

Origin of Mass

Lect. 3: Approach

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Dynamical Chiral Symmetry Breaking and Confinement in QCD

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The diagram illustrates the Schwinger-Dyson equation for the quark propagator in QCD. On the left, a horizontal teal line with a teal circle in the middle is labeled with a superscript -1 . This is followed by an equals sign. To the right of the equals sign is another horizontal teal line with a teal circle in the middle, also labeled with a superscript -1 . This is followed by a minus sign and a third term. The third term consists of a horizontal teal line with a teal circle in the middle, followed by a yellow circle. A red wavy line (gluon) connects the teal circle to a red circle above it, which then connects to the yellow circle.

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$$S_F^{-1}(p) = Z_2 S_F^{(0)-1}(p) + g^2 Z_{1F} C_F \int \frac{d^4 k}{16\pi^4} \gamma_\mu S_F(k) \Gamma_\nu(k, p) \Delta_{\mu\nu}(k - p)$$

Dynamical Mass Generation in QCD



$$S_F^{-1}(p) = Z_2 S_F^{(0)-1}(p) + g^2 Z_{1F} C_F \int \frac{d^4 k}{16\pi^4} \gamma_\mu S_F(k) \Gamma_\nu(k, p) \Delta_{\mu\nu}(k - p)$$

The solution is of the form

$$S_F(p) = \frac{F(p^2)}{\not{p} - M(p^2)}$$

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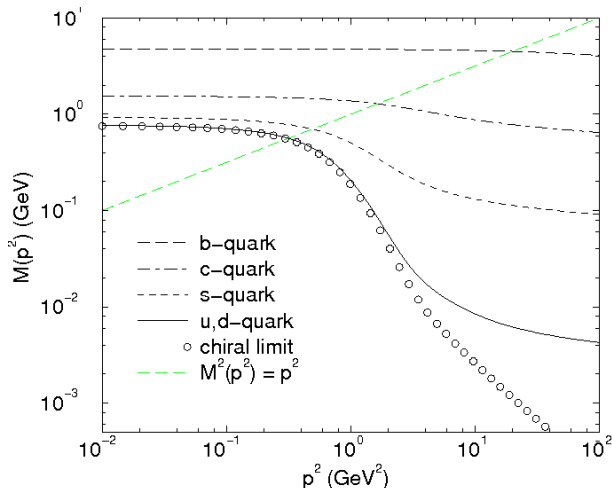
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Adapted from nucl-th/0007054.

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$$g^2 \int d^4k \gamma_\mu S_F(k) \Gamma_\nu(k, p) \Delta_{\mu\nu}(k - p)$$

should have an enormous support.

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Dynamical Mass Generation in QCD

$$g^2 \int d^4k \gamma_\mu S_F(k) \Gamma_\nu(k, p) \Delta_{\mu\nu}(k - p)$$

should have an enormous support.

- Strength of interaction $g \rightarrow g_{\text{eff}}(p)$

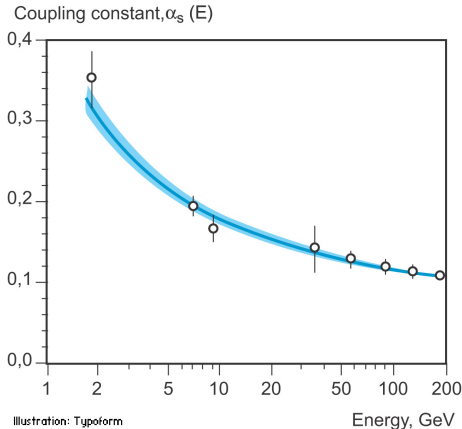


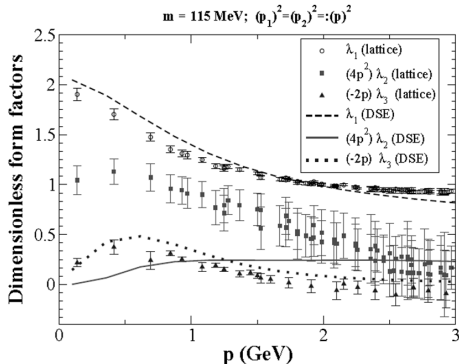
Illustration: Typoform

Dynamical Mass Generation in QCD

$$g^2 \int d^4k \gamma_\mu S_F(k) \Gamma_\nu(k, p) \Delta_{\mu\nu}(k - p)$$

should have an enormous support.

► The Quark-Gluon Vertex

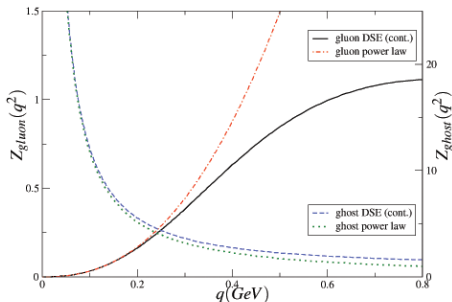


Adapted from Nucl. Phys. Proc. Suppl. **152** 43, (2006).

$$g^2 \int d^4k \gamma_\mu S_F(k) \Gamma_\nu(k, p) \Delta_{\mu\nu}(k - p)$$

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► The Ghost and Gluon Propagators



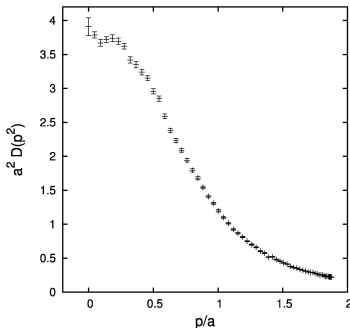
Adapted from Braz. J. Phys. **37** 201 (2007).

Dynamical Mass Generation in QCD

$$g^2 \int d^4k \gamma_\mu S_F(k) \Gamma_\nu(k, p) \Delta_{\mu\nu}(k - p)$$

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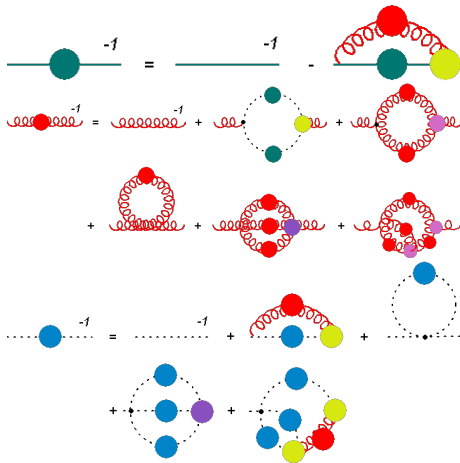
- The Gluon Propagator is IR finite!



Adapted from PoS LAT2007, 297 (2007).

Confinement in QCD

Confinement can be studied through the IR properties of Green's functions



Confinement in QCD

Kugo-Ojima criterion:

- ▶ Ghost-Gluon vertex is IR finite

Confinement in QCD

Kugo-Ojima criterion:

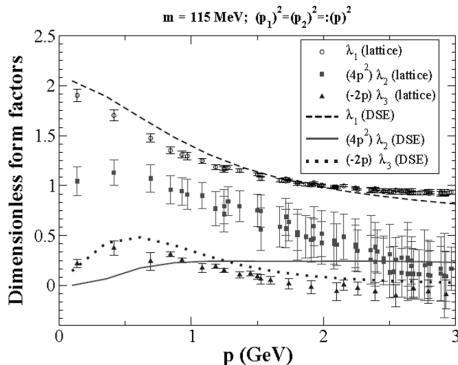
- ▶ Ghost-Gluon vertex is IR finite
- ▶ Ghost propagator is IR divergent

Kugo-Ojima criterion:

- ▶ Ghost-Gluon vertex is IR finite
- ▶ Ghost propagator is IR divergent
- ▶ Gluon propagator is IR suppressed

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Kugo-Ojima criterion:



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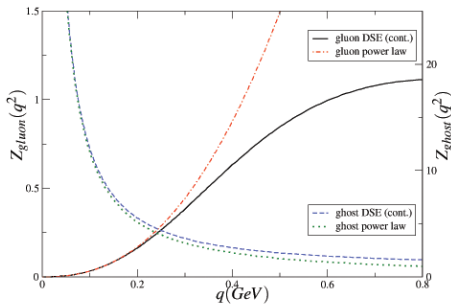
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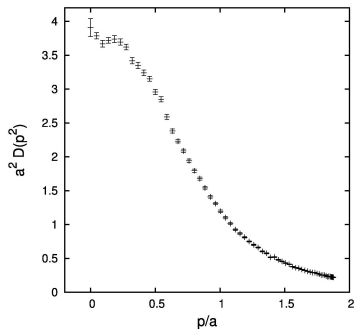
Kugo-Ojima criterion:



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Confinement in QCD

Kugo-Ojima criterion:



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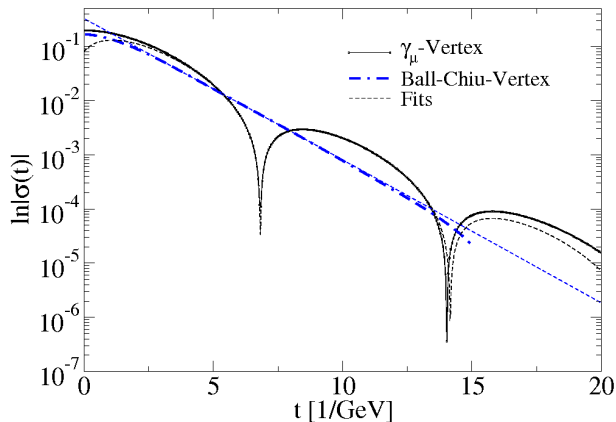
Axiom of Reflexion Positivity

$$\begin{aligned}\Delta(t) &= \int d^3x \int \frac{d^4p}{(2\pi)^4} e^{i(tp_4 + \vec{x} \cdot \vec{p})} \sigma(p^2) \\ &= \frac{1}{\pi} \int_0^\infty dp_4 \cos(tp_4) \sigma(p_4^2) \geq 0,\end{aligned}$$

with

$$\sigma(p^2) = \frac{F(p^2)M(p^2)}{p^2 + M^2(p^2)}.$$

Confinement in QCD



Adapted from J. Phys. G32, R253 (2006).

A Toy Model

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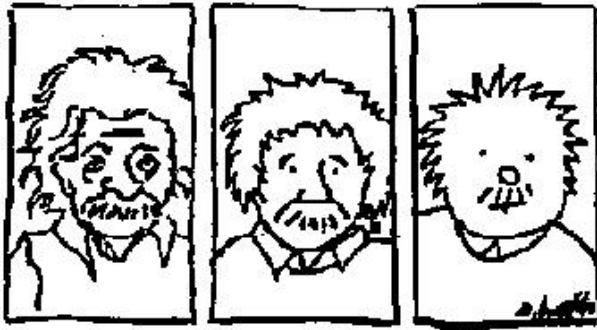
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Why QED₃?

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- Is super renormalizable

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Why QED₃?

- ▶ Is super renormalizable
- ▶ Exhibits DCSB and Confinement

Why QED₃?

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- ▶ Is super renormalizable
- ▶ Exhibits DCSB and Confinement
 - ▶ Provides a popular battleground for lattice and continuum studies

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 - ▶ Quantum Hall Effect
 - ▶ Graphene

SDE in QED₃

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The diagram illustrates the Dyson-Schwinger Equation (SDE) for the fermion propagator in QED₃. It is represented as an equation between two diagrams:

- Left side:** A horizontal teal line with a teal circle vertex. Above the circle is the label -1 .
- Right side:** A horizontal teal line with a teal circle vertex, labeled -1 above it, followed by a minus sign and a loop diagram.
- Loop diagram:** A teal line with a teal circle vertex and a yellow circle vertex, connected by a red wavy line with a red circle vertex.

The equation is: $\text{Teal line with teal circle}^{-1} = \text{Teal line with teal circle}^{-1} - \text{Teal line with teal and yellow circles connected by a red wavy line}$



Corresponds to

$$S_F^{-1}(p) = S_F^{(0)-1}(p) + e^2 \int \frac{d^3k}{(2\pi)^3} \gamma_\mu S_F(k) \Gamma_\nu(k, p) \Delta_{\mu\nu}(k - p)$$

Rainbow Truncation

- We start by neglecting fermion loops, $G(q) = 1$

The diagram shows a fermion propagator with a self-energy insertion. On the left, a horizontal teal line has a teal circle (self-energy) attached to it, with a superscript -1 above the circle. This is followed by an equals sign. On the right, there is a horizontal teal line with a superscript -1 above it, followed by a minus sign. This is followed by a diagram where a horizontal teal line has a teal circle attached to it, which is then connected to a yellow circle by a red wavy line (representing a photon). The entire expression is: $\text{Teal line} \cdot \text{Teal circle}^{-1} = \text{Teal line}^{-1} - \text{Teal line} \cdot \text{Teal circle} \cdot \text{Red wavy line} \cdot \text{Yellow circle}$

- We start by neglecting fermion loops, $G(q) = 1$



The diagram shows an equation for the fermion propagator. On the left is a horizontal line with a teal circle in the middle, labeled with -1 above it. This is equal to a horizontal line labeled with -1 above it, minus a diagram where a horizontal line has a teal circle and a yellow circle connected by a red wavy line (photon propagator) above them.

- In Landau gauge, it corresponds to a photon propagator

$$\Delta_{\mu\nu}^{(0)}(q) = \frac{1}{q^2} \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right)$$

- ▶ We start by neglecting fermion loops, $G(q) = 1$

The diagram shows the rainbow truncation of the electron propagator. On the left, a horizontal teal line with a teal circle in the middle is labeled with -1 above it. This is followed by an equals sign. To the right of the equals sign is another horizontal teal line with -1 above it, followed by a minus sign. To the right of the minus sign is a diagram consisting of a horizontal teal line with a teal circle and a yellow circle connected by a red wavy line (photon propagator) forming a loop on top.

- ▶ In Landau gauge, it corresponds to a photon propagator

$$\Delta_{\mu\nu}^{(0)}(q) = \frac{1}{q^2} \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right)$$

- ▶ With a suitable choice of the electron-photon vertex, the electron propagator can be found self-consistently

Rainbow Truncation

- Possible the simplest choice for the vertex is
 $\Gamma^\nu(k, p) = \gamma^\nu$

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Solving the SDE

► In Rainbow Approximation

$$S_F^{-1}(p) = S_F^{(0)-1}(p) + 4\pi\alpha \int \frac{d^3k}{(2\pi)^3} \gamma_\mu S_F(k) \gamma_\nu \Delta_{\mu\nu}^{(0)}(k-p)$$

► In Rainbow Approximation

$$S_F^{-1}(p) = S_F^{(0)-1}(p) + 4\pi\alpha \int \frac{d^3k}{(2\pi)^3} \gamma_\mu S_F(k) \gamma_\nu \Delta_{\mu\nu}^{(0)}(k-p)$$

► Starting with massless fermions, $m_0 = 0$, multiplying by 1 and \not{p} and taking trace and contracting with $\Delta_{\mu\nu}^{(0)}$

$$\frac{1}{F(p)} = 1 + \frac{\alpha}{2\pi^2 p^2} \int d^3k \frac{F(k)}{k^2 + M^2(k)} \frac{1}{(k-p)^4} \times$$
$$\left[-2(k \cdot p)^2 + (2 - \xi)(k^2 + p^2)k \cdot p - 2(1 - \xi)k^2 p^2 \right]$$
$$\frac{M(p)}{F(p)} = \frac{\alpha(2 + \xi)}{2\pi^2} \int d^3k \frac{F(k)M(k)}{k^2 + M^2(k)} \frac{1}{(k-p)^2}$$

Solving the SDE

► Performing angular integrations

$$\frac{1}{F(p)} = 1 - \frac{\alpha\xi}{\pi^2 p} \int_0^\infty dk \frac{k^2 F(k)}{k^2 + M^2(k)} \times$$
$$\left[1 - \frac{k^2 + p^2}{2kp} \ln \left| \frac{k+p}{k-p} \right| \right]$$
$$\frac{M(p)}{F(p)} = \frac{\alpha(\xi+2)}{\pi p} \int_0^\infty dk \frac{k F(k) M(k)}{k^2 + M^2(k)} \ln \left| \frac{k+p}{k-p} \right|$$

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► In Landau gauge ($\xi = 0$)

$$M(p) = \frac{2\alpha}{\pi p} \int_0^\infty dk \frac{k M(k)}{k^2 + M^2(k)} \ln \left| \frac{k+p}{k-p} \right|$$

Numerical Techniques

- We have an expression of the form

$$M(p) = \int_0^\infty dk f(k, M(k); p, M(p))$$

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$$\begin{aligned} M(p) &= \int_0^\infty dk f(k, M(k); p, M(p)) \\ &\approx \int_\kappa^\lambda dk f(k, M(k); p, M(p)) \end{aligned}$$

for $\kappa \rightarrow 0$ and $\lambda \rightarrow \infty$

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- ▶ Using some quadrature rule, we have

$$M(p) = \sum_{j=1}^{N_{max}} w_j f(k_j, M(k_j); p, M(p))$$

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- ▶ Using some quadrature rule, we have

$$\begin{aligned} M(p) &= \sum_{j=1}^{N_{max}} w_j f(k_j, M(k_j); p, M(p)) \\ &= \sum_{j=1}^{N_{max}} w_j f(k_j, M_j; p, M(p)) \end{aligned}$$

where w_j are the weights of the quadrature, k_j the nodes and $M_j = M(k_j)$

Numerical Techniques

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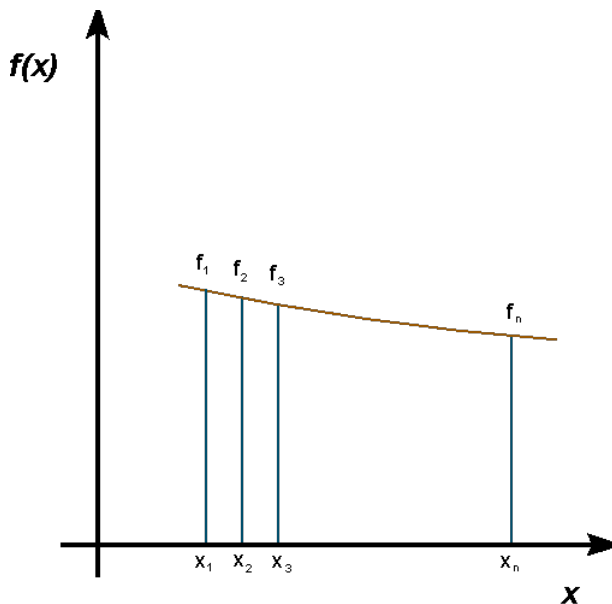
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- ▶ Instead of solving the equation over an entire domain of p , we decide that it is enough to know the mass function only in a discrete set of points

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- ▶ Instead of solving the equation over an entire domain of p , we decide that it is enough to know the mass function only in a discrete set of points
 - ▶ We can use the same points of the quadrature nodes

$$p \rightarrow p_j = k_j$$

Numerical Techniques

- ▶ Instead of solving the equation over an entire domain of p , we decide that it is enough to know the mass function only in a discrete set of points
 - ▶ We can use the same points of the quadrature nodes

$$p \rightarrow p_j = k_j$$

- ▶ We are then left with a system of nonlinear algebraic equations

$$M_1 = \sum_{j=1}^{N_{max}} w_j f(k_j, M_j; k_1, M_1)$$

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- ▶ We are then left with a system of nonlinear algebraic equations

$$M_1 = \sum_{j=1}^{N_{max}} w_j f(k_j, M_j; k_1, M_1)$$
$$M_2 = \sum_{j=1}^{N_{max}} w_j f(k_j, M_j; k_2, M_2)$$

- ▶ Instead of solving the equation over an entire domain of p , we decide that it is enough to know the mass function only in a discrete set of points
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$$M_2 = \sum_{j=1}^{N_{max}} w_j f(k_j, M_j; k_2, M_2)$$

$$M_k = \sum_{j=1}^{N_{max}} w_j f(k_j, M_j; k_k, M_k)$$

Numerical Techniques

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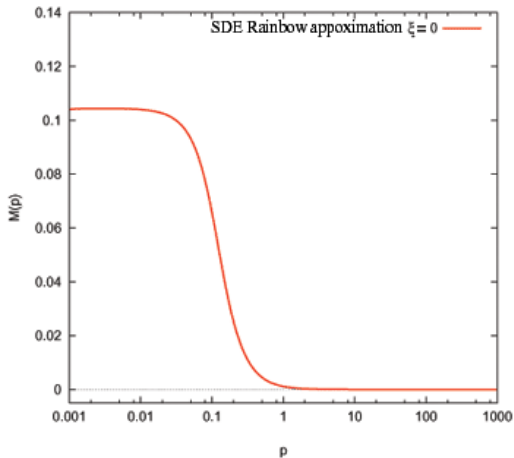
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Analytical Insight

- To have an analytical insight, let us go back to

$$M(p) = 2\alpha \int \frac{d^3k}{2\pi^2} \frac{M(k)}{k^2 + M^2(k)} \frac{1}{(k-p)^2}$$

Analytical Insight

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$$M(p) = 2\alpha \int \frac{d^3k}{2\pi^2} \frac{M(k)}{k^2 + M^2(k)} \frac{1}{(k-p)^2}$$

- Linearize this expression substituting $M^2(k) = m^2$

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$$M(p) = \frac{2\alpha}{2\pi^2} \int d^3k \frac{M(k)}{k^2 + m^2} \frac{1}{(k-p)^2}$$

- ▶ Next, define

$$M(p) = (p^2 + m^2)\chi(p), \quad \chi(r) = \int \frac{d^3k}{(2\pi)^3} \chi(k) e^{ikr}$$

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- ▶ Next, define

$$M(p) = (p^2 + m^2)\chi(p), \quad \chi(r) = \int \frac{d^3k}{(2\pi)^3} \chi(k) e^{ikr}$$

- ▶ It is straightforward to see that $\chi(r)$ verifies

$$\frac{d^2}{dr^2} \chi(r) + \frac{2}{r} \frac{d}{dr} \chi(r) + \left(m^2 - \frac{2\alpha}{r} \right) \chi(r) = 0$$

- ▶ A solution to this equation is

$$\chi(r) = Ce^{-mr}$$

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- ▶ The Fourier transform of $\chi(r)$ yields

$$M(p) = \frac{m^3}{p^2 + m^2}$$

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- ▶ Expectedly, $M(p \rightarrow 0) \sim m$ and $M(p \rightarrow \infty) \sim 1/p^2$.

Confinement in QED₃

- The potential between two static charges in QED₃ is

$$V(r) = \frac{e^3 \mathcal{G}(0)}{8\pi} \ln(e^2 r) + cte + \mathcal{O}\left(\frac{1}{r}\right)$$

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Confinement in QED₃

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- ▶ Quenched approximation $\mathcal{G}(0) = 1$
 - ▶ There is confinement

- ▶ The potential between two static charges in QED₃ is

$$V(r) = \frac{e^3 \mathcal{G}(0)}{8\pi} \ln(e^2 r) + cte + \mathcal{O}\left(\frac{1}{r}\right)$$

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- ▶ Including loops of massless fermions

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- ▶ Including loops of massive fermions, $\mathcal{G}(0)$ finite
 - ▶ Confinement is reinstated

Confinement in QED₃

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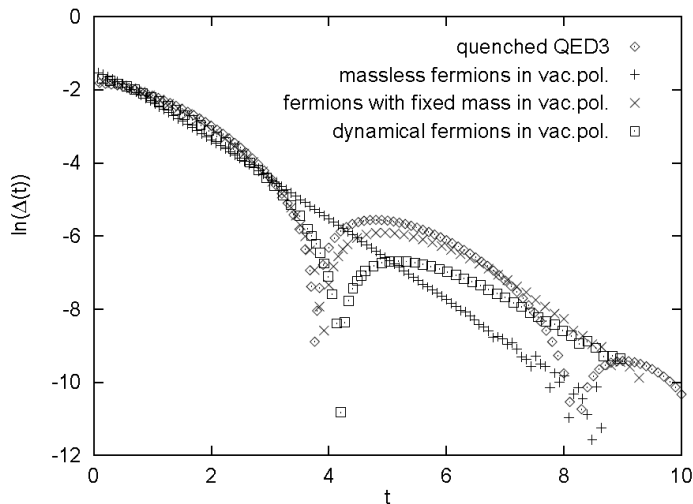
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Vacuum Polarization Effects

Let us consider vacuum polarization effects into the SDE for the fermion propagator

The diagram shows the Schwinger-Dyson equation for the fermion propagator. On the left, a horizontal green line with a teal circle in the middle is labeled with a superscript -1 . This is followed by an equals sign. To the right of the equals sign is another horizontal green line with a teal circle in the middle, also labeled with a superscript -1 . This is followed by a minus sign and a diagram representing a vacuum polarization insertion. The vacuum polarization insertion consists of a horizontal green line with a teal circle in the middle, with a red wavy line (photon) loop attached to the teal circle. A red circle is placed at the top of the loop.

Vacuum Polarization Effects

Let us consider vacuum polarization effects into the SDE for the fermion propagator

The diagram shows the Schwinger-Dyson equation for the fermion propagator. On the left, a horizontal green line with a teal circle in the middle, labeled with -1 above it. This is equal to the sum of two terms. The first term is a horizontal green line labeled with -1 above it. The second term is a horizontal green line with a teal circle in the middle, with a red wavy line loop (representing a fermion loop) attached to the top of the teal circle.

Consider N_f massless fermion families

The diagram shows the vacuum polarization of a gluon. On the left, a wavy line with a red circle in the middle. This is equal to the sum of two terms. The first term is a wavy line with a red circle in the middle, with a fermion loop (brown circle with arrows) attached to the top. The second term is a wavy line with a red circle in the middle, with two fermion loops (brown circles with arrows) attached to the top.

Vacuum Polarization Effects

Let us consider vacuum polarization effects into the SDE for the fermion propagator

$$\text{Fermion line with self-energy}^{-1} = \text{Bare fermion line}^{-1} + \text{Fermion line with self-energy and fermion loop}$$

Consider N_f massless fermion families

$$\text{Photon line with vacuum polarization} = \text{Bare photon line} + \text{Photon line with vacuum polarization and fermion loop} + \dots$$

This amounts to

$$\frac{\mathcal{G}(q)}{q^2} = \frac{1}{q^2[1 + \Pi(q)]} \rightarrow \frac{1}{q^2 + \frac{e^2 N_f q}{8}}$$

Vacuum Polarization Effects

The resulting equation in this case is, setting $e^2 = 1$,

$$M(p) = \frac{1}{2\pi^2 p} \int_0^\infty dk \frac{kM(k)}{k^2 + M^2(k)} \ln \left[\frac{k + p + N_f/8}{|k - p| + N_f/8} \right]$$

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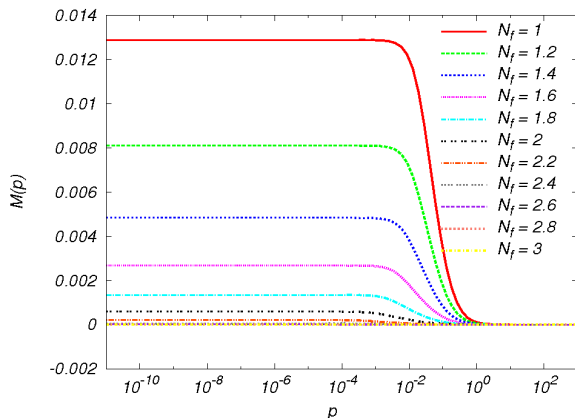
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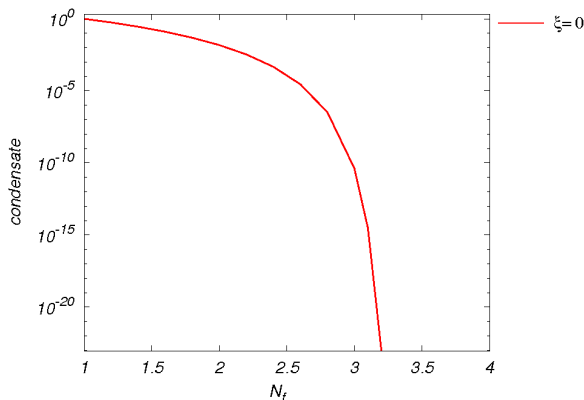
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- ▶ A more realistic situation would consider effective screening from fermion loops
- ▶ There will be a feed back between the amount of DGM and the screening

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- ▶ A more realistic situation would consider effective screening from fermion loops
- ▶ There will be a feed back between the amount of DGM and the screening
- ▶ Analyse the behavior of

$$e^2 \int d^3k \gamma_\mu S_F(k) \Gamma_\nu(k, p) \Delta_{\mu\nu}(k - p)$$

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► Ward identity

$$\text{► } (k - p)_\nu \Gamma_\nu = S_F^{-1}(k) - S_F^{-1}(p)$$

Vacuum Polarization Effects

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- ▶ $(k - p)_\nu \Gamma_\nu = S_F^{-1}(k) - S_F^{-1}(p)$

- ▶ Restricts $\Pi(q)$ to be gauge invariant

- ▶ We end up with

$$M(p) \sim \int dk \frac{F(k)M(k)}{k^2 + M^2(k)} \frac{(F(k), F(p))}{1 + \Pi(k - p)}$$

DMG and Confinement

- ▶ Assume that the effective screening leads to chiral symmetry restoration

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- ▶ There is an *infrared collusion*

DMG and Confinement

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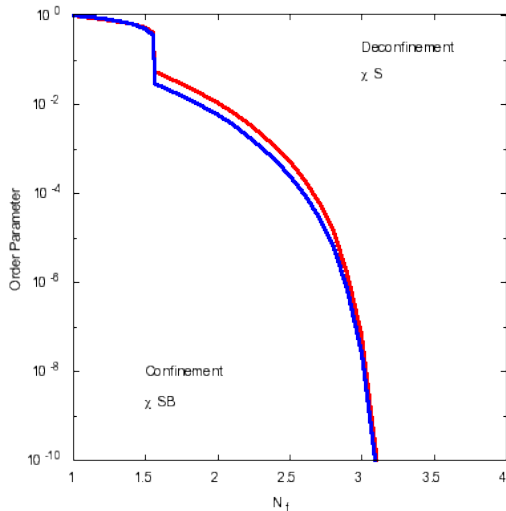
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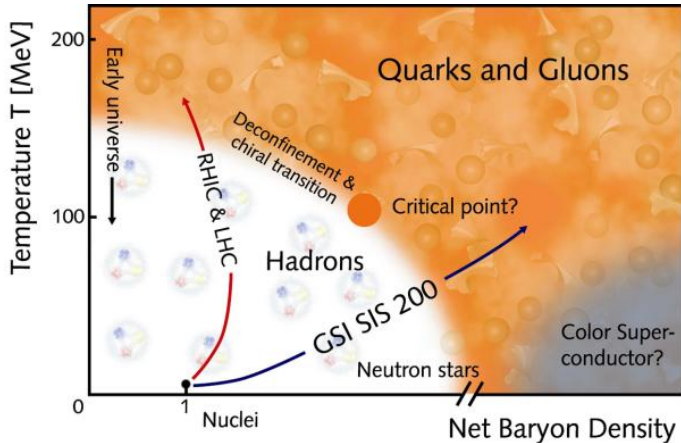
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Veranos Científicos

Sociedad Mexicana de Física

en Laboratorios Extranjeros 2009

Las Divisiones de Partículas y Campos y de Información Cuántica de la Sociedad Mexicana de Física (SMF) convocan a estudiantes de Física o áreas afines a concursar por una beca patrocinada por instituciones mexicanas conjuntamente en el área de física experimental de altas energías con **CERN, DESY, FERMI LAB** y **AUGER**

OBJETIVOS

Los estudiantes becados trabajarán en un grupo de investigación experimental durante 2 meses en el verano 2009.

Esta experiencia en laboratorios de alto nivel permitirá a los estudiantes continuar una carrera de investigación en Física Experimental de Altas Energías

VACANTES

FERMI LAB	1
DESY	2
CERN	1
AUGER	1

REQUISITOS

1. Carta solicitando participar. Indicar área de interés. Incluir datos personales, email y teléfono.
2. Tener más del 60% de créditos de la licenciatura o menos del 50% de la Maestría y promedio mínimo de 8.5.
3. Adjuntar copia de Kardex, Curriculum Vitae y una carta de recomendación.
4. Ensayo redactado en inglés (una cuartilla) sobre su interés en Física de Altas Energías.
5. Participar en el proceso de selección a realizarse en la Universidad de Colima antes del 8 de diciembre de 2008.

Toda la documentación deberá llegar antes del 24 de noviembre a:
Elena Cáceres - elencac@ucol.mx
<http://dpyc.smf.mx/Verano2009>

SMF

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Enjoy the Conference

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Many Thanks