Electroweak Theory Basics

Building the Standard Model

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- Intorduction. Weak Interactions
  - Beta decay
  - Fermi Theory
  - Universality: Pion decay
  - Muon decay
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- A Brief on Gauge Field Theories
  - Gauge Symmetry
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  - Sponaneous Breaking of Local Symmetries
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Outline

- Basics of the Electroweak Theory
  - The Standard Model for leptons
  - Charged currents
  - Neutral currents
  - Adding quarks
  - Yukawa couplings: Masses and Mixings
  - Parameter Counting
  - Model Building rules
- Beyond Standard Model?
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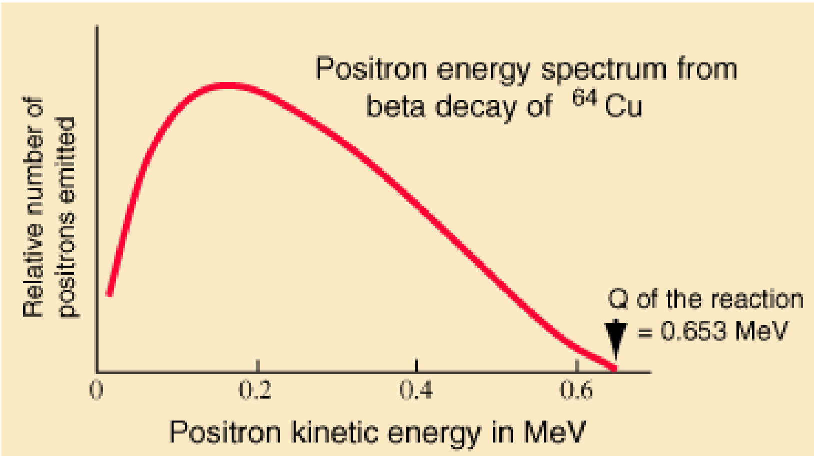
Chapter 1

Introduction.

Weak interactions

Beta Decay



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beta decay: a nucleus emits an electron increasing its charge:

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However: $E_{^{64}Cu} \neq E_{^{64}Zn} + E_e$

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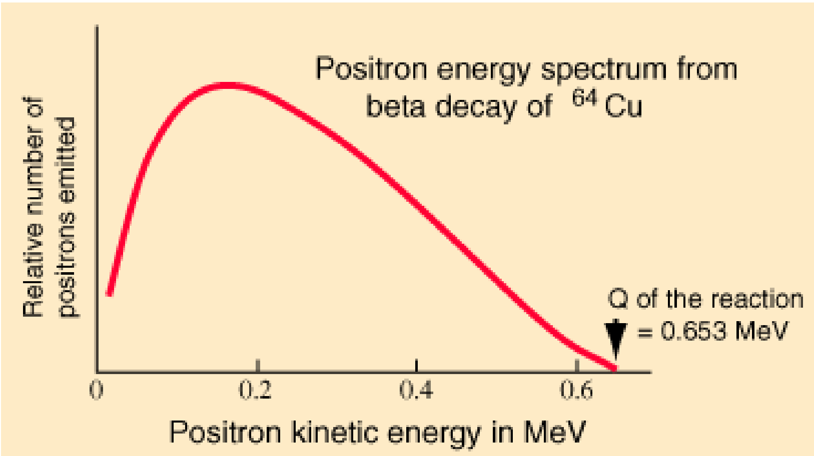

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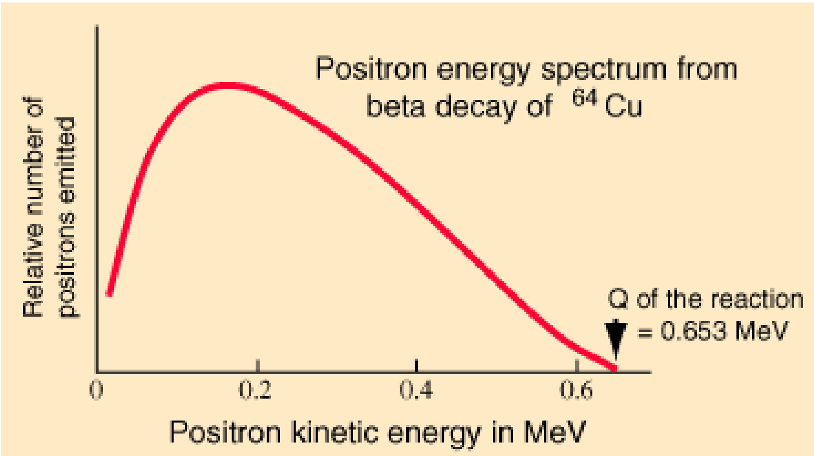

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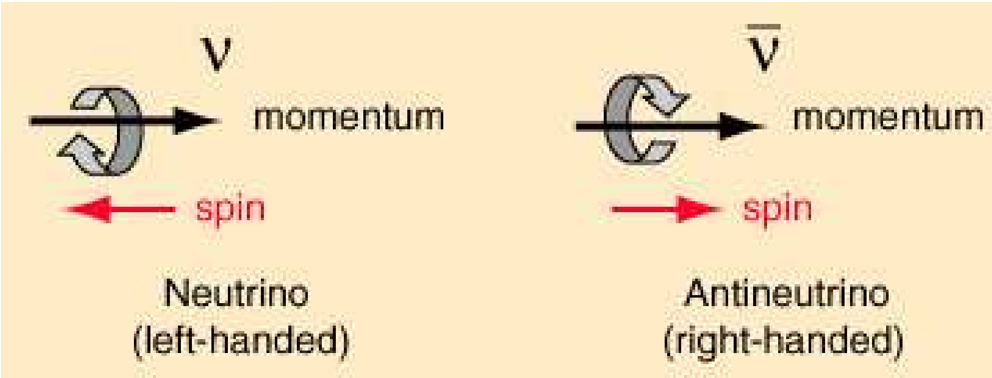
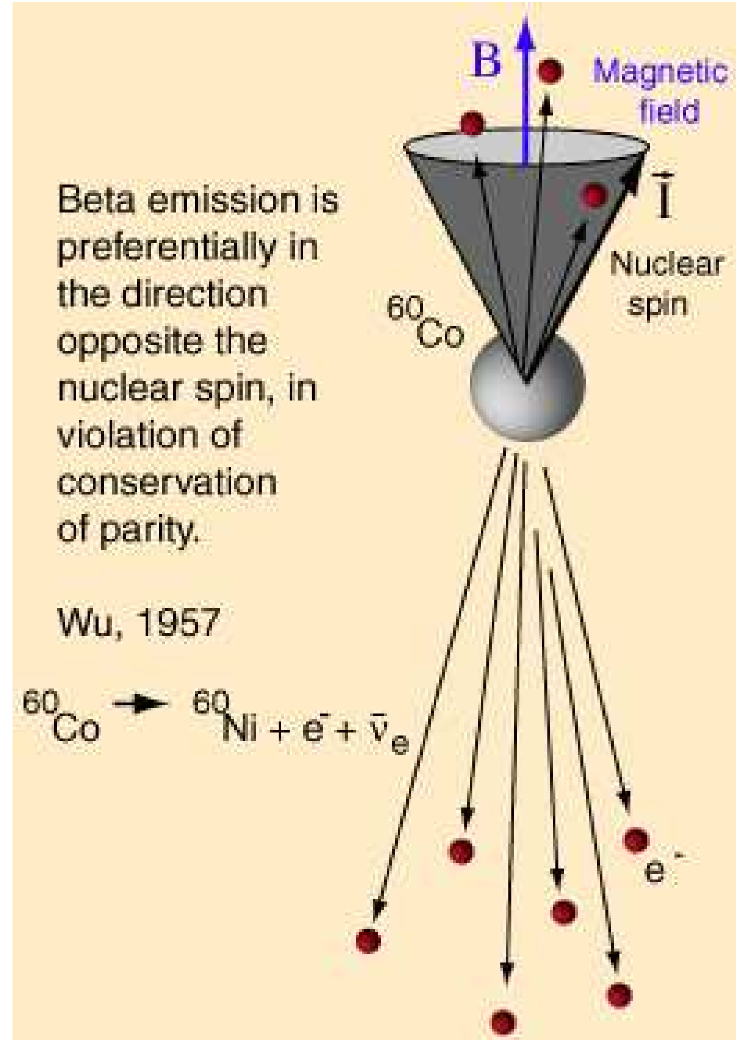
Pauli new particle was first detected in 1956 by Cowan y Reines comming from a nuclear reactor at Savannah River, South Carolina.

Beta Decay



Beta decay violates parity.

Only left components of both, the electron and the neutrino, are involved by the interaction that mediates beta decay.



Fermi Theory

With the discovery of the neutron it was suggested that beta decay was actually produced by the process: $n \rightarrow p + e^- + \bar{\nu}_e$

Nevertheless, the characteristic life times range from few minutes to years:
$\tau_n \approx 15\ min$ vs. $\tau_{\pi^0 \rightarrow \gamma\gamma} \approx 10^{-16}\ s.$

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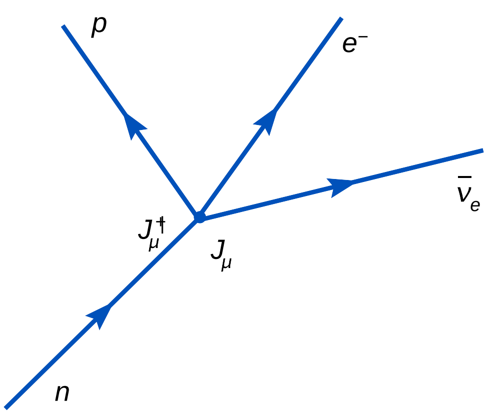

In 1934 Enrico Fermi presented an effective theory that describes beta decay based on the Hamiltonian (in natural units)

$$H = \frac{G_F}{\sqrt{2}} J^\dagger_\mu J^\mu$$

with the charged courrent $J^\dagger_\mu = \bar{p}\gamma_\mu n + \bar{\nu}_e \gamma_\mu e$

$G_F = 1.16637(1) \times 10^{-5}\ GeV^{-2}$

Parity violation requires the V-A current: $\gamma_\mu \rightarrow \gamma_\mu(1 - \gamma_5)$

Universality: Pion decay

$\pi^+ \rightarrow \mu^+ \nu_\mu$; $\pi^- \rightarrow \mu^- \bar{\nu}_\mu$; $\Gamma = 2.53 \times 10^{-14}\ MeV$.

Illustrate parity violation and Universality of weak interactions.

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Fermi theory is easily extended to this case:

$$j_\ell^\mu = \bar{e}\gamma^\mu(1-\gamma_5)\nu_e + \bar{\mu}\gamma^\mu(1-\gamma_5)\nu_\mu + \bar{\tau}\gamma^\mu(1-\gamma_5)\nu_\tau$$

$$H = \frac{\alpha_\pi}{2}\left[j_\ell^\mu \partial_\mu \Phi_\pi + j_\ell^{\mu\dagger}\partial_\mu \Phi_\pi^\dagger\right]$$

To the lower order: $\Gamma_{\pi\to\ell\nu_\ell} = \frac{\alpha_\pi^2}{4\pi}\,(1-v_\ell)\,p_\ell^2 E_\ell\ ;$

therefore: $\frac{\tau(\pi\to\mu\nu_\mu)}{\tau(\pi\to e\nu_e)} = \frac{m_e^2\left(m_\pi^2 - m_e^2\right)^2}{m_\mu^2\left(m_\pi^2 - m_\mu^2\right)^2} = 1.28\times 10^{-4}$

From observations: $\Gamma_{\pi\to e\nu_e} = 3.11\times 10^{-18}\ MeV;\ \Gamma_{\pi\to\mu\nu_\mu} = 2.53\times 10^{-14}\ MeV;$

thus $\alpha_\pi = 2.09\times 10^{-9}\ MeV^{-1}$;

and one gets $\Gamma_{\tau\to\pi\nu_\tau} = \frac{\alpha_\pi^2}{32\pi} m_\tau^3[1-(m_\pi/m_\tau)^2]^2 = 2.42\times 10^{-10} MeV$

vs. $(2.6\pm 0.1)\times 10^{-10} MeV$

Muon decay

The analysis of the decays $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$; and $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$; has played an important role in the understanding of weak interactipons.

From : $H = \frac{G_F}{\sqrt{2}} j^\dagger_{\ell\nu} J^\nu_\ell$ one gets $\frac{1}{\tau(\mu \rightarrow e \nu_e \nu_\mu)} \approx \frac{m_\mu^5 G_F^2}{192\pi^3}$

The measured life time $\tau_\mu = (2.19703 \pm 0.00004) \times 10^{-6}\ s$ provides one of the best stimates for G_F

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From the same theory one also gets good stimates for other processes, as $\tau \to e\bar{\nu}_e\nu_\tau$ and $\mu \to e\bar{\nu}_e\nu_\mu$, with the ratio $\frac{\tau(\tau \to e\bar{\nu}_e\nu_\tau)}{\tau(\mu \to e\bar{\nu}_e\nu_\mu)} \approx (\frac{m_\mu}{m_\tau})^5$.

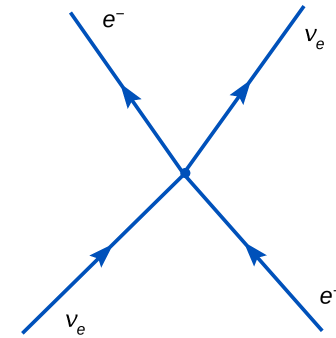
Mass ratio gives 7.43×10^{-7} ; vs. the observed 7.36×10^{-7}

Furthermore:

- $K^+ \to \mu^+\nu_\mu$; $\pi^0 e\nu_e$; ...
- Hyperones decay: $\Lambda \to p\pi^-$; $\Sigma^- \to n\pi^-$; $\Sigma^+ \to \Lambda e^+\nu_e -$;...
- Neutrino scattering: $\nu_\mu e \to \nu_\mu e$; $\nu_\mu n \to \mu p$; $\nu_\mu n \to \mu X$;...

From Fermi Theory to the Standard Model

Although successful, Fermi theory is incomplete. Consider $\nu_e e \to \nu_e e$



$H = \frac{G_F}{\sqrt{2}} j^\dagger_{e\mu} j^\mu_e$; indicates that $\sigma \sim \frac{G_F^2 s}{\pi}$; $s = E^2_{cm}$.

Unitarity requires that $\sigma < \frac{16\pi}{s}$.

$\Rightarrow$ Thus, at $\frac{1}{2} E_{cm} > \sqrt{\frac{\pi}{G_F}} \sim 500\ GeV$; σ violates unitarity

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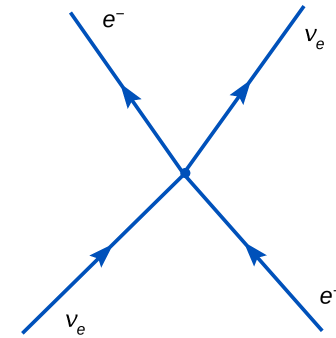

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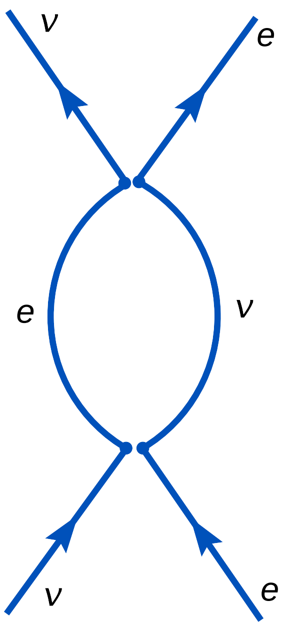
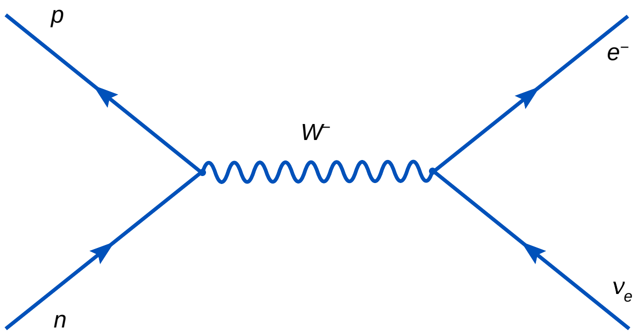


Usually, non unitarity of the amplitude in Born approximation is reestablished by high order corrections, however, Fermi theory involves divergent diagrams in the second order.

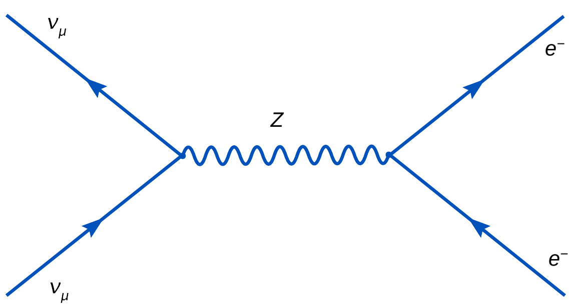
Problem: we deal with a non non renormalizable theory: G_F is dimensionful.

From Fermi Theory to the Standard Model



Yukawa (1935) suggested the concept of intermediary bosons. An idea retaken by Schwinger in 1957.

Assuming a massive boson, $W^{\pm}$, in the low energy regime, $m_W^2 \gg Q^2$; one identifies $\frac{G_F}{\sqrt{2}} \sim \frac{g^2}{8m_w^2}$. Moreover σ will be well behaved at high energies.



Nevertheless in $ee \to WW$; processes σ violates unitarity again unless a neutral current is included.

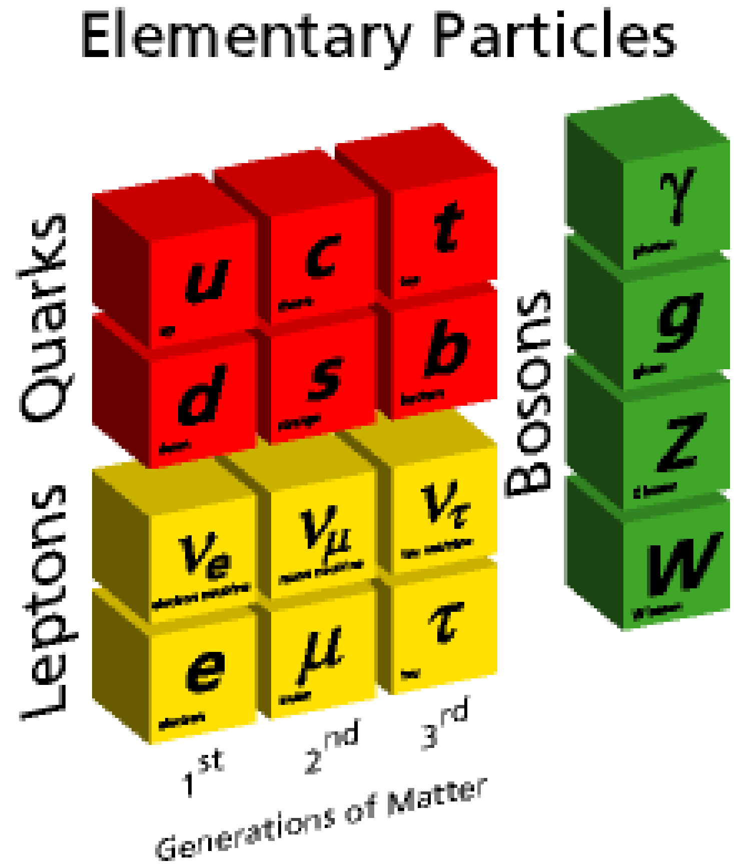
1961: Glashow developed the $SU(2) \times U(1)$ gauge model including QED.

Standard Model for leptons arose finally after adding the Higgs mechanism – Weinberg (1967) y Salam (1968)–

From Fermi Theory to the Standard Model

GIM mechanism allowed to extend the model to include quarks.

Currently we know there are three families of fundamental fermions.



’t Hoof & Veltman proved renormalizability (1971).

W and Z where first observed in CERN.

High precision physics has been provided since then by LEP (CERN) and SLC (SLAC) .

The Higgs remains elusive... waiting for LHC

Chapter 2

A Brief on Gauge Field Theories

Lagrangian Densities

Fundamental quantity for any QFT is the action

$$S = \int d^4x \, \mathcal{L}(x) \, ,$$

where the Lagrangian density $\mathcal{L}$ is a Poincaré invariant, local and real function of fields and their derivatives, with non explicit dependence on space coordinates.

- Complex scalar field: $\mathcal{L}_\phi = \partial^\mu \phi^*(x) \partial_\mu \phi(x) - m^2 \phi^* \phi - \frac{\lambda}{4}(\phi^* \phi)^2$
- EM field: $\mathcal{L}_{EM} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$; where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
- Fermion field: $\mathcal{L}_\psi = \bar{\psi} \left(i \gamma^\mu \partial_\mu - m \right) \psi$

Lagrangian Densities

Remainder on fermion theory:

ψ is solution to $(i\gamma^\mu \partial_\mu - m)\,\psi = 0$; $\bar{\psi} = \psi^\dagger \gamma^0$.

Dirac matrices obey Clifford algebra: $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$
Chiral representation:

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}$$

where $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ are the Pauli matrices.

Defining $\gamma_5 = i\gamma^0\gamma_1\gamma^2\gamma^3 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ such that $\{\gamma_\mu, \gamma_5\} = 0$,

one has $\psi = \psi_L + \psi_R \equiv \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$ where $\gamma_5 \psi_{R,L} = \pm\psi_{R,L}$

Notice that: $\bar{\psi}\psi = \bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L$ but $\bar{\psi}\gamma_\mu\psi = \bar{\psi}_L \gamma_\mu \psi_L + \bar{\psi}_R \gamma_\mu \psi_R$

(exercise)

Gauge Symmetry

Consider the interaction Lagrangian:

$$\mathcal{L}_I = -qA_\mu \bar{\psi}\gamma^\mu \psi \quad = -A_\mu j^\mu_{EM}$$

which describes the coupling of a fermion to an electromagnetic potential.

Derived from the minimal coupling rule: $p^\mu \to p^\mu - qA_\mu$, provides

$$\mathcal{L}_{QED} = \bar{\psi}\left[\gamma^\mu\left(i\partial_\mu - qA_\mu\right) - m\right]\psi - \frac{1}{4}F^2$$

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Then, realizing that, when promoted to be local, $\psi \to e^{-iq\alpha(x)}\psi$, then

$$\mathcal{L}_{QED} \to \bar{\psi}\left[\gamma^\mu\left(i\partial_\mu - q\left(A_\mu - \partial_\mu\alpha\right)\right) - m\right]\psi - \frac{1}{4}F^2$$

remains invariant provided one simultaneously performs the
gauge transformation

$$A_\mu \to A_\mu + \partial_\mu\alpha(x)$$

Gauge Symmetry

Lesson I: One could start with the globally invariant Lagrangian ($\mathcal{L}_\psi$) and "force" it to be locally invariant.

In order to accomplish this:

- Add a gauge field, A_μ, associated to the group symmetry - $U(1)$.
- Change ∂_μ by the covariant derivate $D_\mu = \partial_\mu + iqA_\mu$

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Then, consider: $\phi \to e^{-i\alpha(x)}\phi\,; \quad \Rightarrow \quad \partial_\mu \phi \to e^{-i\alpha}\left(\partial_\mu - iq\partial_\mu \alpha\right)\phi$
$\Rightarrow$ $\mathcal{L}_\phi$ is not any more invariant.

However, $D_\mu \phi \to D_\mu e^{-i\alpha}\phi = e^{-i\alpha}\left[\partial_\mu + iq\left(A_\mu - \partial_\mu \alpha\right)\right] \to e^{-i\alpha} D_\mu \phi$
so provided that $A_\mu \to A_\mu + \partial_\mu \alpha$.

$$\mathcal{L} = (D^\mu \phi)^* (D_\mu \phi) - \tfrac{1}{4} F^2$$ Is Gauge invariant.

However, $A_\mu A^\mu$ mass term is not...

Non Abelian Gauge Symmetries

We may extend previous concepts to non Abelian Lie groups. Consider

$SU(N)$, whose generators, T_a: $[T_a, T_b] = if_{abc}T_c$; $TrT_aT_b = \frac{1}{2}\delta_{ab}$

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Given a set of N scalars, Φ, with the free Lagrangian: $\mathcal{L} = \partial^\mu \Phi^\dagger \cdot \partial_\mu \Phi$
globally invariant under: $\Phi \to U(x)\Phi \equiv e^{-ig\lambda_a T_a}\Phi$.

The locally invariant theory includes the general covariant derivative:

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Yang-Mills fields: $T_aA^{a\prime}_\mu = U^{-1}T_aA^a_\mu U + iU^{-1}\partial_\mu U$

Besides, we must add $\mathcal{L}_{YM} = -\frac{1}{2}Tr\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}$

where $\mathcal{F}_{\mu\nu} = T_aF^a_{\mu\nu}$ and $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - gf_{abc}A^b_\mu A^c_\nu$

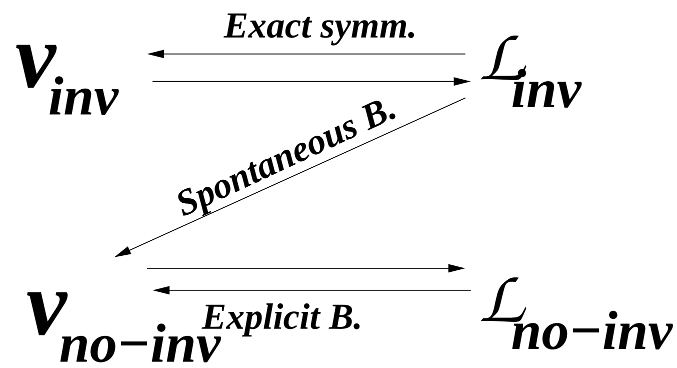
exercise: Verify gauge invariance of $(D^\mu\Phi)^\dagger(D_\mu\Phi)$

Spontaneous Symmetry Breaking

vacuum = Minimal energy state. It can be degenerated.

Coleman's Theorem: If the vacuum is invariant under a given symmetry group, G, so will be the Lagrangian

This describes an exact symmetry



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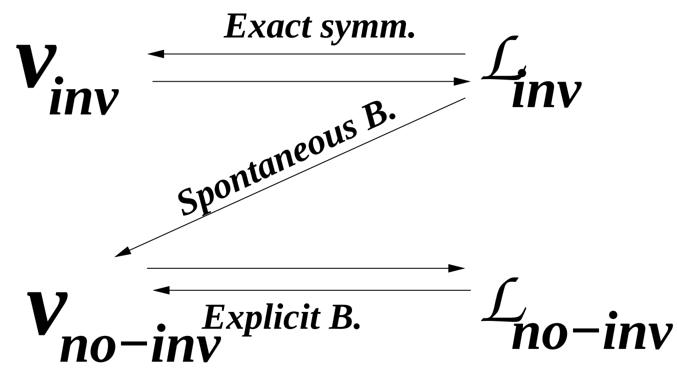

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If vacuum is not invariant, this does not determine what it should happen for the Lagrangian In any case, the symmetry would be broken as a whole.

- $\mathcal{L}$ non invariant indicates an explicitely broken symmetry
- When $\mathcal{L}$ remains invariant we have a Spontaneously Broken symmetry

There is a close connection among SSB and gauge boson masses.

Spontaneous Breaking of Global Symmetries

Consider the Lagrangian:

$$\mathcal{L}_\phi = \partial^\mu \phi^*(x)\partial_\mu \phi(x) - m^2 \phi^* \phi - \frac{\lambda}{4}(\phi^* \phi)^2 \qquad \lambda > 0$$

which is invariant under the global $U(1)$ transformations: $\phi \rightarrow e^{-i\alpha}\phi$

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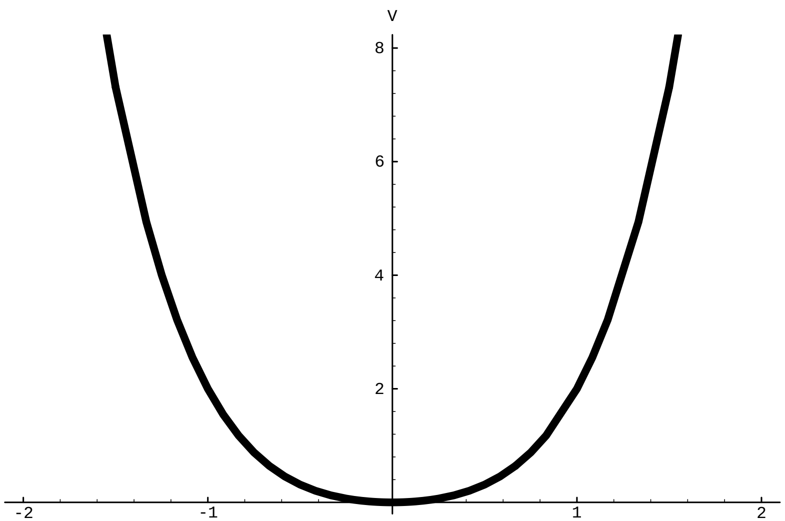
Vacuum corresponds to the field configuration which minimizes the potential (the vacuum expectation value). In this case $\langle \phi \rangle = 0$.

Vacuum is non degenerated and it is invariant $\to$ Symmetry is exact.



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In contrast, consider:

$$V(\phi) = -\mu^2\phi^*\phi + \frac{\lambda}{4}(\phi^*\phi)^2$$

The minimum now fulfills (exercise):

$$|\langle\phi\rangle|^2 = \frac{2\mu^2}{\lambda} \quad \equiv v$$

Degeneracy: $U(1)$ maps any given value into another with a different phase.

Symmetry is spontaneously broken.

Spontaneous Breaking of Global Symmetries

It is convenient to consider the redefinition of field variables over the classical vacuum: $\phi \to \langle\phi\rangle + \phi(x)$

Setting this into the potential we get (exercise):

$$V(\phi) = -\frac{\mu^4}{\lambda} + 2\mu^2\,(Re\,\phi)^2 + \sqrt{\frac{\lambda}{2}}\mu\,Re\,\phi|\phi^2 + \frac{\lambda}{4}|\phi|^4$$

Thus, the theory describes:

- A massive scalar: $\phi_1 = \sqrt{2}\,Re\,\phi$ with mass $\sqrt{2}\mu$.
- A massless scalar: $\phi_1 = \sqrt{2}\,Im\,\phi$ the Goldstone boson.

Spontaneous Breaking of Local Symmetries

Consider instead: $\mathcal{L} = (D^\mu \phi)^* (D_\mu \phi) - V(\phi) - \frac{1}{4} F^2$

Under redefinition: $\phi \rightarrow \left(v + \frac{1}{\sqrt{2}} \varphi(x) \right) e^{-i\chi(x)/v}$

we get $V(\phi) = V(\varphi)$, and (exercise):

$$|D_\mu \phi|^2 \rightarrow \frac{1}{2} \left| \partial_\mu \varphi + iq\varphi \left(A_\mu - \frac{1}{qv} \partial_\mu \chi \right) + i\sqrt{2} qv \left(A_\mu - \frac{1}{qv} \partial_\mu \chi \right) \right|^2$$

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Under redefinition: $\phi \to \left(v + \frac{1}{\sqrt{2}} \varphi(x) \right) e^{-i\chi(x)/v}$

we get $V(\phi) = V(\varphi)$, and (exercise):

$$|D_\mu \phi|^2 \to \frac{1}{2} \left| \partial_\mu \varphi + iq\varphi \left(A_\mu - \frac{1}{qv} \partial_\mu \chi \right) + i\sqrt{2} qv \left(A_\mu - \frac{1}{qv} \partial_\mu \chi \right) \right|^2$$

Now, a gauge transformation allows to remove the Goldstone boson:

$B_\mu = A_\mu - \frac{1}{qv} \partial_\mu \chi$ and one gets the mass term: $q^2 v^2 B_\mu B^\mu$

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Lesson II: Under Spontaneous Breaking gauge fields acquire mass by absorbing the Goldstone bosons.

This is called The Higgs Mechanism (Anderson, Kibble, Guralnik, Hagen, Brout, and Englert)

Higgs Mechanism

Mass generation in non Abelian gauge theories follows a similar path

Consider again a given representation of scalar fields, with

$$D_\mu \Phi = (\partial_\mu + igT_a A^a_\mu)\Phi$$

It is easy to see that the sole contribution of vacuum, $\langle \Phi \rangle$, that comes from the kinetic term, $|D_\mu \Phi|^2$, generates the mass term

$$g^2 \left(T_a \langle \Phi \rangle\right)^\dagger \left(T_b \langle \Phi \rangle\right) A^a_\mu A^{b\mu}$$

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- In general, only some field combinations would get mass:
- All gauge fields associated to T_a, such that $T_a\langle\Phi\rangle = 0$, remain massless $\Rightarrow$ Those T_a's generate the residual symmetry (unbroken): $G' \subset G$.
- To get the massive ones we must diagonalize $m^2_{ab} = g^2 \left(T_a \langle\Phi\rangle\right)^\dagger \left(T_b \langle\Phi\rangle\right)$

Lets explore an interesting case...

Spontaneous Breaking of $SU(2) \times U(1)$

Consider the Lagrangian

$$\mathcal{L}_\Phi = \partial^\mu \Phi^\dagger(x) \partial_\mu \Phi(x) + \mu^2 \Phi^\dagger \Phi - \frac{\lambda}{4} (\Phi^\dagger \Phi)^2$$

where Φ the scalar doublet: $\Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$

The model is invariant under global $SU(2)$:

$$\Phi \to e^{-ig\alpha^a \tau_a} \Phi \qquad \text{with} \qquad \tau_a = \frac{1}{2}\sigma_a \qquad a = 1, 2, 3$$

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The local theory $\partial_\mu \Phi \to D_\mu \Phi = (\partial_\mu + igA^a_\mu \tau_a)\,\Phi$ si $\langle \Phi \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}$

Thus:
$$\begin{aligned} |D_\mu \langle \Phi \rangle|^2 &= g^2 \begin{pmatrix} 0 & v \end{pmatrix} \tau_a \tau_b \begin{pmatrix} 0 \\ v \end{pmatrix} A^a_\mu A^{b\mu} \\ &= \frac{1}{2} g^2 \begin{pmatrix} 0 & v \end{pmatrix} \{\tau_a, \tau_b\} \begin{pmatrix} 0 \\ v \end{pmatrix} A^a_\mu A^{b\mu} = \frac{1}{4} g^2 v^2 A^a_\mu A^{a\mu} \end{aligned}$$

All gauge fields acquire mass

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For the local theory we the then get: $D_\mu \Phi = \left(\partial_\mu + igA^a_\mu \tau_a + ig' \frac{1}{2} B_\mu\right) \Phi$

$$\begin{aligned} \Rightarrow |D_\mu \langle \Phi \rangle|^2 &= \begin{pmatrix} 0 & v \end{pmatrix} \left(gA^a_\mu \tau_a + \frac{1}{2} g' B_\mu\right) \left(gA^{b\mu} \tau_b + \frac{1}{2} g' B^\mu\right) \begin{pmatrix} 0 \\ v \end{pmatrix} \\ &= \frac{v^2}{4}\left[g^2 \left(A^1_\mu\right)^2 + g^2 \left(A^2_\mu\right)^2 + \left(gA^3_\mu - g' B_\mu\right)^2\right] \qquad \text{(exercise)} \end{aligned}$$

Spontaneous Breaking of $SU(2) \times U(1)$

$$\frac{v^2}{4}\left[g^2\left(A_\mu^1\right)^2 + g^2\left(A_\mu^2\right)^2 + \left(gA_\mu^3 - g'B_\mu\right)^2\right]$$

we then have:

- Three massive gauge bosons:

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\left(A_\mu^1 \mp iA_\mu^2\right) \qquad \text{with mass} \qquad m_W = \frac{gv}{\sqrt{2}}$$

$$Z_\mu = \frac{1}{\sqrt{g^2 + g'^2}}\left(gA_\mu^3 - g'B_\mu\right) \qquad \text{with mass} \qquad m_Z = \sqrt{g^2 + g'^2}\frac{v}{\sqrt{2}}$$

  (For weak interactions PDG (2008):
  $m_W = 80.398S \pm 0.025\, GeV$; $m_Z = 91.1876 \pm 0.0021\, GeV$).

- A massless boson (the photon)

$$A_\mu = \frac{1}{\sqrt{g^2 + g'^2}}\left(g'A_\mu^3 + gB_\mu\right)$$

Spontaneous Breaking of $SU(2) \times U(1)$

It is convenient to define the mixing:

$$\cos\theta_W = \frac{g}{\sqrt{g^2 + g'^2}} = \frac{m_W}{m_Z}\ ; \qquad \sin\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}$$

to write:

$$\begin{pmatrix} Z \\ A \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} A^3 \\ B \end{pmatrix}$$

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Finally, we write the covariant derivative in terms of massive fields:

exercise: From $D_\mu = \partial_\mu + igA^a_\mu T_a + ig'\frac{1}{2}YB_\mu$ and defining

$$Q = T_3 + \frac{1}{2}Y \quad \text{y} \quad e = \frac{gg'}{\sqrt{g^2+g'^2}} = g\sin\theta_W = g'\cos\theta_W$$

we get:

$$D_\mu = \partial_\mu + i\frac{g}{\sqrt{2}}\left(W^+_\mu T^+ + W^-_\mu T^-\right) + i\frac{g}{\cos\theta_W}Z_\mu\left(T_3 - \sin^2\theta_W Q\right) + ieA_\mu Q$$

where $T^\pm = T^1 \pm iT^2$.

Three parameters: $e\ ;\ \theta_W\ ;\ m_W$.

Chapter 3

Basics

of the

Electroweak Theory

The Standard Model for Leptons

Let us consider a single lepton flavor: e, ν_e.

$$\mathcal{L} = i\bar{e}_L \,\partial\!\!\!/ e_L + i\bar{\nu}_{eL} \,\partial\!\!\!/ \nu_{eL} + i\bar{e}_R \,\partial\!\!\!/ e_R$$

where by construction we assume no ν_R.

- e_L and ν_{eL} are naturally arranged in a $SU(2)$ doublet:

$$L = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \quad \Rightarrow \quad \mathcal{L} = i\bar{L} \,\partial\!\!\!/ L + i\bar{e}_R \,\partial\!\!\!/ e_R$$

e_R is a singlet, so, it should have not weak interactions, as required.

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- To fix the hypercharge, Y, we use the electric charge: $Q = T_3 + \frac{1}{2}Y$.

$$\Rightarrow \quad \begin{matrix} L(2, Y = -1) \\ e_R(1, Y = -2) \end{matrix}$$

The Standard Model for Leptons

- The covariant derivatives for $L(2,-1)$ and $e_R(1,-2)$:

$$\begin{aligned} D_\mu L &= \left(\partial_\mu + igA_\mu^a\tau_a - ig'\frac{1}{2}B_\mu\right)L \\ D_\mu e_R &= \left(\partial_\mu - ig'B_\mu\right)e_R \end{aligned}$$

thus, the local theory $\mathcal{L}_e = i\bar{L}\gamma^\mu D_\mu L + i\bar{e}_R\gamma^\mu D_\mu e_R$.

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- For SSB we "minimally" have to consider a scalar doublet Φ:

$$\langle\Phi\rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

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For the three families: L_e ; L_μ ; L_τ ; e_R ; μ_R ; τ_R ;

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$$\mathcal{L}_{YM} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

where $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g \epsilon^{abc} A^b_\mu A^c_\nu$; and $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$

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Finally: $$\mathcal{L}_Y = \sum_{\ell=e,\mu,\tau} \left[h_\ell \bar{L}_\ell \Phi \ell_R + h.c\right]$$

SSB: $\langle \Phi \rangle \ldots$

The Standard Model for Leptons

- Next, for the spontaneous breaking:

$$\Phi = \begin{pmatrix} 0 \\ v + \frac{h(x)}{\sqrt{2}} \end{pmatrix}$$

Massive Gauge Bosons:

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Remainder: $g A_\mu^a T_a + g' \frac{1}{2} Y B_\mu$ also goes as

$$\frac{g}{\sqrt{2}} \left(W_\mu^+ T^+ + W_\mu^- T^-\right) + \frac{g}{\cos\theta_W} Z_\mu \left(T_3 - \sin^2\theta_W Q\right) + e A_\mu Q$$

Charged Currents

$\frac{g}{\sqrt{2}}\left(W_\mu^+ T^+ + W_\mu^- T^-\right) \Rightarrow$ Only L_ℓ will coupled to $W^\pm$.

Interaction Lagrangian:

$$\mathcal{L}_C = -\frac{g}{\sqrt{2}} W_\mu^+ \bar{L}_\ell \gamma^\mu \left(\tau_1 + i\tau_2\right) L_\ell + h.c$$

$\tau_1 = \frac{1}{2}\sigma_i$

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$$\implies \quad \mathcal{L}_C = -\frac{g}{\sqrt{2}} W_\mu^+ \left(\bar{\nu}_{\ell L} \gamma^\mu \ell_L\right) + h.c \equiv -\frac{g}{2\sqrt{2}} W_\mu^+ j_c^\mu + h.c$$

where $\quad j_c^\mu = \bar{\nu}_e \gamma^\mu (1-\gamma_5) e + \bar{\nu}_\mu \gamma^\mu (1-\gamma_5)\mu + \bar{\nu}_\tau \gamma^\mu (1-\gamma_5)\tau$

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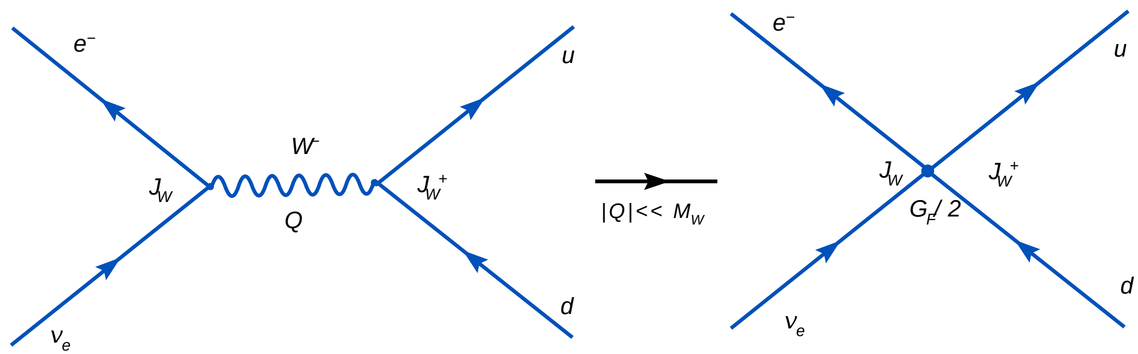

$$Q^2 \ll M_W^2 \quad \Rightarrow \qquad \mathcal{L}_W \approx M_W^2 W^-_\mu W^{+\mu} - \frac{g}{2\sqrt{2}} \left[W^+_\mu j^\mu_c + W^-_\mu j^{\mu\dagger}_c \right]$$

The equation of motion for W^- goes then as: $\quad M_W^2 W^-_\mu \approx \frac{g}{2\sqrt{2}} j^\mu_c$

$$\Longrightarrow \qquad \mathcal{L}_W \approx -\frac{g^2}{8M_W^2} j^{\mu\dagger}_c j_{c\,\mu} \qquad \Longrightarrow \qquad \frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{e^2}{8M_W^2 \sin^2\theta_W}$$

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Next, consider: $\frac{g}{\cos\theta_W} Z_\mu \left(T_3 - \sin^2\theta_W Q\right) + e A_\mu Q$

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Since $L_\ell(2,-1)$; $\ell_R(1,-2)$ y $Q = T_3 + \frac{1}{2}Y$:

$\Longrightarrow$ The electromagnetic interaction:

$$\mathcal{L}_{EM} = eA_\mu \left(\bar{\ell}_L \gamma^\mu \ell_L + \bar{\ell}_R \gamma^\mu \ell_R\right) = eA_\mu \, \bar{\ell}\gamma^\mu \ell$$

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$$\mathcal{L}_{EM} = eA_\mu J^\mu_{EM} \qquad \Rightarrow \quad J^\mu_{EM} = \bar{e}\gamma^\mu e + \bar{\mu}\gamma^\mu \mu + \bar{\tau}\gamma^\mu \tau$$

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Next, consider: $\frac{g}{\cos\theta_W} Z_\mu \left(T_3 - \sin^2\theta_W Q\right) + eA_\mu Q$

Z couplings

$$\mathcal{L}_Z = -\frac{g}{\cos\theta_W} Z_\mu \Big\{ \bar{L}_\ell \left[\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix} - \sin^2\theta_W \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \right] \gamma^\mu L_\ell$$

$$-\bar{\ell}_R \left(\sin^2\theta_W Q\right) \gamma^\mu \ell_R \Big\}$$

Neutral Currents

Next, consider: $\frac{g}{\cos\theta_W} Z_\mu \left(T_3 - \sin^2\theta_W Q\right) + eA_\mu Q$

Z couplings

$$\mathcal{L}_Z = -\frac{g}{\cos\theta_W} Z_\mu \left\{ \bar{L}_\ell \left[\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix} - \sin^2\theta_W \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \right] \gamma^\mu L_\ell \right.$$

$$\left. -\bar{\ell}_R \left(\sin^2\theta_W Q\right) \gamma^\mu \ell_R \right\}$$

$$\mathcal{L}_Z = -\frac{g}{\cos\theta_W} Z_\mu \left[\frac{1}{2} \bar{\nu}_{\ell\, L} \gamma^\mu \nu_{\ell\, L} + \left(-\frac{1}{2} + \sin^2\theta_W\right) \bar{\ell}_L \gamma^\mu \ell_L + \sin^2\theta_W \bar{\ell}_R \gamma^\mu \ell_R \right]$$

Neutral Currents

$$\mathcal{L}_Z = -\frac{g}{\cos\theta_W} Z_\mu \left[\frac{1}{2} \bar{\nu}_{\ell\,L} \gamma^\mu \nu_{\ell\,L} + \left(-\frac{1}{2} + \sin^2\theta_W \right) \bar{\ell}_L \gamma^\mu \ell_L + \sin^2\theta_W \bar{\ell}_R \gamma^\mu \ell_R \right]$$

Neutral currents: $\mathcal{L}_Z = -\frac{e}{\sin(2\theta_W)} Z_\mu \left(j^\mu_{n,\nu} + j^\mu_{n,\ell} \right)$ where

$$j^\mu_{n,\nu} = \tfrac{1}{2} \left[\bar{\nu}_e \gamma^\mu (1-\gamma_5)\nu_e + \bar{\nu}_\mu \gamma^\mu (1-\gamma_5)\nu_\mu + \bar{\nu}_\tau \gamma^\mu (1-\gamma_5)\nu_\tau \right]$$

$$j^\mu_{n,\ell} = \left(-\tfrac{1}{2} + \xi\right) \left[\bar{e}\gamma^\mu(1-\gamma_5)e + \bar{\mu}\gamma^\mu(1-\gamma_5)\mu + \bar{\tau}\gamma^\mu(1-\gamma_5)\tau \right]$$

$$+\xi \left[\bar{e}\gamma^\mu(1+\gamma_5)e + \bar{\mu}\gamma^\mu(1+\gamma_5)\mu + \bar{\tau}\gamma^\mu(1+\gamma_5)\tau \right]$$

$$\xi = \sin^2\theta_W$$

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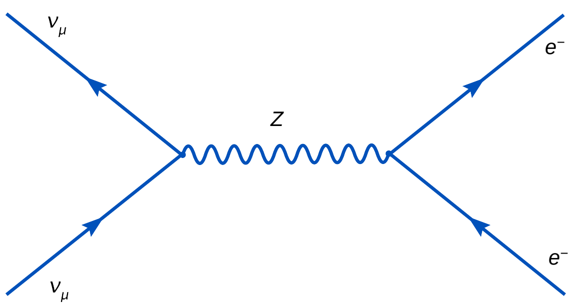
$$j^\mu_{n,\ell} = \left(-\tfrac{1}{2} + \xi\right) \left[\bar{e} \gamma^\mu (1-\gamma_5) e + \bar{\mu} \gamma^\mu (1-\gamma_5) \mu + \bar{\tau} \gamma^\mu (1-\gamma_5) \tau \right]$$

$$+\xi \left[\bar{e} \gamma^\mu (1+\gamma_5) e + \bar{\mu} \gamma^\mu (1+\gamma_5) \mu + \bar{\tau} \gamma^\mu (1+\gamma_5) \tau \right]$$

$$\xi = \sin^2 \theta_W$$



$\nu_\mu e \rightarrow \nu_\mu e$ $\quad \mathcal{L}_{eff} = -\frac{G_F}{\sqrt{2}} j_{\ell,\mu} j^\mu_\ell$

where $j^\mu_\ell = \bar{\nu}_\ell \gamma^\mu \frac{1}{2} (1-\gamma^5) \nu_\ell + \bar{\ell} \gamma^\mu (c_V - c_A \gamma_5) \ell$

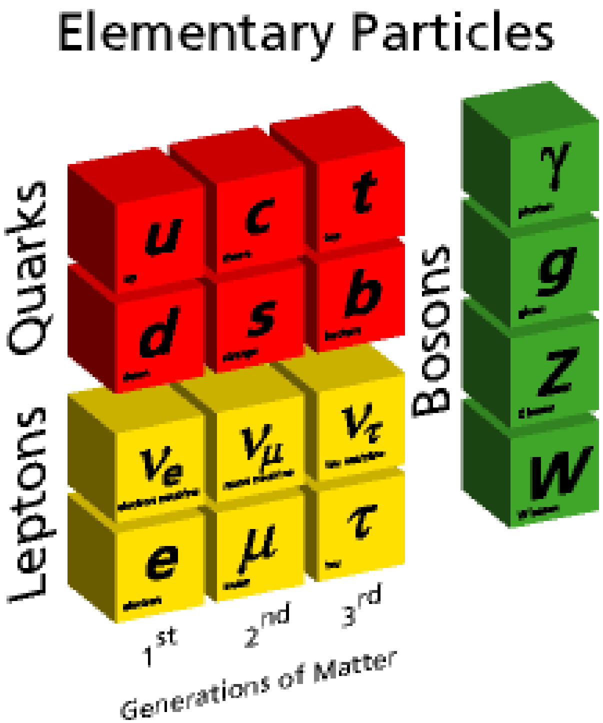
$$\sigma(\nu_\mu e \rightarrow \nu_\mu e) = \frac{G_F^2 s}{\pi} \left[\tfrac{4}{3} \sin^4 \theta_W - \sin^2 \theta_W + \tfrac{1}{4} \right]$$

$$\sin^2 \theta_W = 0.2324 \pm 0.0083$$

$$\sigma(\bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e) = \frac{G_F^2 s}{\pi} \left[\tfrac{4}{3} \sin^4 \theta_W - \tfrac{1}{3} \sin^2 \theta_W + \tfrac{1}{12} \right]$$

Adding Quarks to the Model

- Quarks have both weak and color interactions [unbroken QCD $SU(3)$].
- At low energy the model should describe beta decay: $n \rightarrow pe\nu$



- Hadron Model indicates:

$$p = (uud) \; ; \quad n = (udd)$$

- Electric charges: $u(2/3) \; ; \quad d(-1/3)$
- Three Colors: q_α ; $\alpha = 1, 2, 3$.
- Three Families: $(u, d) \; ; \; (c, s) \; ; \; (t, b)$.
- Electroweak Model includes:

$$Q\left(2, \frac{1}{3}\right)_{\alpha, iL} = \begin{pmatrix} u_{\alpha, iL} \\ d_{\alpha, iL} \end{pmatrix} \; ; \qquad u_{\alpha, iR}\left(1, \frac{4}{3}\right) \; ; \qquad d_{\alpha, iR}\left(1, -\frac{2}{3}\right)$$

Adding Quarks to the Model

$$Q\left(2,\frac{1}{3}\right)_{\alpha,iL} = \begin{pmatrix} u_{\alpha,iL} \\ d_{\alpha,iL} \end{pmatrix} \ ; \qquad u_{\alpha,iR}\left(1,\frac{4}{3}\right) \ ; \qquad d_{\alpha,iR}\left(1,-\frac{2}{3}\right)$$

Covariant Derivatives for weak interactions:

$$\begin{aligned} D_\mu Q_{iL} &= \left(\partial_\mu + igA_\mu^a\tau_a + ig'\frac{1}{6}B_\mu\right)Q_{iL} \\ D_\mu u_{iR} &= \left(\partial_\mu + ig'\frac{2}{3}B_\mu\right)u_{iR} \\ D_\mu d_{iR} &= \left(\partial_\mu - ig'\frac{1}{3}B_\mu\right)d_{iR} \end{aligned}$$

Adding Quarks to the Model

$$Q\left(2,\frac{1}{3}\right)_{\alpha,iL} = \left(\begin{array}{c} u_{\alpha,iL} \\ d_{\alpha,iL} \end{array}\right) \;;\qquad u_{\alpha,iR}\left(1,\frac{4}{3}\right) \;;\qquad d_{\alpha,iR}\left(1,-\frac{2}{3}\right)$$

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More convenient:

$$D_\mu = \partial_\mu + i\frac{g}{\sqrt{2}}\left(W^+_\mu T^+ + W^-_\mu T^-\right) + i\frac{g}{\cos\theta_W} Z_\mu \left(T_3 - \sin^2\theta_W Q\right) + ieA_\mu Q$$

Adding Quarks to the Model

More convenient:

$$D_\mu = \partial_\mu + i\frac{g}{\sqrt{2}}\left(W_\mu^+ T^+ + W_\mu^- T^-\right) + i\frac{g}{\cos\theta_W} Z_\mu \left(T_3 - \sin^2\theta_W Q\right) + ieA_\mu Q$$

After the algebra (exercise):

$$\mathcal{L}_{int} = -eJ_q^\mu A_\mu - \frac{g}{2\sqrt{2}}\left(W_\mu^- J_{c,q}^\mu + h.c\right) - \frac{e}{\sin(2\theta_W)} Z_\mu J_{n,q}^\mu$$

$$J_q^\mu = \frac{1}{3}\left(\bar{d}\gamma^\mu d + \bar{s}\gamma^\mu s + \bar{b}\gamma^\mu b\right) - \frac{2}{3}\left(\bar{u}\gamma^\mu u + \bar{c}\gamma^\mu c + \bar{t}\gamma^\mu t\right)$$

$$J_{c,q}^\mu = \bar{u}\gamma_L^\mu d + \bar{c}\gamma_L^\mu s + \bar{t}\gamma_L^\mu b \qquad \gamma_L^\mu \equiv \gamma^\mu(1-\gamma_5);\ \ \gamma_R^\mu \equiv \gamma^\mu(1+\gamma_5)$$

$$J_{n,q}^\mu = \left(\tfrac{1}{2} - \tfrac{2}{3}\xi\right)\left[\bar{u}\gamma_L^\mu u + \bar{c}\gamma_L^\mu c + \bar{t}\gamma_L^\mu t\right] - \tfrac{2}{3}\xi\left[\bar{u}\gamma_R^\mu u + \bar{c}\gamma_R^\mu c + \bar{t}\gamma_R^\mu t\right]$$

$$+\left(-\tfrac{1}{2} + \tfrac{1}{3}\xi\right)\left[\bar{d}\gamma_L^\mu d + \bar{s}\gamma_L^\mu s + \bar{b}\gamma_L^\mu b\right] + \tfrac{1}{3}\xi\left[\bar{d}\gamma_R^\mu d + \bar{s}\gamma_R^\mu s + \bar{b}\gamma_R^\mu b\right]$$

Yukawa Couplings: Masses and Mixings

$h_{\ell\ell'}\bar{L}_\ell \Phi \ell'_R$ can always be written such that: $h_{\ell\ell'} = h_\ell \delta_{\ell\ell'} \Rightarrow m_\ell = h_\ell \langle \Phi \rangle$

Yukawa Couplings: Masses and Mixings

That is not the case for the quark sector!!:

$$f_{ab}\bar{Q}_{aL}\Phi d_{bR} + h_{ab}\bar{Q}_{aL}\tilde{\Phi}u_{bR}$$

where $\tilde{\Phi}$ is the charged conjugated field, $\tilde{\Phi} = i\sigma_2\Phi^*$;

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$$\bar{d}_{aL}\,(M_d)_{ab}\,d_{aR} + \bar{u}_{aL}\,(M_u)_{ab}\,u_{aR}$$

where $(M_d)_{ab} \equiv v f_{ab}$; $\qquad (M_u)_{ab} \equiv v h_{ab}$

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To diagonalize: $M^2_{uL} = M_u \cdot M_u^\dagger \rightarrow \left(M^2_{uL}\right)_{diag} = U_L \cdot M^2_{uL} \cdot U_L^\dagger$

Similarly: $M^2_{dL} = M_d \cdot M_d^\dagger \rightarrow \left(M^2_{dL}\right)_{diag} = V_L \cdot M^2_{dL} \cdot V_L^\dagger$

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Mass eigenstates: $u_{\alpha,L} = \left(U_L\right)_{\alpha a} \cdot u_{aL}$; $d_{\alpha,L} = \left(V_L\right)_{\alpha a} \cdot d_{aL}$

$$W_\mu \bar{u}_{aL}\gamma^\mu d_{aL} = W_\mu \bar{u}_{\alpha L}\left(U_{CKM}\right)_{\alpha\beta}\gamma^\mu d_{\beta L} \qquad U_{CKM} = U_L V_L^\dagger$$

Yukawa Couplings: Masses and Mixings

$$U_{CKM} = \begin{pmatrix} U_{ud} & U_{us} & U_{ub} \\ U_{cd} & U_{cs} & U_{cb} \\ U_{td} & U_{ts} & U_{tb} \end{pmatrix} = \begin{pmatrix} 0.97419(22) & 0.2257(10) & 0.00359(16) \\ 0.2256(10) & 0.97334(23) & 0.0415(^{+10}_{-11}) \\ 0.00874(^{+26}_{-37}) & 0.0407(10) & 0.9990(^{+44}_{-43}) \end{pmatrix}$$

Standard Parameterization: Three angles and one phase:

$$U_{CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\varphi} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\varphi} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\varphi} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\varphi} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\varphi} & c_{23}c_{13} \end{pmatrix} ;$$

$$\theta_{12} = \theta_C \simeq 12.9^o \; ; \quad \theta_{23} \simeq 2.4^o \; ; \quad \theta_{13} \simeq 0.2^o \; ; \quad \varphi \simeq 59^o \pm 13$$

Wolfenstein Parameterization:

$$U_{CKM} = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} ;$$

$$\lambda = 0.2257 \; ; \quad A = 0.814 \; ; \quad \rho = 0.135 \; ; \quad \eta = 0.349$$

Counting Parameters

3: Coupling constants: $g\ ;\ g'\ ;\ g_s$ or equivalently $\alpha_s = \frac{g_s^2}{4\pi}$; α_{EM} ; $\sin^2\theta_W$.

1: Number of Families $=\ 3$.

9: Fermion masses (Yukawa couplings).

4: U_{CKM} parameters.

2: Parameters in the Higgs sector: $\mu^2\ ;\quad \lambda$; or equivalently $\quad m_H\ ;\quad M_W$.

Total: 19 free parameters

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Total: 19 free parameters

Additionally:

- Λ_{QCD}
- θ_{QCD}: QCD global anomaly $\quad \partial_\mu j_5^\mu = \frac{\theta}{32\pi^2} g_s^2 F \cdot \tilde{F}$

Final Lesson: Model Building

General Lessons for building models:

- Choose a symmetry. *vgr.* a gauge group G
- Choose proper representations that accommodate fermion fields
- Take an appropriated number of scalar multiplets in an adequate representations that provide the required SSB.
- Write down the locally invariant Lagrangian. Include all renormalizable terms that are permitted by the symmetry: $\mathcal{L} = \mathcal{L}_\ell + \mathcal{L}_\Phi + \mathcal{L}_{YM} + \mathcal{L}_Y$
- Determine the vacuum configuration that breaks the symmetry
- Insert vev and diagonalize mass matrices.
- Finally, rewrite the Lagrangian in terms of mass eigenstates.
- Put your model to the test.

Chapter 4

Beyond Standard Model?

Open Questions

- **The Gauge problem**: $G_{MS} = SU(3) \times SU(2) \times U(1)$ where $g_s \neq g \neq g'$ from phenomenological considerations.

  Partial Unification: $U(1)_Q \subset SU(2)_L \times U(1)_Y$: $\quad Q = T_L + \frac{1}{2}Y$

  Color remains as a separated sector: $SU(3)$

- **Charge Quantization**: Why hypercharges are as they are? $q = \pm 1, \frac{2}{3}, -\frac{1}{3}$

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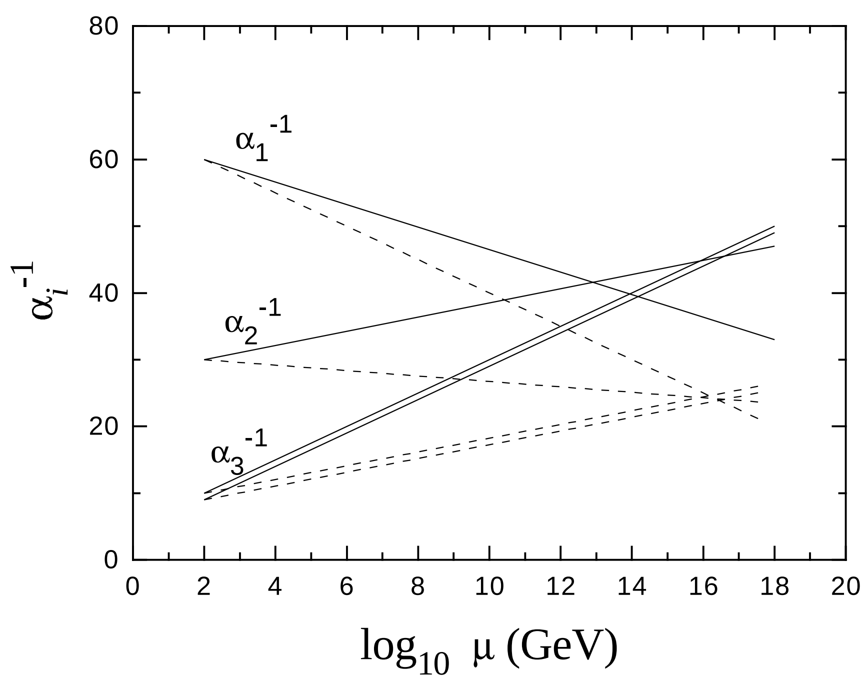


- RGE:

$$\frac{d\alpha_i}{d\ln\mu} = \frac{1}{2\pi} b_i \alpha_i^2 \quad \text{for} \quad \alpha_i = \frac{g_i^2}{4\pi}$$

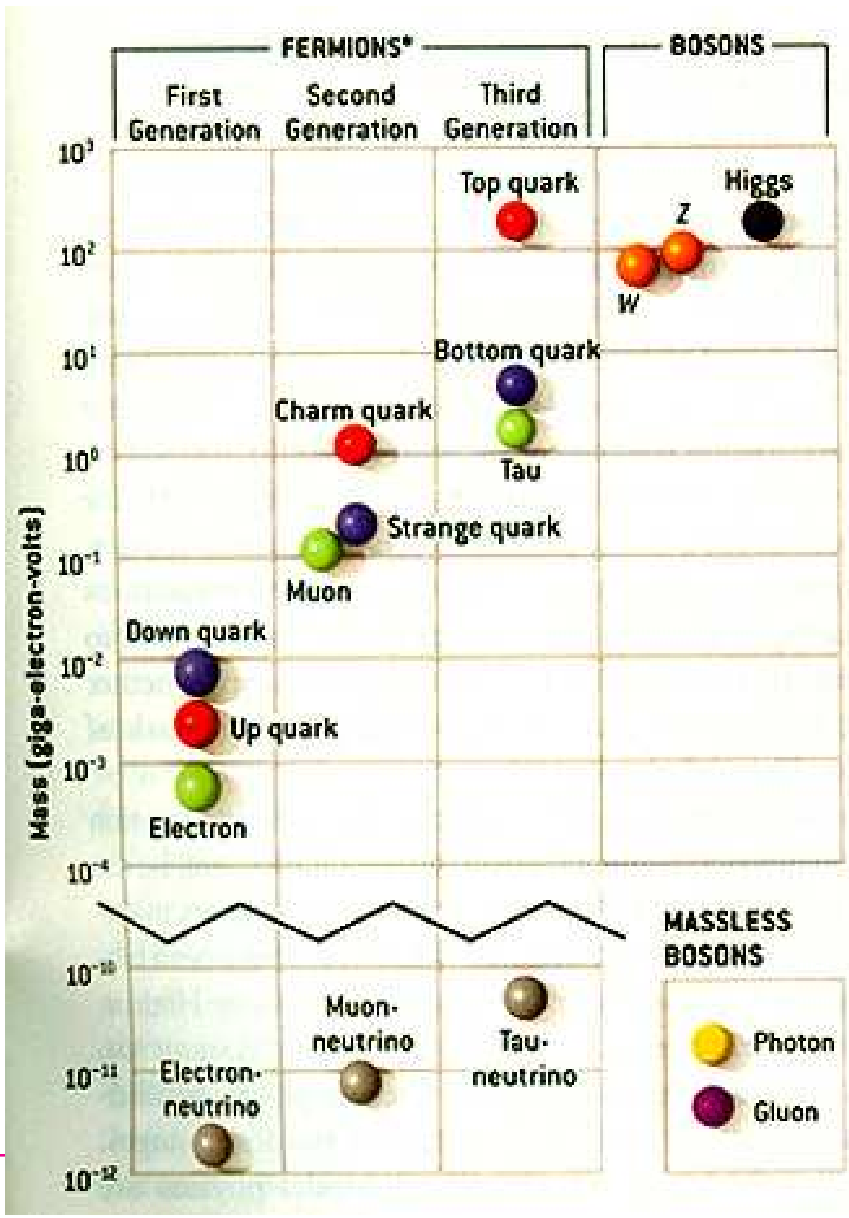
$M_{GUT} \approx 10^{16}$ GeV

Needs supersymmetry

A unique simple gauge group?... $SU(5)$, $SO(10)$, $E_6 \ldots$?

Open Questions

- **The Flavor Problem**



- There is no a priori reason for fermion mass spectrum.

$$m_t \gg m_q > m_\ell$$

In general $m_{down} > m_{up}$

but $m_u > m_d$.

Is there a Flavor symmetry?

- Neutrinos are massive !!

Is neutrino Dirac or Majorana?

$$\bar{L}\tilde{\Phi}N_R + \frac{1}{2}M_R\bar{N}_R^c N_R$$

See-saw $m_\nu \sim m_D^2/M_R$

Open Questions

- **The Flavor Problem**: Neutrino Oscillations

  Compelling evidence that neutrinos oscillate

  - Solar neutrinos (Clorine, Gallex, Kamiokande, SAGE, SuperKamiokande, SNO)

    $$\nu_e \longrightarrow \nu_\mu, \quad \nu_\tau$$

  - Atmospheric neutrinos (Kamiokande, MACRO, Soudan, SuperKamiokande)

    $$\nu_\mu \longrightarrow \nu_\tau$$

  - Accelerator and Power Plant neutrinos (KamLAND, K2K, CHOOZ, Palo-Verde, MINOS, MiniBoon,...), confirm evidence.

Open Questions

- **The Flavor Problem**: Neutrino Oscillations

See-saw: $M_{\alpha\beta}\bar{\nu}_\alpha \nu_\beta$; $M \approx -m_D^T M_R^{-1} m_D$ non diagonal.

$$\nu_\alpha = \sum_i U_{\alpha i}\nu_i \quad \text{for} \quad \alpha = e, \mu, \tau; i = 1, 2, 3$$

where $U = U_{PMNS} \cdot K$; $K = \mathrm{diag}\{1, e^{i\phi_1}, e^{i\phi_2}\}$

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\varphi} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\varphi} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\varphi} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\varphi} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\varphi} & c_{23}c_{13} \end{pmatrix} ;$$

Flavor oscillations

$$P_{\alpha\beta} = |\langle\nu_\alpha|\nu_\beta(L)\rangle|^2 = \delta_{\alpha\beta} - 4\sum_{a<b} U^*_{\alpha a}U^*_{\beta b}U^*_{\alpha b}U^*_{\beta a} \sin 2\frac{\Delta m^2_{ab}}{4E}L$$

Open Questions

- **The Flavor Problem**: Neutrino Oscillations

Flavor oscillations

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- Solar scale $\Delta m^2_\odot = \Delta m^2_{12} = 7.6\,^{+0.5}_{-0.3} \times 10^{-5}\ \mathrm{eV}^2$;
- ATM scale $\Delta m^2_{ATM} = |\Delta m_{23}|^2 \approx |\Delta m_{13}|^2 = 2.4 \pm 0.3 \times 10^{-3}\ \mathrm{eV}^2$,
- Mixings

$$\sin^2 \theta_\odot = \sin^2 \theta_{12} = 0.32\,^{+0.05}_{-0.04}$$

$$\sin^2 \theta_{ATM} = \sin^2 \theta_{23} = 0.5\,^{+0.13}_{-0.12}$$

$$\sin^2 \theta_{13} \leq 0.033$$

- Phases had not been measured yet

Open Questions

- **The Higgs Problem**
  - Higgs interactions: $\frac{1}{4}\lambda(\Phi^\dagger\Phi)^2 + \lambda\bar{\psi}_L\Phi\psi_R$
  - m_H^2 in unstable (Hierarchy Problem)

$$\rightarrow \delta m_H^2 \approx \Lambda^2(\lambda - f^2)$$

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What should be Λ?

- See-saw mass scale: $M_R \approx 10^{13}\ GeV$
- GUT scale: $M_{GUT} = 10^{16}$ GeV
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Open Questions

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wait for LHC...

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- **The Gravity Problem**: How to include Gravity?

Early Universe

Cosmological Model: SM + GR (FRW Model)

- **Matter Content**
  - Known Standard matter: $\approx 3\%$
  - Dark Matter: $\approx 27\%$ perhaps in LHC
  - Dark Energy: $\approx 70\%$

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Cosmological Model: SM + GR (FRW Model)

- **Matter Content**
  - Known Standard matter: $\approx 3\%$
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  - Dark Energy: $\approx 70\%$
- **Other Initial conditions for BBN**
  - Matter asymmetry: $\eta_B \approx 10^{-10}$

    $\longrightarrow$ C and CP violation; B or L violation
  - Flatness problem: Inflation

    $\longrightarrow$ Inflaton: φ ; $m_\varphi \lesssim 10^{14}$ GeV A new sector?

Concluding Remark

"There are too many points at which the conventional picture may be wrong or incomplete. The $SU(3)_c \times SU(2)_L \times U(1)_Y$ *gauge theory with three families is certainly a good beginning, not to accept but to attack, and exploit."*

Sheldon Lee Glashow, 1979

The Standard Model is perhaps just one more step on our way towards a better and deeper understanding of Nature