MODERN ASPECTS OF SCATTERING AMPLITUDES

Bryan Larios

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Facultad de Ciencias Físico Matemáticas

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Outline

- Introduction and Motivation,
- The Spinor Helicity Formalism,
- Application,
- Final Comments.

Motivation

“Scattering experiments are crucial to understand the fundamental blocks in nature”.

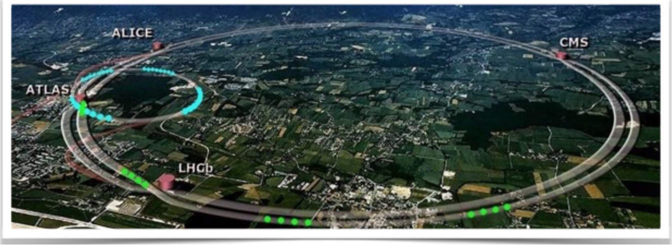


Figure: CERN (Geneve, Switzerland)

Motivation

The Standard Model of elementary particles was developed largely because scattering experiments, (the discovery the gauge bosons $W^{\pm}$ y Z^{0} and the gluons and quarks and more recently the Higgs bosons).

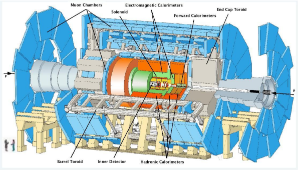


Figure: Detector ATLAS-CERN

Motivation

The principal physical observable in the scattering experiments is the differential cross section (DCS) $\frac{d\sigma}{d\Omega}$.

Interpretation of DATA from scattering experiments is *based* mainly in the theoretical predictions of the cross section.

Motivation

It is well know (theoretically and experimentally) that **QFT** describes the elementary particles and the fundamental forces of nature.
The DCS (calculated with **QFT**) that *connect* theory with experiments is proportional to the modulus squared of the amplitude.

$$\frac{d\sigma}{d\Omega} \propto |\mathcal{A}|^2$$

Motivation

We have a lot of programs that undoubtedly help computing the scattering amplitudes (*MadGraph*, *Form*, *FeynRules, FeynCalc*, among others), most of them numerically. For example let me show you the computation $\tilde{t} \to b\ W\ G$ (with the LSP 3/2-spin gravitino particle):

$\tilde{t}_1 \rightarrow b + W + \tilde{\Psi}_\mu$

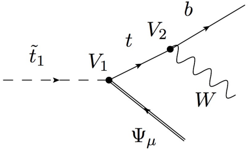


Figure 1. top diagram

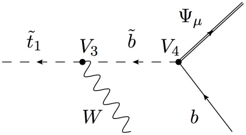


Figure 2. sbotom diagram

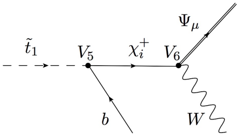
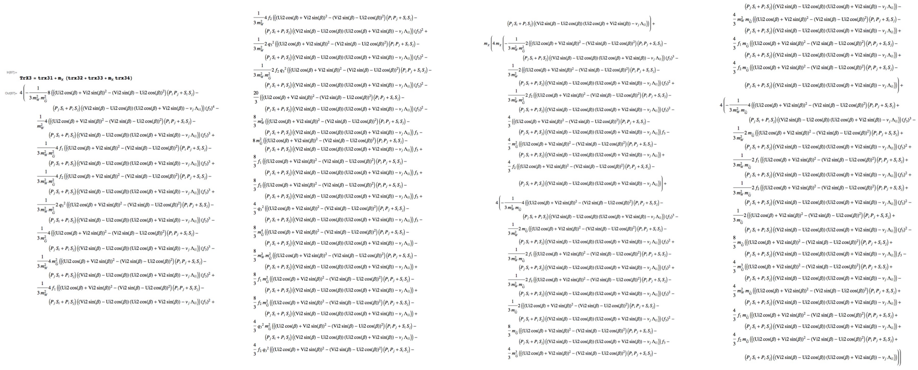


Figure 3. chargino diagram

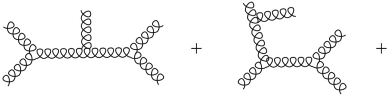
Lorenzo Diaz-Cruz & Bryan Larios-Lopez, EPJC (2016) ***76****:157.*

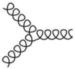
Considering just one channel and using Mathematica one obtain for the chargino $|\overline{\mathcal{M}_{1\chi}}|^2$.



Pure YM theory

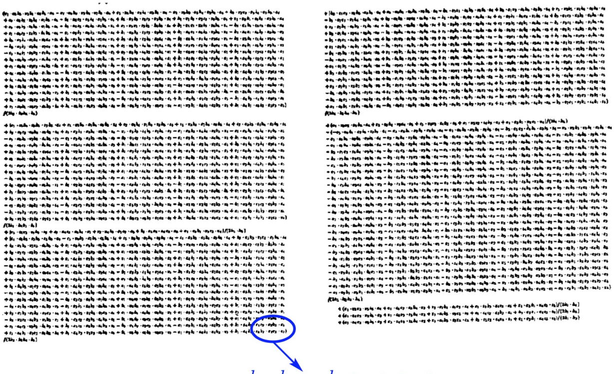
If we consider a pure Yang - Mills theory, computing the scattering amplitude (modulus square) at three level i.e. for 5 gluons, we have:





$$\rightarrow g f^{abc}[(p_1 - p_2)_\rho \eta_{\mu\nu} + (p_2 - p_3)_\mu \eta_{\nu\rho} + (p_3 - p_1)_\nu \eta_{\rho\mu}]$$

After brute force computing, part of the modulus square SA is shown



$k_1 \cdot k_4 \, \varepsilon_2 \cdot k_1 \, \varepsilon_1 \cdot \varepsilon_3 \, \varepsilon_4 \cdot \varepsilon_5$

But using modern techniques, we obtain a very simple and compact expression for the 5 gluons HA

$$A_5(1^{\pm},2^{+},3^{+},4^{+},5^{+}) = 0$$

$$A_5(1^{-},2^{-},3^{+},4^{+},5^{+}) = i\frac{\langle 1\,2\rangle^4}{\langle 1\,2\rangle\langle 2\,3\rangle\langle 3\,4\rangle\langle 4\,5\rangle\langle 5\,1\rangle}$$

Spinor Helicity Formalism (a pragmatic point of view)

The SHF is based in the following observation:

Fields with spin-1 transform in the $(\frac{1}{2}, \frac{1}{2})$ representation of the Lorentz group.

So we are able to express 4-momentum of any particle as a biespinor: $p_\mu \to p_{a\dot{a}}$

$$p_{a\dot{a}} = p_\mu \sigma^\mu_{a\dot{a}} \tag{1}$$

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}$$

$$\sigma^\mu_{a\dot{a}} = (I, \vec{\sigma}),\ \bar{\sigma}^{\mu\dot{a}a} = (I, -\vec{\sigma})$$

4-component spinors

4-Component Dirac spinors ($u_s(\vec{p})$ and $v_s(\vec{p})$) are solutions of the following EOMs:

$$(\not{p} + m)u_s(\vec{p}) = 0, \tag{2}$$

$$(-\not{p} + m)v_s(\vec{p}) = 0, \tag{3}$$

with $s = \pm$.

Massless Case (*textbooks*)

For massless Dirac spinors we know that the following equation exist: $u_s(\vec{p})\bar{u}_s(\vec{p}) = \frac{1}{2}(1+s\gamma_5)(-\not{p})$, for instance if $s=-$, we have:

$$u_-(\vec{p})\bar{u}_-(\vec{p}) = \frac{1}{2}(1-\gamma_5)(-\not{p}) = \begin{pmatrix} 0 & -p_{a\dot{a}} \\ 0 & 0 \end{pmatrix}$$

where

$$u_-(\vec{p}) = \begin{pmatrix} \phi_a \\ 0 \end{pmatrix}$$

ϕ_a is a 2-component numerical spinor,

also $\bar{u}_-(\vec{p}) = (0, \phi^*_{\dot{a}})$, all this lead to $\boxed{p_{a\dot{a}} = -\phi_a\phi^*_{\dot{a}}}$

The key of the SHF is to considerate ϕ_a as the fundamental object and express the momenta of the particles in terms of ϕ_a.

Highly convenient and powerful notation

If p and k are the momenta and ϕ_a, κ_a their associated spinors, we can define the following products of spinors:

$$[pk] = \phi^a \kappa_a = \bar{u}_+(\vec{p})u_-(\vec{k}) = -[kp] \quad (4)$$

$$\langle pk \rangle = \phi^*_{\dot{a}} \kappa^{*\dot{a}} = \bar{u}_-(\vec{p})u_+(\vec{k}) = -\langle kp \rangle \quad (5)$$

Notation for 4-components Dirac Spinor

$$|p] = u_-(\mathbf{p}) = v_+(\mathbf{p}) \;,$$

$$|p\rangle = u_+(\mathbf{p}) = v_-(\mathbf{p}) \;,$$

$$[p| = \overline{u}_+(\mathbf{p}) = \overline{v}_-(\mathbf{p}) \;,$$

$$\langle p| = \overline{u}_-(\mathbf{p}) = \overline{v}_+(\mathbf{p}) \;.$$

where spinor products as $\bar{u}_s(\vec{p})\, u_s(\vec{k}) = 0 \;\forall\; s = \pm$, or more nice, we have:

$$\boxed{\langle pk] = [pk\rangle = 0} \tag{6}$$

Polarizations 4-vectores

If we want to apply the SHF to spinor electrodynamics, we need to write polarization vectors in term of 2-component numerical spinors.

$$\varepsilon_+^\mu(k) = -\frac{\langle q|\gamma^\mu|k]}{\sqrt{2}\,\langle q\,k\rangle}\,,$$

$$\varepsilon_-^\mu(k) = -\frac{[q|\gamma^\mu|k\rangle}{\sqrt{2}\,[q\,k]}\,,$$

where they satisfy $p \cdot \mathcal{E}_s = 0$ and $\mathcal{E}_s^2 = 1\ \forall\ s = \pm$.

- **The contraction with γ matrix is as follows:** :

$$\not{\epsilon}_+(k,q) = \frac{1}{\sqrt{2}\langle qk\rangle}\langle q|\gamma^\mu|k]\gamma_\mu = \frac{2}{\sqrt{2}}(|k]\langle q| + |q\rangle[k|) = \frac{\sqrt{2}}{\langle qk\rangle}(|k]\langle q| + |q\rangle[k|), \quad (7)$$

$$\not{\epsilon}_-(k,q) = \frac{1}{\sqrt{2}[qk]}[q|\gamma^\mu|k\rangle\gamma_\mu = \frac{1}{\sqrt{2}[qk]}\langle k|\gamma^\mu|q]\gamma_\mu = \frac{\sqrt{2}}{[qk]}(|k]\langle q| + |q\rangle[k|). \quad (8)$$

Some of the most important formulas that are needed to compute scattering amplitudes :

$$[ij] = -[ji],$$

$$\langle ij\rangle = [ji]^*,$$

$$\langle ij\rangle[ji] = \langle ij\rangle\langle ij\rangle^* = |\langle ij\rangle|^2,$$

$$\langle ij\rangle[ji] = -2k_i \cdot k_j = s_{ij},$$

$$\langle i|\gamma_\mu|j] = [j|\gamma_\mu|i\rangle,$$

$$\langle i|\gamma_\mu|j]\langle k|\gamma^\mu|l] = 2\langle ik\rangle[lj],$$

$$\langle ab\rangle\langle cd\rangle = \langle ac\rangle\langle bd\rangle + \langle ad\rangle\langle cb\rangle,$$

$$\sum_{k=1}^{n}\langle ik\rangle[kj] = 0,$$

What about massive particles ?

SHF for massive particles

The SHF is a highly convenient and powerful notational tool for amplitudes of massless particles in 4-dimensional theories. But we would like to calculate amplitudes taking into account the mass of the particles.

All the massive spinors are as follows

$$u_- = |r] + \frac{m}{\langle rq\rangle}|q\rangle \quad , \quad u_+ = \frac{m}{[rq]}|q] + |r\rangle;$$

$$v_+ = |r] - \frac{m}{\langle rq\rangle}|q\rangle \quad , \quad v_- = -\frac{m}{[rq]}|q] + |r\rangle;$$

$$\bar{u}_- = \frac{m}{[qr]}[q| + \langle r| \quad , \quad \bar{u}_+ = [r| + \frac{m}{\langle qr\rangle}\langle q|;$$

$$\bar{v}_+ = -\frac{m}{[qr]}[q| + \langle r| \quad , \quad \bar{v}_- = [r| - \frac{m}{\langle qr\rangle}\langle q|.$$

In order to apply the SHF (massive or massless) to the standard model of particle physics, we just need to have the massive polarization vector in term of 2-component numerical spinors.

We already know the massive polarization vector in terms of spinors.

$$\epsilon_+^\mu = \frac{\langle q|\gamma^\mu|r]}{\sqrt{2}\langle rq\rangle},$$
$$\epsilon_-^\mu = \frac{\langle r|\gamma^\mu|q]}{\sqrt{2}[qr]},$$

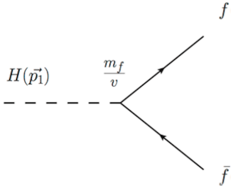
$$\epsilon_0^\mu = \frac{1}{2m}\left(\langle r|\gamma^\mu|r] - \alpha\langle q|\gamma^\mu|q]\right)$$
$$= \frac{1}{m}r^\mu + \frac{m}{2p\cdot q}q^\mu.$$

Let us see some examples.

Computing Scattering Amplitudes

2-body Higgs decay $h(p_1) \to f(p_2)\bar{f}(p_3)$

From the Feynman diagram we get the helicity amplitudes (HAs) (as is usual)



$$\mathcal{M}_{\lambda_2\lambda_3}(p_1,p_2,p_3) = \frac{m_f}{v}\bar{u}_{\lambda_2}(p_2)v_{\lambda_3}(p_3), \quad (9)$$

$\lambda_2\lambda_3$	$\mathcal{M}_{\lambda_2\lambda_3}$
$--$	$\frac{m_f}{v[q_2q_3]}(s_{q_2q_3} - m_f^2)$
$++$	$\frac{m_f}{v\langle q_2q_3\rangle}(s_{q_2q_3} - m_f^2)$

2-body Higgs decay $h(p_1) \to f(p_2)\bar{f}(p_3)$

Let me compute these HAs step by step

- $\bar{u}_-(p_2),\ \bar{u}_+(p_2)$

$$\mathcal{M}_{\lambda_2\lambda_3}(p_1,p_2,p_3) = \frac{m_f}{v}\,\bar{u}_{\lambda_2}(p_2)\,v_{\lambda_3}(p_3) \tag{10}$$

- $v_-(p_3),\ v_+(p_3)$

We see that there are 4 HAs, these are $-+, --, +-, ++$ (In the massless case there are just 2, +- and -+).

2-body Higgs decay $h(p_1) \to f(p_2)\bar{f}(p_3)$

$$\mathcal{M}_{--} = \frac{m_f}{v}\bar{u}_-(2)v_-(3) \tag{11}$$

$$= \frac{m_f}{v}\left(\frac{m_f}{[q_2 r_2]}[q_2| + \langle r_2|\right)\left(-\frac{m_f}{[r_3 q_3]}|q_3] + |r_3\rangle\right) \tag{12}$$

$$= \frac{m_f}{v}\left(-\frac{m_f^2[q_2 q_3]}{[q_2 r_2][r_3 q_3]} + \frac{m_f}{[q_2 r_2]}\cancelto{0}{[q_2 r_3\rangle} - \frac{m_f\cancelto{0}{\langle r_2 q_3]}}{[r_3 q_3]} + \langle r_2 r_3\rangle\right) \tag{13}$$

$$= \frac{m_f}{v}\left(-\frac{m_f^2[q_2 q_3]}{[q_2 r_2][r_3 q_3]} + \langle r_2 r_3\rangle\right) \tag{14}$$

For this special example we will use simultaneous LCD (SLCD),

$$p_2 = r_2 - \frac{m_f^2}{2r_2 \cdot q_2} q_2 \tag{15}$$

$$p_3 = r_3 - \frac{m_f^2}{2r_3 \cdot q_3} q_3 \tag{16}$$

For this special example we will use simultaneous LCD (SLCD),

$$p_2 = r_2 - \frac{m_f^2}{2r_2 \cdot q_2} q_2 \tag{17}$$

$$p_3 = q_2 - \frac{m_f^2}{2q_2 \cdot r_2} r_2 \tag{18}$$

We choose $r_3 = q_2$ and $q_3 = r_2$, this will reduce our calculations.

2-body Higgs decay $h(p_1) \to f(p_2)\bar{f}(p_3)$

Returning to the HA $\mathcal{M}_{--}$ and taking into account SLCD, we have

$$\mathcal{M}_{--} = \frac{m_f}{v}\left(-\frac{m_f^2 \cancel{[q_2 r_2]}}{\cancel{[q_2 r_2]}[q_2 r_2]} + \langle r_2 q_2 \rangle\right) \tag{19}$$

$$= \frac{m_f}{v}\left(-\frac{m_f^2}{[q_2 r_2]} + \langle r_2 q_2 \rangle\right) = \frac{m_f}{v[q_2 r_2]}(-m_f^2 + \langle r_2 q_2 \rangle [q_2 r_2]) \tag{20}$$

$$= \frac{m_f}{v[q_2 r_2]}(-m_f^2 + s_{q_2 r_2}) \tag{21}$$

We do not need to compute $\mathcal{M}_{++}$, we just complex conjugate $\mathcal{M}_{--}$ (one of the greatest advantages of the SHF).

$$\mathcal{M}_{++} = \frac{m_f}{v\langle q_2 r_2\rangle}(-m_f^2 + s_{q_2 r_2}) \tag{22}$$

As you can guess we shall compute just one HA ($\mathcal{M}_{-+}$)

$$\mathcal{M}_{-+} = \frac{m_f}{v}\bar{u}_-(2)v_+(3) = \frac{m_f}{v}\left(\frac{m_f}{[q_2 r_2]}[q_2| + \langle r_2|\right)\left(|q_2] - \frac{m_f}{\langle r_2 q_2\rangle}|r_2\rangle\right) \tag{23}$$

$$\propto \cancelto{0}{[q_2 q_2]} - \cancelto{0}{[q_2 r_2\rangle} + \cancelto{0}{\langle r_2 q_2]} - \cancelto{0}{\langle r_2 r_2\rangle} \tag{24}$$

$$= 0 \tag{25}$$

Finally the averaged squared amplitude is then:

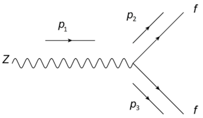
$$\langle|\mathcal{M}|^2\rangle = 2|\mathcal{M}_{--}|^2 = \frac{2m_f^2}{v^2 s_{q_2q_3}}(s_{q_2q_3} - m_f^2)^2 = \frac{y^2}{v^2}(1-4y^2), \tag{26}$$

with $y = \frac{m_f}{M_h}$. Then the decay width Γ goes as follows

$$\Gamma(h \to f\bar{f}) = \frac{\alpha_W M_h y^2}{8}(1-4y^2)^{3/2}. \tag{27}$$

2-body Z boson decay $Z(p_1) \to f(p_2)\bar{f}(p_3)$

Again from the Feynman diagram we get the helicity amplitudes



$$\mathcal{M} = \frac{1}{2} g_Z \epsilon_\mu(p_1)\bar{u}(p_2)\gamma^\mu(v_f - a_f\gamma^5)v(p_3) \tag{28}$$

$\lambda_1\lambda_2\lambda_3$	$\mathcal{M}_{\lambda_1\lambda_2\lambda_3}$
$++-$	$\frac{1}{2}g_Z\frac{\langle p_2\|\gamma_\mu\|r_1]}{\sqrt{2}\langle r_1p_2\rangle}(v_f-a_f)[p_2\|\gamma^\mu\|p_3\rangle = \frac{g_Z(v_f-a_f)}{\sqrt{2}}\frac{\langle p_2p_3\rangle[r_1p_2]}{\langle r_1p_2\rangle}$
$0+-$	$\frac{1}{2}g_Z\left(\frac{1}{M}r_{1\mu}+\frac{M}{2p_{12}}p_{2\mu}\right)(v_f-a_f)[p_2\|\gamma^\mu\|p_3\rangle = \frac{g_Z(v_f-a_f)}{\sqrt{2}M}\langle r_1p_3\rangle[r_1p_2] = 0$
$-+-$	$\frac{1}{2}g_Z\frac{\langle r_1\|\gamma_\mu\|p_2]}{\sqrt{2}[p_2r_1]}(v_f-a_f)[p_2\|\gamma^\mu\|p_3\rangle = 0$

Finally the squared averaged scattering amplitude is as follows

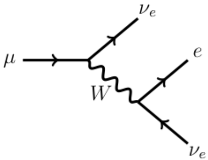
$$\langle|\mathcal{M}|^2\rangle = \frac{1}{3}\left(|\mathcal{M}_{++-}|^2 + |\mathcal{M}_{--+}|^2\right) = \frac{g_Z^2 M^2}{3}\left(|v_f|^2 + |a_f|^2\right) \tag{29}$$

The decay width for this channel is then:

$$\Gamma\left(Z \to f\bar{f}\right) = \frac{g_Z^2 M}{48\pi}\left(|v_f|^2 + |a_f|^2\right). \tag{30}$$

3-body Muon Decay $\mu(p_1) \to \bar{\nu}_{e^-}(p_2)\,\nu_\mu(p_3)\,e^-(p_4)$

From the Feynman diagram we get the helicity amplitudes (HAs) (as is usual)



$$\mathcal{M}_{\lambda_1\lambda_2\lambda_3\lambda_4} = \left(\frac{g_W}{\sqrt{8}M_W}\right)^2 [\bar{u}_{\lambda_3}(p_3)\gamma^\mu(1-\gamma_5)u_{\lambda_1}(p_1)]\,[\bar{u}_{\lambda_4}(p_4)\gamma_\mu(1-\gamma_5)v_{\lambda_2}(p_2)]\,,$$
$$= \left(\frac{g_W}{\sqrt{8}M_W}\right)^2 \mathcal{A}^\mu_{\lambda_3\lambda_1}\mathcal{B}_{\mu\lambda_4\lambda_2},$$

$\lambda_1\lambda_2\lambda_3\lambda_4$	$\mathcal{A}^{\mu\lambda_3\lambda_1}$	$\mathcal{B}^{\lambda_4\lambda_2}_\mu$	$\mathcal{A}^{\mu\lambda_3\lambda_1}\mathcal{B}^{\lambda_4\lambda_2}_\mu$	$\mathcal{M}$ $(p_2 = q_1,\, p_3 = q_4)$
$+-+-$	$2[p_3\vert\gamma^\mu\vert r_1\rangle$	$2[r_4\vert\gamma_\mu\vert p_2\rangle$	$4\langle p_2 r_1\rangle\langle p_3 r_4\rangle$	$\left(\frac{g_W}{\sqrt{2}m_\mu}\right)^2\langle p_2 r_1\rangle[p_3 r_4]$
$+++-$	$2[p_3\vert\gamma^\mu\vert r_1\rangle$	$\frac{2m_e}{[q_4 r_4]}[q_4\vert\gamma_\mu\vert p_2\rangle$	$4\frac{m_e}{[q_4 r_4]}\langle p_2 r_1\rangle\langle p_3 q_4\rangle$	0
$--++$	$\frac{2m_\mu}{\langle r_1 q_1\rangle}[p_3\vert\gamma^\mu\vert q_1\rangle$	$2[r_4\vert\gamma_\mu\vert p_2\rangle$	$4\frac{m_\mu}{\langle r_1 q_1\rangle}\langle p_2 q_1\rangle[p_3 r_4]$	0
$-++-$	$\frac{2m_\mu}{\langle r_1 q_1\rangle}[p_3\vert\gamma^\mu\vert q_1\rangle$	$\frac{2m_e}{[q_4 r_4]}[q_4\vert\gamma_\mu\vert p_2\rangle$	$4\frac{m_\mu m_e}{\langle r_1 q_1\rangle[q_4 r_4]}\langle p_2 q_1\rangle[p_3 q_4]$	0

The squared and averaged amplitude for the muon decay is:

$$\langle|\mathcal{M}^{+-+-}|^2\rangle = \frac{1}{2}|\mathcal{M}^{+-+-}|^2 = 2\left(\frac{g_W}{M_W}\right)^4 (p_1 \cdot p_2)(p_3 \cdot p_4) \tag{31}$$

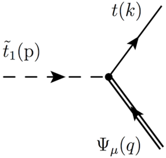
From this result we can arrive to the decay width, which agrees with result of textbooks.

Gravitino wave functions

We will consider a model in a local supersymmetric field theory ($\mathcal{N} = 1$ SUGRA) where the gravitino is the lightest supersymmetric particle (LSP), therefore a good DM candidate.

$$\tilde{\Psi}^{\mu}_{++}(p) = \epsilon^{\mu}_{+}(p)u_{+}(p),$$
$$\tilde{\Psi}^{\mu}_{--}(p) = \epsilon^{\mu}_{-}(p)u_{-}(p)$$
$$\tilde{\Psi}^{\mu}_{+}(p) = \sqrt{\frac{2}{3}}\epsilon^{\mu}_{0}(p)u_{+}(p) + \frac{1}{\sqrt{3}}\epsilon^{\mu}_{+}(p)u_{-}(p),$$
$$\tilde{\Psi}^{\mu}_{-}(p) = \sqrt{\frac{2}{3}}\epsilon^{\mu}_{0}(p)u_{-}(p) + \frac{1}{\sqrt{3}}\epsilon^{\mu}_{-}(p)u_{+}(p).$$

Two body NLSP Stop decay ($\tilde{t} \to t\,\tilde{G}$)



$$\mathcal{M}_{\lambda_2\lambda_3} = -\frac{1}{\sqrt{2}M}\bar{\psi}^{\mu}_{\lambda_2}(p_2)\gamma_\alpha\gamma_\mu p_1^\alpha \hat{P}_R u_{\lambda_3}(p_3) \quad (32)$$

$\lambda_2\lambda_3$	$\mathcal{M}_{\lambda_2\lambda_3}$
$-+$	$-\frac{\langle r_2 q_2\rangle}{\sqrt{3}M m_{\tilde{G}} s_{r_2 q_2}}(s^2_{r_2 q_2} - m^2_{\tilde{G}} m^2_t)$
$+-$	$-\frac{[r_2 q_2] m_t}{\sqrt{3}M s^2_{r_2 q_2}}(s^2_{r_2 q_2} - m^2_{\tilde{G}} m^2_t)$

Squaring the helicity amplitudes of the last Table, we obtain the following expression for the total squared amplitude

$$\langle|\mathcal{M}|^2\rangle = |\mathcal{M}_{-+}|^2 + |\mathcal{M}_{+-}|^2, \tag{33}$$

$$= \frac{\left(s_{r_2q_2}^4 - (m_t m_{\tilde{G}})^4\right)\left(s_{r_2q_2}^2 - (m_t m_{\tilde{G}})^2\right)}{3M^2 m_{\tilde{G}}^2 s_{r_2q_2}^3}, \tag{34}$$

$$= \frac{(m_{\tilde{t}}^2 - m_{\tilde{G}}^2 - m_t^2)\left((m_{\tilde{t}}^2 - m_t^2 - m_{\tilde{G}}^2)^2 - 4m_t^2 m_{\tilde{G}}^2\right)}{3M^2 m_{\tilde{G}}^2}, \tag{35}$$

To appreciate the power and efficiency of the SHF, we can remember the completeness relation for the spin-3/2 Gravitino Field.

$$\sum_{\tilde{\lambda}=1}^{3} \Psi_\mu(\vec{p}_1, \tilde{\lambda})\overline{\Psi}_\nu(\vec{p}_1, \tilde{\lambda}) = -(\not{p}_1 + m_{\tilde{G}}) \times \left\{ \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{m_{\tilde{G}}^2} \right) - \frac{1}{3}\left(g_{\mu\sigma} - \frac{p_\mu p_\sigma}{m_{\tilde{G}}^2} \right)\left(g_{\nu\lambda} - \frac{p_\nu p_\lambda}{m_{\tilde{G}}^2} \right)\gamma^\sigma\gamma^\lambda \right\}$$

And we still need to take into account other fields to compute the trace.

$e^+e^- \to \gamma \tilde{G} \tilde{G}$

Using the SUSY QED model constructed by Mawatari and Oexl (arXiv:1402.3223v2), and applying the **SHF** we will compute the scattering amplitude for the $e^+e^- \to \gamma \tilde{G} \tilde{G}$ reaction. It is important to mention that the cross sections for this reaction has been computed numerically, so it is interesting have an analytical result.

Feynman Diagrams

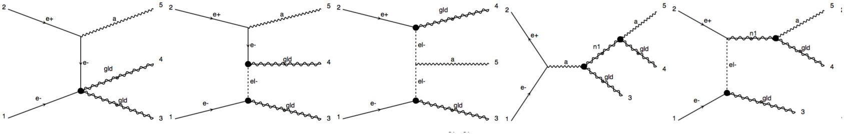


Figure: Feynman Diagrams for $e^+e^- \to \gamma\,\tilde{G}\,\tilde{G}$

$e^+e^- \to \gamma\,\tilde{G}\,\tilde{G}$

We have 5 Feynman diagrams each on them with 5 external (massless) particles, each particle have two helicity states $(\pm)$, in principle we need to compute 2^5 helicity amplitudes for each diagram.

Feynman Diagrams

	+	+	+	+
Ampb[-1, -1, -1, -1, -1]	Ampc[-1, -1, -1, -1, -1]	Ampe[-1, -1, -1, -1, -1]	A[-1, -1, -1, -1, -1]	A[-1, -1, -1, -1, -1]
Ampb[-1, -1, -1, -1, 1]	Ampc[-1, -1, -1, -1, 1]	Ampe[-1, -1, -1, -1, 1]	Ampa[-1, -1, -1, -1, 1]	A[-1, -1, -1, -1, 1]
Ampb[-1, -1, -1, 1, -1]	Ampc[-1, -1, -1, 1, -1]	Ampe[-1, -1, -1, 1, -1]	Ampa[-1, -1, -1, 1, -1]	Ampd[-1, -1, -1, 1, -1]
Ampb[-1, -1, -1, 1, 1]	Ampc[-1, -1, -1, 1, 1]	Ampe[-1, -1, -1, 1, 1]	A[-1, -1, -1, 1, 1]	Ampd[-1, -1, -1, 1, 1]
Ampb[-1, -1, 1, -1, -1]	Ampc[-1, -1, 1, -1, -1]	Ampe[-1, -1, 1, -1, -1]	A[-1, -1, 1, -1, -1]	Ampd[-1, -1, 1, -1, -1]
Ampb[-1, -1, 1, -1, 1]	Ampc[-1, -1, 1, -1, 1]	Ampe[-1, -1, 1, -1, 1]	Ampa[-1, -1, 1, -1, 1]	Ampd[-1, -1, 1, -1, 1]
Ampb[-1, -1, 1, 1, -1]	Ampc[-1, -1, 1, 1, -1]	Ampe[-1, -1, 1, 1, -1]	Ampa[-1, -1, 1, 1, -1]	A[-1, -1, 1, 1, -1]
Ampb[-1, -1, 1, 1, 1]	Ampc[-1, -1, 1, 1, 1]	Ampe[-1, -1, 1, 1, 1]	A[-1, -1, 1, 1, 1]	A[-1, -1, 1, 1, 1]
Ampb[-1, 1, -1, -1, -1]	Ampc[-1, 1, -1, -1, -1]	Ampe[-1, 1, -1, -1, -1]	Ampa[-1, 1, -1, -1, -1]	Ampd[-1, 1, -1, -1, -1]
A[-1, 1, -1, -1, 1]	A[-1, 1, -1, -1, 1]	A[-1, 1, -1, -1, 1]	Ampa[-1, 1, -1, -1, 1]	Ampd[-1, 1, -1, -1, 1]
Ampb[-1, 1, -1, 1, -1]	Ampc[-1, 1, -1, 1, -1]	Ampe[-1, 1, -1, 1, -1]	Ampa[-1, 1, -1, 1, -1]	Ampd[-1, 1, -1, 1, -1]
Ampb[-1, 1, -1, 1, 1]	Ampc[-1, 1, -1, 1, 1]	Ampe[-1, 1, -1, 1, 1]	Ampa[-1, 1, -1, 1, 1]	Ampd[-1, 1, -1, 1, 1]
Ampb[-1, 1, 1, -1, -1]	Ampc[-1, 1, 1, -1, -1]	Ampe[-1, 1, 1, -1, -1]	Ampa[-1, 1, 1, -1, -1]	Ampd[-1, 1, 1, -1, -1]
A[-1, 1, 1, -1, 1]	A[-1, 1, 1, -1, 1]	Ampe[-1, 1, 1, -1, 1]	Ampa[-1, 1, 1, -1, 1]	Ampd[-1, 1, 1, -1, 1]
Ampb[-1, 1, 1, 1, -1]	Ampc[-1, 1, 1, 1, -1]	Ampe[-1, 1, 1, 1, -1]	Ampa[-1, 1, 1, 1, -1]	Ampd[-1, 1, 1, 1, -1]
Ampb[-1, 1, 1, 1, 1]	Ampc[-1, 1, 1, 1, 1]	A[-1, 1, 1, 1, 1]	Ampa[-1, 1, 1, 1, 1]	Ampd[-1, 1, 1, 1, 1]
Ampb[1, -1, -1, -1, -1]	Ampc[1, -1, -1, -1, -1]	A[1, -1, -1, -1, -1]	Ampa[1, -1, -1, -1, -1]	Ampd[1, -1, -1, -1, -1]
Ampb[1, -1, -1, -1, 1]	Ampc[1, -1, -1, -1, 1]	Ampe[1, -1, -1, -1, 1]	Ampa[1, -1, -1, -1, 1]	Ampd[1, -1, -1, -1, 1]
A[1, -1, -1, 1, -1]	A[1, -1, -1, 1, -1]	Ampe[1, -1, -1, 1, -1]	Ampa[1, -1, -1, 1, -1]	Ampd[1, -1, -1, 1, -1]
Ampb[1, -1, -1, 1, 1]	Ampc[1, -1, -1, 1, 1]	Ampe[1, -1, -1, 1, 1]	Ampa[1, -1, -1, 1, 1]	Ampd[1, -1, -1, 1, 1]
Ampb[1, -1, 1, -1, -1]	Ampc[1, -1, 1, -1, -1]	Ampe[1, -1, 1, -1, -1]	Ampa[1, -1, 1, -1, -1]	Ampd[1, -1, 1, -1, -1]
Ampb[1, -1, 1, -1, 1]	Ampc[1, -1, 1, -1, 1]	Ampe[1, -1, 1, -1, 1]	Ampa[1, -1, 1, -1, 1]	Ampd[1, -1, 1, -1, 1]
A[1, -1, 1, 1, -1]	A[1, -1, 1, 1, -1]	A[1, -1, 1, 1, -1]	Ampa[1, -1, 1, 1, -1]	Ampd[1, -1, 1, 1, -1]
Ampb[1, -1, 1, 1, 1]	Ampc[1, -1, 1, 1, 1]	Ampe[1, -1, 1, 1, 1]	Ampa[1, -1, 1, 1, 1]	Ampd[1, -1, 1, 1, 1]
Ampb[1, 1, -1, -1, -1]	Ampc[1, 1, -1, -1, -1]	Ampe[1, 1, -1, -1, -1]	A[1, 1, -1, -1, -1]	A[1, 1, -1, -1, -1]
Ampb[1, 1, -1, -1, 1]	Ampc[1, 1, -1, -1, 1]	Ampe[1, 1, -1, -1, 1]	Ampa[1, 1, -1, -1, 1]	A[1, 1, -1, -1, 1]
Ampb[1, 1, -1, 1, -1]	Ampc[1, 1, -1, 1, -1]	Ampe[1, 1, -1, 1, -1]	Ampa[1, 1, -1, 1, -1]	Ampd[1, 1, -1, 1, -1]
Ampb[1, 1, -1, 1, 1]	Ampc[1, 1, -1, 1, 1]	Ampe[1, 1, -1, 1, 1]	A[1, 1, -1, 1, 1]	Ampd[1, 1, -1, 1, 1]
Ampb[1, 1, 1, -1, -1]	Ampc[1, 1, 1, -1, -1]	Ampe[1, 1, 1, -1, -1]	A[1, 1, 1, -1, -1]	Ampd[1, 1, 1, -1, -1]
Ampb[1, 1, 1, -1, 1]	Ampc[1, 1, 1, -1, 1]	Ampe[1, 1, 1, -1, 1]	Ampa[1, 1, 1, -1, 1]	Ampd[1, 1, 1, -1, 1]
Ampb[1, 1, 1, 1, -1]	Ampc[1, 1, 1, 1, -1]	Ampe[1, 1, 1, 1, -1]	Ampa[1, 1, 1, 1, -1]	A[1, 1, 1, 1, -1]
Ampb[1, 1, 1, 1, 1]	Ampc[1, 1, 1, 1, 1]	Ampe[1, 1, 1, 1, 1]	A[1, 1, 1, 1, 1]	A[1, 1, 1, 1, 1]

Figure: All the helicity amplitudes

$e^+e^- \to \gamma\,\tilde{G}\,\tilde{G}$

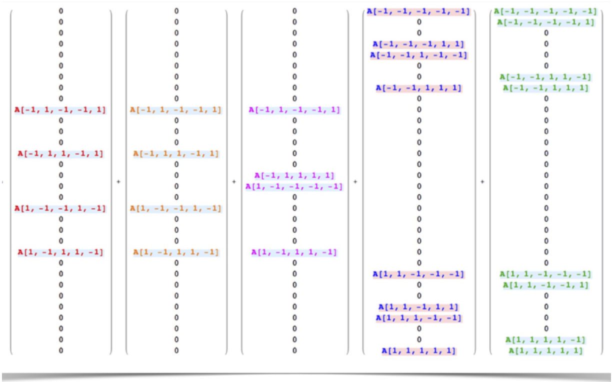
One of the several and marvelous advantages of the helicity amplitudes is that it is easy to identify the symmetries as well as the null helicity amplitudes. We already know that the terms $\langle xy]$ and $[xy\rangle$ are zero, a small program could help us to find which helicity amplitude is zero.

$$\mathcal{A}^a_{\lambda_1\lambda_1\lambda_2\lambda_3\lambda_4\lambda_5} \approx \left(\bar{v}_{\lambda_2}(2)\not{\epsilon}_{\lambda_3}(3)\not{q}u_{\lambda_1}(1)\bar{u}_{\lambda_5}(5)v_{\lambda_4}(4)\right), \quad (36)$$

$$\mathcal{A}^a_{-+--+} \approx \left(\bar{v}_+(2)\not{\epsilon}_-(3)\not{q}u_-(1)\bar{u}_+(5)v_-(4)\right), \quad (37)$$

$$\mathcal{A}^a_{-+--+} \approx [54\rangle = 0. \quad (38)$$

Counting helicity amplitudes



We started our problem with **160** helicity amplitudes, but just looking at the possible helicity states of the external particles we found that there are only **28** helicity amplitudes to compute. Applying complex conjugation we really need to compute half of the final helicity amplitudes. *At the end, just the **10%** of the work will be done and without the help of any machine if you desired*.

The total squared amplitud is as follows:

$$
\begin{aligned}
|\mathcal{M}|^2 &= \sum_{perm} |A_{\lambda_1\lambda_2\lambda_3\lambda_4\lambda_5}|^2 \\
&= 2\Big(|\mathcal{A}^i_{-+--+}|^2 + |\mathcal{A}_{-++-+}|^2 + |\mathcal{A}_{-++++}|^2 \\
&+ |\mathcal{A}_{-----}|^2 + |\mathcal{A}_{----+}|^2 + |\mathcal{A}_{---++}|^2 \\
&+ |\mathcal{A}_{--+--}|^2 + |\mathcal{A}_{--++-}|^2 + |\mathcal{A}^i_{--+++}|^2\Big) \qquad (39)
\end{aligned}
$$

Each partial squared helicity amplitude is as follows:

$$|\mathcal{A}^{i}_{-+--+}|^2 = 2\frac{s_{15}s_{34}}{s_{23}}\Big((B-2Em_{\chi 0})^2 s_{23}^2 + 4C^2 s_{24}^2 + 4(B-2Em_{\chi 0})C s_{23}s_{24}\Big) \tag{40}$$

$$|\mathcal{A}_{-++-+}|^2 = \frac{8C^2 s_{24}^2 s_{15}s_{34}}{s_{23}} \tag{41}$$

$$|\mathcal{A}_{-++++}|^2 = 8E^2 s_{34}^2 s_{12}s_{35} \tag{42}$$

$$|\mathcal{A}_{-----}|^2 = 2D^2 s_{34}^3 s_{12}s_{35} \tag{43}$$

$$|\mathcal{A}_{----+}|^2 = 2D^2 s_{34}s_{12}s_{35}m_{\chi 0}^2 \tag{44}$$

$$|\mathcal{A}_{---++}|^2 = 2A^2 s_{34} s_{25} s_{15} \tag{45}$$

$$|\mathcal{A}_{--+--}|^2 = |\mathcal{A}_{---++}|^2 \tag{46}$$

$$|\mathcal{A}_{--++-}|^2 = 2D^2 s_{34}^2 s_{12} s_{54} m_{\chi 0}^2 \tag{47}$$

$$|\mathcal{A}^i_{--+++}|^2 = 2\Big(s_{45} s_{25} s_{15} A^2 + D^2 s_{34}^3 s_{12} s_{35} - AD(s_{34} s_{45})\Big) \tag{48}$$

where we have to remember that:

$$s_{ij} = -(p_i + p_j)^2$$

Conclusions

- We **analytically** compute the total scattering amplitude for the reaction $e^+e^- \rightarrow \gamma\tilde{G}\tilde{G}$, we have found that our results match with the previous work done numerically.
- It was show that the **SHF** is a powerful method, in fact in several cases is much more economic than the traditional approach.
- We are exploring how to implement more sophisticate methods to our calculation with gravitinos, namely **KLT** and **BCFW** relations. We would like to apply this techniques to some relevant process in modern Cosmology.

Thank you

In spite of the tremendous difficulties lying ahead, I feel that S-matrix theory is far from dead and that . . . much new interesting mathematics will be created by attempting to formalize it.

"Tullio Regge"

Light Cone Decomposition (LCD)

Let $\mathbf{p}^\mu$ be any time-like 4-momentum, we can decompose it into 2 light-like four momenta as follows. Let $\mathbf{q}^\mu$ be an arbitrary light-like four momentum, and define

$$\boxed{r^\mu \equiv p^\mu - \alpha q^\mu}. \tag{49}$$

We want that r be light-like too, so we impose $r^2 = 0$; then

$$0 = (p^\mu - \alpha q^\mu)(p_\mu - \alpha q_\mu) = p^2 - 2\alpha p^\mu q_\mu + \alpha^2 q^2, \tag{50}$$

but $q^2 = 0$, therefore $\alpha = \frac{p^2}{2p\cdot q}$.

Now we can see how LCD applies to massive spinors. Remember Dirac equation:

$$\boxed{(\not{p} + m)u_s(\vec{p}) = 0}, \tag{51}$$

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when one consider the 4-component Dirac spinor in terms of two 2-component spinors

$$u = \begin{pmatrix} \chi_a \\ \xi^{\dot{a}} \end{pmatrix}, \tag{52}$$

Dirac equation is equivalent to the following system

$$p_{a\dot{a}}\xi^{\dot{a}} + m\chi_a = 0, \tag{53}$$

$$p^{\dot{a}a}\chi_a + m\xi^{\dot{a}} = 0. \tag{54}$$

We have the last two equation in such a way that is easy applies LCD, this reads as

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- $-\left(|r]_a\langle r|_{\dot{a}} + \alpha|q]_a\langle q|_{\dot{a}}\right)$

$$p_{a\dot{a}}\xi^{\dot{a}} + m\chi_a = 0 \tag{55}$$

$$p^{\dot{a}a}\chi_a + m\xi^{\dot{a}} = 0 \tag{56}$$

- $-\left(|r\rangle^{\dot{a}}[r|^a + \alpha|q\rangle^{\dot{a}}[q|^a\right)$

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We have the last two equation in such a way that is easy applies LCD, this reads as

$$\left(|r]_a\langle r|_{\dot{a}} + \alpha|q]_a\langle q|_{\dot{a}}\right)\xi^{\dot{a}} = m\chi_a \tag{57}$$

$$\left(|r\rangle^{\dot{a}}[r|^a + \alpha|q\rangle^{\dot{a}}[q|^a\right)\chi_a = m\xi^{\dot{a}} \tag{58}$$

The solutions for the last 2 equations are as follows:

$$u = \begin{pmatrix} \frac{m}{[rq]}|q]_a \\ |r\rangle^{\dot{a}} \end{pmatrix} \tag{59}$$

with the spinors $\chi_a = \frac{m}{[rq]}|q]_a$ and $\xi^{\dot{a}} = |r\rangle^{\dot{a}}$.