Origin of Mass
Lect. 2: Framework

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The dynamics of a particle moving in 1D can be derived from the action

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The *Principle of Least Action*,

$$\delta S = 0,$$

yields the equation of motion of the particle

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = 0 \; .$$

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A functional has the generic form

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The notion of derivatives can be generalized

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Functional derivative verifies all the properties of ordinary derivatives.

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$$S[q] = \int_{t_i}^{t_f} dt\ L(q, \dot{q})$$

The Euler-Lagrange equation is a functional extremum of the action:

$$\frac{\delta S[q]}{\delta q(t)} = -\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) + \frac{\partial L}{\partial q} = 0\ .$$

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- Fields ϕ and their derivatives $\partial^\mu \phi$ generalize q and $\dot{q}$

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$$S[\phi, J] = S[\phi] + \int d^4x \; J(x)\phi(x)$$

- Equations of motion follow from the extremum condition for the action

$$\frac{\delta S[\phi, J]}{\delta \phi(x)} = 0 \; .$$

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Quantum Fields

- Elementary particles are described by relativistic quantum fields
- Fields can be quantized in the canonical formalism or within the path integral approach
- The Euler-Lagrange equations are valid only as expectation value

$$\langle 0 \left| \frac{\delta S[\phi, J]}{\delta \phi(x)} \right| 0 \rangle^J.$$

Quantum Fields

- The full quantum equation of motion (Schwinger-Dyson equation) has the generic form

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- F depends on the details of the specific theory.

Gauge Field Theories

- Particle Physics is described in terms of quantum gauge field theories

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$$\mathcal{L} = \text{Matter fields} + \text{Gauge fields} + \text{Interactions.}$$

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- Interactions are derived from the gauge principle

Matter Fields

- Matter fields are Dirac fermions

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  - Global $\Rightarrow$ Charge conservation
  - Local $\Rightarrow$ Interactions with gauge fields

Gauge fields

- In Classical Electrodynamics

$$\mathcal{L}_{em} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \qquad F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu .$$

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- In the quantum theory, A_μ is the photon wavefunction

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$$\mathcal{L}_{int} = -e\bar{\psi}\gamma^\mu\psi A_\mu$$

Interactions

- In QCD,
  - $\psi \rightarrow \psi' = e^{i\frac{1}{2}\sum_j \lambda_j \alpha_j(x)}\psi$

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QCD Lagrangian

The gauge invariant gluon field-strength tensor is defined as

$$G^i_{\mu\nu} \equiv \partial_\mu G^i_\nu - \partial_\nu G^i_\mu - g \sum_{j,k} f_{ijk} G^j_\mu G^k_\nu$$

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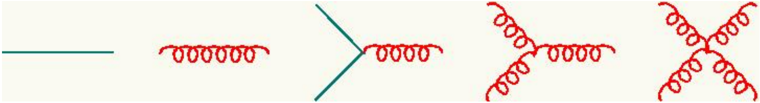

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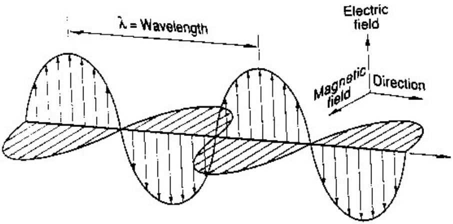
Photon and Gauge Freedom

- Electromagnetic waves are transverse



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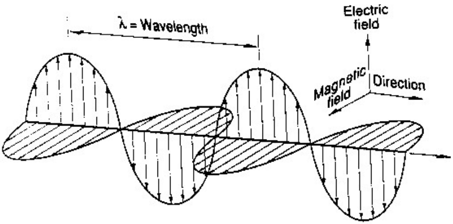

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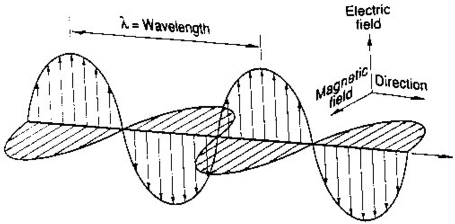

- Photons are transverse
- Yet, their wavefunction seems too big...

$$A^\mu = \begin{pmatrix} A^0 \\ A^1 \\ A^2 \\ A^3 \end{pmatrix} \begin{array}{l} \longrightarrow \text{Scalar} \\ \longrightarrow \text{Transverse} \\ \longrightarrow \text{Transverse} \\ \longrightarrow \text{Longitudinal} \end{array}$$

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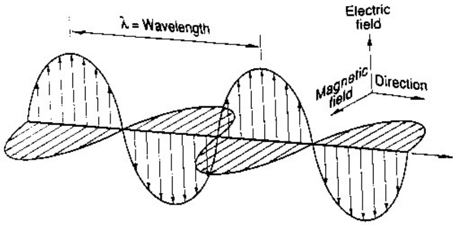

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- Arbitrary amounts of energy by creating scalar photons
- Scalar an longitudinal photons are unphysical
- Gupta-Bleuler: Lorentz condition on state vectors in Hilbert space
  - $A_\mu = A_\mu^{(+)} + A_\mu^{(-)}$

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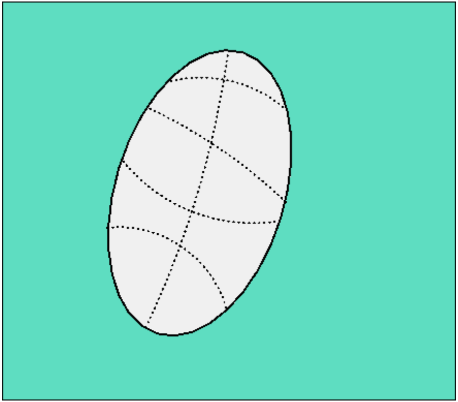
- Expectation values for the number of scalar and longitudinal photons are equal in admissible physical states
- All states having the same configuration of transverse photons are physically equivalent

Photon and Gauge Freedom

Photon Fock space



Photon and Gauge Freedom

- To obtain the propagator

$$\mathcal{L}_{em} = A_\nu \left[g^{\mu\nu}\Box - \partial^\mu \partial^\nu \right] A_\mu \equiv A_\nu D^{\nu\mu} A_\mu$$

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$$\mathcal{L}_{em} = A_\nu \left[g^{\mu\nu} \Box - \partial^\mu \partial^\nu \right] A_\mu + \frac{1}{2\xi} \left(\partial_\mu A^\mu \right)^2$$

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&\equiv A_\nu \Delta^{\nu\mu} A_\mu
\end{aligned}$$

Photon and Gauge Freedom

- The bare propagator is

$$\Delta^{(0)}_{\mu\nu}(q) = \frac{1}{q^2}\left(g_{\mu\nu} + (\xi - 1)\frac{q_\mu q_\nu}{q^2}\right)$$

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Gluons and Gauge Freedom

- A similar reasoning can be applied to gluons

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$$\mathcal{L}_{ghost} = \partial_\mu c_i^* (\partial^\mu \delta^{ij} + g f^{ijk} G^\mu_k) c_j .$$

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- Ghosts are anticommuting scalar fields.

Gluons and Gauge Freedom

- We need to include ghost propagators and ghost-gluon vertex

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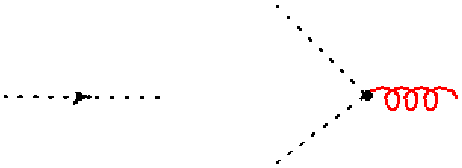

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  - $H^\dagger = H \quad \Rightarrow \quad \mathcal{L}^\dagger = \mathcal{L}$

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  - Quarks and gluons are colored, but we only see colorless hadrons
- We need to isolate the colored states from the physical state space $\mathcal{V}_{phys}$
  - $H^\dagger = H \quad \Rightarrow \quad \mathcal{L}^\dagger = \mathcal{L}$
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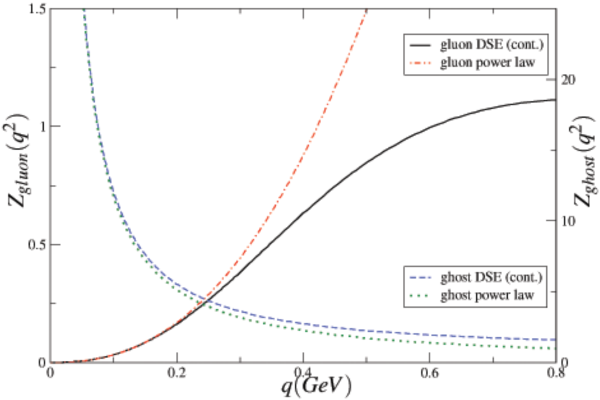
Aspects of Confinement



Adapted from Braz. J. Phys. 37 201 (2007).

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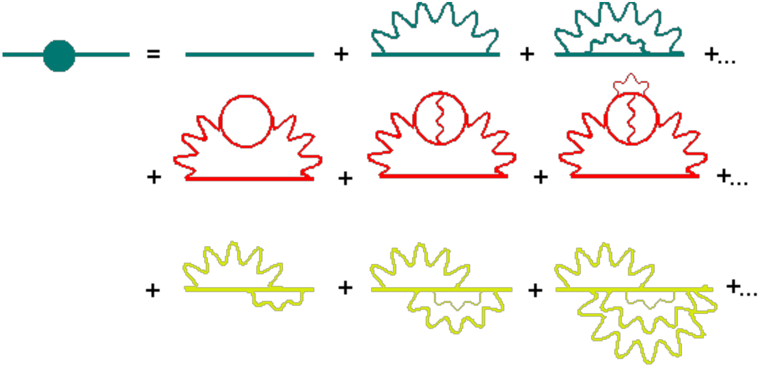
- These relations are non perturbative in nature
- They provide a natural platform to study DCSB and Confinement in gauge theories

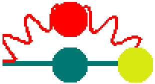
Electron Propagator in QED

In perturbation theory



Electron Propagator in QED

Define the self-energy $\Sigma(p)$



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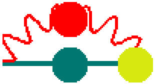

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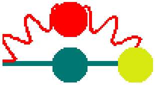

In terms of $\Sigma(p)$

This corresponds to the expression

$$
\begin{aligned}
S_F(p) &= S_F^{(0)}(p) + S_F^{(0)}(p)\Sigma(p)S_F^{(0)}(p) \\
&\quad + S_F^{(0)}(p)\Sigma(p)S_F^{(0)}(p)\Sigma(p)S_F^{(0)}(p) + \ldots
\end{aligned}
$$

Electron Propagator in QED

This is a geometric series, which can be written as

$$S_F(p) = \frac{S_F^{(0)}(p)}{1 - \Sigma(p) S_F^{(0)}(p)}.$$

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Combining these two expressions

$$S_F(p) = S_F^{(0)}(p) + S_F^{(0)}(p)\Sigma(p)S_F(p)$$

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It is customary to re-write this expression for the inverse electron propagator

$$S_F^{-1}(p) = S_F^{(0)\ -1}(p) - \Sigma(p)$$

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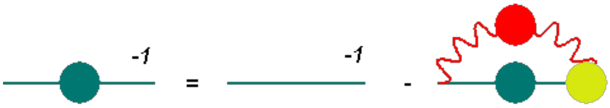
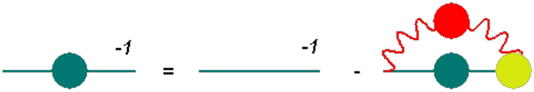
Diagrammatically, it corresponds to



SDE in QED

- Electron propagator



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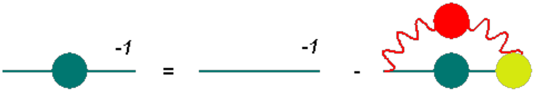
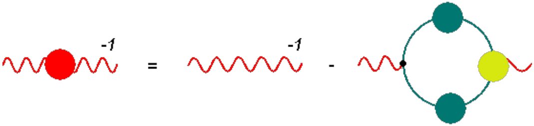


- Photon propagator



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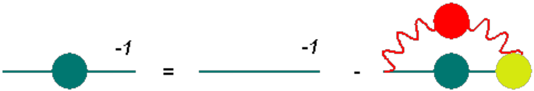

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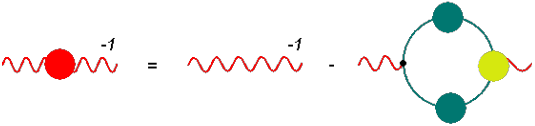
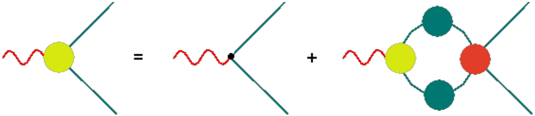


- Electron-photon vertex



SDE in QCD

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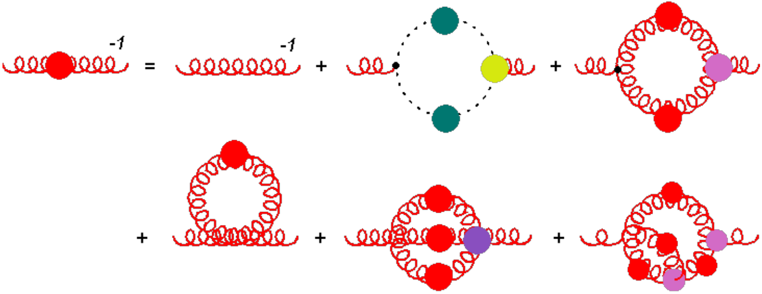

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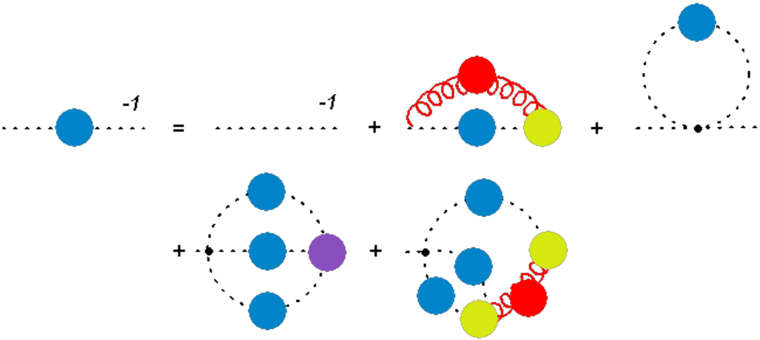
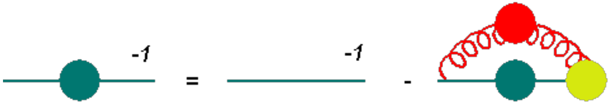


Gap equation in QCD



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\begin{aligned}
S_F^{-1}(p) &= Z_2\, S_F^{(0)\ -1}(p) \\
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\end{aligned}
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Gap equation in QCD

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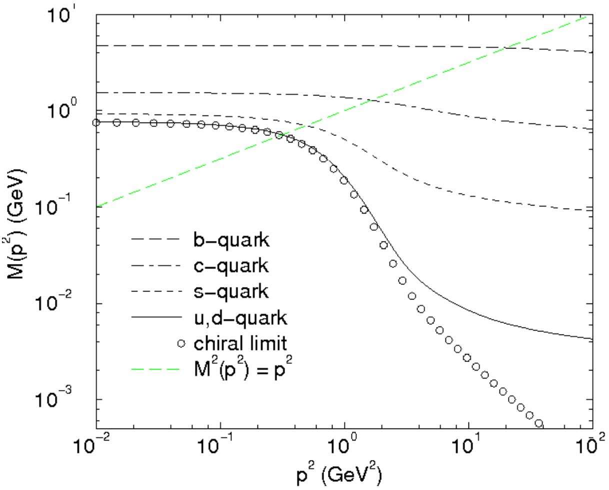
$$+g^2\, Z_{1F}\, C_F \int \frac{d^4k}{16\pi^4}\, \gamma_\mu\, S_F(k)\, \Gamma_\nu(k,p)\Delta_{\mu\nu}(k-p)$$

The most general form of the quark propagator is

$$S_F(p) = \frac{1}{\not p A(p^2) + B(p^2)} = \frac{F(p^2)}{\not p - M(p^2)}$$

Dynamical Mass Generation



Adapted from nucl-th/0007054.

Confinement and the Axiom of Reflexion Positivity

- For the quark propagator, the Axiom implies that

$$\begin{aligned}\Delta(t) &= \int d^3x \int \frac{d^4p}{(2\pi)^4} e^{i(tp_4+\vec{x}\cdot\vec{p})}\sigma(p^2)\\ &= \frac{1}{\pi}\int_0^\infty dp_4 \cos(tp_4)\sigma(p_4^2) \;\geq 0\,,\end{aligned}$$

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with

$$\sigma(p^2) = \frac{F(p^2)M(p^2)}{p^2+M^2(p^2)}.$$

Confinement and Axiom of Reflexion Positivity

- If there exists an asymptotic stable state associated to this propagator,

$$\Delta(t) \sim e^{-mt} \Rightarrow -\lim_{t\to\infty} \frac{d}{dt} \ln |\Delta(t)| = m$$

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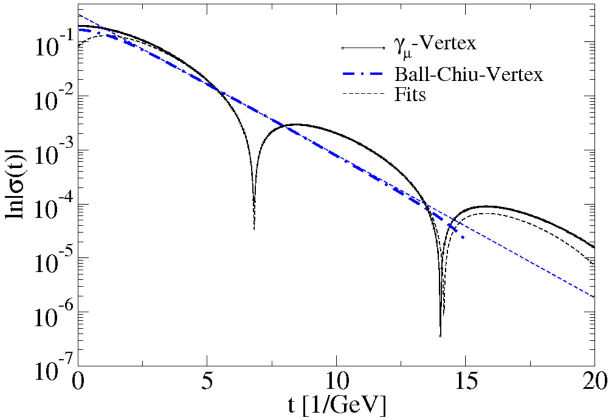
- For complex mass-like singularities $\mu = a \pm ib$

$$\Delta(t) \sim e^{-at}\cos(bt + \delta)$$

Confinement Test



Adapted from J. Phys. G32, R253 (2006).

In Summary

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