Origin of Mass
Lect. 3: Approach

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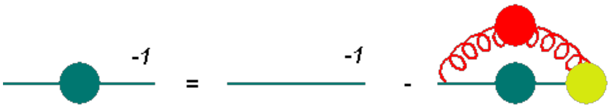
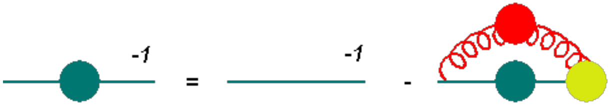


$$S_F^{-1}(p) = Z_2\, S_F^{(0)\ -1}(p) + g^2\, Z_{1F}\, C_F \int \frac{d^4k}{16\pi^4}\, \gamma_\mu\, S_F(k)\, \Gamma_\nu(k,p) \Delta_{\mu\nu}(k-p)$$

Dynamical Mass Generation in QCD



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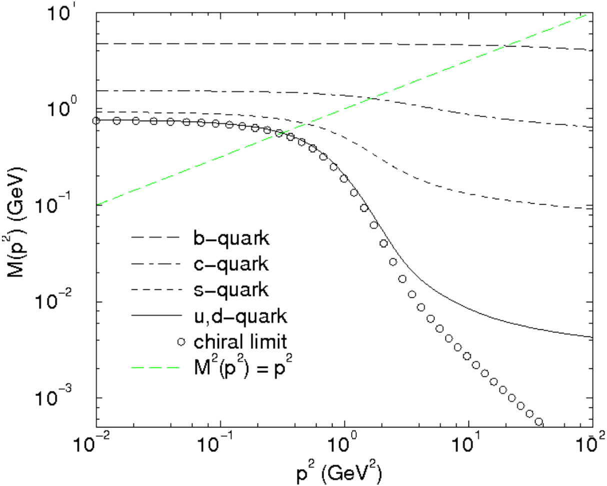
The solution is of the form

$$S_F(p) = \frac{F(p^2)}{\not{p} - M(p^2)}$$

Dynamical Mass Generation in QCD



Adapted from nucl-th/0007054.

Dynamical Mass Generation in QCD

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should have an enormous support.

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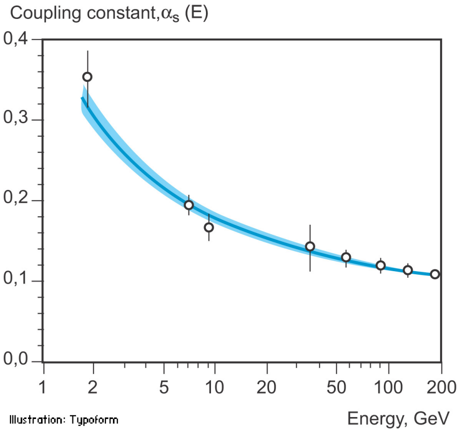
- Strength of interaction $g \longrightarrow g_{eff}(p)$



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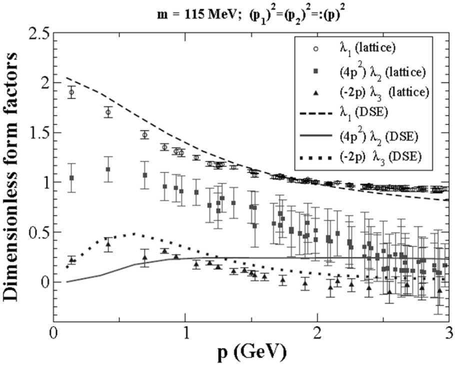
- The Quark-Gluon Vertex



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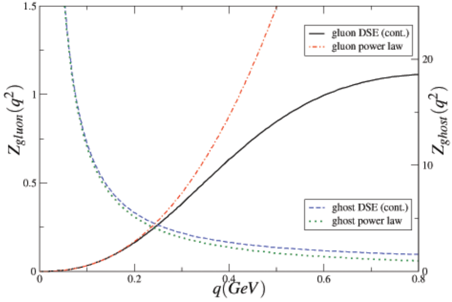
- The Ghost and Gluon Propagators



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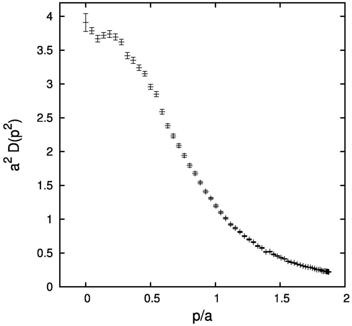
- The Gluon Propagator is IR finite!



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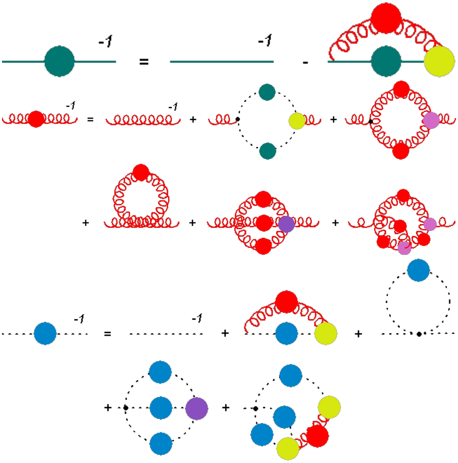
Confinement in QCD

Confinement can be studied through the IR properties of Green's functions



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Kugo-Ojima criterion:

- Ghost-Gluon vertex is IR finite

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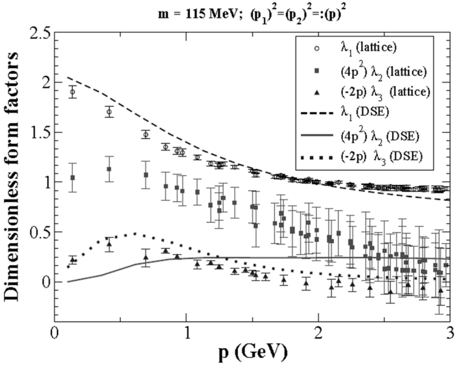

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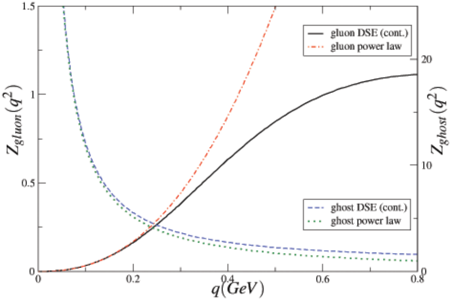

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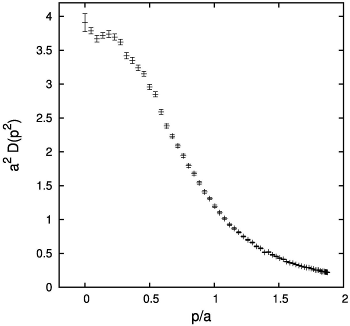

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Confinement in QCD

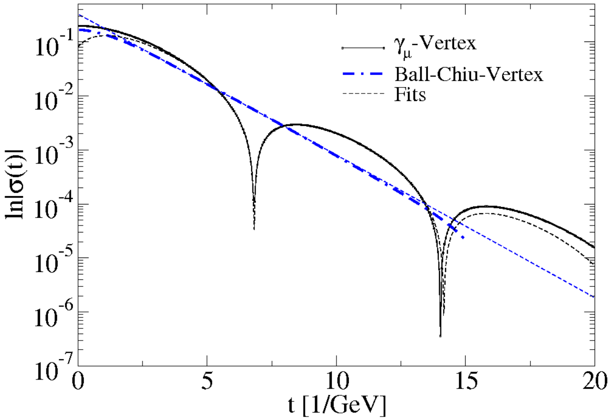
Axiom of Reflexion Positivity

$$\Delta(t) = \int d^3x \int \frac{d^4p}{(2\pi)^4} e^{i(tp_4+\vec{x}\cdot\vec{p})}\sigma(p^2)$$
$$= \frac{1}{\pi}\int_0^\infty dp_4 \cos(tp_4)\sigma(p_4^2) \geq 0\,,$$

with

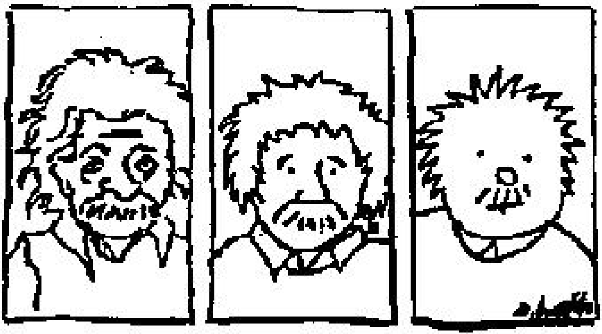
$$\sigma(p^2) = \frac{F(p^2)M(p^2)}{p^2+M^2(p^2)}.$$

Confinement in QCD



Adapted from J. Phys. G32, R253 (2006).

A Toy Model



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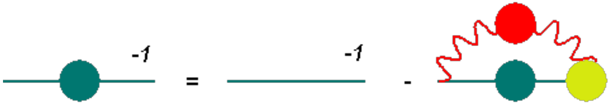
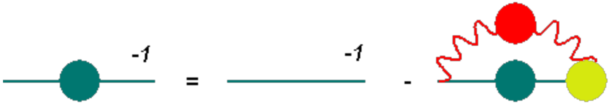
  - Quantum Hall Effect
  - Graphene

SDE in QED_3



SDE in QED$_3$



Corresponds to

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\begin{aligned}
S_F^{-1}(p) &= S_F^{(0)\ -1}(p) \\
&+e^2 \int \frac{d^3k}{(2\pi)^3}\, \gamma_\mu\, S_F(k)\, \Gamma_\nu(k,p) \Delta_{\mu\nu}(k-p)
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Rainbow Truncation

- We start by neglecting fermion loops, $G(q) = 1$

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$$\Delta^{(0)}_{\mu\nu}(q) \;=\; \frac{1}{q^2}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)$$

- With a suitable choice of the electron-photon vertex, the electron propagator can be found self-consistently

Rainbow Truncation

- Possible the simplest choice for the vertex is $\Gamma^{\nu}(k,p) = \gamma^{\nu}$

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- This corresponds to the diagram

Solving the SDE

- In Rainbow Approximation

$$S_F^{-1}(p) = S_F^{(0)\ -1}(p) + 4\pi\alpha \int \frac{d^3k}{(2\pi)^3}\,\gamma_\mu\, S_F(k)\,\gamma_\nu \Delta^{(0)}_{\mu\nu}(k-p)$$

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- Starting with massless fermions, $m_0 = 0$, multiplying by 1 and $\not{p}$ and taking trace and contracting with $\Delta^{(0)}_{\mu\nu}$

$$\frac{1}{F(p)} = 1 + \frac{\alpha}{2\pi^2 p^2} \int d^3k \frac{F(k)}{k^2 + M^2(k)} \frac{1}{(k-p)^4} \times \left[-2(k\cdot p)^2 + (2-\xi)(k^2+p^2)k\cdot p - 2(1-\xi)k^2p^2 \right]$$

$$\frac{M(p)}{F(p)} = \frac{\alpha(2+\xi)}{2\pi^2} \int d^3k \frac{F(k)M(k)}{k^2+M^2(k)} \frac{1}{(k-p)^2}$$

Solving the SDE

- Performing angular integrations

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\frac{1}{F(p)} &= 1 - \frac{\alpha\xi}{\pi^2 p}\int_0^\infty dk \frac{k^2 F(k)}{k^2+M^2(k)} \times \\
&\quad \left[1 - \frac{k^2+p^2}{2kp}\ln\left|\frac{k+p}{k-p}\right|\right] \\
\frac{M(p)}{F(p)} &= \frac{\alpha(\xi+2)}{\pi p}\int_0^\infty dk \frac{kF(k)M(k)}{k^2+M^2(k)}\ln\left|\frac{k+p}{k-p}\right|
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- In Landau gauge ($\xi = 0$)

$$M(p) = \frac{2\alpha}{\pi p}\int_0^\infty dk \frac{kM(k)}{k^2+M^2(k)}\ln\left|\frac{k+p}{k-p}\right|$$

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$$M(p) = \int_0^\infty dk \; f(k, M(k); p, M(p))$$

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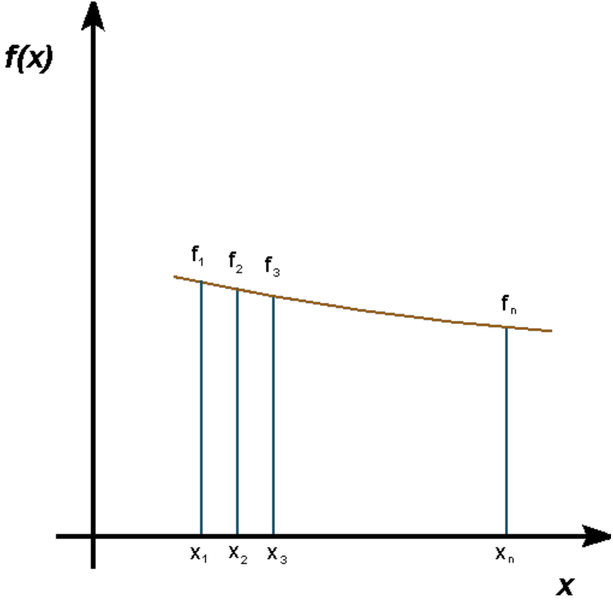
$$M(p) = \sum_{j=1}^{N_{max}} w_j f(k_j, M(k_j); p, M(p)) = \sum_{j=1}^{N_{max}} w_j f(k_j, M_j; p, M(p))$$

where w_j are the weights of the quadrature, k_j the nodes and $M_j = M(k_j)$

Numerical Techniques



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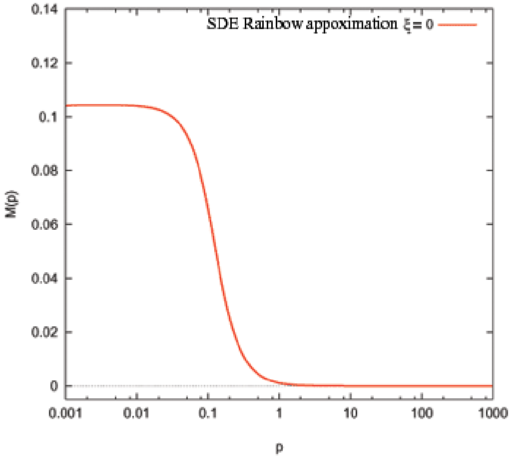

Analytical Insight

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$$M(p) \quad = \quad 2\alpha \int \frac{d^3k}{2\pi^2} \frac{M(k)}{k^2 + M^2(k)} \frac{1}{(k-p)^2}$$

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- Next, define

$$M(p) = (p^2 + m^2)\chi(p), \quad \chi(r) = \int \frac{d^3k}{(2\pi)^3} \chi(k) e^{ikr}$$

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$$M(p) = (p^2 + m^2)\chi(p), \quad \chi(r) = \int \frac{d^3k}{(2\pi)^3} \chi(k) e^{ikr}$$

- It is straightforward to see that $\chi(r)$ verifies

$$\frac{d^2}{dr^2}\chi(r) + \frac{2}{r}\frac{d}{dr}\chi(r) + \left(m^2 - \frac{2\alpha}{r}\right)\chi(r) = 0$$

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- Expectedly, $M(p \to 0) \sim m$ and $M(p \to \infty) \sim 1/p^2$.

Confinement in QED_3

- The potential between two static charges in QED_3 is

$$V(r) = \frac{e^3 \mathcal{G}(0)}{8\pi} \ln(e^2 r) + cte + \mathcal{O}\left(\frac{1}{r}\right)$$

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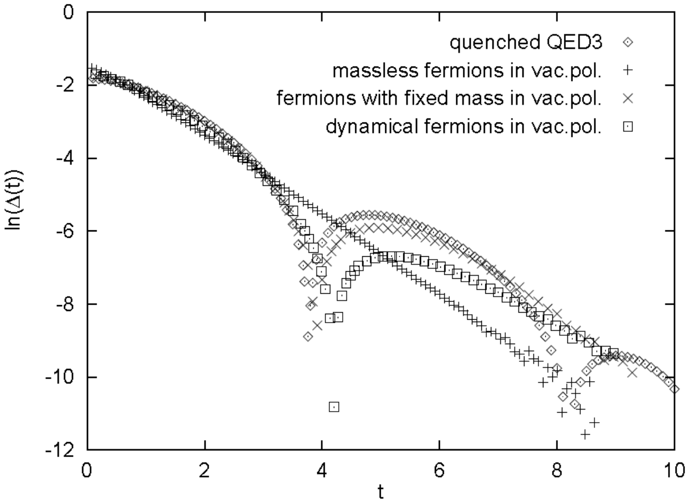
- Including loops of massive fermions, $\mathcal{G}(0)$ finite
  - Confinement is reinstated

Confinement in QED$_3$



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Vacuum Polarization Effects

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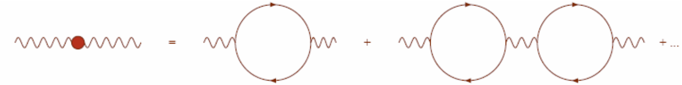
Consider N_f massless fermion families



Vacuum Polarization Effects

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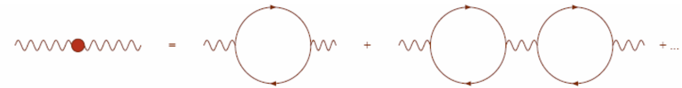

This amounts to

$$\frac{\mathcal{G}(q)}{q^2} = \frac{1}{q^2[1+\Pi(q)]} \to \frac{1}{q^2 + \frac{e^2 N_f q}{8}}$$

Vacuum Polarization Effects

The resulting equation in this case is, setting $e^2 = 1$,

$$M(p) = \frac{1}{2\pi^2 p}\int_0^\infty dk \frac{kM(k)}{k^2 + M^2(k)} \ln\left[\frac{k + p + N_f/8}{|k-p| + N_f/8}\right]$$

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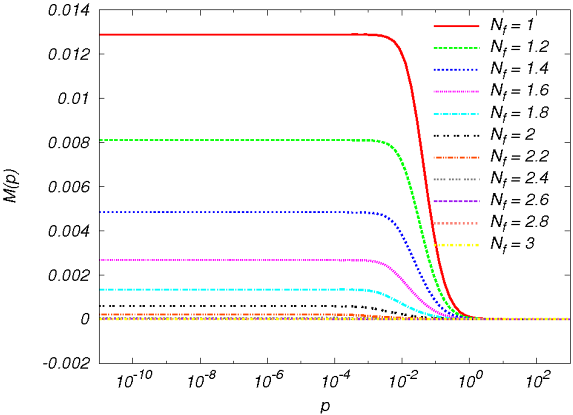

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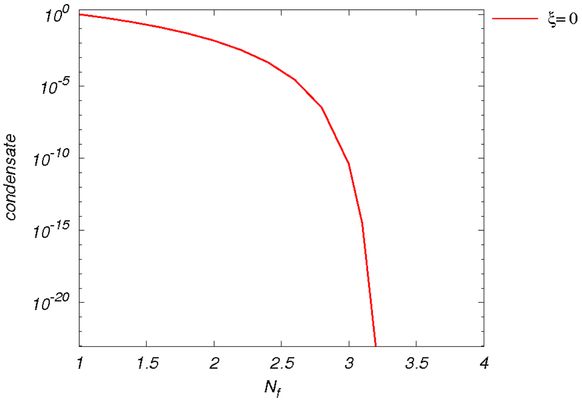

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- Analyse the behavior of

$$e^2 \int d^3k \, \gamma_\mu \, S_F(k) \, \Gamma_\nu(k,p) \Delta_{\mu\nu}(k-p)$$

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  - Restricts $\Pi(q)$ to be gauge invatiant
- We end up with

$$M(p) \sim \int dk \frac{F(k)M(k)}{k^2 + M^2(k)} \frac{(F(k), F(p))}{1 + \Pi(k-p)}$$

DMG and Confinement

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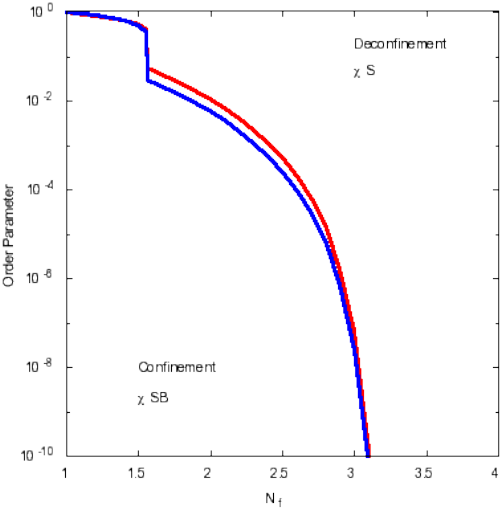

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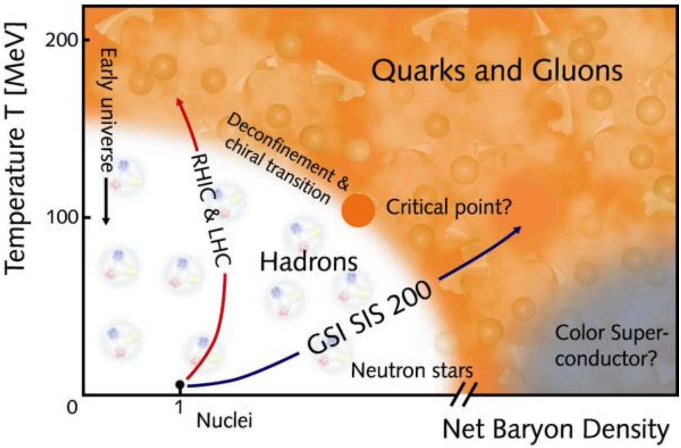

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Many Thanks