K_0^2(\varepsilon\, r)\right\}$$

$$\varepsilon^2 = [\alpha^2 + (1-\alpha)^2]Q^2 + m_f^2$$

DIS cross section



$$\partial_Y \langle T_{\mathbf{xy}} \rangle = \frac{\bar{\alpha}}{2\pi} \int d^2z \, \frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{z})^2(\mathbf{z}-\mathbf{y})^2} \left[\langle T_{\mathbf{xz}} \rangle + \langle T_{\mathbf{zy}} \rangle - \langle T_{\mathbf{xy}} \rangle - \langle T_{\mathbf{xz}} \rangle \langle T_{\mathbf{zy}} \rangle \right]$$

$$r_t \ll 1/Q_s \qquad T \sim (r_t \, Q_s)^{\gamma_s}$$

$$r_t \gg 1/Q_s \qquad T \sim ln^2(r_t \, Q_s)$$

- Assume translation and rotation invariance, and define :

$$N(Y,k_\perp) \equiv 2\pi \int d^2\vec{x}_\perp\, e^{i\vec{k}_\perp\cdot\vec{x}_\perp}\, \frac{\langle T(0,\vec{x}_\perp)\rangle_Y}{x_\perp^2}$$

- From the Balitsky-Kovchegov equation for $\langle T\rangle$, we obtain the following equation for N :

$$\frac{\partial N(Y,k_\perp)}{\partial Y} = \frac{\alpha_s N_c}{\pi}\Big[\chi(-\partial_L)N(Y,k_\perp) - N^2(Y,k_\perp)\Big]$$

with

$$L \equiv \ln(k_\perp^2/k_0^2)$$
$$\chi(\gamma) \equiv 2\psi(1) - \psi(\gamma) - \psi(1-\gamma)$$

- Expand the function $\chi(\gamma)$ to second order around its minimum $\gamma = 1/2$

- Introduce new variables :

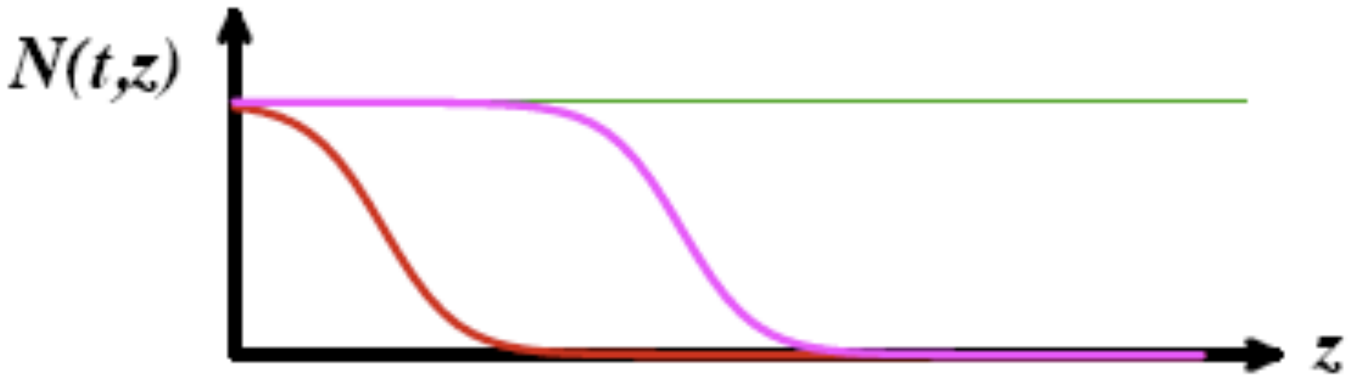
$$t \sim Y$$
$$z \sim L + \frac{\alpha_s N_c}{2\pi} \chi''(1/2)\, Y$$

- The equation for N becomes :

$$\partial_t N = \partial_z^2 N + N - N^2$$

(known as the Fisher-Kolmogorov-Petrov-Piscounov equation)

- Assume an initial condition $N(t_0, z)$ that goes smoothly from 1 at $z = -\infty$ to 0 at $z = +\infty$, and behaves like $\exp(-\beta z)$ when $z \gg 1$



- The solution of the F-KPP equation is known to behave like a traveling wave at asymptotic times (Bramson, 1983) :

$$N(t,z) \underset{t\to+\infty}{\sim} N(z - m_\beta(t))$$

with $\quad m_\beta(t) = 2t - 3\ln(t)/2 + \mathcal{O}(1) \quad$ if $\quad \beta > 1$

▷ universal front velocity for a large class of initial conditions

- In QCD, the initial condition is of the required form, with $\beta > 1$
  ▷ front velocity independent of the initial condition

- Going back to the original variables, one gets :

$$N(Y, k_\perp) = N\left(k_\perp / Q_s(Y)\right)$$

with

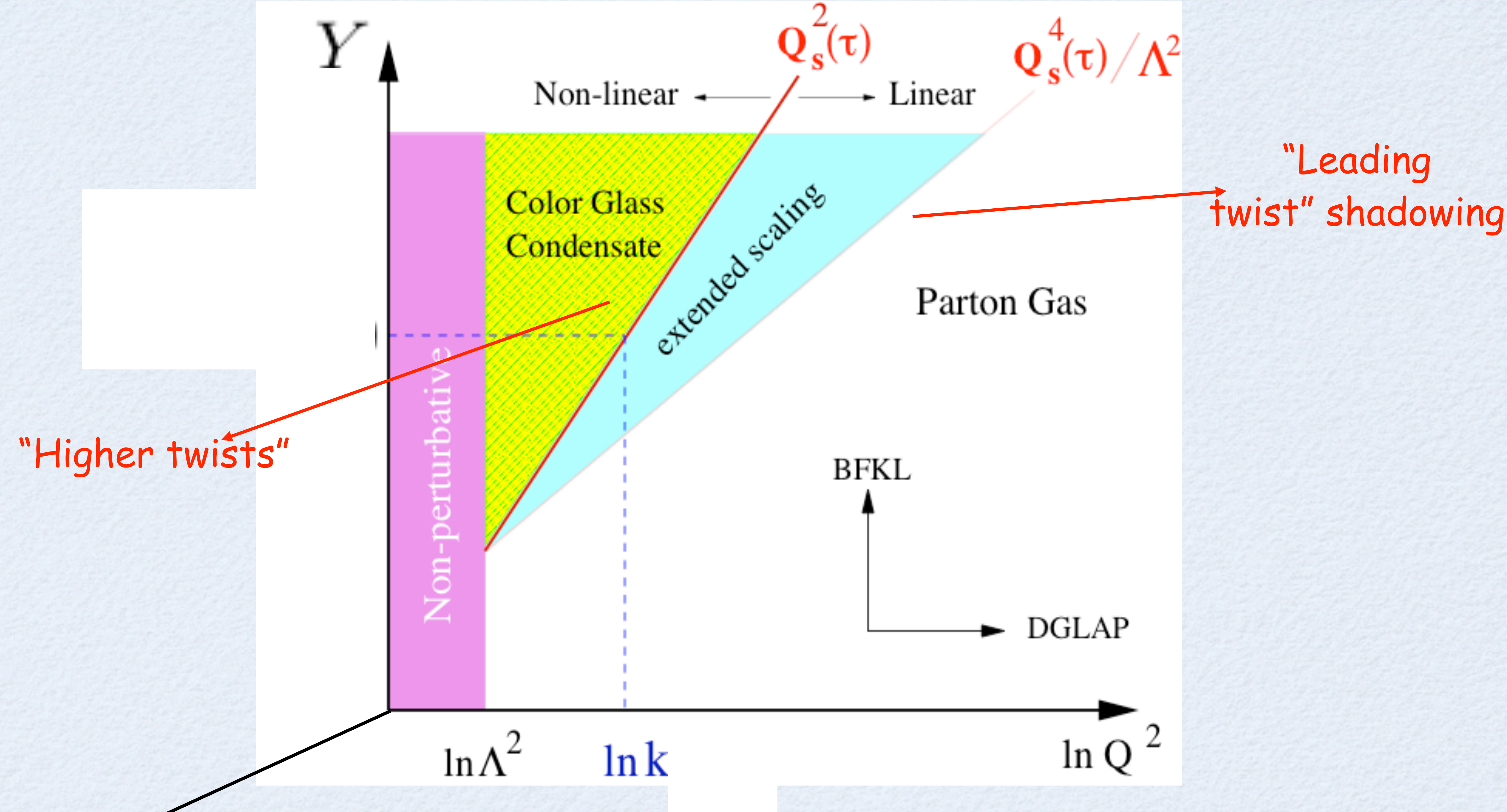
$$Q_s^2(Y) = k_0^2 \, Y^{-\delta} \, e^{\lambda Y}$$

- Going from $N(Y, k_\perp)$ to $\langle T(0, \vec{x}_\perp)\rangle_Y$, we obtain :

$$\langle T(0, \vec{x}_\perp)\rangle_Y = T(Q_s(Y) x_\perp)$$

"geometric scaling"

Road map of strong interactions



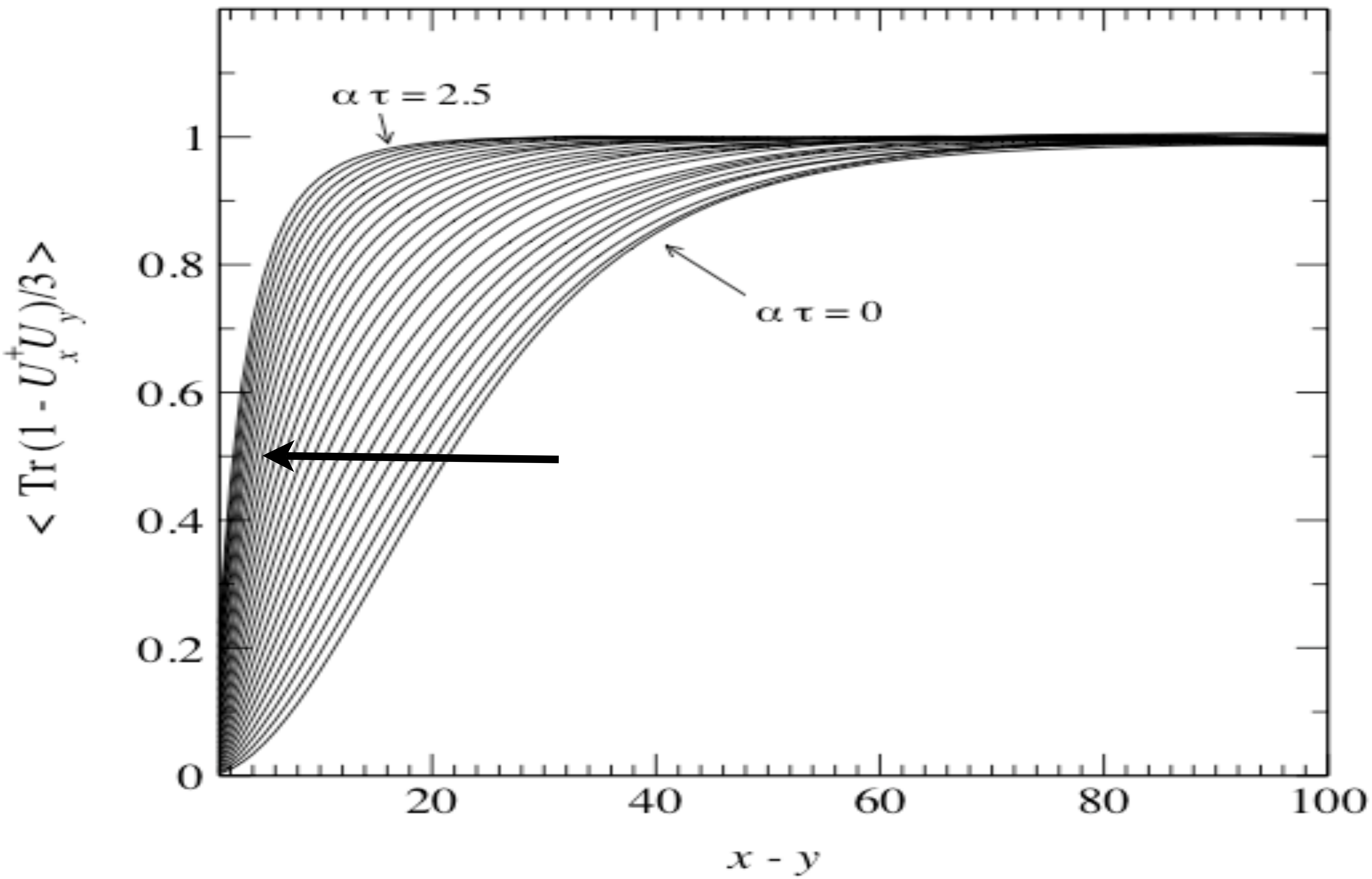
$$Q_s^2(x) = Q_0^2 \left(\frac{x_0}{x}\right)^{\lambda} A^{1/3}$$

Q_0 = 1 GeV; λ = 0.3; x_0 = 3* 10^{-4}

Solving the BK equation

the 2-point function $T(x_t, y_t) = 1/N_c \, Tr\,[1 - U^+(x_t)\, U(y_t)]$

(probability for scattering of a quark-anti-quark dipole on a target)



define $Q_s = 1/r_t$

when $T(r_t) = 1/2$

it grows with energy

pQCD:

color transparency

$$T \sim r_t^2 \, xG(x, \frac{1}{r_t})$$

non-linearities unitarize the scattering probability

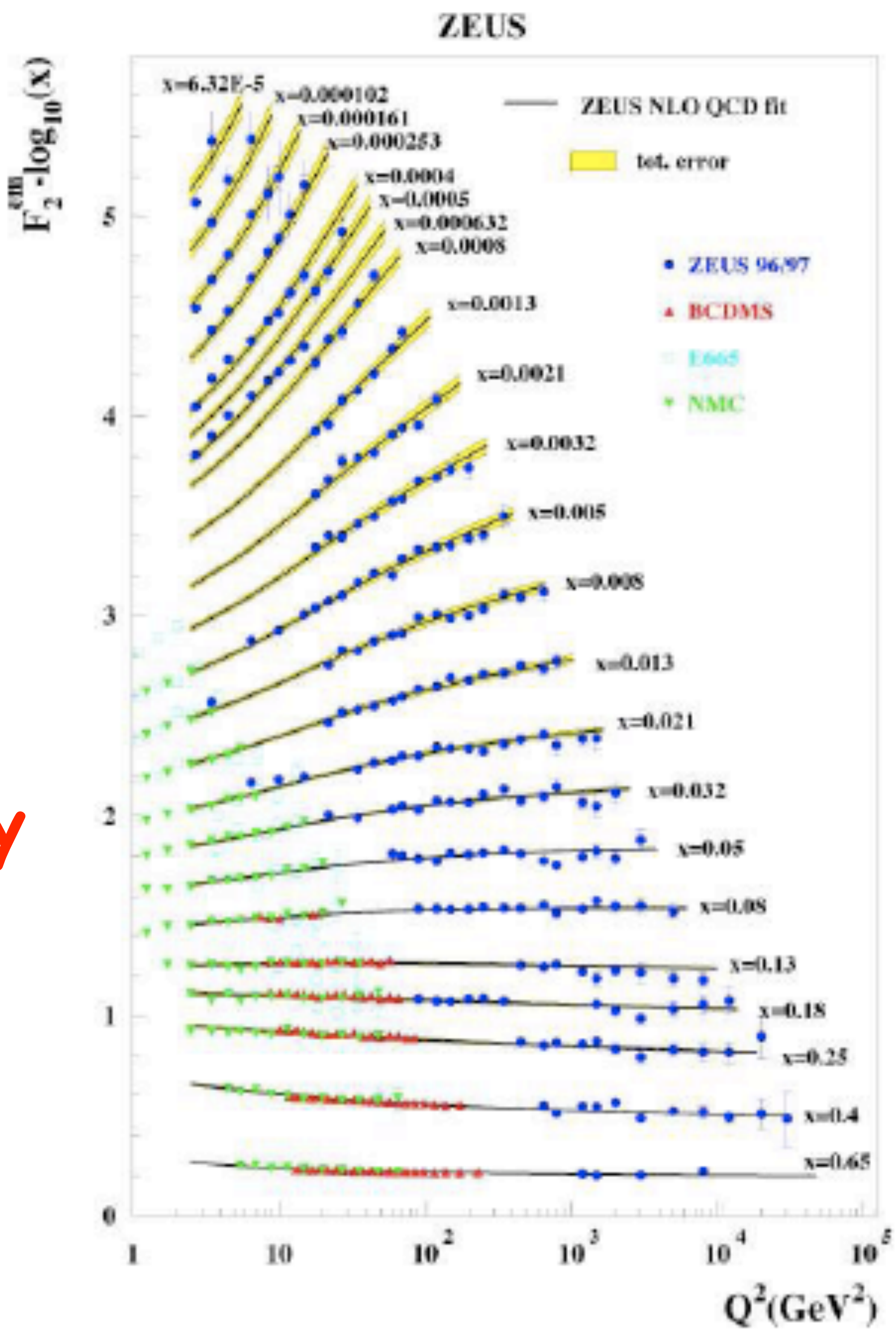
Iancu-Itakura-Munier model

- This model of the dipole cross-section is derived from LO BFKL :

$$\begin{cases} Q_s r_\perp \le 2 \;: \quad \sigma_{\rm dip}(\vec{r}_\perp, Y) = \dfrac{\sigma_0}{2} \left(\dfrac{Q_s(Y) r_\perp}{2} \right)^{2(\gamma_s + \ln(2/Q_s r_\perp)/\kappa\lambda Y)} \\ Q_s r_\perp \ge 2 \;: \quad \sigma_{\rm dip}(\vec{r}_\perp, Y) = \sigma_0 \left[1 - e^{a \ln^2(b Q_s r_\perp)} \right] \\ \\ Q_s^2(Y) = Q_0^2 \, e^{\lambda(Y - Y_0)} \end{cases}$$

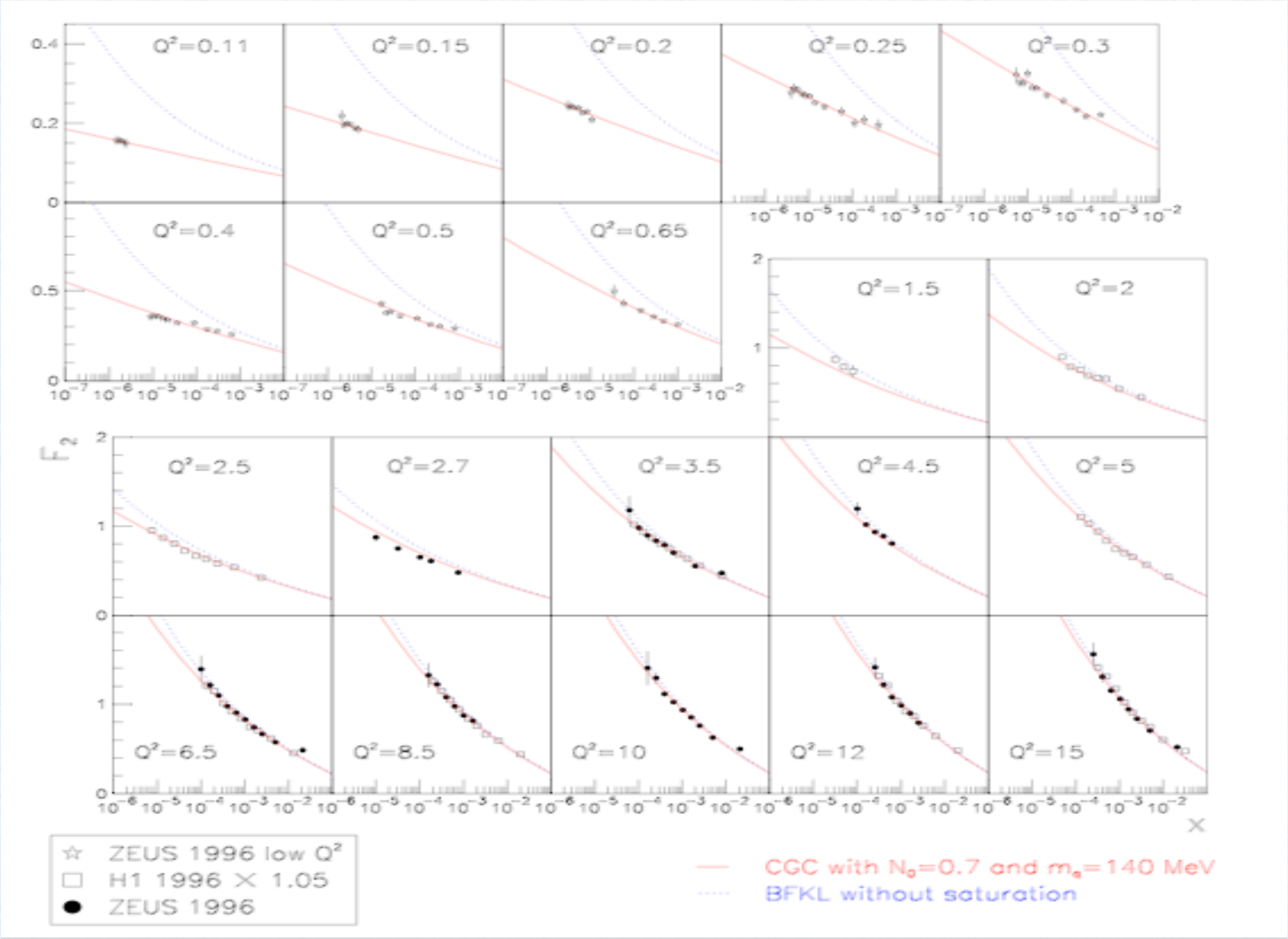
  - Some parameters are fixed from LO BFKL : $\gamma_s = 0.63, \kappa = 9.9$
  - σ_0, Q_0 and λ must be fitted
  - a and b are adjusted for a smooth transition at $Q_s r_\perp = 2$

- HERA data as a function of Q^2 and x :

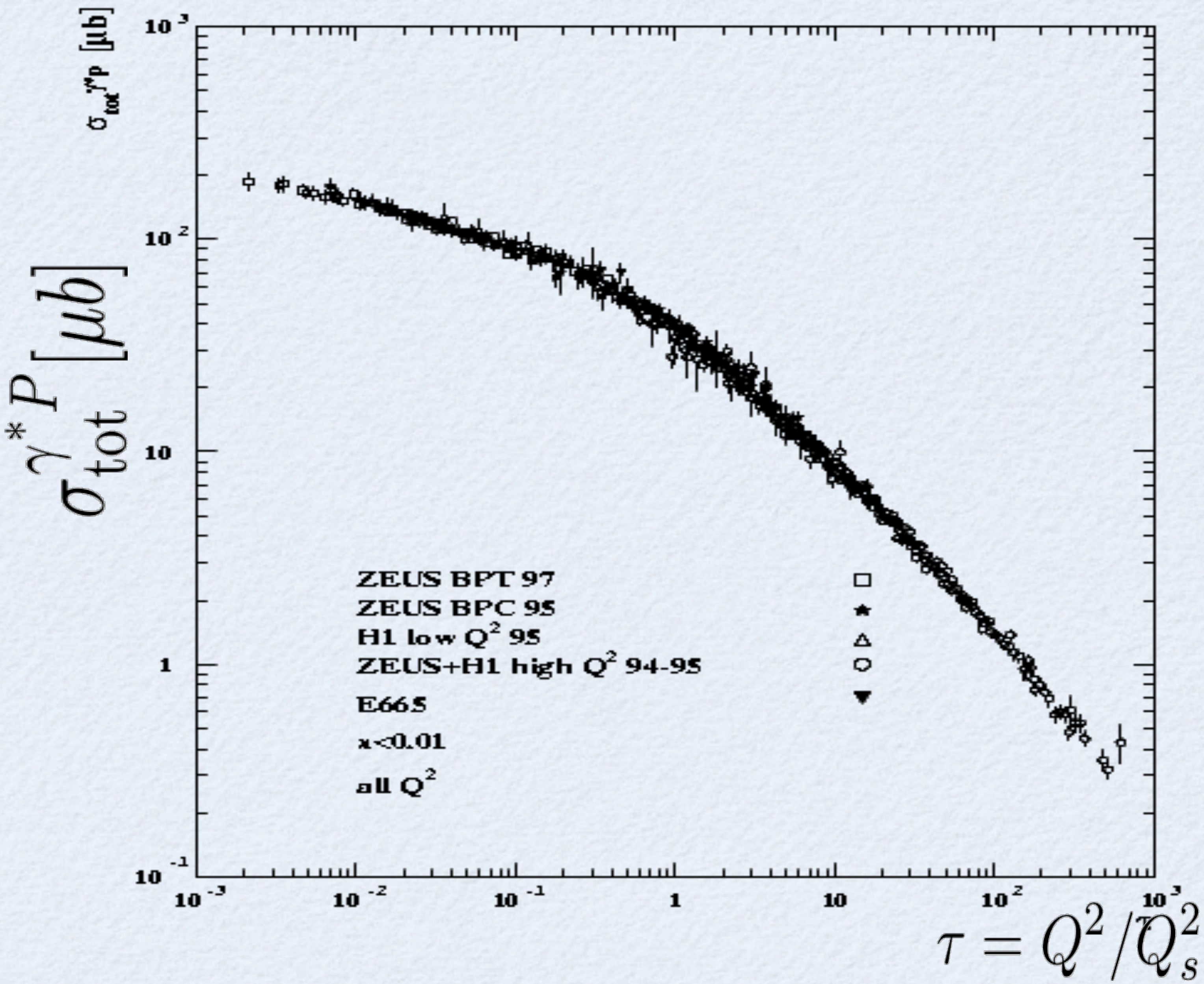


pQCD (DGLAP) work very nicely

CGC at HERA (ep: √S = 310 GeV)



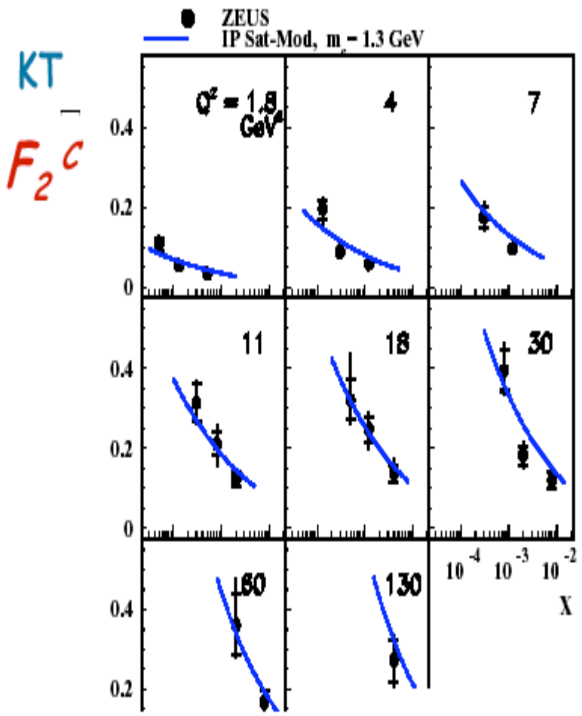
Geometrical Scaling



scaling seen for all data with x < 0.01

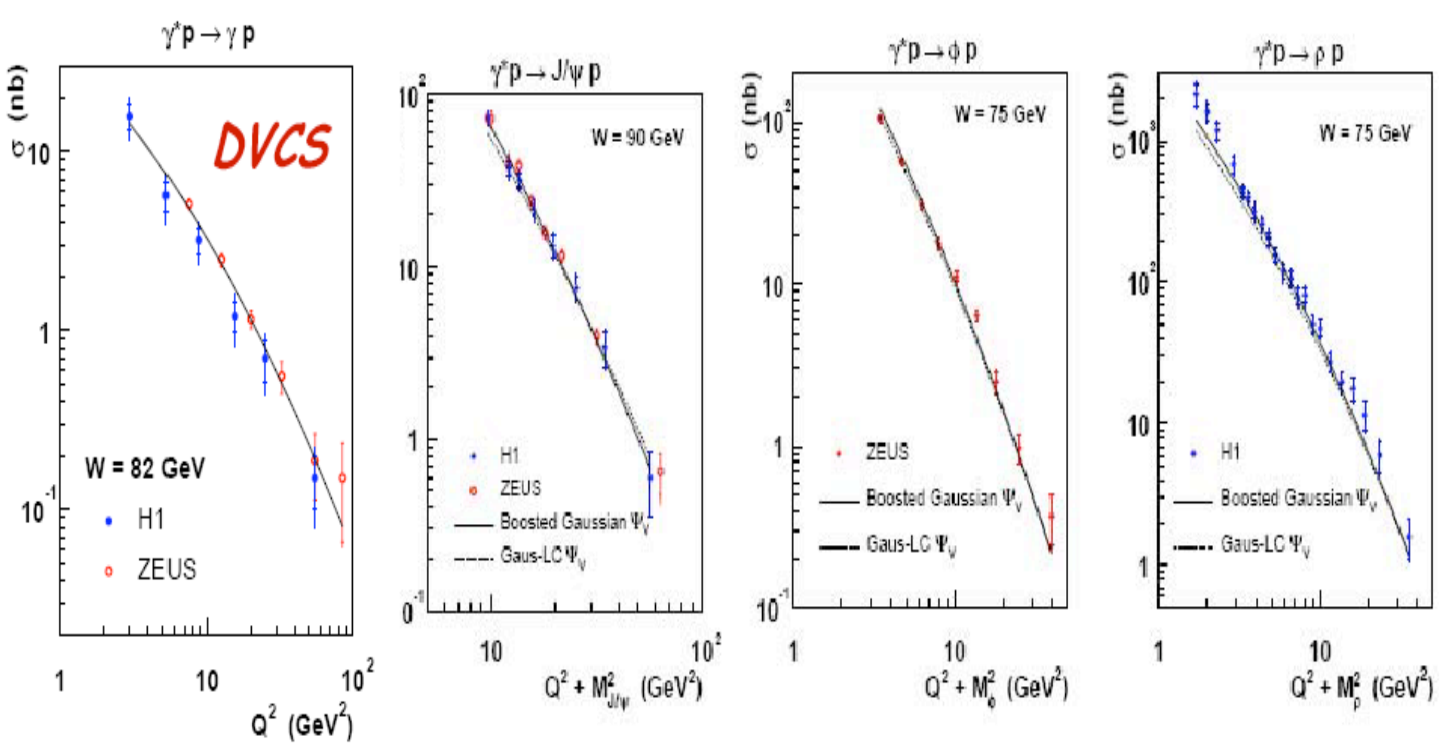
also seen for less inclusive quantities

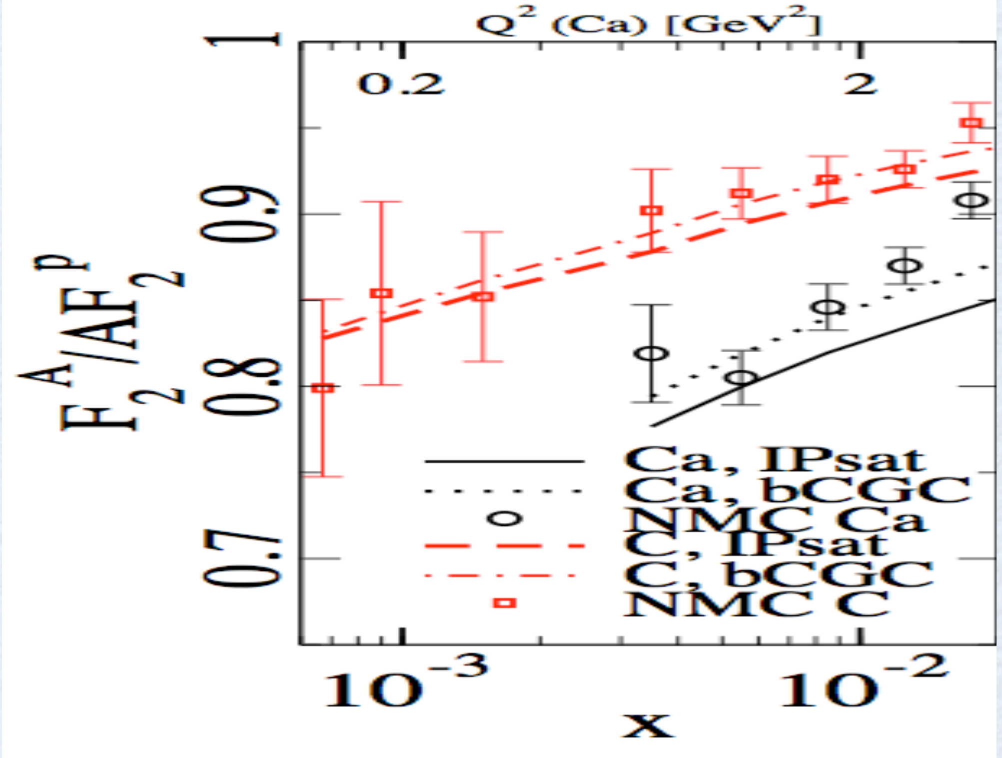
Dipole models & DIS data

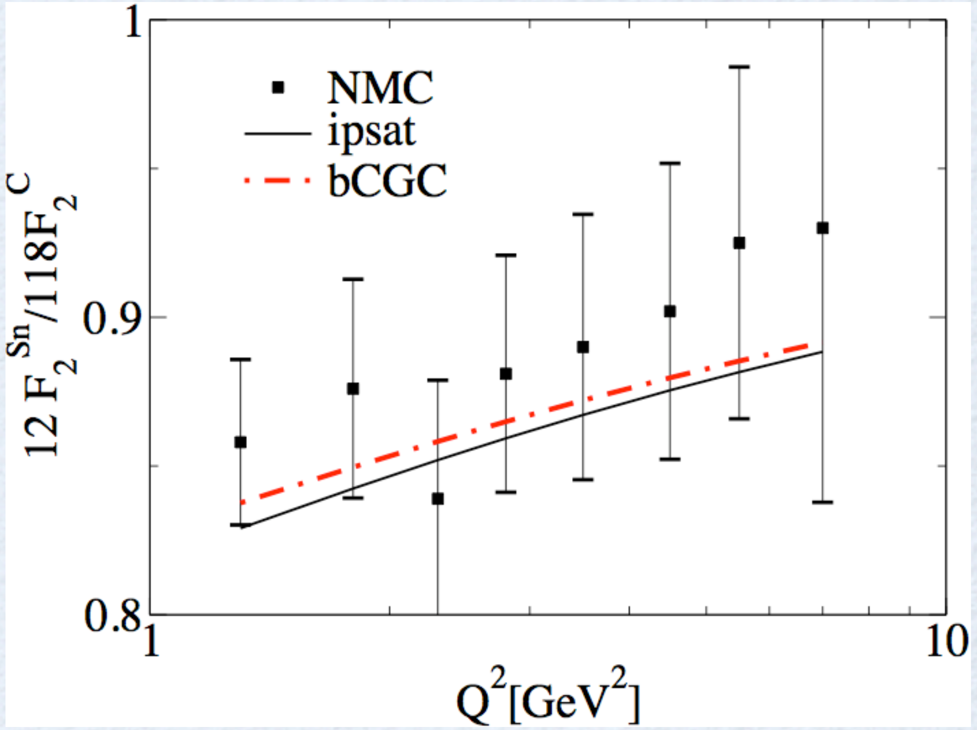


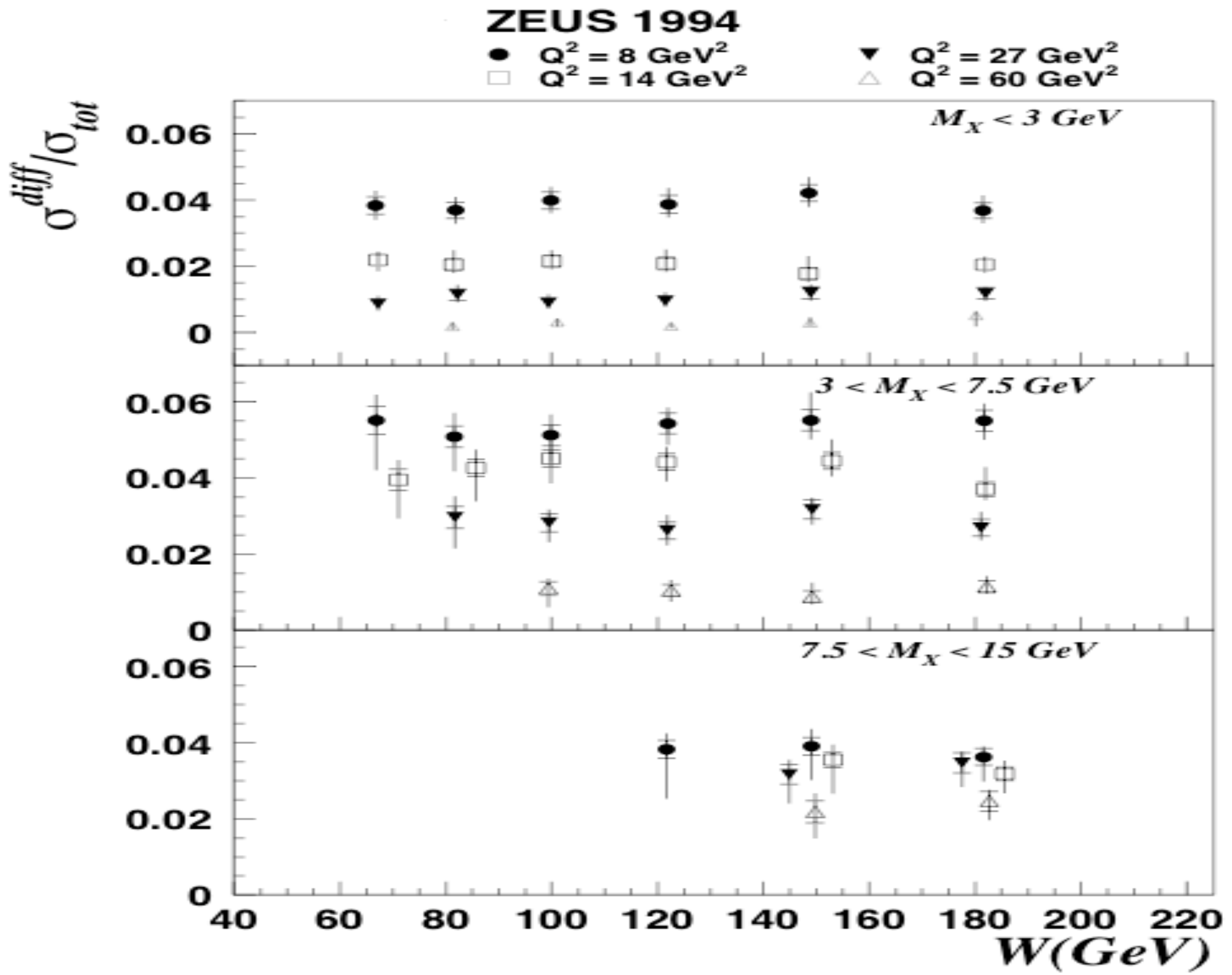
Dipole cross section determined by fit to F_2 ⟹
Simultaneous description of many reactions

Vector Mesons







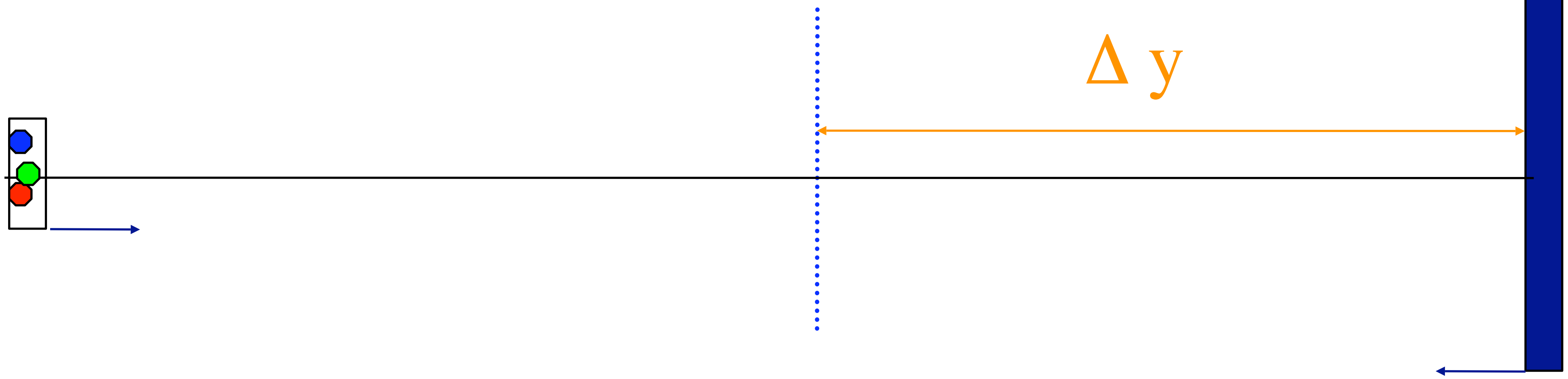


ZEUS 1994
Q² = 8 GeV²
Q² = 14 GeV²
Q² = 27 GeV²
Q² = 60 GeV²
σ^diff/σ_tot
M_X < 3 GeV
3 < M_X < 7.5 GeV
7.5 < M_X < 15 GeV
0.06
0.04
0.02
0
40 60 80 100 120 140 160 180 200 220
W(GeV)

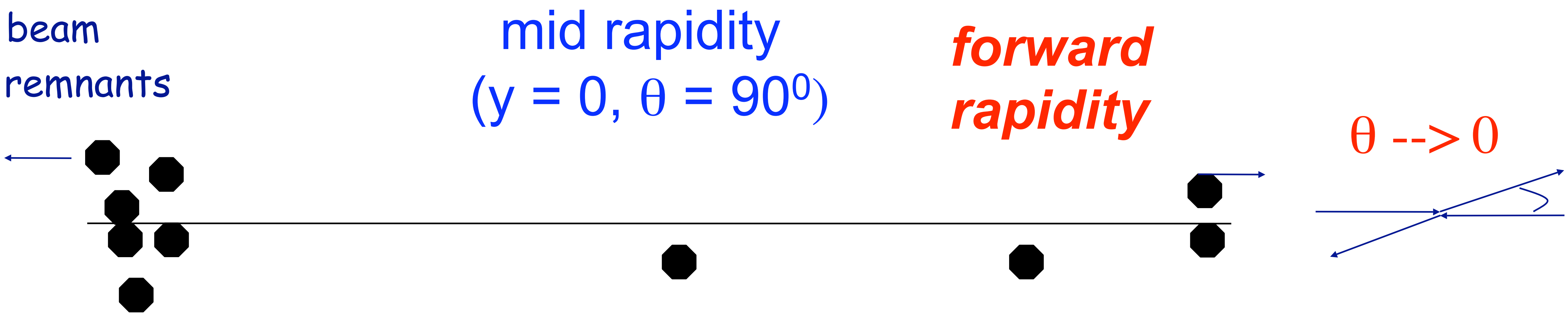
DGLAP ??

shattering the Color Glass condensate in high energy nuclear collisions

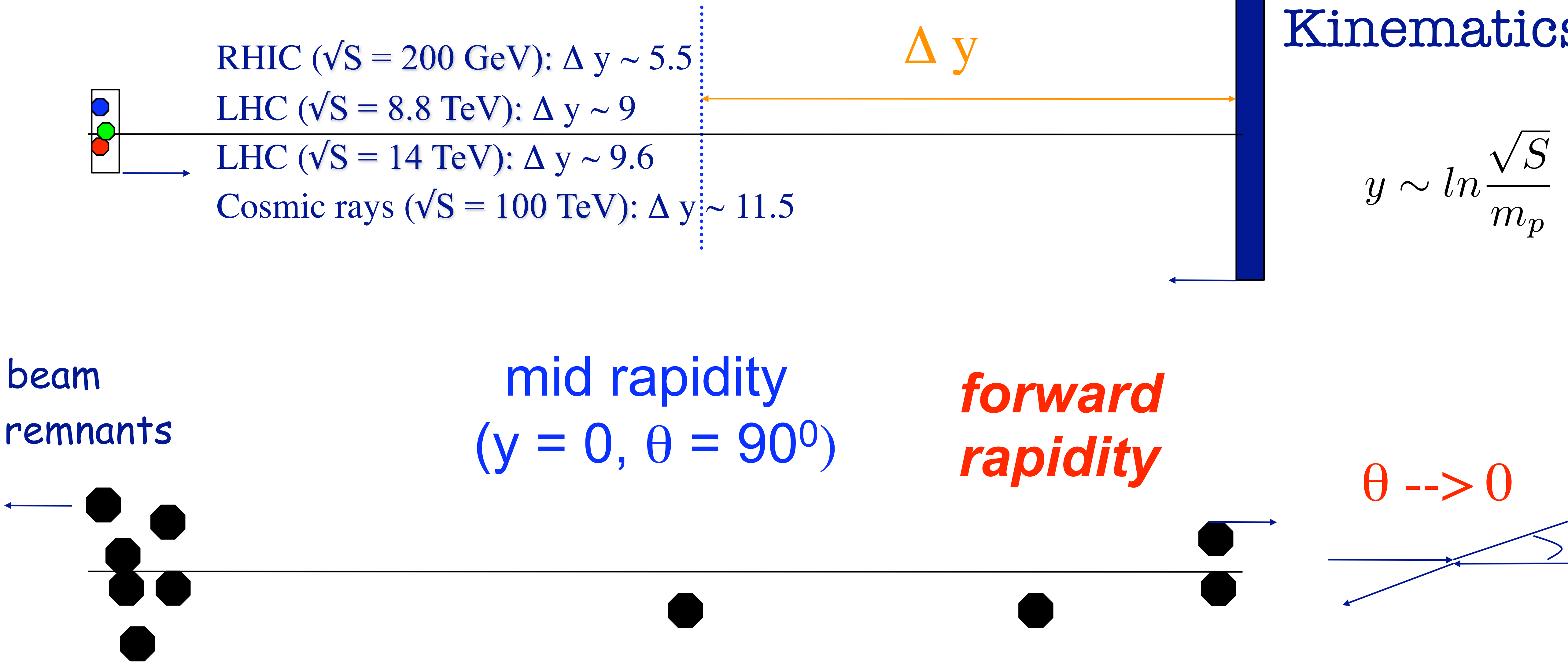
Kinematics



$$y \sim ln \frac{\sqrt{S}}{m_p}$$



Kinematics



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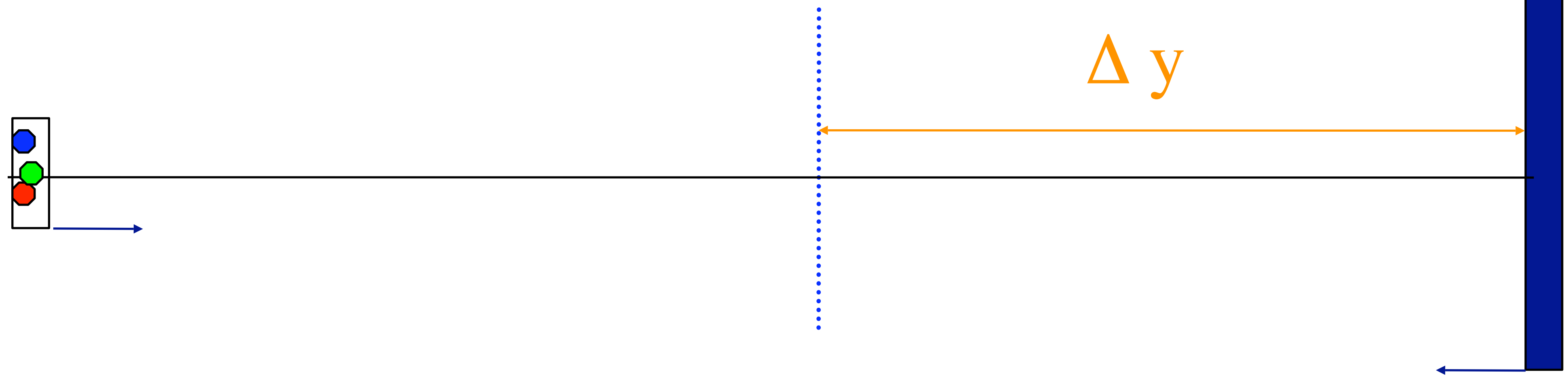
Kinematics



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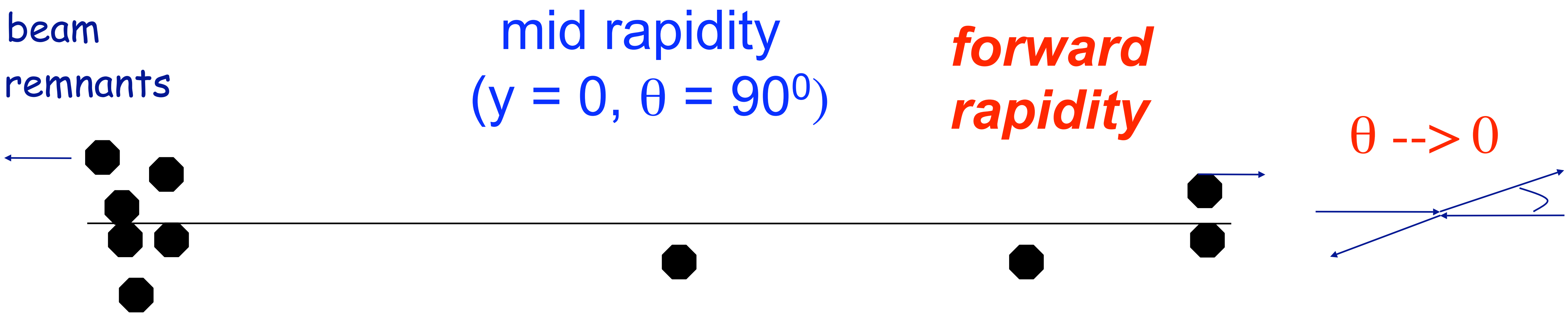

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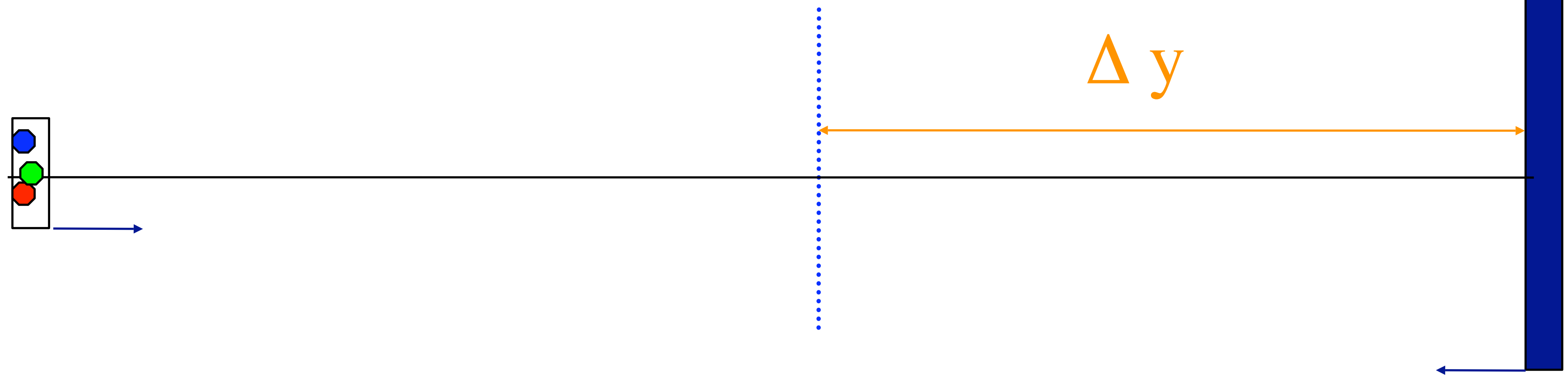

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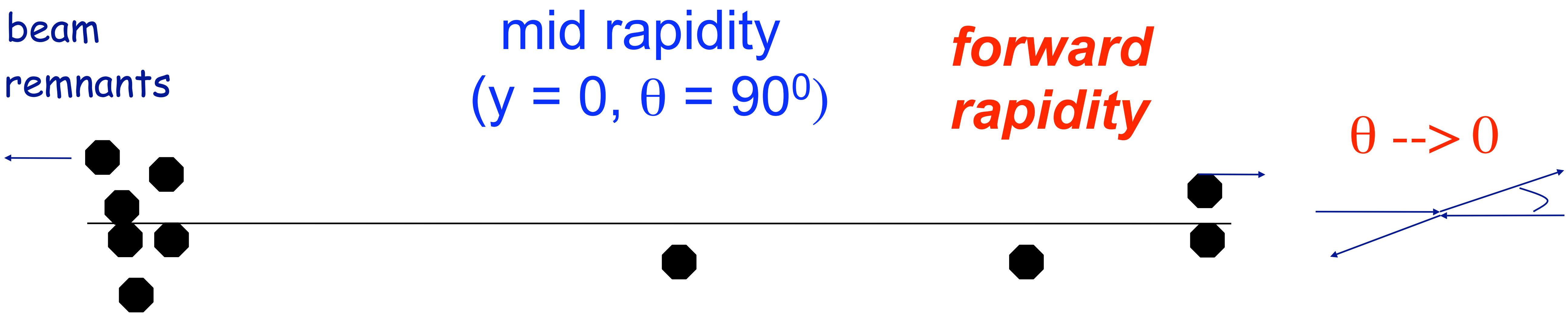


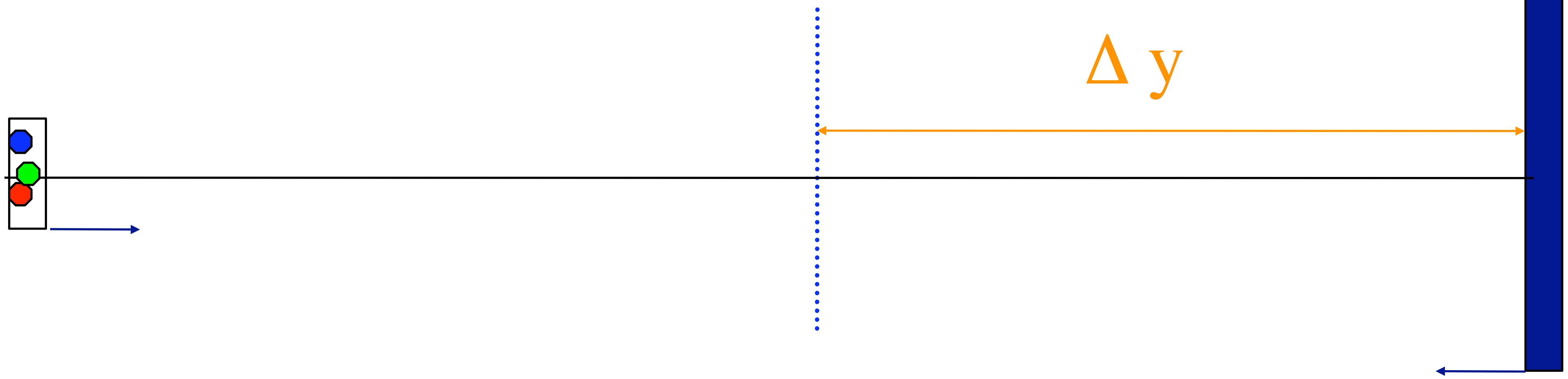
$y = 0$: x_1 = x_2 = 10^{-2}

$y \sim 4$: $x_1 \sim 0.5$, $x_2 \sim 10^{-4}$

(RHIC: for p_t = 2 GeV)

$$x_{1,2} = \frac{p_t}{\sqrt{S}} e^{\pm y}$$

Kinematics

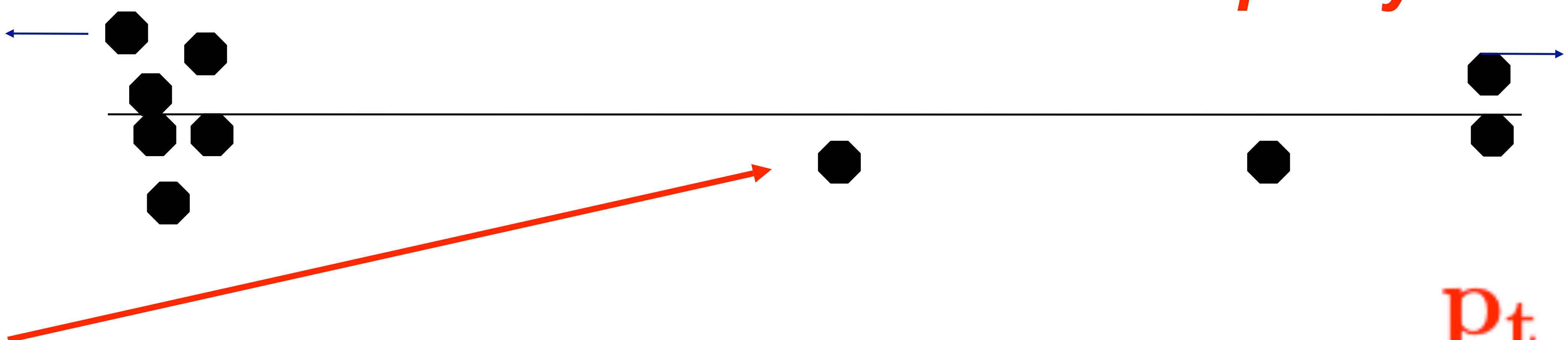


$$y \sim ln \frac{\sqrt{S}}{m_p}$$

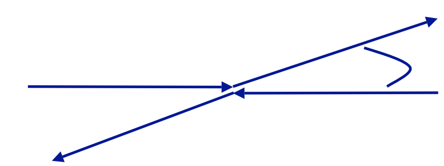
beam remnants

mid rapidity
($y = 0, \theta = 90^0$)

forward rapidity



θ --> 0



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(RHIC: for p_t = 2 GeV)

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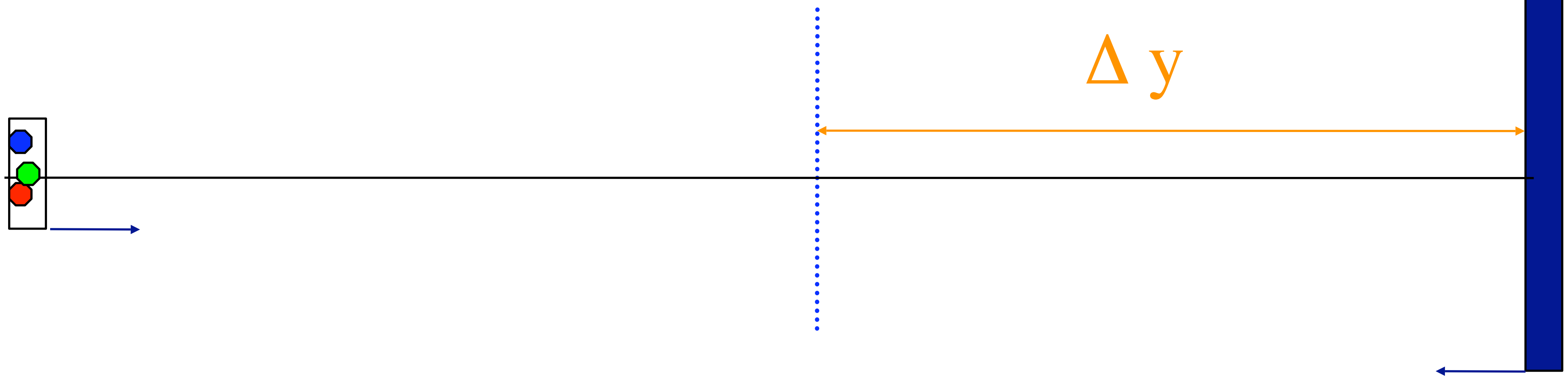
Kinematics

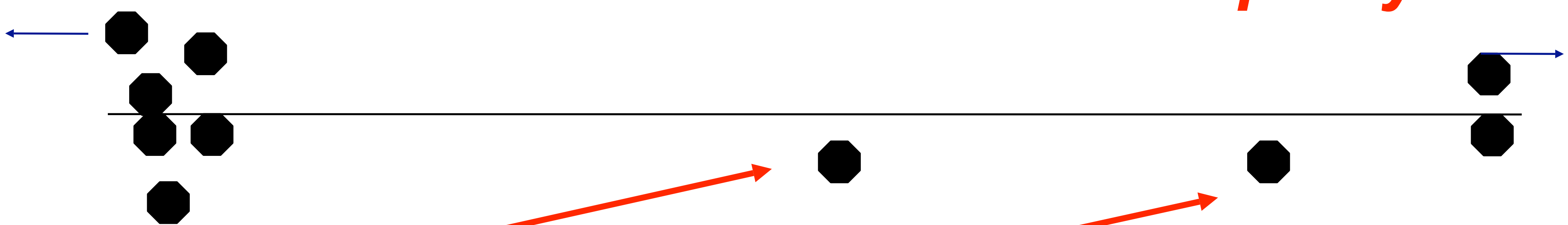


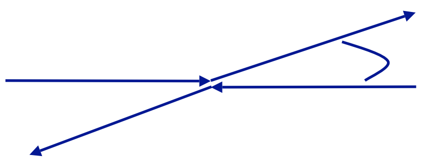
$$y \sim ln \frac{\sqrt{S}}{m_p}$$

beam remnants

mid rapidity
(y = 0, θ = 90⁰)

forward rapidity





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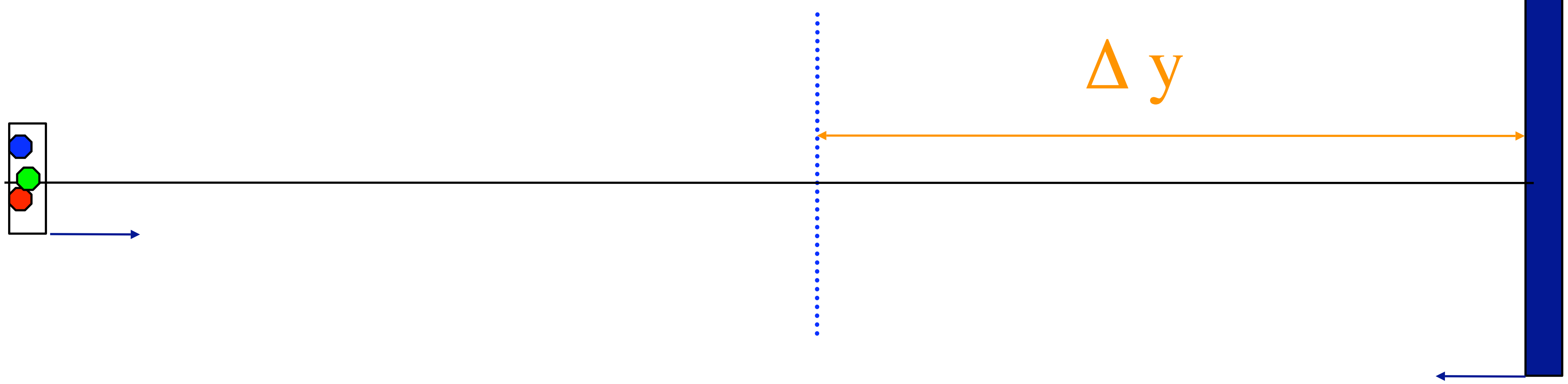
(RHIC: for p_t = 2 GeV)

$$x_{1,2} = \frac{p_t}{\sqrt{S}} e^{\pm y}$$

Kinematics

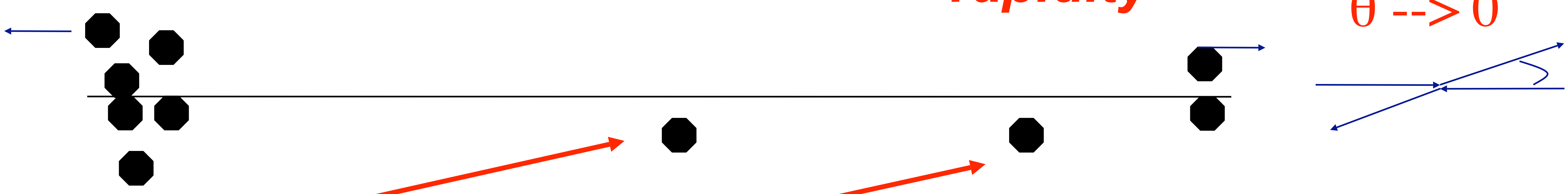


$$y \sim ln\frac{\sqrt{S}}{m_p}$$

beam
remnants

mid rapidity
$(y = 0, \theta = 90^0)$

forward rapidity



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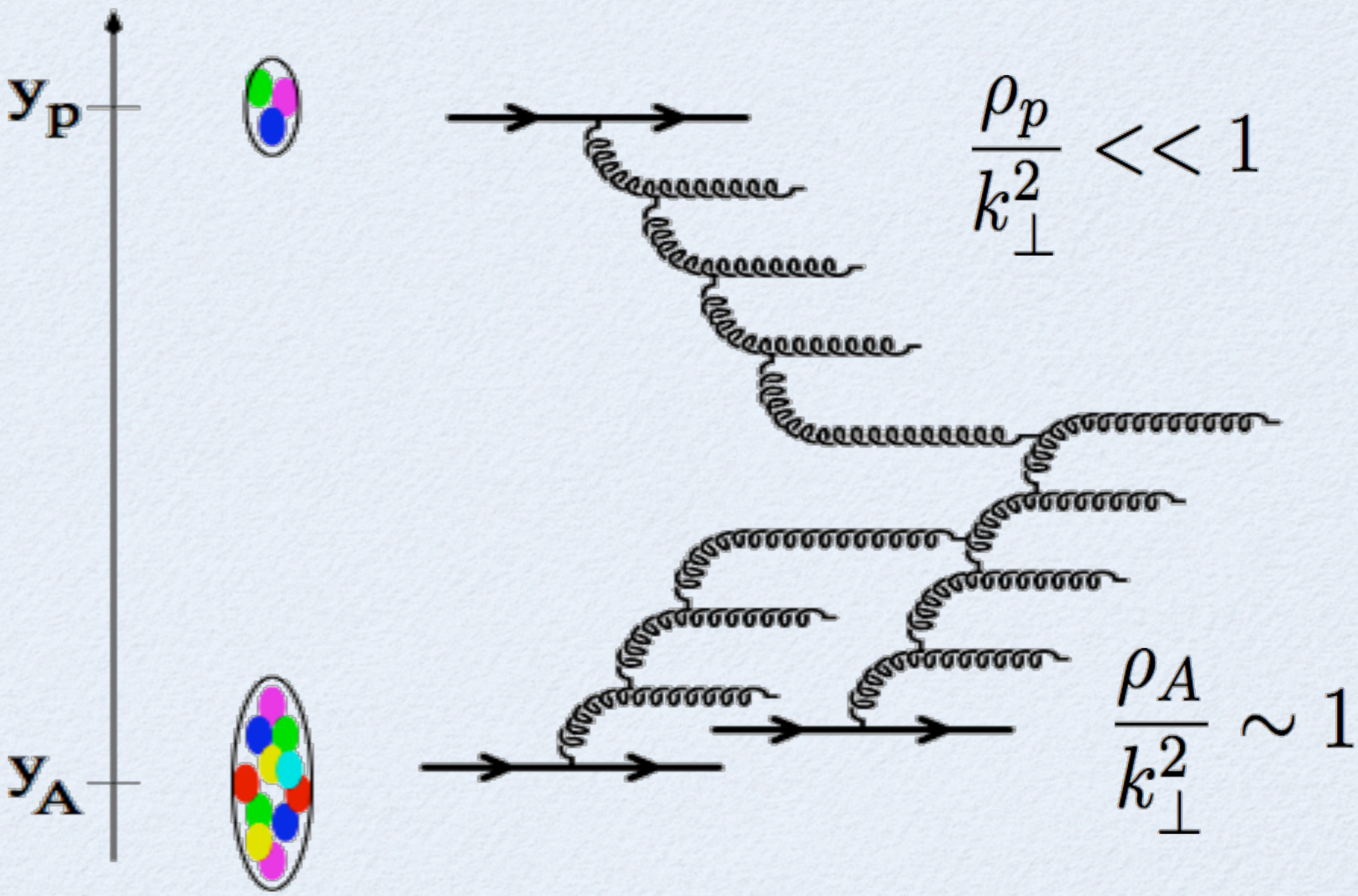
(RHIC: for p_t = 2 GeV)

$$x_{1,2} = \frac{p_t}{\sqrt{S}} e^{\pm y}$$

Cosmic rays: $y \sim 8$ --> $x_2 \sim 10^{-8}$

Orders of magnitude evolution in x

Proton-Nucleus collisions in CGC formalism



Solving classical Yang-Mills equations

$$[D_\mu, F^{\mu\nu}] = J^\nu \;;\; [D_\nu, J^\nu] = 0$$

with two light cone sources

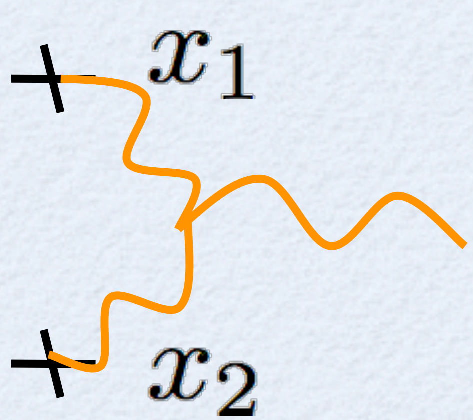
$$J^{\nu,a} = \delta^{\nu+} \rho_p^a(x_\perp)\delta(x^-) + \delta^{\nu-} \rho_A^a(x_\perp)\delta(x^+)$$

Proton source

Nuclear source

obtain k_T factorization formula:

$$\frac{dN_g}{dy d^2p_\perp} = \frac{\alpha_S S_{AB}}{2\pi^4 C_F (\pi R_A^2)(\pi R_B^2)} \frac{1}{p_\perp^2} \int \frac{d^2k_\perp}{(2\pi)^2} \phi_A(x_1, k_\perp)\, \phi_B(x_2, |p_\perp - k_\perp|)$$



$$x_1 = \frac{p_\perp}{m_N} e^{(y - y_{\text{beam}})}$$

$$x_2 = \frac{p_\perp}{m_N} e^{-(y + y_{\text{beam}})}$$

Unintegrated distribution:

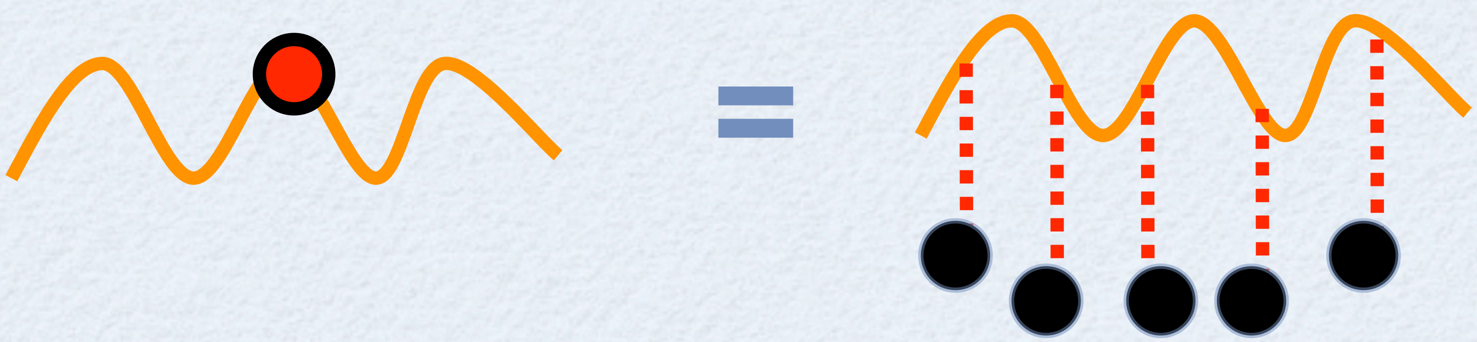
Compute in CGC EFT

$$\phi_{A,B}(x,k_\perp) = \frac{\pi R_A^2 k_\perp^2}{4\alpha_S N_c} \int d^2x_\perp e^{ik_\perp \cdot x_\perp} \langle \mathrm{Tr} \left(U^\dagger(0) U(x_\perp) \right) \rangle_Y$$

$$Y = \ln\left(\frac{1}{x}\right)$$

with the path ordered exponentials in the adjoint representation

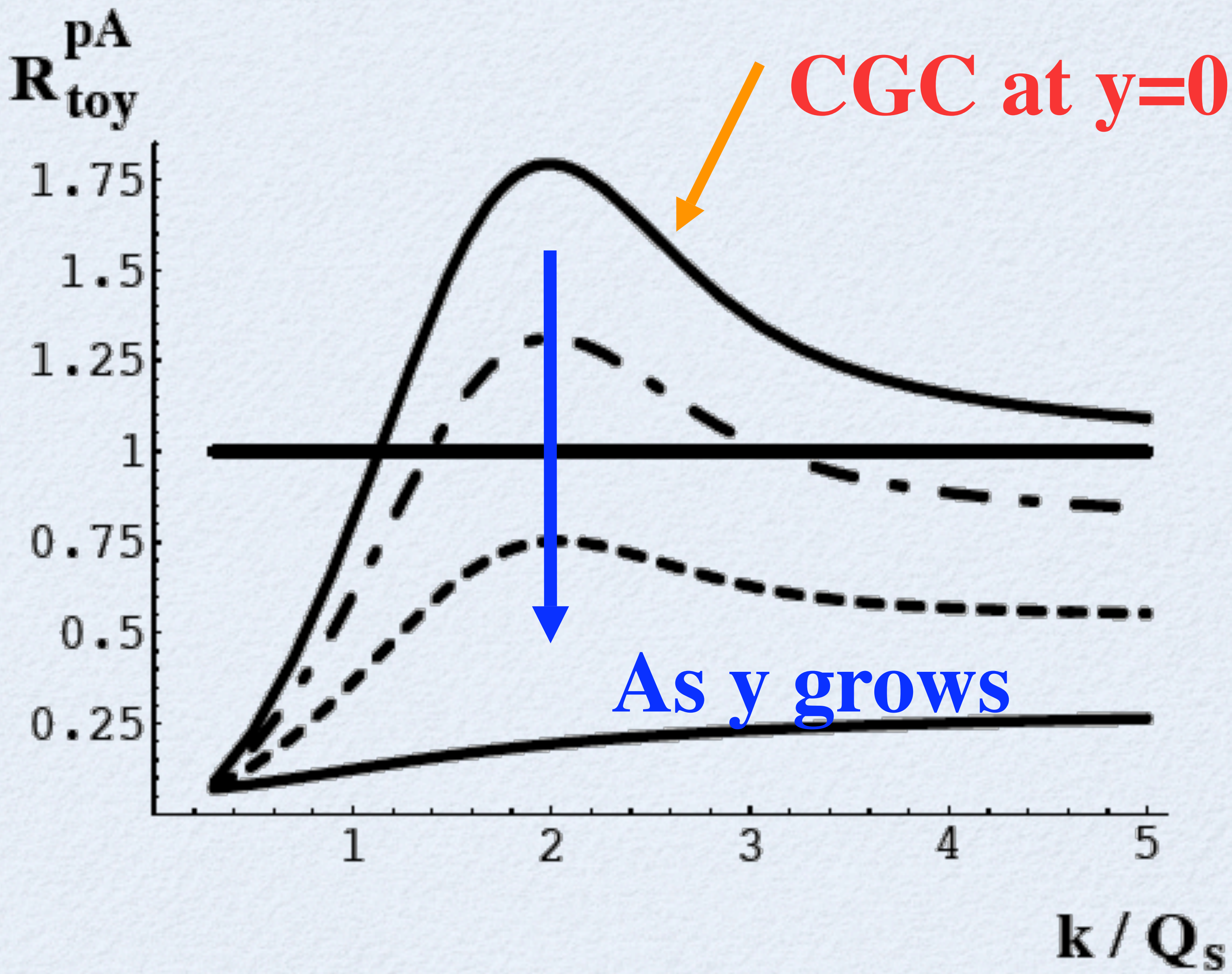
$$U(x_\perp) = P_+ \exp\left(-ig^2 \int dz^+ \frac{1}{\nabla_\perp^2} \rho_A^a(z^+, x_\perp) T^a \right)$$

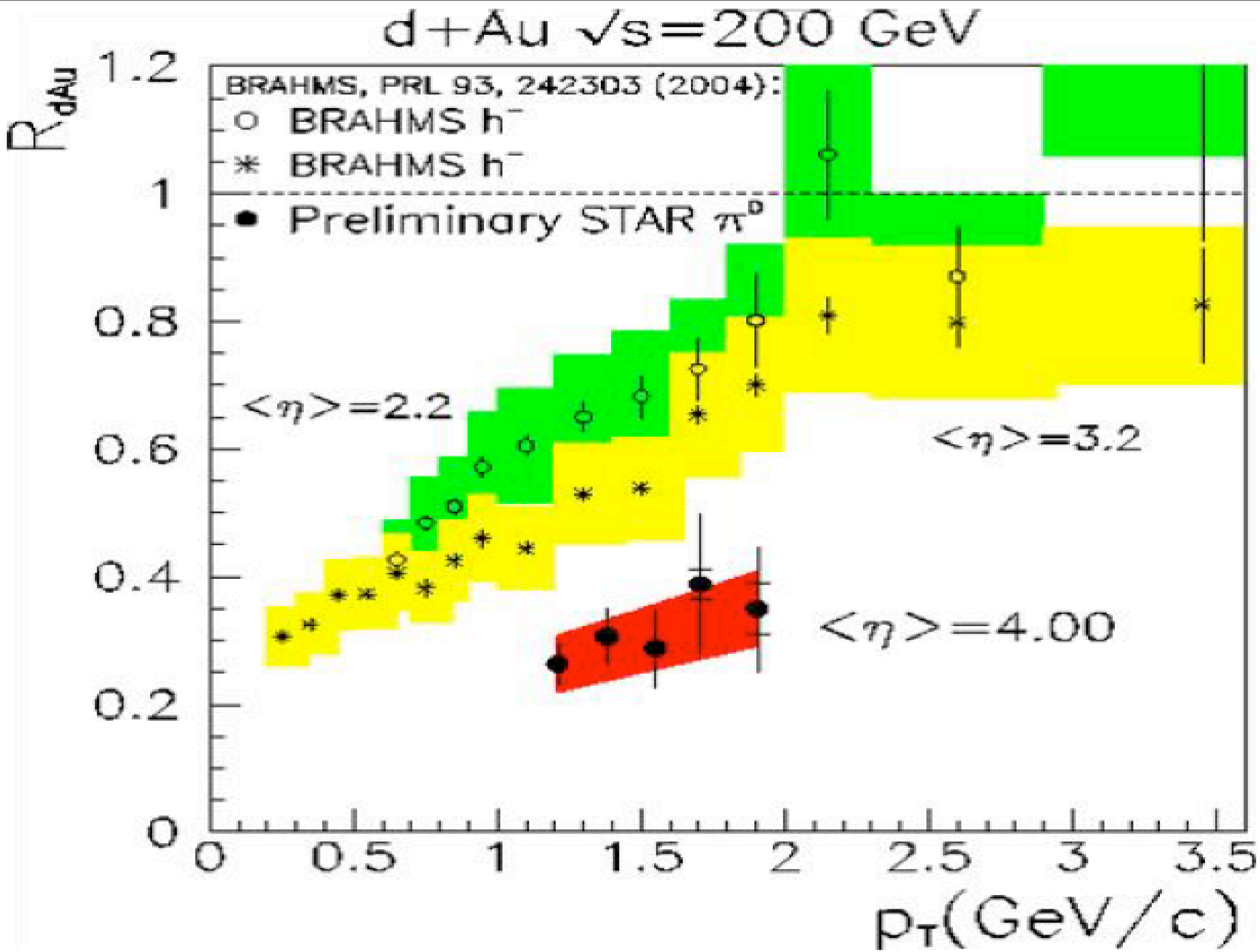


Encodes both multiple scattering and shadowing effects

proton-nucleus collisions at RHIC

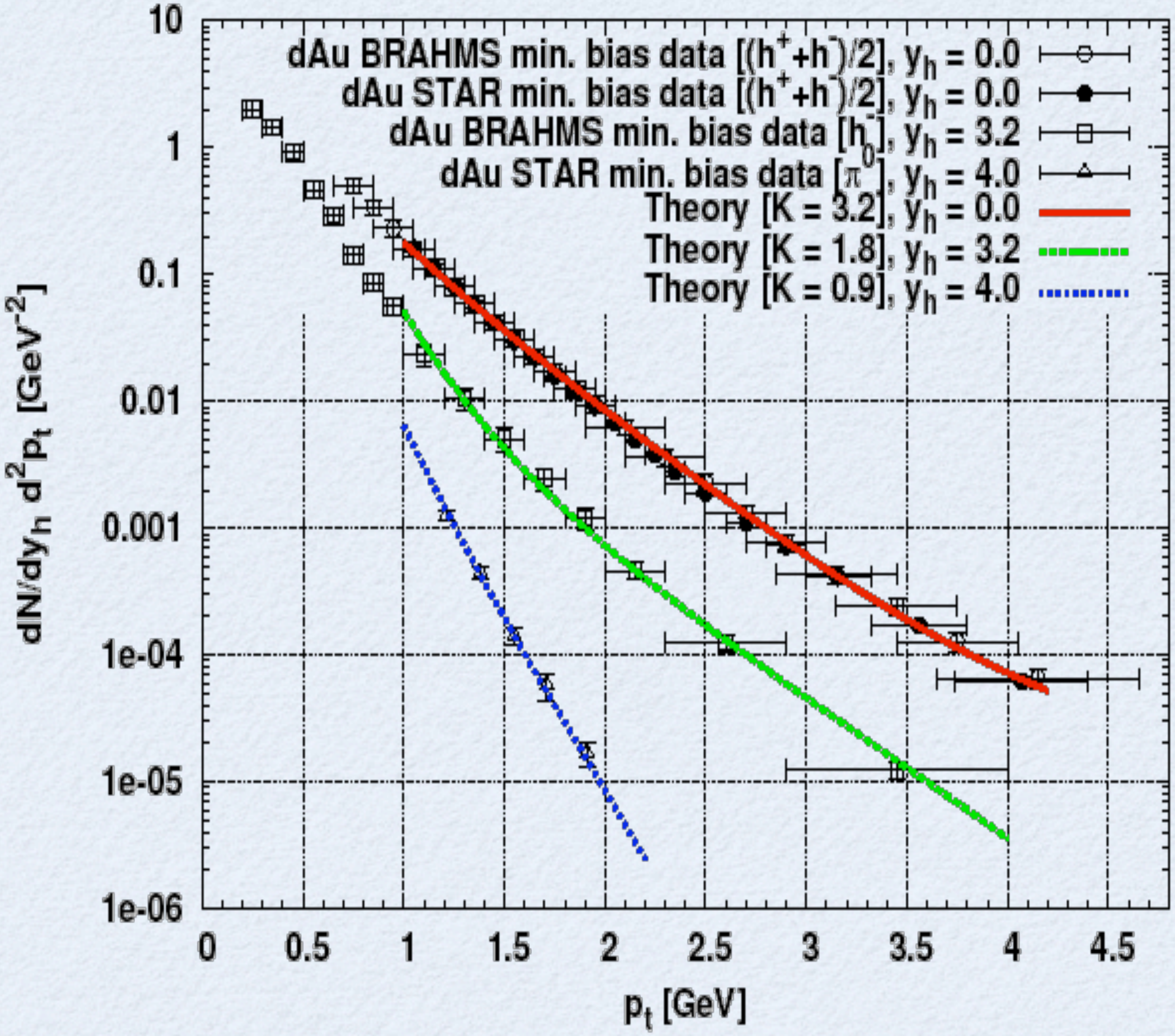
$$R_{pA} \equiv \frac{1}{A} \frac{\frac{d\sigma^{pA\to h\,X}}{dy\,d^2p_t}}{\frac{d\sigma^{pp\to h\,X}}{dy\,d^2p_t}}$$





d+Au √s=200 GeV
BRAHMS, PRL 93, 242303 (2004):
BRAHMS h⁻
BRAHMS h⁻
Preliminary STAR π⁰
<η>=2.2
<η>=3.2
<η>=4.00
R_dAu
p_T(GeV/c)
0
0.2
0.4
0.6
0.8
1
1.2
0
0.5
1
1.5
2
2.5
3
3.5

proton-nucleus collisions at RHIC



predictions to be verified/falsified at LHC

Signatures of CGC at a collider

★ Multiplicities (dominated by $p_t < Q_s$):

energy, rapidity, centrality dependence

★ Single particle production: hadrons, photons, dileptons

rapidity, p_t, centrality dependence

i) Fixed p_t: vary rapidity (evolution in x)

ii) Fixed rapidity: vary p_t (transition from dense to dilute)

★ Two particle production:

back to back correlations

Conclusions

★ *The CGC offers a systematic way to think about a wide range of problems in high energy QCD.*

★ *Its been very successful phenomenologically,in a wide range of e+p, e+A DIS, p/D+A and A+A processes.*

★ *Understanding the plasma created from melting CGC in A+A collisions offers the prospect of a first principles understanding of thermalization...*

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challenge: can one unify large x with small x ?