Heavy quarks within electroweak multiplet

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J. Besprosvany y R. Romero “Representation of quantum field theory in an extended spin space and fermion mass hierarchy " *Int. J. Mod. Phys. A* **29**, No. 29 1450144 (17 pp.) (2014), arXiv:1408.4066[hep-th].

Ricardo Romero and Jaime Besprosvany, “Quark horizontal flavor hierarchy and two-Higgs- doublet model in a (7+1)-dimensional extended spin space ", arXiv:1611.07446[hep-ph]

Jaime Besprosvany and Ricardo Romero, “Heavy quarks within electroweak multiplet", arXiv:11701.01191[hep-ph]

Argument summary

- Electroweak conventional fields and their Lagrangian can be written in a spin-extended space.
- Scalar-vector term, invariant under conjugate scalar parametrization.
- *Same* scalar field within SV and SF terms connects V and F; after the Higgs mechanism, it constrains quark masses.
- Yukawa constants are reinterpreted as geometrical.
- Multiplet structure suggested for heavy standard-model particles.

Motivation: multiplet structure

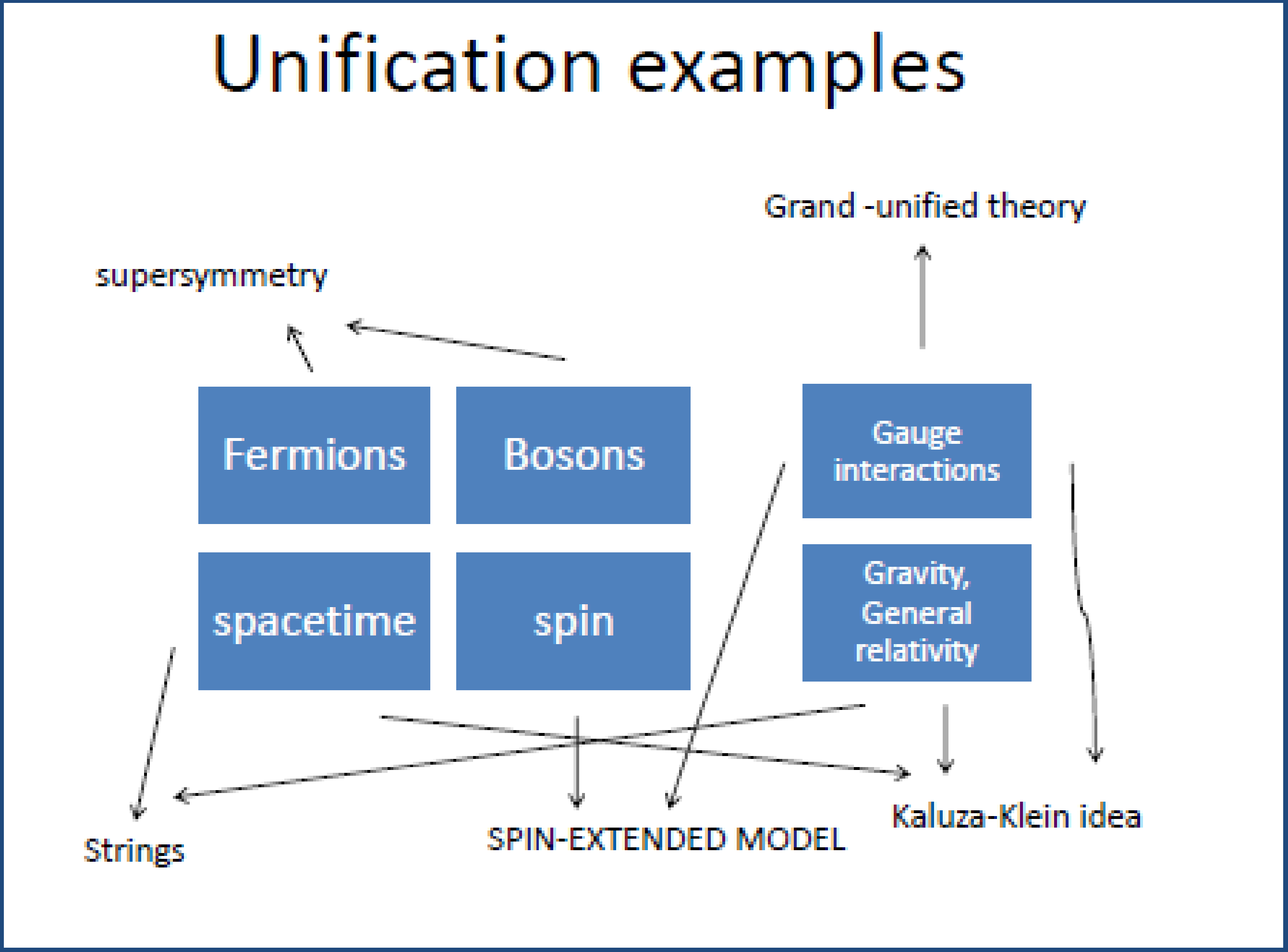
Puzzles in the standard model:

- Fermion-mass parameters; Yukawa sector independent of scalar-vector.
- Origin of electroweak symmetry breaking (Higgs mechanism).

	Masses (GeV)	Spin	Weak I^2	Hypercharge Y
$W^{+/-}$	80.4	1	1	0
Z	91.2	1	0	0
H	126	0	½	1
t	173	½	½ ,0	1/3, 4/3
b	4	½	½ ,0	1/3, -2/3

Composite multiplet structure suggested

Spin-extended model within standard-model extensions



Use of conventional and spin bases

spin basis → conventional basis

- Constrain representations and interactions at given dimension.
- Finite number of possible partitions.

spin basis ← conventional basis

Reinterpretation of fields:

- SV: scalar operator acting over vectors
- SF: scalar operator acting over fermions
- Standard-model projection.

LORENTZ AND MAXIMAL SCALAR SYMMETRY AT D DIMENSION

$$\underbrace{\gamma_0 \quad \gamma_1 \quad \gamma_2 \quad \gamma_3} \qquad \underbrace{\gamma_4, \dots \gamma_{D-1}}$$

$\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$ $\mu,\nu = 0,\dots,3$ $\qquad \gamma_a \quad a = 4,\dots,D-1$

4-D Lorentz symmetry $\otimes$ Scalar symmetry

unitary: $U(2^{(D-4)/2})$

$$[\sigma_{\mu\nu}, \gamma_a] = 0$$

$$[\tilde{\gamma}_5, \gamma_a] = 0 \qquad \tilde{\gamma}_5 = -i\gamma_0\gamma_1\gamma_2\gamma_3$$

$$[H, \gamma_a] = 0 \qquad H = i\gamma_0 \bar{\nabla}\cdot\bar{\gamma}$$

maximal scalar symmetry

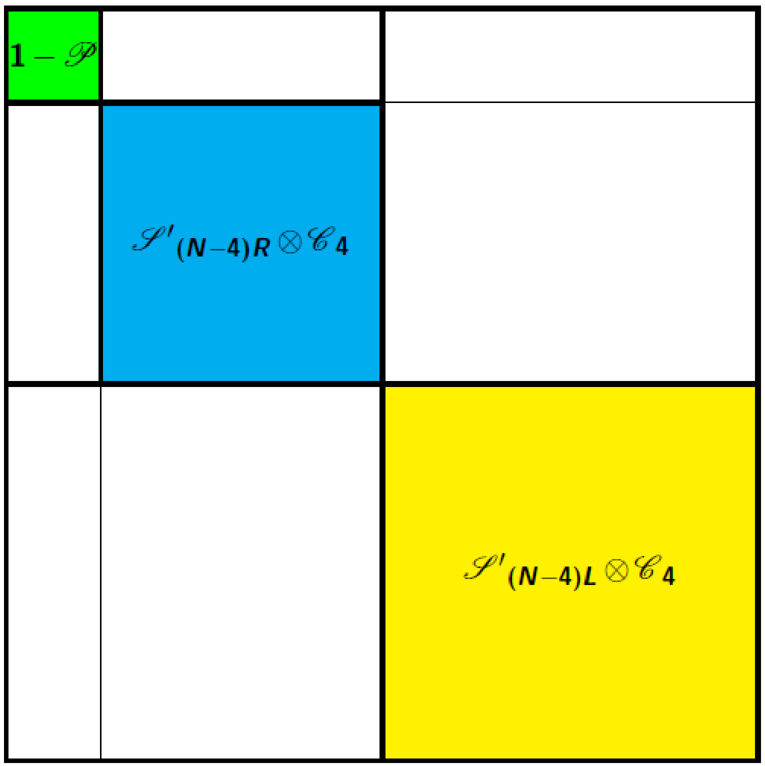
$$U_R \otimes U_L \qquad U_R = \frac{1}{2}(1+\tilde{\gamma}_5)\, U(2^{(D-4)/2})$$

$$U_L = \frac{1}{2}(1-\tilde{\gamma}_5)\, U(2^{(D-4)/2})$$

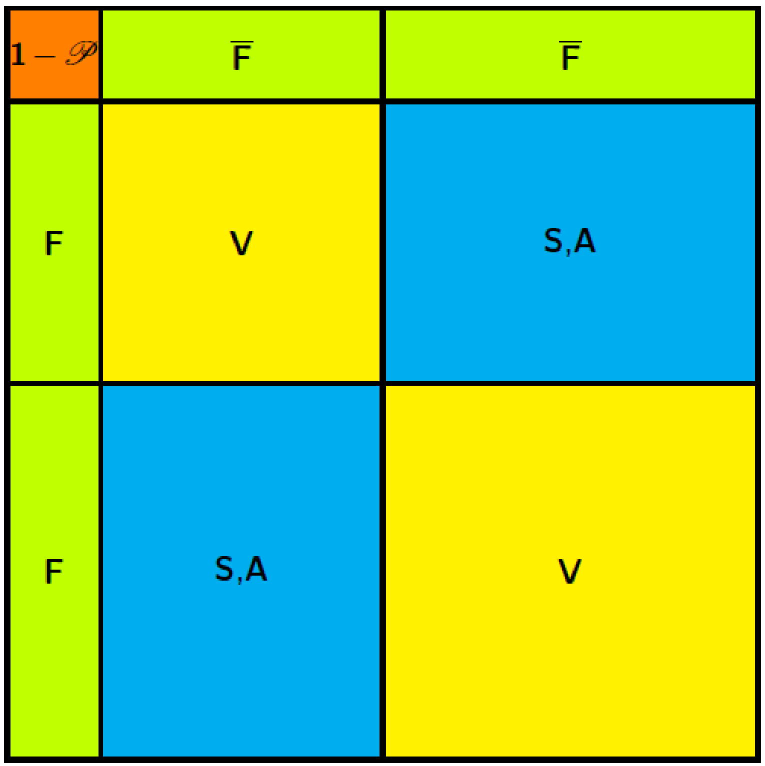
Coleman-Mandula OK

Esquema del espacio matricial

Operadores



Estados



Conventional and spin-extended bases, Lagrangian equivalence: fermion-vector

conventional basis

spin-extended basis

Field formulation: $A_\mu(x) = g_\mu{}^\nu A_\nu(x)$

$A_\mu(x)\gamma_0\gamma^\mu$

$$\mathcal{L}_{FV} = \bar{\mathrm{q}}_L(x)[i\partial_\mu + \frac{1}{2}g\tau^a W^a_\mu(x) + \frac{1}{6}g' B_\mu(x)]\gamma^\mu \mathrm{q}_L(x) +$$

$$\bar{t}_R(x)[i\partial_\mu + \frac{2}{3}g' B_\mu(x)]\gamma^\mu t_R(x) + \bar{b}_R(x)[i\partial_\mu - \frac{1}{3}g' B_\mu(x)]\gamma^\mu b_R(x)$$

$$\mathrm{q}_L(x) = \begin{pmatrix} t_L(x) \\ b_L(x) \end{pmatrix}$$

$$t_L(x) = \begin{pmatrix} \psi^1_{tL}(x) \\ \psi^2_{tL}(x) \end{pmatrix}$$

$$\mathcal{L}_{FV} = \mathrm{tr}\{\boldsymbol{\Psi}^\dagger_{qL}(x)[i\partial_\mu + gI^a W^a_\mu(x) + \frac{1}{2}g' Y_o B_\mu(x)]\gamma^0\gamma^\mu \boldsymbol{\Psi}_{qL}(x) +$$

$$\Psi^\dagger_{tR}(x)[i\partial_\mu + \frac{1}{2}g' Y_o B_\mu(x)]\gamma^0\gamma^\mu \Psi_{tR}(x) + \Psi^\dagger_{bR}(x)[i\partial_\mu + \frac{1}{2}g' Y_o B_\mu(x)]\gamma^0\gamma^\mu \Psi_{bR}(x)\}P_f$$

$$\boldsymbol{\Psi}_{qL}(x) = \textstyle\sum_\alpha \psi^\alpha_{tL}(x) T^\alpha_L + \psi^\alpha_{bL}(x) B^\alpha_L$$

Conjugate SU(2) property

- For a set of generators $G_{i,}$ conjugate $-G_i^*$ satisfy the same Lie algebra.
- SU(2) property: conjugate representation obtained by similarity transformation:

$\sigma_2 \sigma_i \sigma_2 = -\sigma_i^*$ σ_i : Pauli matrices

$\sigma_2 \psi^*$ transforms as ψ

Scalar-vector Lagrangian representations

Single scalar representation

$$\mathbf{F}_\mu(x) = i\partial_\mu + \tfrac{1}{2}g\boldsymbol{\tau}\cdot\mathbf{W}_\mu(x) + \tfrac{1}{2}g'B_\mu(x)$$

$$\mathbf{W}_\mu(x) = (W_\mu^1(x), W_\mu^2(x), W_\mu^3(x))$$

$$\mathbf{H}(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} \eta_1(x) + i\eta_2(x) \\ \eta_3(x) + i\eta_4(x) \end{pmatrix}$$

$$\mathcal{L}_{SV} = \mathbf{H}^\dagger(x)\mathbf{F}^{\mu\dagger}(x)\mathbf{F}_\mu(x)\mathbf{H}(x)$$

Scalar and conjugate representation

$$\mathbf{F'}_\mu\bar{\mathbf{H}}_{\chi_t\chi_b}(x) = (i\partial_\mu + \tfrac{1}{2}g\boldsymbol{\tau}\cdot\mathbf{W}_\mu(x))\bar{\mathbf{H}}_{\chi_t\chi_b}(x) + g'\bar{\mathbf{H}}_{\chi_t\chi_b}(x)B_\mu(x)\tau_3$$

Sikivie et al. [80], Chivukula [98] $\bar{\mathbf{H}}_{\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}}(x)$

$$\bar{\mathbf{H}}_{\chi_t,\chi_b}(x) = (\chi_t\mathbf{H}(x), \chi_b\tilde{\mathbf{H}}(x))$$

$$\mathcal{L}_{SV} = \mathrm{tr}[\mathbf{F'}_\mu\bar{\mathbf{H}}_{\chi_t\chi_b}(x)]^\dagger\mathbf{F'}^\mu\bar{\mathbf{H}}_{\chi_t\chi_b}(x) \quad \alpha \quad |\chi_t|^2 + |\chi_b|^2$$

Scalar-field normalization requires $|\chi_t|^2 + |\chi_b|^2 = 1$

SV Lagrangian and scalar t-b spin representation

Scalar correspondence

$$\mathbf{H(x)} \to \phi_1(x) - \phi_2(x)$$

$$\tilde{\mathbf{H}}^\dagger(x) \to \phi_1(x) + \phi_2(x).$$

$$\mathbf{H}_t(x) = \phi_1(x) + \phi_2(x),\ \mathbf{H}_b(x) = \phi_1(x) - \phi_2(x)$$

$$R_5 = \tfrac{1}{2}(1+\tilde{\gamma}_5),\ \text{e. g., } R_5\mathbf{H}_t(x)L_5 = \mathbf{H}_t(x)$$

$$L_5\mathbf{H}_t(x)R_5 = 0,\ R_5\mathbf{H}_b(x)L_5 = 0$$

$$\mathbf{H}_{af}(x) = a\phi_1(x) + f\phi_2(x)$$

$$\mathbf{H}_{af}(x) = \frac{1}{\sqrt{2}}(\chi_t\mathbf{H}_t(x) + \chi_b\mathbf{H}_b(x))$$

$$\chi_t = \frac{1}{\sqrt{2}}(a+f),\ \chi_b = \frac{1}{\sqrt{2}}(a-f)$$

SV spin representation

$$\mathbf{F}''(x) = [i\partial_\mu + gW^i_\mu(x)I^i + \tfrac{1}{2}g'B_\mu(x)Y_o]\gamma_0\gamma^\mu$$

$$\mathcal{L}_{SV} = \mathrm{tr}\{[\mathbf{F}''(x), \mathbf{H}_{af}(x)]^\dagger_\pm[\mathbf{F}''(x), \mathbf{H}_{af}(x)]_\pm\}_{\mathrm{sym}}$$

Spin-space: connection between scalar-vector and Yukawa terms

$$\mathcal{L}_{SV} = \mathrm{tr}\{[\mathbf{F}''(x), \mathbf{H}_{af}(x)]^{\dagger}_{\pm}[\mathbf{F}''(x), \mathbf{H}_{af}(x)]_{\pm}\}_{\mathrm{sym}}$$

$$\mathbf{H}_{af}(x) = \frac{1}{\sqrt{2}}(\chi_t \mathbf{H}_t(x) + \chi_b \mathbf{H}_b(x))$$

$$-\mathcal{L}_{SF} = \mathrm{tr}\frac{\sqrt{2}}{v}[m_t \Psi^{\dagger}_{tR}(x)\mathbf{H}^{\dagger}_t(x)\mathbf{\Psi}_{qL}(x) + m_b \mathbf{\Psi}^{\dagger}_{qL}(x)\mathbf{H}^{\dagger}_b(x)\Psi_{bR}(x)] + \{hc\}$$

$$\mathbf{H}_m(x) = \frac{\sqrt{2}}{v}(m_t \mathbf{H}_t(x) + m_b \mathbf{H}_b(x))$$

Scalar-vector scalar-fermion comparison

$$\langle \eta_3(x) \rangle = v, \ \langle \mathbf{H}(x) \rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Z-vector mass

Higgs mechanism

$$\langle \mathbf{H}_{af}(x) \rangle = H_n = \frac{v}{2}(\chi_t H_t^0 + \chi_b H_b^0),$$

$$\begin{aligned} \mathcal{L}_{SZm0} &= \mathrm{tr}[H_n, W_0^3(x) g I^3 + B_0(x) \frac{1}{2} g' Y_o]^\dagger [H_n, W_0^3(x) g I^3 + B_0(x) \frac{1}{2} g' Y_o] \qquad (11) \\ &= Z_0^2(x) \frac{1}{g^2 + g'^2} \mathrm{tr}[H_n, g^2 I^3 - \frac{1}{2} g'^2 Y_o]^\dagger [H_n, g^2 I^3 - \frac{1}{2} g'^2 Y_o] = \frac{1}{2} Z_0^2(x) m_Z^2, \end{aligned}$$

Top-quark mass

Higgs mechanism

$$H_m = \langle \mathbf{H}_m(x) \rangle = m_t H_t^0 + m_b H_b^0$$

$$H_m^h T_M^1 = m_t T_M^1, \quad H_m^h T_M^{c1} = -m_t T_M^{c1},$$

$$H_m^h B_M^1 = m_b B_M^1, \quad H_m^h B_M^{c1} = -m_b B_M^{c1}, \qquad (13)$$

where $H_m^h = H_m + H_m^\dagger$, and T_M^{c1}, B_M^{c1} correspond to negative-energy solution states

Quark-mass relation

The "punchline:" $|\langle Z|\sqrt{2}H_n|Z\rangle|^2 = m_Z^2$ and $\langle t|H_m + H_m^\dagger|t\rangle = m_t$

Higgs mechanism

$$\langle \mathbf{H}_{af}^\dagger(x)\mathbf{H}_{af}(x)\rangle = (|a|^2+|f|^2)v^2/2 = (|\chi_t|^2+|\chi_b|^2)v^2/2 = v^2/2$$

$$(|a|^2+|f|^2)v^2/2 = |m_t|^2+|m_b|^2 = v^2/2$$

$$m_t = \sqrt{v^2/2 - m_b^2} \simeq 173.90\ \text{GeV}$$

$v = 246$ GeV

$m_b = 4$ GeV

Quark-mass relation

The "punchline:" $|\langle Z|\sqrt{2}H_n|Z\rangle|^2 = m_Z^2$ and $\langle t|H_m + H_m^\dagger|t\rangle = m_t$

Higgs mechanism

$$\langle \mathbf{H}_{af}^\dagger(x)\mathbf{H}_{af}(x)\rangle = (|a|^2 + |f|^2)v^2/2 = (|\chi_t|^2 + |\chi_b|^2)v^2/2 = v^2/2$$

$$(|a|^2 + |f|^2)v^2/2 = |m_t|^2 + |m_b|^2 = v^2/2$$

$$m_t = \sqrt{v^2/2 - m_b^2} \simeq 173.90 \text{ GeV}$$

$v = 246$ GeV

$m_b = 4$ GeV

Correspondence's physical interpretation

- Dynamical: action of scalar on fermion and vector share the same effect: common Hamiltonian H.

  $[H+H^{\dagger},F]$ vs $[H,V]^{\dagger}[H,V]$

- Symmetry: e. g., $SU(2)_L xU(1)$ fundamental-adjoint representation connection.

- Compositeness: Standard-model gauge structure. No information on whether this a formal or physical feature.

from gauge invariance. Formally, a boson field $B_o(x)$ expansion may be obtained using $B_o(x) = \sum_{lk} F_l(x)F_k(x) + [B_o(x) - \sum_{lk} F_l(x)F_k(x)]$, where $F_l(x)$, $F_k(x)$ are fermion fields reproducing $B_o(x)$'s quantum numbers, and the last two terms give corrections.