MODERN ASPECTS OF SCATTERING AMPLITUDES

Bryan Larios arXiv:1510.01447, arXiv:1511.07477, arXiv:1608.04129, arXiv:1612.04331v2



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- Introduction and Motivation,
- The Spinor Helicity Formalism,
- Application,
- Final Comments.

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Theory and Experiments Scattering amplitude as principal subject

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Motivation

"Scattering experiments are crucial to understand the fundamental blocks in nature".



Figure: CERN (Geneve, Switzerland)

Theory and Experiments Scattering amplitude as principal subject

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Motivation

The Standard Model of elementary particles was developed largely because scattering experiments, (the discovery the gauge bosons W^{\pm} y Z^{0} and the gluons and quarks and more recently the Higgs bosons).



Figure: Detector ATLAS-CERN

Motivation

Theory and Experiments Scattering amplitude as principal subject

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The principal physical observable in the scattering experiments is the differential cross section (DCS) $\frac{d\sigma}{d\Omega}$.

Interpretation of DATA from scattering experiments is *based* mainly in the theoretical predictions of the cross section.

It is well know (theoretically and experimentally) that **QFT** describes the elementary particles and the fundamental forces of nature.

The DCS (calculated with QFT) that *connect* theory with experiments is proportional to the modulus squared of the <u>amplitude</u>.

$$\frac{d\sigma}{d\Omega} \propto |\mathcal{A}|^2$$

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Motivation

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We have a lot of programs that undoubtedly help computing the scattering amplitudes (*MadGraph, Form, FeynRules, FeynCalc*, among others), most of them numerically. For example let me show you the computation $\tilde{t} \rightarrow b W G$ (with the LSP 3/2-spin gravitino particle):

Theory and Experiments Scattering amplitude as principal subject





Figure 1. top diagram

Figure 2. sbotom diagram

 ${\bf Figure \ 3.}\ {\rm chargino\ diagram}$

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Lorenzo Diaz-Cruz & Bryan Larios-Lopez, EPJC (2016) 76:157.

Theory and Experiments Scattering amplitude as principal subject

Considering just one channel and using Mathematica one obtain for the chargino $|\overline{\mathcal{M}_{1\chi}}|^2$.

Tr83 = trx31 + m_x (trx32 + trx33 + m_x trx34)

- $\frac{1}{3 \text{ m}_{1}^{2}} \frac{4 f_{1} \left[\left[(312 \cos(\beta) + V12 \sin(\beta))^{2} (V12 \sin(\beta) 132 \cos(\beta))^{2} \right] \left[P_{1} P_{2} + S_{1} S_{2} \right] \frac{1}{3 \text{ m}_{1}^{2}} \frac{1}{3 \text{ m}_{1}^$
- $\left(P_{1} S_{1}+P_{1} S_{1}\right)\left(\left(\text{V12} \sin(\beta)-\text{U12} \cos(\beta)\right)(\text{U12} \cos(\beta)+\text{V12} \sin(\beta))-r_{1} \Lambda_{1}\right)(f_{1})^{2}+\right.$

$$\begin{split} &\frac{1}{1+q_0^2} \left(f_1(y)(2)\cos(q_0) + V(2)\sin(q_0)^2 - (V(2)\sin(q_0) - U(2)\cos(q_0)^2) (f_1, F_1 + g, F_1) \right) - \\ &\frac{1}{1+q_0^2} \left(f_1, S_1 + F_1, S_1 \right) (V(2)\sin(q_0) - U(2)\cos(q_0) + V(2)\sin(q_0) - Y, S_1 = (f_1(f_1)^2 + g, F_2) (f_1(f_2)^2 + g, F_2) (f_1(f_2)^2 - (V(2)\sin(q_0) - U(2)\cos(q_0)^2) (f_1, F_1 + S_1) - \\ &\frac{1}{1+q_0^2} \left(2q_0^{-1} \right) (V(2)\cos(q_0) + V(2)\sin(q_0) - U(2)\cos(q_0) - U(2)\cos(q_0) - V(2)\sin(q_0) - Y, S_1 = (f_1(f_2)^2 - (f_1(f_2)^2 - (f_2)^2 - (f_2)$$

 $\frac{1}{3m_{W}^{2}}m_{V}^{2}f_{1}g_{V}^{2}[(1)\Omega^{2}\cos(\beta) + V\Omega^{2}\sin(\beta)^{2} - (V\Omega^{2}\sin(\beta) - U\Omega^{2}\cos(\beta)^{2}](P, P_{1} + \xi, \xi_{1}) - (V_{2}\cos(\beta) + V\Omega^{2}\sin(\beta) - V_{1}) + (\xi_{1})g_{1}^{2}(\xi_{1}) + (\xi_{1})g_{1}$

 $\begin{array}{c} & \\ \left(P_{1}, S_{1}+P_{1}, S_{2}\right) \left\{ (V12 \sin(g)-112 \cos(g)) \left(112 \cos(g)-V12 \sin(g)-V_{1}, S_{2}\right) \right\} \\ \left\{ R_{1}, S_{2}+P_{1}, S_{2}\right\} \left\{ (V12 \sin(g)-112 \cos(g)) \left(112 \cos(g)-V_{1}, S_{2}\right) \right\} \\ \left\{ R_{2}, S_{2}+P_{2}, S_{2}\right\} \left\{ (V12 \sin(g)-112 \cos(g)) \left(112 \cos(g)-V_{1}, S_{2}\right) \right\} \\ \left\{ R_{2}, S_{2}+P_{2}, S_{2}\right\} \left\{ (V12 \sin(g)-112 \cos(g)) \left(112 \cos(g)-V_{2}\right) \right\} \\ \left\{ R_{2}, S_{2}+P_{2}, S_{2}\right\} \\ \left\{ R_{2}, S_{2}+P_{2}, S_{2}+$

 $(P, S, + P, S_i)$ [Vi2 sin(P) - U2 cos(P) (U2 cos(P) + V2 sin(P) - $v_i A_0$] $F_i = \frac{1}{2}$, [)12 cos(P) + V2 sin(P) - $v_i A_0$] $F_i = \frac{1}{2}$, [)12 cos(P) + V2 sin(P) - (V2 sin(P) - U2 cos(P) + P + $S_i S_i$] -

 $\{P_1, S_1 + P_1, S_2\}$ [VVI2 sing[0 - UV2 energits (UV2 energits + VV2 sing[0) - r_1, S_2]] $f_1 + \frac{6}{2}f_2$](UV2 cos(β) + VV2 sing[0] - (VV2 sing[0) - UV2 cos(β) ($P_1, P_1 + S_1, S_2$) -

 $\begin{array}{l} \left\{P_{1},S_{1}+P_{1},S_{2}\right\}\left[\left(V12\sin(p)-132\cos(p)\left(132\cos(p)+V12\sin(p)-r_{1},S_{2}\right)\right]f_{1}+\\ & q_{1}^{-1}\left(\left(512\cos(p)+V12\sin(p)^{2}-(V12\sin(p)-U12\cos(p)^{2}\right)\right)(P,P_{1}+S,T_{1})-\\ \end{array}\right. \end{array}$

 $\{P_1, S_1 + P_1, S_2\}$ [V12 sin(β) – U12 em(β) (U12 em(β) + V12 sin(β) – r_1, A_0]] f_1 – m_{21}^{21} ([U12 cos(β) + V12 sin(β)² – (V12 sin(β) – U12 cos(β)²] ($P_1 P_1 + S_1 P_2$) –

 $(P, S, +P, S_i)$ [V12 m(β) - 112 m(β) (112 m(β) + V12 m(β)) - $r_i A_i$]) = $m_{\mu}^2 m_i^2$ [0.12 m(β) - V12 m(β)² - (V12 m(β) - U12 m(β)² (β , $P_i + S, T_i$) -

$$\begin{split} & \sup_{\theta} \sup_{\theta} \sup_{\theta} \left[(U(2\cos(\theta) + V(2\sin(\theta) - (V(2\sin(\theta) - U(2\cos(\theta) + V(2\sin(\theta) - v_i, h_i)) + h_i)_i - V_i + h_i)_i - (P_i, S_i + P_i, S_i) \right] (V(2\cos(\theta) - U(2\cos(\theta) + V(2\sin(\theta) - v_i, h_i))_i + \frac{h_i}{2} f_i = \sup_{\theta} (U(2\cos(\theta) + V(2\sin(\theta) - V(2\sin(\theta) - U(2\cos(\theta) + V_i))_i)_i + h_i)_i - (V_i, h_i)_i + \frac{h_i}{2} f_i = \int_{\theta} (U(2\cos(\theta) + V(2\sin(\theta) - V(2\sin(\theta) - U(2\cos(\theta) + V(2\sin(\theta) + V(2\sin(\theta$$

 $(P_1, S_1 + P_1, S_2)$ {Vi2 sin(P) - U2 cos(P) (U2 cos(P) + V2 sin(P) - $\nu_1 A_2$] + $\frac{8}{7} f_1 m_1^2$ ()U2 cos(P) + V2 sin(P)² - (V12 sin(P) - U2 cos(P)²) (P, P_1 + S_1 S_2) -

 $\frac{(P_1, S_1 + P_1, S_2)}{(P_1, S_1 + P_1, S_2)} [(V12 \sin(\beta) - U12 \cos(\beta)) (U12 \cos(\beta) + V12 \sin(\beta)) - v_1, \delta_0]] + \frac{1}{2} \frac{(V_1^{-1} \omega_{12}^{-1})}{(V_1^{-1} \omega_{12}^{-1})} [(U12 \cos(\beta) + V12 \sin(\beta)) - (U12 \cos(\beta))^{-1}](P_1, P_1 + S_1, S_1) - U12 \cos(\beta)^{-1}](P_1, P_1 + S_1) - U12 \cos(\beta)^{-1}](P_1, P_1 + S_1, S_1) - U12 \cos(\beta)^{-1}](P_1, P_1 + S_1) - U12 \cos(\beta)^{-$

 $\left(P_{i}A_{i}+P_{i}A_{j}\right)\left[\left(Vi2\sin(\beta)-Ui2\cos(\beta)\right)Ui2\cos(\beta)+Vi2\sin(\beta)-r_{i}A_{i}\right]\right]$

 $\frac{1}{2} f_2 q_2^{-2} ([(U12\cos(\beta) + V12\sin(\beta))^2 - (V12\sin(\beta) - U12\cos(\beta))^2) (P_1P_2 + S_1S_2)$

 $\left(P_{1}X_{1}+P_{1}X_{1}^{2}\right)\left(\nabla \Omega \sin(\beta)-U\Omega \cos(\beta)\left(U\Omega \cosh(\beta)-\nabla \Omega \sin(\beta)-v_{1}A_{1}\right)\right)=0$

 $-2.[(5.12 \cos(\theta) + Vi2 \sin(\theta))^2 - (Vi2 \sin(\theta) - Ui2 \cos(\theta))^2](P, P, + S, R) -$ [P.5.+P.5.] [VI2 sin(P - U2 cos(P)-U2 cos(P + V2 sin(P) - v.A.]] (V)² $\frac{1}{1-2}2\left[(512\cos(\beta) + V12\sin(\beta)^2 - (V12\sin(\beta) - U12\cos(\beta)^2)\right](P, P_1 + S, F_1)$ $\{P_1,S_1+P,S_2\}\{\{V|2\sin(\beta)-U|2\cos(\beta),|U|2\cos(\beta)+V|2\sin(\beta))-r_1,S_2\}\{(f_2)^2+$ $\frac{1}{2 \sin^2_{0} m_0^2} 2 \int_{U} [(112 \cos(\beta) + V12 \sin(\beta))^2 - (V12 \sin(\beta) - U12 \cos(\beta))^2) [P_1 P_2 + S_1 S_1] + S_2 S_2] + S_1 S_2] + S_2 S_2 S_2 + S_1 S_2 + S_2 + S_2 S_2 + S_2$ $[P_1,S_1+P_1X_1][(V|2 singR - U|2 cos(20)U|2 cos(21 + V|2 sin(20 - r_1,S_1])(A)]^2$ $-\frac{4}{2}\left[\left[U(2 \operatorname{conj} R)^{2} + V(2 \operatorname{sinj} R)^{2} - (V(2 \operatorname{sinj} R) - U(2 \operatorname{conj} R)^{2}\right](P, P) + \chi(P_{1})\right]$ (P, S + P, S) (V(2 sin(P) - U(2 cos(P))) (U(2 cos(P) + V(2 sin(P)) - r, A_1)) f_1 m^2 (60)2 cos(D + V(2 sin(D(\tilde{r} - (V(2 sin(D) - U(2 cos(D(\tilde{r}))(P, P, + S, S)) - $\{P_1, S_1 + P_1, S_2\}$ { $\{V|2 \sin qR - U|2 \cos qR + V|2 \sin qR - r_1, A_1\}$ } 4 - 6 (03)2 empth + V(2 similar) - (V(2 similar - U(2 empth/) (0, 0, + 5, 2)) $\{P, S, + P, S\}$ [(Vi2 singl) - Ui2 congle (Ui2 congl) + Vi2 single) - $r_1 A_2$] + $4\left[-\frac{1}{3\,m_{\rm p}^2\,m_{\rm s}}4\left[\right](132\,\cos(\beta+V12\,\sin(\beta))^2-(V12\,\sin(\beta)-132\,\cos(\beta))^2\right](P,P_{\rm s}+2,S_{\rm s})-122\,\cos(\beta)^2\left[(P,P_{\rm s}+2,S_{\rm s})+2(S_{\rm s}+2)\right](P,P_{\rm s}+2,S_{\rm s})\right]$ $[P_1S_1 + P_1S_2]$ [V12 single - U12 couple d112 couple + V12 single - r_1A_2] $(S_2)^2$ [P.5.+P.5.] [(V[2 sim(R - U[2 cos(R) /U[2 cos(R + V[2 sim(R) - r. A_1])(5)]² -2.6.(1)132cost@+V32cost@2-(V32cost@-U32cost@2)(P.P.+5.5.) 202.00 [2.5.+P.5.] [W2 step] - U2 cos(2) U2 cos(2 + W2 step]) - r. A. [U5]² + -2.61)102cm68 + V02m681² - (V02m68 - 02cm681²)(A.A.+5.5) -17-5 + P.5.1 (VI2 sin(R - UI2 cos(R) / UI2 cos(R + VI2 sin(R) - r. A.1)(5)² $\frac{1}{1-2}2[(112\cos(p_1^2+V12\sin(p_1^2-(V12\sin(p_1^2)-U12\cos(p_1^2)(P,P_1+S,S_1)-V12\cos(p_1^2)(P,P_1+S,S_1)-V12\cos(p_1^2)(P,P_1+S,S_1)-V12\sin(p_1^2)(P,P_1+S,S_1)-V12\cos(p_1^2)(P,P_1+S,S_1)-V12\cos(p_1^2))-V12\sin(p_1^2)(P,P_1+S,S_1)-V12\cos(p_1^$ $\{P_1,S_1+P_1,S_2\}\{(\forall 12\sin(\beta)-U12\cos(\beta))|U12\cos(\beta)+V12\sin(\beta))-\nu_1,S_2\}(1/j)\}^2$ $m_{1/2}[|U|2\cos(\beta) + V|2\sin(\beta)|^2 - (V|2\sin(\beta) - U|2\cos(\beta)|^2](P, P_1 + S, S_2)$ [P.5.+P.5.] [(V[2 sim(P] - U[2 cos(P) /U[2 cos(P] + V[2 sim(P] - r, A_2]) f. ⁴ m², ((3))2 cos(p) + V(2 sin(p)² - (V(2 sin(p) - U(2 cos(p)²))(P, P, + 5, 2))

 $-\int_{T}w_{11}\left[\left|(U_{1}^{2}\cos(\beta)+V_{2}^{2}\sin(\beta)\right|^{2}-(V_{1}^{2}\sin(\beta)-U_{2}^{2}\cos(\beta)^{2})\left(P_{1}P_{1}+S,S\right)\right]$ $(P_1S_1 + P_1S_2)(|V|2 \sin(\beta) - U|2 \cos(\beta))(U|2 \cos(\beta) + V|2 \sin(\beta)) - v_1A_2|) +$ $\frac{\alpha}{\pi} f_{1} m_{\tilde{Q}} \left[\left[3 \text{UR} \cos(\beta t + \text{VR} \sin(\beta t)^{2} - (\text{VR} \sin(\beta t) - \text{UR} \cos(\beta t)^{2}) \right] (P, P_{1} + S, S_{1} - (\text{VR} \sin(\beta t) - \text{UR} \cos(\beta t)^{2}) \right]$ $(P_1S_1 + P_1S_2)/(Vi2 \sin(\beta) - Ui2 \cos(\beta))(Ui2 \cos(\beta) + Vi2 \sin(\beta) - v_1A_2))$ $+ [(112 \cos(\beta) + Vi2 \sin(\beta))^2 - (Vi2 \sin(\beta) - Ui2 \cos(\beta))^2)(\beta; P_1 + S; S_1) +$ $(P_1, S_2 + P_1, S_2)$ [(V)[2 sim(2) - U)[2 cm(2)] (U)[2 cm(2) + V)[2 sim(2)] - v_1 A_2 [1((7))] $\frac{1}{3 m_{\odot}^2} 2 m_{\odot} \left[(132 \cos(\beta) + V12 \sin(\beta))^2 - (V12 \sin(\beta) - U12 \cos(\beta))^2 (\beta, F_1 + 5, F_1) + 5 \beta + 5 \beta$ $-2f_1[(U2\cos(\beta) + V12\sin(\beta))^2 - (V12\sin(\beta) - U12\cos(\beta))^2)[F_1F_1 + S_1S_2] +$ -2 /- 1/102 cost(0 + V(2 sint(0)² - (V(2 sint(0) - U(2 cost(0)²))(0, 0, +5, 5))+ 3 m_2 m_2 (P.5 + P.5.)(V2 sinch - U2 cost/h) (U2 cost/h + V2 sinch - v. A. [)(6)? -24(1032 cost(0) + Vi2 sin(0)² - (Vi2 sin(0) - Ui2 cost(0)²)(P, P, + 5, S,) + (P.5. + P.5.)/(V[2sin(2) - U[2cos(2))(U[2cos(3) + V[2sin(2) - y, A_2])(6)^2 $\frac{8}{2}m_{\tilde{h}}[[3Ul2\cos(p)+Vl2\sin(p)]^2 - (Vl2\sin(p)-Ul2\cos(p)^2)[P, P_1 + S, S_1] +$ (P.5.+P.5)/(V/2 sin(2) - U/2 cos(2))(U/2 cos(2) + V/2 sin(2)) - y, A, T(5.- $-m_{1}^{2}$ [[UI2 cm(β + VI2 sin(β ()² - (VI2 sin(β) - UI2 cm(β ()²) [$P, P_{1} + S, S_{2}$] + $(P, S_i + P, S_i)(|\nabla i2 \operatorname{single} - Ui2 \operatorname{cost}(l) | (Ui2 \operatorname{cost}(l) + \nabla i2 \operatorname{single}) - v_j | S_i |) = (P, S_i + P, S_i)(|\nabla i2 \operatorname{single}) - v_j | S_i |)$ $-m_{H}^{2}m_{\Lambda}(\beta)U(2\cos(\beta) + V(2\sin(\beta))^{2} - (V(2\sin(\beta) - U(2\cos(\beta))^{2})(P, P_{1} + S, S_{1}) +$ $(P_1A_1 + P_1S_2)$ $(Vi2 single - Ui2 cost(0) (Ui2 cost(0) + Vi2 single) - v_2A_2)$

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$$\begin{split} & \frac{1}{2} \int d_{1} w_{0}\left(||U||^{2} \cos(||t| + V||^{2} \sin(|t|^{2} - (V||^{2} \sin(|t|) - U||^{2} \cos(|t|^{2} + ||t|) + (t_{1}^{2} - t_{1}^{2} - t_{2}^{2} - t_{1}^{2} - t_{1}^{2}$$

(P.5. + P.5.23(V)2 sin(R - U)2 cos(R)(U)2 cos(R + V)2 sin(R) - v; A₂[]

(P.S.+P.S.)(V/2 similt - U/2 cost (0) (U/2 cost (0) + V/2 similt) - v. A.())-

 $(P_1S_1 + P_1S_2)(Vi2 \operatorname{single} - Ui2 \operatorname{cosc}(0)(Ui2 \operatorname{cosc}(0) + Vi2 \operatorname{single}) - v_2S_2))$

 $= m_{1}^{2} m_{2} \left[(U12 \cos(\beta) + V12 \sin(\beta))^{2} - (V12 \sin(\beta) - U12 \cos(\beta))^{2} \right] (P_{1}P_{1} + S, S_{1})^{2}$

Pure YM theory

If we consider a pure Yang - Mills theory, computing the scattering amplitude (modulus square) at three level i.e. for 5 gluons, we have:



$$\rightarrow g f^{abc} [(p_1 - p_2)_{\rho} \eta_{\mu\nu} + (p_2 - p_3)_{\mu} \eta_{\nu\rho} + (p_3 - p_1)_{\nu} \eta_{\rho\mu}]$$

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After brute force computing, part of the modulus square SA is shown



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HEP Seminar IF-UNAM

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But using modern techniques, we obtain a very simple and compact expression for the 5 gluons HA

$$\begin{aligned} A_5(1^{\pm}, 2^+, 3^+, 4^+, 5^+) &= 0\\ A_5(1^-, 2^-, 3^+, 4^+, 5^+) &= i \frac{\langle 1 \, 2 \rangle^4}{\langle 1 \, 2 \rangle \, \langle 2 \, 3 \rangle \, \langle 3 \, 4 \rangle \, \langle 4 \, 5 \rangle \, \langle 5 \, 1 \rangle} \end{aligned}$$

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The spinor helicity formalism (SHF) (a pragmatic point of view) Notation SHF for massive particles

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Spinor Helicity Formalism (a pragmatic point of view)



Outline Introduction and Motivation Spinor Helicity Formalism (SHF) (a pragmatic point of view) Applications Gravitiino Monophoton signal in LSP gravitino production Conclusions

The SHF is based in the following observation:

Fields with spin-1 transform in the $(\frac{1}{2}, \frac{1}{2})$ representation of the Lorentz group.

So we are able to express 4-momentum of any particle as a biespinor: $p_{\mu} \rightarrow p_{a \dot{a}}$

$$p_{a\dot{a}} = p_{\mu}\sigma^{\mu}_{a\dot{a}}$$

$$\gamma^{\mu} = \begin{pmatrix} 0 & \sigma^{\mu} \\ \bar{\sigma}^{\mu} & 0 \end{pmatrix}$$

$$\sigma^{\mu}_{a\dot{a}} = (I, \vec{\sigma}), \ \bar{\sigma}^{\mu\dot{a}a} = (I, -\vec{\sigma})$$

$$(1)$$

4-component spinors

4-Component Dirac spinors $(u_s(\vec{p}) \text{ and } v_s(\vec{p}))$ are solutions of the following EOMs:

$$(p + m)u_s(\vec{p}) = 0,$$
 (2)
 $(-p + m)v_s(\vec{p}) = 0,$ (3)

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with $s = \pm$.

Massless Case (textbooks)

For massless Dirac spinors we know that the following equation exist: $u_s(\vec{p})\bar{u}_s(\vec{p}) = \frac{1}{2}(1+s\gamma_5)(-\not p)$, for instance if s = -, we have:

$$u_{-}(\vec{p})\bar{u}_{-}(\vec{p}) = \frac{1}{2}(1-\gamma_{5})(-p) = \begin{pmatrix} 0 & -p_{a\dot{a}} \\ 0 & 0 \end{pmatrix}$$

where

$$u_{-}(\vec{p}) = \left(\begin{array}{c} \phi_a \\ 0 \end{array}\right)$$

 ϕ_a is a 2-component numerical spinor,

also $\bar{u}_{-}(\vec{p}) = (0, \phi_{\dot{a}}^{*})$, all this lead to

$$p_{a\dot{a}}$$
 = $-\phi_a\phi^*_{\dot{a}}$

Outline Introduction and Motivation Spinor Helicity Formalism (SHF) (a pragmatic point of view) Applications Gravitino Monophoton signal in LSP gravitino production Conclusions

The key of the SHF is to considerate ϕ_a as the fundamental object and express the momenta of the particles in terms of ϕ_a .

Highly convenient and powerful notation

If *p* and *k* are the momenta and ϕ_a , κ_a their associated spinors, we can define the following products of spinors:

$$[pk] = \phi^a \kappa_a = \bar{u}_+(\vec{p})u_-(\vec{k}) = -[kp]$$
(4)

(5)

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$$\langle pk \rangle = \phi_{\dot{a}}^* \kappa^{*\dot{a}} = \bar{u}_-(\vec{p})u_+(\vec{k}) = -\langle kp \rangle$$

The spinor helicity formalism (SHF) (a pragmatic point of view) Notation SHF for massive particles

Notation for 4-components Dirac Spinor

$$\begin{split} |p] &= u_{-}(\mathbf{p}) = v_{+}(\mathbf{p}) ,\\ |p\rangle &= u_{+}(\mathbf{p}) = v_{-}(\mathbf{p}) ,\\ [p| &= \overline{u}_{+}(\mathbf{p}) = \overline{v}_{-}(\mathbf{p}) ,\\ \langle p| &= \overline{u}_{-}(\mathbf{p}) = \overline{v}_{+}(\mathbf{p}) . \end{split}$$

where spinor products as $\bar{u}_s(\vec{p}) u_s(\vec{k}) = 0 \forall s = \pm$, or more nice, we have:

$$\langle pk] = [pk\rangle = 0 \tag{6}$$

Polarizations 4-vectores

If we want to apply the SHF to spinor electrodynamics, we need to write polarization vectors in term of 2-component numerical spinors.

$$\begin{split} \varepsilon^{\mu}_{+}(k) &= -\frac{\langle q|\gamma^{\mu}|k]}{\sqrt{2}\,\langle q\,k\rangle} \;, \\ \varepsilon^{\mu}_{-}(k) &= -\frac{[q|\gamma^{\mu}|k\rangle}{\sqrt{2}\,[q\,k]} \;, \end{split}$$

where they satisfy $p \cdot \mathcal{E}_s = 0$ and $\mathcal{E}_s^2 = 1 \ \forall \ s = \pm$.

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• The contraction with γ matrix is as follows: :

$$\pounds_{+}(k,q) = \frac{1}{\sqrt{2}\langle qk \rangle} \langle q|\gamma^{\mu}|k] \gamma_{\mu} = \frac{2}{\sqrt{2}} (|k] \langle q| + |q\rangle [k|)$$

$$= \frac{\sqrt{2}}{\langle qk \rangle} (|k] \langle q| + |q\rangle [k|), \qquad (7)$$

$$\pounds_{-}(k,q) = \frac{1}{\sqrt{2}[qk]} [q|\gamma^{\mu}|k\rangle \gamma_{\mu} = \frac{1}{\sqrt{2}[qk]} \langle k|\gamma^{\mu}|q] \gamma_{\mu}$$

$$= \frac{\sqrt{2}}{[qk]} (|k] \langle q| + |q\rangle [k|). \qquad (8)$$

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Some of the most important formulas that are needed to compute scattering amplitudes :



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What about massive particles ?

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SHF for massive particles

The SHF is a highly convenient and powerful notational tool for amplitudes of massless particles in 4-dimensional theories. But we would like to calculate amplitudes taking into account the mass of the particles.

All the massive spinors are as follows

$$\begin{split} u_{-} &= |r] + \frac{m}{\langle rq \rangle} |q \rangle \quad , \quad u_{+} = \frac{m}{[rq]} |q] + |r \rangle; \\ v_{+} &= |r] - \frac{m}{\langle rq \rangle} |q \rangle \quad , \quad v_{-} = -\frac{m}{[rq]} |q] + |r \rangle; \\ \bar{u}_{-} &= \frac{m}{[qr]} [q] + \langle r | \quad , \quad \bar{u}_{+} = [r] + \frac{m}{\langle qr \rangle} \langle q |; \\ \bar{v}_{+} &= -\frac{m}{[qr]} [q] + \langle r | \quad , \quad \bar{v}_{-} = [r] - \frac{m}{\langle qr \rangle} \langle q |. \end{split}$$

In order to apply the SHF (massive or massless) to the standard model of particle physics, we just need to have the massive polarization vector in term of 2-component numerical spinors.

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We already know the massive polarization vector in terms of spinors.

$$\begin{split} \epsilon^{\mu}_{+} &= \frac{\langle q | \gamma^{\mu} | r]}{\sqrt{2} \langle rq \rangle}, \\ \epsilon^{\mu}_{-} &= \frac{\langle r | \gamma^{\mu} | q]}{\sqrt{2} [qr]}, \end{split}$$

$$\begin{split} \epsilon_0^\mu &= \frac{1}{2m} \left(\langle r | \gamma^\mu | r] - \alpha \langle q | \gamma^\mu | q] \right) \\ &= \frac{1}{m} r^\mu + \frac{m}{2p \cdot q} q^\mu. \end{split}$$

Let us see some examples.

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Computing Scattering Amplitudes

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2-body Higgs decay $h(p_1) \rightarrow f(p_2)f(p_3)$

From the Feynman diagram we get the helicity amplitudes (HAs) (as is usual)



$$\mathcal{M}_{\lambda_2\lambda_3}(p_1,p_2,p_3) = \frac{m_f}{v} \bar{u}_{\lambda_2}(p_2) v_{\lambda_3}(p_3),$$

(9

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$$\begin{array}{|c|c|c|c|c|c|c|}\hline \lambda_2 \lambda_3 & \mathcal{M}_{\lambda_2 \lambda_3} \\ \hline & -- & \frac{m_f}{v[q_2 q_3]}(s_{q_2 q_3} - m_f^2) \\ \hline & ++ & \frac{m_f}{v(q_2 q_3)}(s_{q_2 q_3} - m_f^2) \\ \hline \end{array}$$

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2-body Higgs decay $h(p_1) \rightarrow f(p_2)f(p_3)$

Let me compute these HAs step by step

•
$$\bar{u}_{-}(p_2), \bar{u}_{+}(p_2)$$

 $\mathcal{M}_{\lambda_2\lambda_3}(p_1, p_2, p_3) = \frac{m_f}{v} \bar{u}_{\lambda_2}(p_2) v_{\lambda_3}(p_3)$ (10)
• $v_{-}(p_3), v_{+}(p_3)$
We see that there are 4 HAs, these are -+, --, +-, ++ (In the massless case there are just 2, +- and -+).

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 $\begin{array}{l} \textbf{2-body Higgs decay } h(p_1) \rightarrow f(p_2)\bar{f}(p_3) \\ \textbf{3} & 2\text{-body } Z \text{ boson decay } Z(p_1) \rightarrow f(p_2)\bar{f}(p_3) \\ \textbf{3-body Muon Decay } \mu(p_1) \rightarrow \bar{\nu}_{e^-}(p_2) \, \nu_{\mu}(p_3) \, e^-(p_4) \end{array}$

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2-body Higgs decay $h(p_1) \rightarrow f(p_2)f(p_3)$

$$\mathcal{M}_{--} = \frac{m_f}{v} \bar{u}_{-}(2) v_{-}(3) \tag{11}$$

$$= \frac{m_f}{v} \left(\frac{m_f}{[q_2 r_2]} [q_2| + \langle r_2| \right) \left(-\frac{m_f}{[r_3 q_3]} |q_3] + |r_3 \rangle \right) \tag{12}$$

$$= \frac{m_f}{v} \left(-\frac{m_f^2 [q_2 q_3]}{[q_2 r_2] [r_3 q_3]} + \frac{m_f}{[q_2 r_2]} [q_2 r_3) - \frac{0}{[r_3 q_3]} - \frac{0}{[r_3 q_3]} + \langle r_2 r_3 \rangle \right) \tag{13}$$

$$= \frac{m_f}{v} \left(-\frac{m_f^2 [q_2 q_3]}{[q_2 r_2] [r_3 q_3]} + \langle r_2 r_3 \rangle \right) \tag{14}$$

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3-body Muon Decay $\mu(p_1) \rightarrow \bar{\nu}_{e^-}(p_2) \nu_{\mu}(p_3) e^-(p_4)$

For this special example we will use simultaneous LCD (SLCD),

$$p_{2} = r_{2} - \frac{m_{f}^{2}}{2r_{2} \cdot q_{2}}q_{2}$$
(15)
$$p_{3} = r_{3} - \frac{m_{f}^{2}}{2r_{3} \cdot q_{3}}q_{3}$$
(16)

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For this special example we will use simultaneous LCD (SLCD),

$$p_{2} = r_{2} - \frac{m_{f}^{2}}{2r_{2} \cdot q_{2}}q_{2}$$
(17)
$$p_{3} = q_{2} - \frac{m_{f}^{2}}{2q_{2} \cdot r_{2}}r_{2}$$
(18)

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We choose $r_3 = q_2$ and $q_3 = r_2$, this will reduce our calculations.

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2-body Higgs decay $h(p_1) \rightarrow f(p_2)f(p_3)$

Returning to the HA \mathcal{M}_{--} and taking into account SLCD, we have

$$\mathcal{M}_{--} = \frac{m_f}{v} \left(-\frac{m_f^2 [q_2 r_2]}{[q_2 r_2]} + \langle r_2 q_2 \rangle \right)$$
(19)
$$= \frac{m_f}{v} \left(-\frac{m_f^2}{[q_2 r_2]} + \langle r_2 q_2 \rangle \right) = \frac{m_f}{v [q_2 r_2]} (-m_f^2 + \langle r_2 q_2 \rangle [q_2 r_2])$$
(20)
$$= \frac{m_f}{v [q_2 r_2]} (-m_f^2 + s_{q_2 r_2})$$
(21)

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We do not need to compute \mathcal{M}_{++} , we just complex conjugate \mathcal{M}_{--} (one of the greatest advantages of the SHF).

$$\mathcal{M}_{++} = \frac{m_f}{v \langle q_2 r_2 \rangle} (-m_f^2 + s_{q_2 r_2})$$
(22)

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As you can guess we shall compute just one HA (\mathcal{M}_{-+})

$$\mathcal{M}_{-+} = \frac{m_f}{v} \bar{u}_{-}(2) v_{+}(3) = \frac{m_f}{v} \left(\frac{m_f}{[q_2 r_2]} [q_2| + \langle r_2| \right) \left(|q_2] - \frac{m_f}{\langle r_2 q_2 \rangle} |r_2\rangle \right)$$
(23)
$$\propto [q_2 q_2]^{\bullet 0} [q_2 r_2)^{\bullet 0} + \langle r_2 q_2]^{\bullet 0} (r_2 r_2)^{\bullet 0}$$

$$= 0$$
(24)

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Finally the averaged squared amplitude is then:

$$\langle |\mathcal{M}|^2 \rangle = 2|\mathcal{M}_{--}|^2 = \frac{2m_f^2}{v^2 s_{q_2 q_3}} (s_{q_2 q_3} - m_f^2)^2 = \frac{y^2}{v^2} (1 - 4y^2),$$
(26)

with $y = \frac{m_f}{M_h}$. Then the decay width Γ goes as follows

$$\Gamma(h \to f\bar{f}) = \frac{\alpha_W M_h y^2}{8} (1 - 4y^2)^{3/2}.$$

(27)

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2-body Z boson decay $Z(p_1) \rightarrow f(p_2)\overline{f(p_3)}$

Again from the Feynman diagram we get the helicity amplitudes

$$\mathcal{M} = \frac{1}{2}g_Z \epsilon_\mu(p_1)\bar{u}(p_2)\gamma^\mu(v_f - a_f\gamma^5)v(p_3)$$
(28)

$\lambda_1\lambda_2\lambda_3$	$\mathcal{M}_{\lambda_1\lambda_2\lambda_3}$
++-	$\frac{1}{2}g_Z \frac{\langle p_2 \gamma_\mu r_1 \rangle}{\sqrt{2} \langle r_1 p_2 \rangle} (v_f - a_f) [p_2 \gamma^\mu p_3 \rangle = \frac{g_Z (v_f - a_f)}{\sqrt{2}} \frac{\langle p_2 p_3 \rangle [r_1 p_2]}{\langle r_1 p_2 \rangle}$
0+-	$\frac{1}{2}g_Z\left(\frac{1}{M}r_{1\mu} + \frac{M}{2p_{12}}p_{2\mu}\right)(v_f - a_f)[p_2 \gamma^{\mu} p_3\rangle = \frac{g_Z(v_f - a_f)}{\sqrt{2}M}\langle r_1p_3\rangle[r_1p_2] = 0$
-+-	$rac{1}{2}g_Zrac{\langle r_1 \gamma_\mu p_2]}{\sqrt{2}[p_2r_1]}(v_f-a_f)[p_2 \gamma^\mu p_3 angle=0$

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Finally the squared averaged scattering amplitude is as follows

$$\langle |\mathcal{M}|^2 \rangle = \frac{1}{3} \left(|\mathcal{M}_{++-}|^2 + |\mathcal{M}_{--+}|^2 \right) = \frac{g_Z^2 M^2}{3} \left(|v_f|^2 + |a_f|^2 \right)$$
(29)

The decay width for this channel is then:

$$\Gamma(Z \to f\bar{f}) = \frac{g_Z^2 M}{48\pi} \left(|v_f|^2 + |a_f|^2 \right).$$
 (30)

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3-body Muon Decay $\mu(p_1) \rightarrow \overline{\nu}_{e^-}(p_2) \nu_{\mu}(p_3) e^-(p_4)$

From the Feynman diagram we get the helicity amplitudes (HAs) (as is usual)

$$\mu \xrightarrow{\nu_e} e \qquad \mathcal{M}_{\lambda_1 \lambda_2 \lambda_3 \lambda_4} = \left(\frac{g_W}{\sqrt{8}M_W}\right)^2 \left[\bar{u}_{\lambda_3}(p_3)\gamma^{\mu}(1-\gamma_5)u_{\lambda_1}(p_1)\right] \left[\bar{u}_{\lambda_4}(p_4)\gamma_{\mu}(1-\gamma_5)v_{\lambda_2}(p_2)\right],$$
$$= \left(\frac{g_W}{\sqrt{8}M_W}\right)^2 \mathcal{A}^{\mu}_{\lambda_3 \lambda_1} \mathcal{B}_{\mu \lambda_4 \lambda_2},$$

$\lambda_1\lambda_2\lambda_3\lambda_4$	$\mathcal{A}^{\mu\lambda_{3}\lambda_{1}}$	$\mathcal{B}^{\lambda_4\lambda_2}_\mu$	${\cal A}^{\mu\lambda_3\lambda_1}{\cal B}^{\lambda_4\lambda_2}_\mu$	\mathcal{M} $(p_2 = q_1, p_3 = q_4)$
+-+-	$2[p_3 \gamma^\mu r_1\rangle$	$2[r_4 \gamma_\mu p_2\rangle$	$4\langle p_2r_1 angle\langle p_3r_4 angle$	$\left(rac{g_W}{\sqrt{2}m_\mu} ight)^2 \langle p_2 r_1 angle [p_3 r_4]$
+++-	$2[p_3 \gamma^{\mu} r_1\rangle$	$\frac{2m_e}{[q_4r_4]}[q_4 \gamma_\mu p_2\rangle$	$4 \frac{m_e}{[q_4 r_4]} \langle p_2 r_1 \rangle \langle p_3 q_4 \rangle$	0
++	$\frac{2m_{\mu}}{\langle r_1 q_1 \rangle} [p_3 \gamma^{\mu} q_1 \rangle$	$2[r_4 \gamma_\mu p_2 angle$	$4 \frac{m_{\mu}}{\langle r_1 q_1 \rangle} \langle p_2 q_1 \rangle [p_3 r_4]$	0
-++-	$\frac{2m_{\mu}}{\langle r_1q_1 \rangle} [p_3 \gamma^{\mu} q_1 \rangle$	$\frac{2m_e}{[q_4r_4]}[q_4 \gamma_\mu p_2\rangle$	$4 \frac{m_{\mu}m_{e}}{\langle r_{1}q_{1}\rangle[q_{4}r_{4}]} \langle p_{2}q_{1}\rangle[p_{3}q_{4}]$	0

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The squared and averaged amplitude for the muon decay is:

$$\langle |\mathcal{M}^{+-+-}|^2 \rangle = \frac{1}{2} |\mathcal{M}^{+-+-}|^2 = 2 \left(\frac{g_W}{M_W}\right)^4 (p_1 \cdot p_2) (p_3 \cdot p_4)$$
(31)
This result we can arrive to the decay width, which agrees

From this result we can arrive to the decay width, which agrees with result of textbooks.

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Gravitino wave functions

We will consider a model in a local supersymmetric field theory ($\mathcal{N} = 1$ SUGRA) where the gravitino is the lightest supersymmetric particle (LSP), therefore a good DM candidate.

$$\begin{split} \tilde{\Psi}^{\mu}_{++}(p) &= \epsilon^{\mu}_{+}(p)u_{+}(p), \\ \tilde{\Psi}^{\mu}_{--}(p) &= \epsilon^{\mu}_{-}(p)u_{-}(p) \\ \tilde{\Psi}^{\mu}_{+}(p) &= \sqrt{\frac{2}{3}}\epsilon^{\mu}_{0}(p)u_{+}(p) + \frac{1}{\sqrt{3}}\epsilon^{\mu}_{+}(p)u_{-}(p), \\ \tilde{\Psi}^{\mu}_{-}(p) &= \sqrt{\frac{2}{3}}\epsilon^{\mu}_{0}(p)u_{-}(p) + \frac{1}{\sqrt{3}}\epsilon^{\mu}_{-}(p)u_{+}(p). \end{split}$$

Two body NLSP Stop decay $(\tilde{t} \rightarrow t \, \tilde{G})$

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$$\begin{array}{c} \text{Outline} \\ \text{Introduction and Motivation} \\ \text{Spinor Helicity Formalism} \\ \text{Applications} \\ \text{Gravitino} \\ \text{Monophoton signal in LSP gravitino production} \\ \text{Conclusions} \end{array} \\ \begin{array}{c} \text{Two body NLSP Stop decay } (\tilde{t} \rightarrow t \ \hat{G}) \\ \text{Two body NLSP Stop decay } (\tilde{t} \rightarrow t \ \hat{G}) \\ \text{Two body NLSP Stop decay } (\tilde{t} \rightarrow t \ \hat{G}) \\ \text{Two body NLSP Stop decay } (\tilde{t} \rightarrow t \ \hat{G}) \\ \text{Two body NLSP Stop decay } (\tilde{t} \rightarrow t \ \hat{G}) \\ \text{Two body NLSP Stop decay } (\tilde{t} \rightarrow t \ \hat{G}) \\ \text{Two body NLSP Stop decay } (\tilde{t} \rightarrow t \ \hat{G}) \\ \text{Two body NLSP Stop decay } (\tilde{t} \rightarrow t \ \hat{G}) \\ \text{Two body NLSP Stop decay } (\tilde{t} \rightarrow t \ \hat{G}) \\ \text{Two body NLSP Stop decay } (\tilde{t} \rightarrow t \ \hat{G}) \\ \text{Two body NLSP Stop decay } (\tilde{t} \rightarrow t \ \hat{G}) \\ \text{Two body NLSP Stop decay } (\tilde{t} \rightarrow t \ \hat{G}) \\ \text{Two body NLSP Stop decay } (\tilde{t} \rightarrow t \ \hat{G}) \\ \text{Two body NLSP Stop decay } (\tilde{t} \rightarrow t \ \hat{G}) \\ \text{Two body NLSP Stop decay } (\tilde{t} \rightarrow t \ \hat{G}) \\ \text{Two body NLSP Stop decay } (\tilde{t} \rightarrow t \ \hat{G}) \\ \text{Two body NLSP Stop decay } (\tilde{t} \rightarrow t \ \hat{G}) \\ \text{Two body NLSP Stop decay } (\tilde{t} \rightarrow t \ \hat{G}) \\ \text{Two body NLSP Stop decay } (\tilde{t} \rightarrow t \ \hat{G}) \\ \text{Two body NLSP Stop decay } (\tilde{t} \rightarrow t \ \hat{G}) \\ \text{Two body NLSP Stop decay } (\tilde{t} \rightarrow t \ \hat{G}) \\ \text{Two body NLSP Stop decay } (\tilde{t} \rightarrow t \ \hat{G}) \\ \text{Two body NLSP Stop decay } (\tilde{t} \rightarrow t \ \hat{G}) \\ \text{Two body NLSP Stop decay } (\tilde{t} \rightarrow t \ \hat{G}) \\ \text{Two body NLSP Stop decay } (\tilde{t} \rightarrow t \ \hat{G}) \\ \text{Two body NLSP Stop decay } (\tilde{t} \rightarrow t \ \hat{G}) \\ \text{Two body NLSP Stop decay } (\tilde{t} \rightarrow t \ \hat{G}) \\ \text{Two body NLSP Stop decay } (\tilde{t} \rightarrow t \ \hat{G}) \\ \text{Two body NLSP Stop decay } (\tilde{t} \rightarrow t \ \hat{G}) \\ \text{Two body NLSP Stop decay } (\tilde{t} \rightarrow t \ \hat{G}) \\ \text{Two body NLSP Stop decay } (\tilde{t} \rightarrow t \ \hat{G}) \\ \text{Two body NLSP Stop decay } (\tilde{t} \rightarrow t \ \hat{G}) \\ \text{Two body NLSP Stop decay } (\tilde{t} \rightarrow t \ \hat{G}) \\ \text{Two body NLSP Stop decay } (\tilde{t} \rightarrow t \ \hat{G}) \\ \text{Two body NLSP Stop decay } (\tilde{t} \rightarrow t \ \hat{G}) \\ \text{Two body NLSP Stop decay } (\tilde{t} \rightarrow t \ \hat{G}) \\ \text{Two body NLSP Stop decay } (\tilde{t} \rightarrow t \ \hat{G}) \\ \text{Two body NLSP Stop decay } (\tilde{t} \rightarrow t \$$

Squaring the helicity amplitudes of the last Table, we obtain the following expression for the total squared amplitude

$$\langle |\mathcal{M}|^2 \rangle = |\mathcal{M}_{-+}|^2 + |\mathcal{M}_{+-}|^2, \tag{33}$$

$$=\frac{\left(s_{r_{2}q_{2}}^{4}-(m_{t}m_{\tilde{G}})^{4}\right)\left(s_{r_{2}q_{2}}^{2}-(m_{t}m_{\tilde{G}})^{2}\right)}{3M^{2}m_{\tilde{G}}^{2}s_{r_{2}q_{2}}^{3}},$$
(34)

$$=\frac{(m_{\tilde{t}}^2-m_{\tilde{G}}^2-m_t^2)\left((m_{\tilde{t}}^2-m_t^2-m_{\tilde{G}}^2)^2-4m_t^2m_{\tilde{G}}^2\right)}{3M^2m_{\tilde{G}}^2},$$
 (35)

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To appreciate the power and efficiency of the SHF, we can remember the completeness relation for the spin-3/2 Gravitino Field.

$$\begin{split} \sum_{\tilde{\lambda}=1}^{3} \Psi_{\mu}(\vec{p}_{1},\tilde{\lambda})\overline{\Psi}_{\nu}(\vec{p}_{1},\tilde{\lambda}) &= -(\not\!\!\!p_{1}+m_{\tilde{G}}) \times \left\{ \left(g_{\mu\nu}-\frac{p_{\mu}p_{\nu}}{m_{\tilde{G}}^{2}}\right) \\ &-\frac{1}{3} \left(g_{\mu\sigma}-\frac{p_{\mu}p_{\sigma}}{m_{\tilde{G}}^{2}}\right) \left(g_{\nu\lambda}-\frac{p_{\nu}p_{\lambda}}{m_{\tilde{G}}^{2}}\right) \gamma^{\sigma} \gamma^{\lambda} \right\} \end{split}$$

And we still need to take into account other fields to compute the trace.

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 $e^+e^- \rightarrow \gamma \, G \, G$

Using the SUSY QED model constructed by Mawatari and Oexl (arXiv:1402.3223v2), and applying the SHF we will compute the scattering amplitude for the $e^+e^- \rightarrow \gamma \tilde{G} \tilde{G}$ reaction. It is important to mention that the cross sections for this reaction has been computed numerically, so it is interesting have an analytical result.

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Feynman Diagrams



Figure: Feynman Diagrams for $e^+e^- \rightarrow \gamma \tilde{G} \tilde{G}$

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We have 5 Feynman diagrams each on them with 5 external (massless) particles, each particle have two helicity states (\pm) , in principle we need to compute 2^5 helicity amplitudes for each diagram.

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Feynman Diagrams

	Ampb[-1, -1, -1, -1, -1]	<u> </u>	Ampc[-1, -1, -1, -1, -1]	1	Ampe[-1, -1, -1, -1]	1	(A[-1, -1, -1, -1, -1]	1	[A[-1, -1, -1, -1, -1]	1
	Ampb[-1, -1, -1, -1, 1]		Ampc[-1, -1, -1, -1, 1]		Ampe[-1, -1, -1, -1, 1]		Ampa[-1, -1, -1, -1, 1]		A[-1, -1, -1, -1, 1]	
	Ampb[-1, -1, -1, 1, -1]		Ampc[-1, -1, -1, 1, -1]		Ampe[-1, -1, -1, 1, -1]		Ampa[-1, -1, -1, 1, -1]		Ampd[-1, -1, -1, 1, -1]	
	Ampb[-1, -1, -1, 1, 1]		Ampc[-1, -1, -1, 1, 1]		Ampe[-1, -1, -1, 1, 1]		A[-1, -1, -1, 1, 1]		Ampd[-1, -1, -1, 1, 1]	
	Ampb[-1, -1, 1, -1, -1]		Ampc[-1, -1, 1, -1, -1]		Ampe[-1, -1, 1, -1, -1]		A[-1, -1, 1, -1, -1]		Ampd[-1, -1, 1, -1, -1]	
	Ampb[-1, -1, 1, -1, 1]		Ampc[-1, -1, 1, -1, 1]		Ampe[-1, -1, 1, -1, 1]		Ampa[-1, -1, 1, -1, 1]		Ampd[-1, -1, 1, -1, 1]	
	Ampb[-1, -1, 1, 1, -1]		Ampc[-1, -1, 1, 1, -1]		Ampe[-1, -1, 1, 1, -1]		Ampa[-1, -1, 1, 1, -1]		A[-1, -1, 1, 1, -1]	
	Ampb[-1, -1, 1, 1, 1]		Ampc[-1, -1, 1, 1, 1]		Ampe[-1, -1, 1, 1, 1]		A[-1, -1, 1, 1, 1]		A[-1, -1, 1, 1, 1]	
	Ampb[-1, 1, -1, -1, -1]		Ampc[-1, 1, -1, -1, -1]		Ampe[-1, 1, -1, -1, -1]		Ampa[-1, 1, -1, -1, -1]		Ampd[-1, 1, -1, -1, -1]	
	A[-1, 1, -1, -1, 1]		A[-1, 1, -1, -1, 1]		A[-1, 1, -1, -1, 1]		Ampa[-1, 1, -1, -1, 1]		Ampd[-1, 1, -1, -1, 1]	
	Ampb[-1, 1, -1, 1, -1]		Ampc[-1, 1, -1, 1, -1]		Ampe[-1, 1, -1, 1, -1]		Ampa[-1, 1, -1, 1, -1]		Ampd[-1, 1, -1, 1, -1]	
	Ampb[-1, 1, -1, 1, 1]		Ampc[-1, 1, -1, 1, 1]		Ampe[-1, 1, -1, 1, 1]		Ampa[-1, 1, -1, 1, 1]		Ampd[-1, 1, -1, 1, 1]	
	Ampb[-1, 1, 1, -1, -1]		Ampc[-1, 1, 1, -1, -1]		Ampe[-1, 1, 1, -1, -1]		Ampa[-1, 1, 1, -1, -1]		Ampd[-1, 1, 1, -1, -1]	
	A[-1, 1, 1, -1, 1]		A[-1, 1, 1, -1, 1]		Ampe[-1, 1, 1, -1, 1]		Ampa[-1, 1, 1, -1, 1]		Ampd[-1, 1, 1, -1, 1]	
	Ampb[-1, 1, 1, 1, -1]		Ampc[-1, 1, 1, 1, -1]		Ampe[-1, 1, 1, 1, -1]		Ampa[-1, 1, 1, 1, -1]		Ampd[-1, 1, 1, 1, -1]	
	Ampb[-1, 1, 1, 1, 1]		Ampc[-1, 1, 1, 1, 1]		A[-1, 1, 1, 1, 1]		Ampa[-1, 1, 1, 1, 1]		Ampd[-1, 1, 1, 1, 1]	
- 24	Ampb[1, -1, -1, -1, -1]	•	Ampc[1, -1, -1, -1, -1]		A[1, -1, -1, -1, -1]	•	Ampa[1, -1, -1, -1, -1]	· *	Ampd[1, -1, -1, -1, -1]	
	Ampb[1, -1, -1, -1, 1]		Ampc[1, -1, -1, -1, 1]		Ampe[1, -1, -1, -1, 1]		Ampa[1, -1, -1, -1, 1]		Ampd[1, -1, -1, -1, 1]	
	A[1, -1, -1, 1, -1]		A[1, -1, -1, 1, -1]		Ampe[1, -1, -1, 1, -1]		Ampa[1, -1, -1, 1, -1]		Ampd[1, -1, -1, 1, -1]	
	Ampb[1, -1, -1, 1, 1]		Ampc[1, -1, -1, 1, 1]		Ampe[1, -1, -1, 1, 1]		Ampa[1, -1, -1, 1, 1]		Ampd[1, -1, -1, 1, 1]	
	Ampb[1, -1, 1, -1, -1]		Ampc[1, -1, 1, -1, -1]		Ampe[1, -1, 1, -1, -1]		Ampa[1, -1, 1, -1, -1]		Ampd[1, -1, 1, -1, -1]	
	Ampb[1, -1, 1, -1, 1]		Ampc[1, -1, 1, -1, 1]		Ampe[1, -1, 1, -1, 1]		Ampa[1, -1, 1, -1, 1]		Ampd[1, -1, 1, -1, 1]	
	A[1, -1, 1, 1, -1]		A[1, -1, 1, 1, -1]		A[1, -1, 1, 1, -1]		Ampa[1, -1, 1, 1, -1]		Ampd[1, -1, 1, 1, -1]	
	Ampb[1, -1, 1, 1, 1]		Ampc[1, -1, 1, 1, 1]		Ampe[1, -1, 1, 1, 1]		Ampa[1, -1, 1, 1, 1]		Ampd[1, -1, 1, 1, 1]	
	Ampb[1, 1, -1, -1, -1]		Ampc[1, 1, -1, -1, -1]		Ampe[1, 1, -1, -1, -1]		A[1, 1, -1, -1, -1]		A[1, 1, -1, -1, -1]	
	Ampb[1, 1, -1, -1, 1]		Ampc[1, 1, -1, -1, 1]		Ampe[1, 1, -1, -1, 1]		Ampa[1, 1, -1, -1, 1]		A[1, 1, -1, -1, 1]	
	Ampb[1, 1, -1, 1, -1]		Ampc[1, 1, -1, 1, -1]		Ampe[1, 1, -1, 1, -1]		Апра[1, 1, -1, 1, -1]		Ampd[1, 1, -1, 1, -1]	
	Ampb[1, 1, -1, 1, 1]		Ampc[1, 1, -1, 1, 1]		Ampe[1, 1, -1, 1, 1]		A[1, 1, -1, 1, 1]		Ampd[1, 1, -1, 1, 1]	
	Ampb[1, 1, 1, -1, -1]		Ampc[1, 1, 1, -1, -1]		Ampe[1, 1, 1, -1, -1]		A[1, 1, 1, -1, -1]		Ampd[1, 1, 1, -1, -1]	
	Ampb[1, 1, 1, -1, 1]		Ampc[1, 1, 1, -1, 1]		Ampe[1, 1, 1, -1, 1]		Ampa[1, 1, 1, -1, 1]		Ampd[1, 1, 1, -1, 1]	
	Ampb[1, 1, 1, 1, -1]		Ampc[1, 1, 1, 1, -1]		Ampe[1, 1, 1, 1, -1]		Ampa[1, 1, 1, 1, -1]		A[1, 1, 1, 1, -1]	
	Ampb[1, 1, 1, 1, 1]	1	Ampc[1, 1, 1, 1, 1]		Ampe[1, 1, 1, 1, 1]	1	A[1, 1, 1, 1, 1])	A[1, 1, 1, 1, 1]	1

Figure: All the helicity amplitudes

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 $e^+e^- \rightarrow \gamma \, G \, G$

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One of the several and marvelous advantages of the helicity amplitudes is that it is easy to identify the symmetries as well as the null helicity amplitudes. We already know that the terms $\langle xy \rangle$ and $[xy \rangle$ are zero, a small program could help us to find which helicity amplitude is zero.

$$\mathcal{A}^{a}_{-+--+} \approx \left(\bar{v}_{+}(2) \not \epsilon_{-}(3) \not q u_{-}(1) \bar{u}_{+}(5) v_{-}(4) \right), \tag{37}$$

$$\mathcal{A}^{a}_{-+--+} \approx [54) = 0.$$
 (38)

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Counting helicity amplitudes

0	0	0		A[-1, -1, -1, -1, -1]	(A[-1, -1, -1, -1, -1]	
0	0	0		0	A[-1, -1, -1, -1, 1]	
0	0	0		0	0	
0	0	0		A[-1, -1, -1, 1, 1]	0	
0	0	0		A[-1, -1, 1, -1, -1]	0	
0	0	0		0	0	
0	0	0		0	A[-1, -1, 1, 1, -1]	
0	0	0		A[-1, -1, 1, 1, 1]	A[-1, -1, 1, 1, 1]	
0	0	0		0	0	
A[-1, 1, -1, -1, 1]	A[-1, 1, -1, -1, 1]	A[-1, 1, -1, -1, 1]		0	0	
0	0	0		0	0	
0	0	0		0	0	
0	0	0		0	0	
A[-1, 1, 1, -1, 1]	A[-1, 1, 1, -1, 1]	0		0	0	
0	0	0		0	0	
0	0	A[-1, 1, 1, 1, 1]		0	0	
0	0	A[1, -1, -1, -1, -1]		0	0	
0	0	0		0	0	
A[1, -1, -1, 1, -1]	A[1, -1, -1, 1, -1]	0		0	0	
0	0	0		0	0	
0	0	0		0	0	
0	0	0		0	0	
A[1, -1, 1, 1, -1]	A[1, -1, 1, 1, -1]	A[1, -1, 1, 1, -1]		0	0	
0	0	0		0	0	
0	0	0		A[1, 1, -1, -1, -1]	A[1, 1, -1, -1, -1]	
0	0	0		0	A[1, 1, -1, -1, 1]	
0	0	0		0	0	
0	0	0		A[1, 1, -1, 1, 1]	0	
0	0	0		A[1, 1, 1, -1, -1]	0	
0	0	0		0	0	
0	0	0		0	A[1, 1, 1, 1, -1]	
(0)	(0)	0	/	A[1, 1, 1, 1, 1]	A[1, 1, 1, 1, 1]	

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We started our problem with **160** helicity amplitudes, but just looking at the possible helicity states of the external particles we found that there are only **28** helicity amplitudes to compute. Applying complex conjugation we really need to compute half of the final helicity amplitudes. *At the end, just the* **10%** *of the work will be done and without the help of any machine if you desired.*

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The total squared amplitud is as follows:

$$|\mathcal{M}|^{2} = \sum_{perm} |A_{\lambda_{1}\lambda_{2}\lambda_{3}\lambda_{4}\lambda_{5}}|^{2}$$

= 2($|\mathcal{A}_{-+--+}^{i}|^{2} + |\mathcal{A}_{-++-+}|^{2} + |\mathcal{A}_{-++++}|^{2}$
+ $|\mathcal{A}_{-----}|^{2} + |\mathcal{A}_{---++-}|^{2} + |\mathcal{A}_{---+++}|^{2}$
+ $|\mathcal{A}_{--+--}|^{2} + |\mathcal{A}_{--++--}|^{2} + |\mathcal{A}_{--++++}|^{2}$ (39)

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Each partial squared helicity amplitude is as follows:

$$\begin{aligned} |\mathcal{A}_{-+-+}^{i}|^{2} &= 2 \frac{s_{15}s_{34}}{s_{23}} \left((B - 2Em_{\chi_{0}})^{2} s_{23}^{2} + 4C^{2} s_{24}^{2} \right. \tag{40} \\ &+ 4 (B - 2Em_{\chi_{0}}) C s_{23} s_{24} \right) \\ |\mathcal{A}_{-++++}|^{2} &= \frac{8C^{2} s_{24}^{2} s_{15} s_{34}}{s_{23}} \\ |\mathcal{A}_{-++++}|^{2} &= 8E^{2} s_{34}^{2} s_{12} s_{35} \\ |\mathcal{A}_{-----}|^{2} &= 2D^{2} s_{34}^{3} s_{12} s_{35} \\ |\mathcal{A}_{----+}|^{2} &= 2D^{2} s_{34} s_{12} s_{35} m_{\chi_{0}}^{2} \end{aligned}$$

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$$|\mathcal{A}_{--++}|^2 = 2A^2 s_{34} s_{25} s_{15} \tag{45}$$

$$|\mathcal{A}_{-++-}|^2 = |\mathcal{A}_{-+++}|^2 \tag{46}$$

$$|\mathcal{A}_{--++-}|^2 = 2D^2 s_{34}^2 s_{12} s_{54} m_{\chi_0}^2 \tag{47}$$

$$|\mathcal{A}_{--+++}^{i}|^{2} = 2(s_{45}s_{25}s_{15}A^{2} + D^{2}s_{34}^{3}s_{12}s_{35} - AD(s_{34}s_{45}))$$

$$(48)$$

where we have to remember that:

$$s_{ij} = -(p_i + p_j)^2$$

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Conclusions	

- We analytically compute the total scattering amplitude for the reaction e⁺e⁻ → γ G̃ G̃, we have found that our results match with the previous work done numerically.
- It was show that the SHF is a powerful method, in fact in several cases is much more economic than the traditional approach.
- We are exploring how to implement more sophisticate methods to our calculation with gravitinos, namely KLT and BCFW relations. We would like to apply this techniques to some relevant process in modern Cosmology.

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Conclusions

Thank you

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Conclusions

In spite of the tremendous difficulties lying ahead, I feel that S-matrix theory is far from dead and that . . . much new interesting mathematics will be created by attempting to formalize it.

"Tullio Regge"

MAAAA

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Light Cone Decomposition (LCD)

Let p^{μ} be any time-like 4-momentum, we can decompose it into 2 light-like four momenta as follows. Let q^{μ} be an arbitrary light-like four momentum, and define

$$r^{\mu} \equiv p^{\mu} - \alpha q^{\mu} \qquad (49)$$

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We want that r be light-like too, so we impose $r^2 = 0$; then

$$0 = (p^{\mu} - \alpha q^{\mu})(p_{\mu} - \alpha q_{\mu}) = p^2 - 2\alpha p^{\mu}q_{\mu} + \alpha^2 q^2,$$
 (50)

but $q^2 = 0$, therefore $\alpha = \frac{p^2}{2p \cdot q}$.

Now we can see how LCD applies to massive spinors. Remember Dirac equation:

$$(\not p + m)u_s(\vec{p}) = 0 \qquad , \tag{5}$$

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Now we can see how LCD applies to massive spinors. Remember Dirac equation:

$$(p + m)u_s(\vec{p}) = 0$$
 , (51)

when one consider the 4-component Dirac spinor in terms of two 2-component spinors

$$u = \left(\begin{array}{c} \chi_a \\ \xi^{\dot{a}} \end{array}\right),\tag{52}$$

Dirac equation is equivalent to the following system

$$p_{a\dot{a}}\xi^{\dot{a}} + m\chi_a = 0, \tag{53}$$

$$p^{\dot{a}a}\chi_a + m\xi^{\dot{a}} = 0.$$
 (54)

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•
$$-(|r]_a \langle r|_{\dot{a}} + \alpha |q]_a \langle q|_{\dot{a}})$$

 $p_{a\dot{a}} \xi^{\dot{a}} + m\chi_a = 0$ (55)
 $p^{\dot{a}a} \chi_a + m\xi^{\dot{a}} = 0$ (56)
• $-(|r)^{\dot{a}} [r|^a + \alpha |q)^{\dot{a}} [q|^a)$

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$$(|r]_{a}(r|_{\dot{a}} + \alpha |q]_{a}(q|_{\dot{a}}) \xi^{\dot{a}} = m\chi_{a}$$

$$(57)$$

$$(|r)^{\dot{a}}[r|^{a} + \alpha |q\rangle^{\dot{a}}[q|^{a}) \chi_{a} = m\xi^{\dot{a}}$$

$$(58)$$

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The solutions for the last 2 equations are as follows:

$$u = \left(\begin{array}{c} \frac{m}{[rq]} |q]_a \\ |r\rangle^{\dot{a}} \end{array}\right)$$

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with the spinors
$$\chi_a = \frac{m}{[rq]} |q]_a$$
 and $\xi^{\dot{a}} = |r\rangle^{\dot{a}}$.