Hadronic light by light contribution to the μ anomalous magnetic moment

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Work done in collaboration with Khépani Raya & Adnan Bashir

QCD WG

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Jegerlehner & Nyffeler, Phys.Rept. 477 (2009) 1-110

Interest of a_μ

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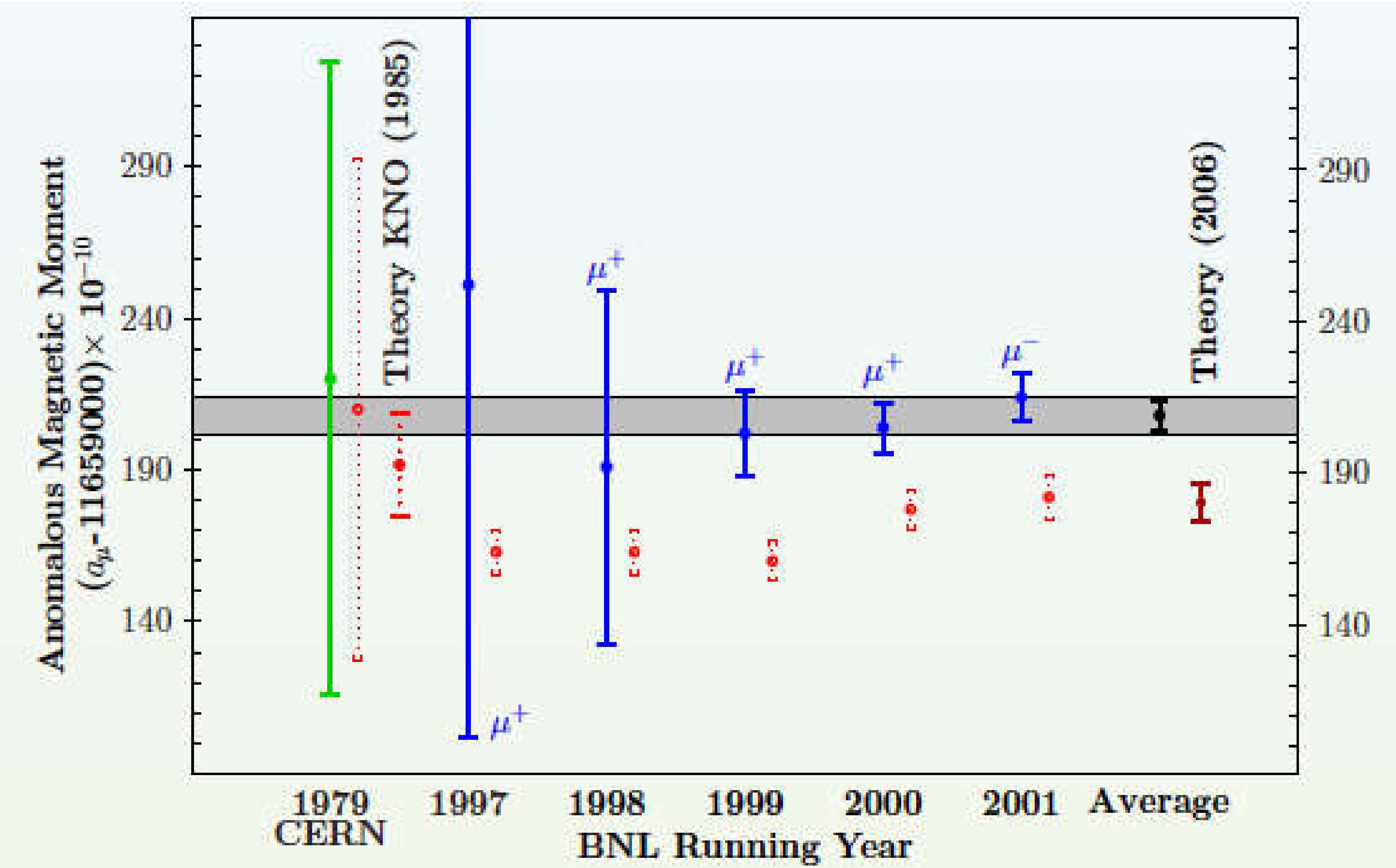

Jegerlehner & Nyffeler, Phys.Rept. 477 (2009) 1-110

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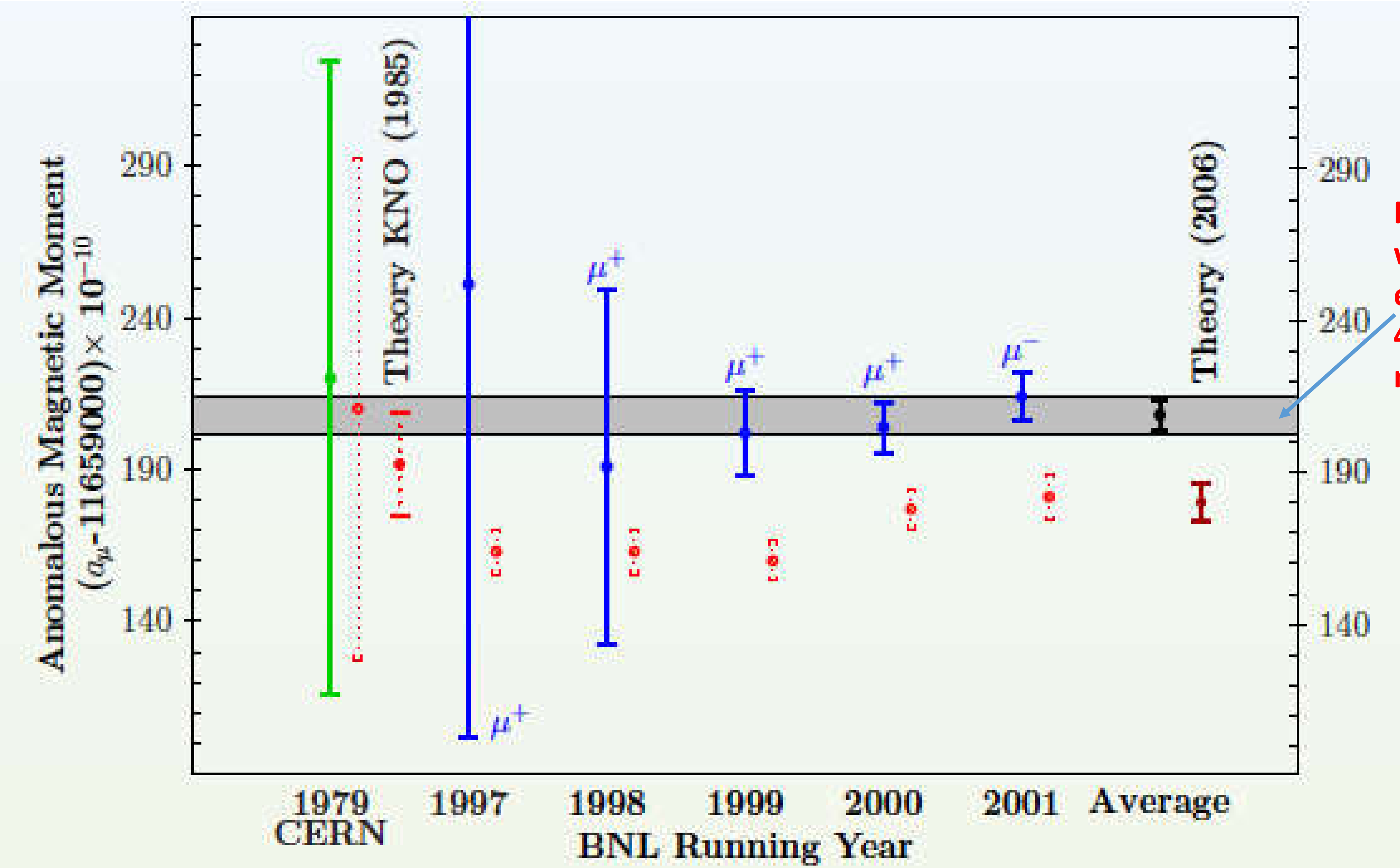
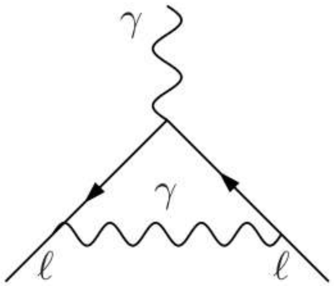


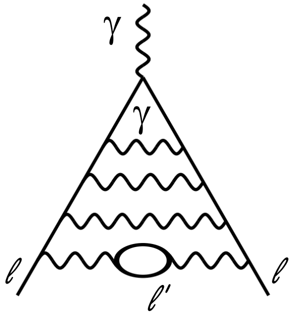
FNAL & J-PARC will bring this error a factor 4 down in the near future

Jegerlehner & Nyffeler, Phys.Rept. 477 (2009) 1-110

Interest of a_μ

FNAL & J-PARC will bring the error down to 16×10^{-11} in the near future





Type of contribution	Value x 10^{11}	Error x 10^{11}
QED	116'584,718.95	0.08
EW	153.6	1.0
HVP	6825	(42)
HLbL	105	(26)
Total	116'591,803	(1)(42)(26)
Exp	116'592,091	(54)(33)

Jegerlehner & Nyffeler, Phys.Rept. 477 (2009) 1-110

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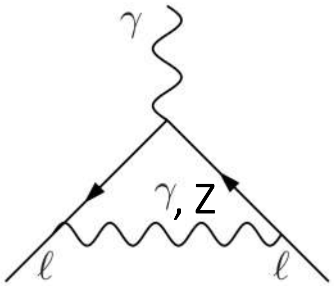

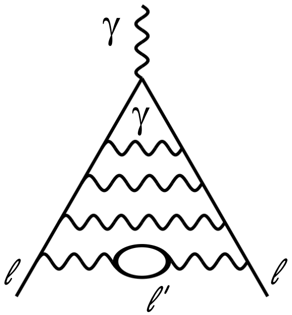

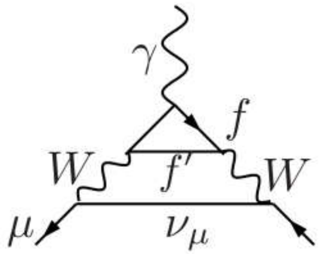

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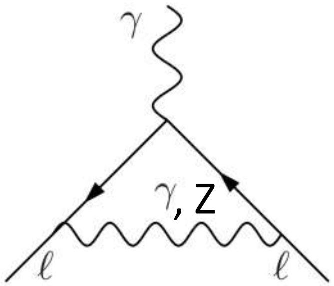

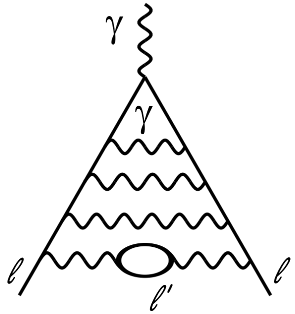

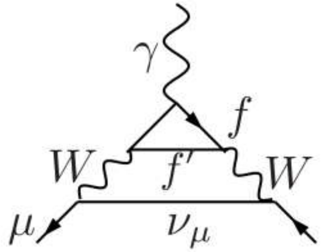

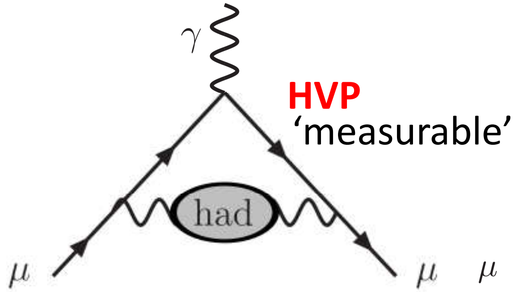

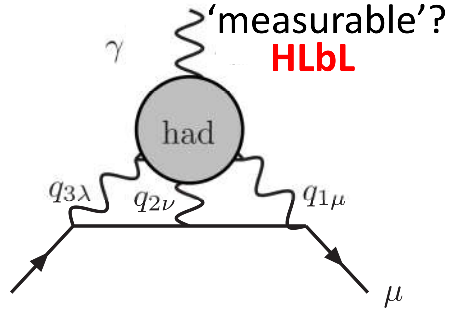

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Recent evaluations of a_μ contributions in the SM

- **5-loop QED**: Aoyama, Hayakawa, Kinoshita & Nio, Phys.Rev.Lett. 109 (2012) 111807, 111808
- **EW contributions after M_H measurement**: Gnedinger, Stockinger & Stockinger-Kim, Phys.Rev. D88 (2013) 053005
- **Higher order QCD effects**:

1. Kurz, Liu, Marquard & Steinhauser, Phys.Lett. B734 (2014) 144-147; $a_\mu^{\mathrm{HVP@NNLO}}=(1.24\pm0.01)10^{-10}$
2. Colangelo, Hoferichter, Nyffeler, Passera, Stoffer, Phys.Lett. B735 (2014) 90-91; $a_\mu^{\mathrm{HLbl@HO}}=(0.3\pm0.2)10^{-10}$

- **Towards a data driven analysis of HLbL**: Colangelo *et. al.* JHEP 1409 (2014) 091, Phys.Lett. B738 (2014) 6-12, JHEP 1509 (2015) 074, e-Print: arXiv:1701.06554 [hep-ph], e-Print: arXiv:1702.07347 [hep-ph].

[See also Pauk & Vanderhaeghen Phys.Rev. D90 (2014) no.11, 113012, Eur.Phys.J. C74 (2014) no.8, 3008]

Jegerlehner & Nyffeler, Phys.Rept. 477 (2009) 1-110

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Interest of a_μ

$$a_\mu^{exp} - a_\mu^{SM} = 288(63)(49) \times 10^{-11} \quad \sim 3.5\sigma$$

C. Patrignani *et al.* (Particle Data Group), Chin. Phys. C**40** (2016)

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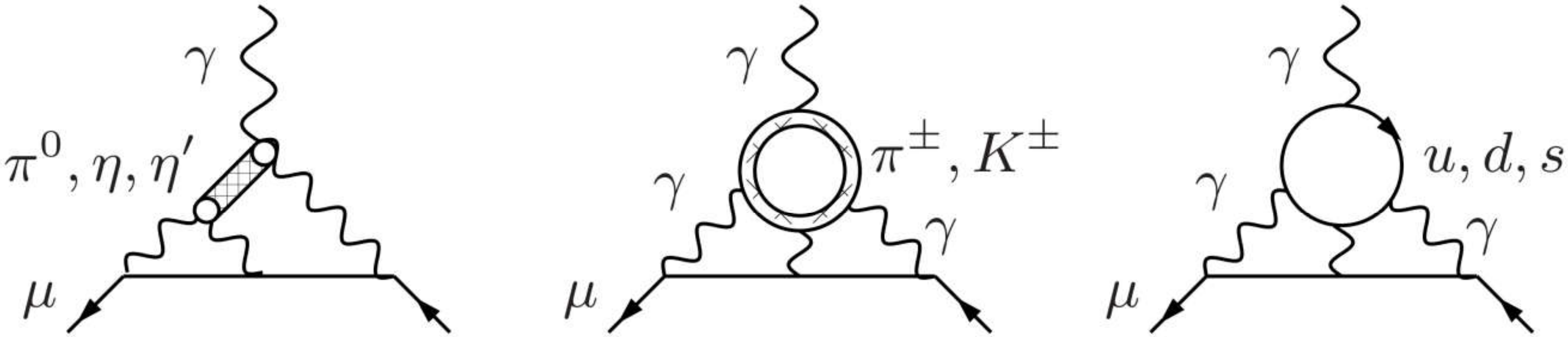
Jegerlehner & Nyffeler, Phys.Rept. 477 (2009) 1-110

(Role of π^0 TFF in) $a_\mu^{(P^0),HLbL}$

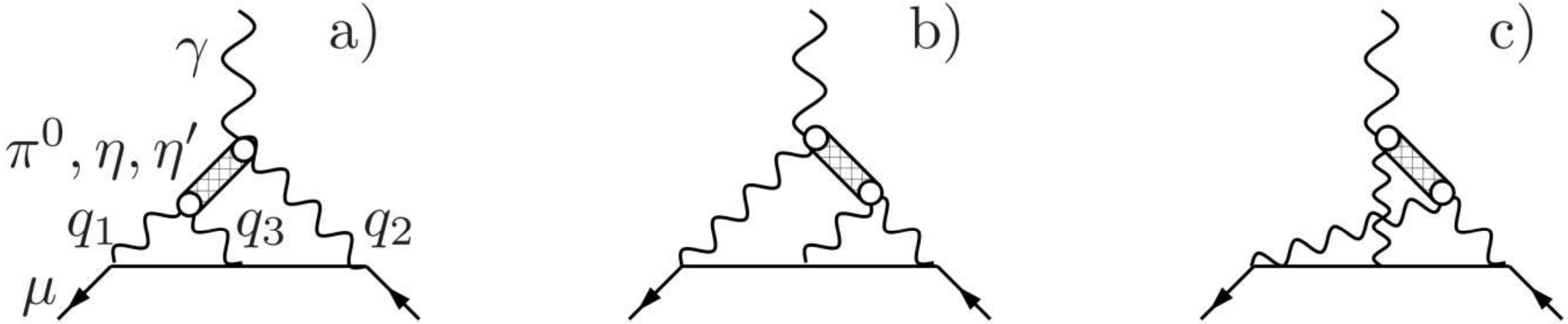


(a) [L.D.] (b) [L.D.] (c) [S.D.]

$a_\mu^{(a)}(\pi^0)$	$a_\mu^{(b+c)}(\pi^+)$
$(6.5 \pm 0.6) \times 10^{-10}$	$(0.9 \pm 0.5) \times 10^{-10}$

But this model-dependent splitting between L.D. & S.D contributions does not need to be made in a framework capable of dealing consistently with both extreme regimes (and intermediate regions) simultaneously, like DSE

Role of π^0 TFF in $a_\mu^{P^0,HLbL}$



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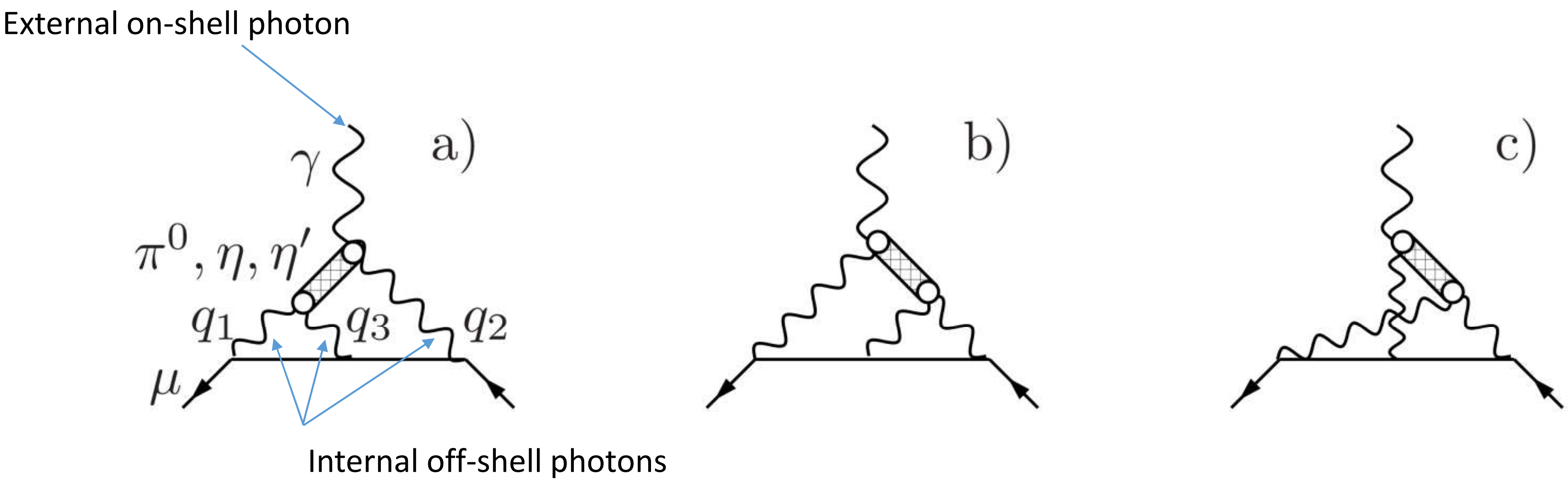


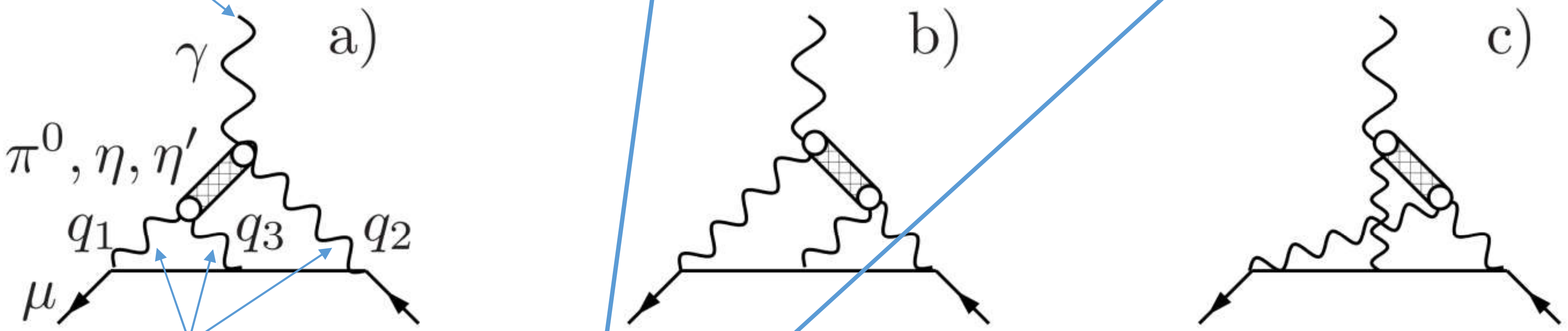
Role of π^0 TFF in $a_\mu^{P^0,HLbL}$

unmeasurable

measured

External on-shell photon

only $\mathcal{F}_{\pi^{0*}\gamma^*\gamma}(-Q^2,-Q^2,0)$ and **not** $\mathcal{F}_{\pi^{0*}\gamma^*\gamma}(m_\pi^2,-Q^2,0)$ can enter at the **external** vertex



Internal off-shell photons

In the TFF measurement one γ is nearly on-shell

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unmeasurable **measured**

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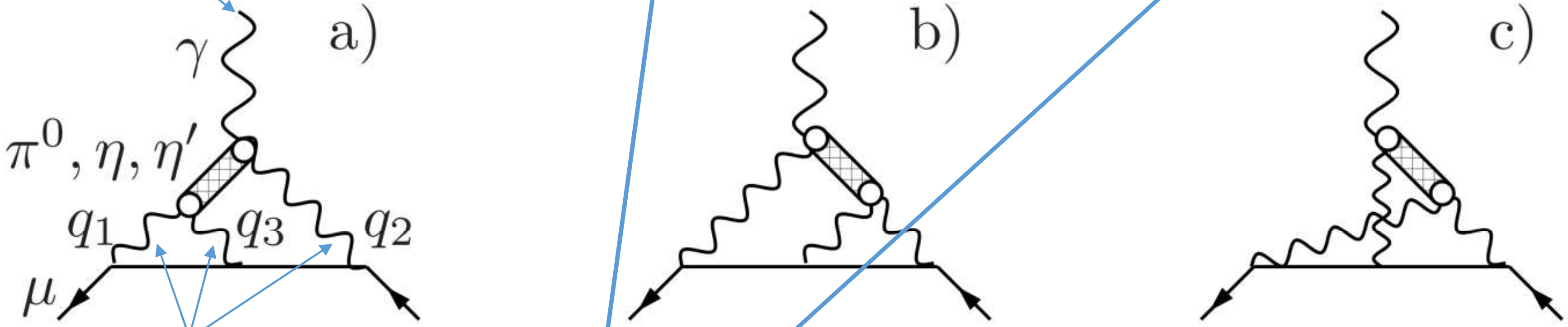


Internal off-shell photons

In the TFF measurement one γ is nearly on-shell

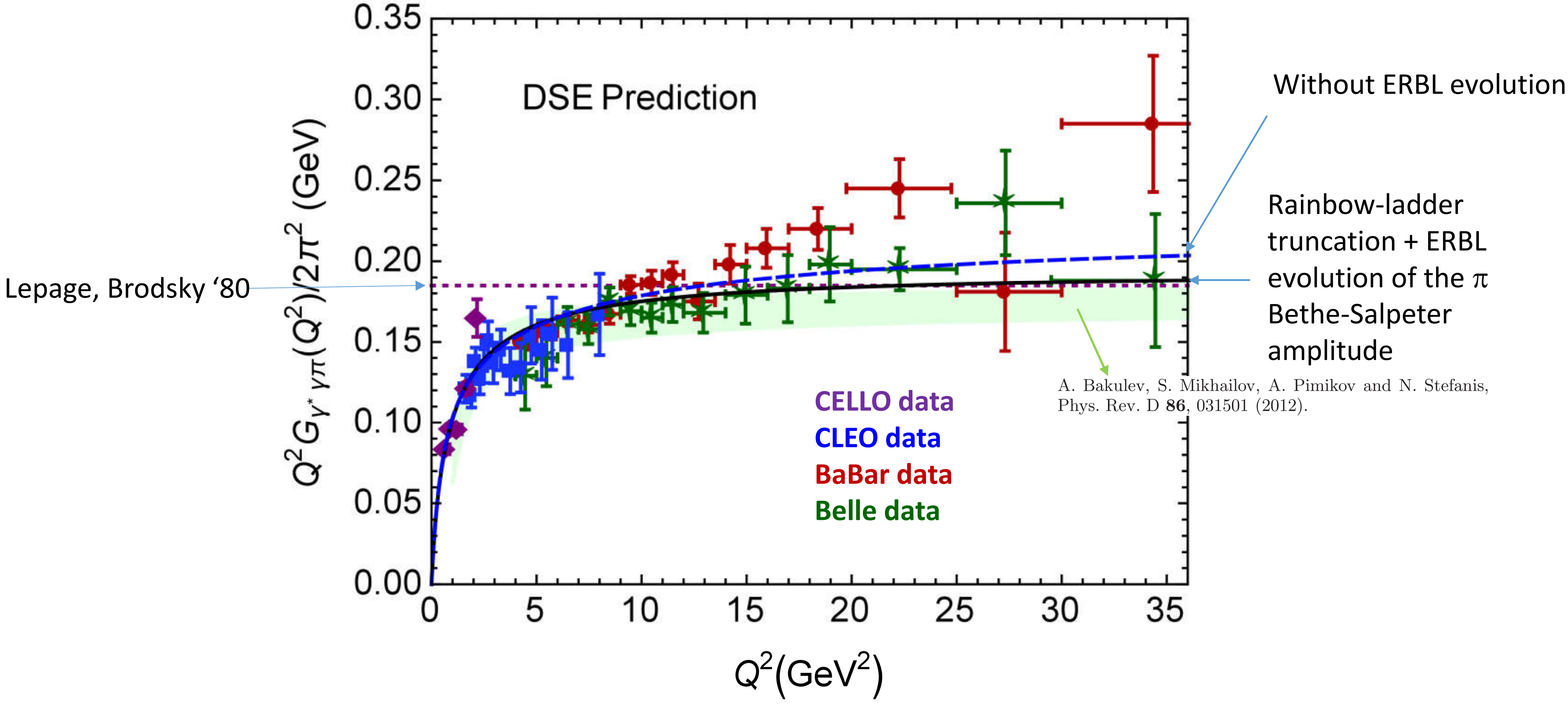
$$\int d^4x\, d^4y\, e^{i(q_1\cdot x+q_2\cdot y)}\,\langle 0|T\{j_\mu(x)j_\nu(y)P^3(0)\}|0\rangle = \varepsilon_{\mu\nu\alpha\beta}\,q_1^\alpha q_2^\beta\,\frac{i\langle\overline{\psi}\psi\rangle}{F_\pi}\,\frac{i}{(q_1+q_2)^2-m_\pi^2}\,\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}((q_1+q_2)^2,q_1^2,q_2^2)$$

$$P^3=\bar{\psi}i\gamma_5\frac{\lambda^3}{2}\psi=\left(\bar{u}i\gamma_5 u-\bar{d}i\gamma_5 d\right)/2$$

Bose-symmetric

π^0 TFF from Dyson-Schwinger equations

K. Raya, L. Chang, A. Bashir, J. J. Cobos-Martínez, L. X. Gutiérrez-Guerrero, C. D. Roberts, P. C. Tandy Phys.Rev. D93 (2016) no.7, 074017



How to parametrize DSE π^0 TFF? (III)

Jegerlehner & Nyffeler, Phys.Rept. 477 (2009) 1-110; Nyffeler, Phys.Rev. D79 (2009) 073012 **Fully off-shell !!**

0

$$\mathcal{P}(q_1^2, q_2^2, p_\pi^2) = q_1^2\, q_2^2\, (q_1^2 + q_2^2 + p_\pi^2) + h_1\, (q_1^2 + q_2^2)^2 + h_2\, q_1^2\, q_2^2 + h_3\, (q_1^2 + q_2^2)\, p_\pi^2 + h_4\, p_\pi^4 + h_5\, (q_1^2 + q_2^2) + h_6\, p_\pi^2 + h_7$$

ABJ: $h_7 = -N_c M_{V_1}^4 M_{V_2}^4 / (4\pi^2 F_\pi^2) - h_6 m_\pi^2 - h_4 m_\pi^4$

BL: → $h_5 = 6\, M_{V_1}^2\, M_{V_2}^2 + \delta_{BL}$

~7.7 GeV^4

$$\frac{\mathcal{F}_{\pi^0\gamma^*\gamma^*}(0, -Q^2, -Q^2)}{\mathcal{F}_{\pi^0\gamma\gamma}(0,0,0)} = \frac{8}{3}\pi^2 F_0^2 \left\{ \frac{1}{Q^2} - \frac{8}{9}\frac{\delta^2}{Q^4} + \cdots \right\} \rightarrow h_2 = -4\,(M_{V_1}^2 + M_{V_2}^2) + (16/9)\,\delta^2 \simeq -10.63\ \mathrm{GeV}^2$$

PDG

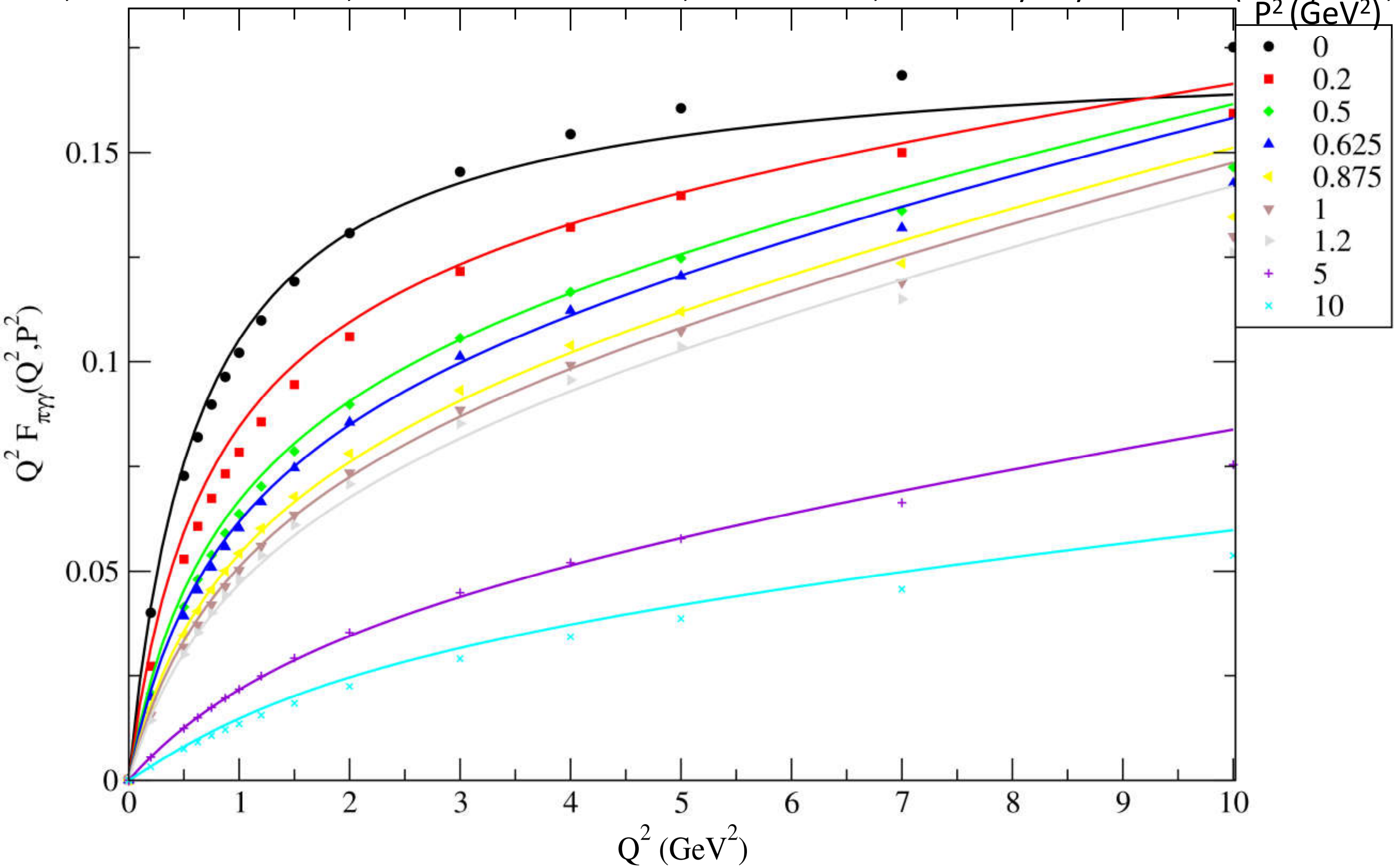
$$\Pi_{\mathrm{VT}}^{\mathrm{LMD+V}}(p^2) = -\,\langle \overline{\psi}\psi \rangle_0\, \frac{p^2 + c_{\mathrm{VT}}}{(p^2 - M_{V_1}^2)(p^2 - M_{V_2}^2)}\,, \quad c_{\mathrm{VT}} = \frac{M_{V_1}^2 M_{V_2}^2 \chi}{2} \rightarrow h_1 + h_3 + h_4 = 2c_{\mathrm{VT}}$$

h_3 (h_4) & h_6 are still free parameters. We will fit δ_{BL} to DSE data in the region relevant for a_μ ($Q_i^2 \leq 10$ GeV^2)

According to different estimates $|h_3|(|h_4|) \leq 10$ GeV & $h_6 \leq 10$ GeV

π^0 TFF from Dyson-Schwinger equations

K. Raya, L. Chang, A. Bashir, J. J. Cobos-Martínez, L. X. Gutiérrez-Guerrero, C. D. Roberts, P. C. Tandy Phys.Rev. D93 (2016) no.7, 074017



Evaluation of $a_\mu^{\pi^0\text{-pole}}$ with DSE input

We have evaluated $a_\mu^{\pi^0\text{-pole}}$ varying parameters in the ranges discussed previously

M_{V_2} (GeV)	δ_{BL}
1.440	-0.57 ± 0.18
1.465	-0.43 ± 0.17
1.490	-0.36 ± 0.18

Table 1: Dependence of δ_{BL} on M_{V_2}.

$|h_3|(|h_4|) \leq 10$ GeV & $h_6 \leq 10$ GeV

$$h_3 + h_4 = M_{V_1}^2 M_{V_2}^2 \chi$$

$\chi = -(3.05 \pm^{0.20}_{1.00})\,\text{GeV}^{-2}$

The only relevant variations are h_3 (h_4) & δ_{BL}

$$a_\mu^{\pi^0\text{-pole}} = (6.26 \pm 0.08)10^{-10}$$

Evaluation of $a_\mu^{\pi^0\text{-pole}}$ with DSE input: Conclusions

We have evaluated $a_\mu^{\pi^0\text{-pole}}$ varying parameters in the ranges discussed previously

$$a_\mu^{\pi^0\text{-pole}} = (6.26 \pm 0.08)10^{-10}$$

What's next? Obtention of on/off-shell $\eta/\eta'/\eta_{b,c}$ TFF and evaluation of the corresponding contributions to a_μ(Khepani Raya , Minghui Ding, Adnan Bashir, Lei Chang, Craig D. Roberts; Phys.Rev. D95 (2017) 074014).

Our result is approx. 8% higher than pion-pole evaluation of Nyffeler. This makes us **optimistic** with respect to a reduction of the exp-theo discrepancy in a_μ.

Bern's group work clarifies that only pole contributions are needed

Some reference values…

Pole contributions

$$a_\mu^{\pi^0,HLbL} = (5.75 \pm 0.06) \cdot 10^{-10}$$

$$a_\mu^{\eta,HLbL} = (1.44 \pm 0.26) \cdot 10^{-10}$$

$$a_\mu^{\eta',HLbL} = (1.08 \pm 0.09) \cdot 10^{-10}$$

Exchange contributions

$$a_\mu^{\pi^0,HLbL} = (6.66 \pm 0.21) \cdot 10^{-10}$$

$$a_\mu^{\eta,HLbL} = (2.04 \pm 0.44) \cdot 10^{-10}$$

$$a_\mu^{\eta',HLbL} = (1.77 \pm 0.23) \cdot 10^{-10}$$

According to Roig, Guevara & López-Castro, Phys.Rev. D89 (2014) no.7, 073016

$$a_\mu^{P,HLbL} = (10.47 \pm 0.54) \cdot 10^{-10}$$

$$a_\mu^{BSM} \lesssim 288 \times 10^{-11}$$