

# **GRUPO DE PRINCIPIOS PRINCIPIOS FUNDAMENTALES**

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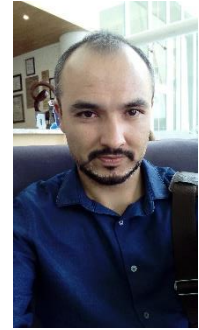
**Reunión general de la Red-FAE**

**De una manera muy general los temas considerados se pueden agrupar de la siguiente manera:**

- (1) Problemas fundamentales de la mecánica cuántica y su relación con la gravitación:** (Y. Bonder, C. Chryssomalakos, T. Koslowski, E. Okon, C. Ramirez, M. Salgado, D. Sudarsky).
- (2) Estudio de simetrías espacio temporales:** (Y. Bonder, D. Delepine, P. Hess, E. Martinez, H Martínez L. Nellen, J. Toscano, L. Urrutia, D. Vergara).
- (3) Teorías no conmutativas:** (C. Chryssomalakos, D. Vergara)
- (4) Violación de CP:** ( R. Juárez, P. Kielanowski, D. Delepine )

- (5) Gravedad clásica y cuántica:** (T. Koslowski, C. Ramirez, P. Hess, B. Juárez).
- (6) Dimensiones extras:** (E. Martínez, J. Toscano, H. Novales).
- (7) Teoría de Cuerdas:** (E. Cáceres, M. Chernicoff, H. García-Compeán, J. A. García Zenteno, A. Güijosa, O. Loaiza-Brito, L. Patiño, S. Ramos-Sánchez, M. Sabido).
- (8) Grafeno y Aislantes topológicos:** (A. Raya, L. Urrutia).
- (9) Espines Altos y Métodos Espinoriales:** (M. Kirchbach, B. Larios).

# PARTICIPANTES



- **Héctor Novales, FCFM-BUAP**



- **Mariana Kirchbach IF-UASLP**



- **Peter Hess, ICN-UNAM**



- **Benito Juárez ICN-UNAM**

- **Chryssomalis Chryssomalakos, ICN-UNAM**



- **Cupatitzio Ramirez, FCFM-BUAP**



- **David Vergara, ICN-UNAM**



- **Bryan Larios, FCFM-BUAP**





# • TEORIAS DE KK Y EL MODELO ESTÁNDAR EN DIMENSIONES EXTRAS

**La acción...**  $S[\mathcal{A}_M^a] = \int d^d x \left( -\frac{1}{4} \mathcal{F}_{MN}^a(x, \bar{x}) \mathcal{F}_a^{MN}(x, \bar{x}) \right)$

- ♦ Curvaturas:  $\mathcal{F}_{MN}^a = \partial_M \mathcal{A}_N^a - \partial_N \mathcal{A}_M^a + g_{(4+n)} f^{abc} \mathcal{A}_M^b \mathcal{A}_N^c$
- ♦ Conexiones de  $SU(N, \mathcal{M}^{4+n})$ :  $\mathcal{A}_M^a(x, \bar{x})$
- ♦ Constante de acoplamiento:  $g_{(4+n)} \rightarrow$  unidades de inverso de masa

$$S = -\frac{1}{4} \int d^d x \left( \mathcal{F}_{\mu\nu}^a \mathcal{F}_a^{\mu\nu} - 2 \mathcal{F}_{\mu\bar{\nu}}^a \mathcal{F}_a^{\mu\bar{\nu}} + \mathcal{F}_{\bar{\mu}\bar{\nu}}^a \mathcal{F}_a^{\bar{\mu}\bar{\nu}} \right)$$

**Masas de norma:**  $\sum_{(m)} \frac{1}{2} m_{(m)}^2 A_{\mu}^{(m)a} A^{(m)a\mu}$

**Masas escalares:**  $-\sum_{(m)} \frac{1}{2} A_{\bar{\mu}}^{(m)a} \underbrace{\mathfrak{M}_{\bar{\mu}\bar{\nu}}^{(m)}} A_{\bar{\nu}}^{(m)a}$

**Las torres de KK:**

$$A_{\mu}^a(x, \bar{x}) = \sum_{(m)} f^{(m)}(\bar{x}) A_{\mu}^{(m)a}(x)$$

$$A_{\bar{\mu}}^a(x, \bar{x}) = \sum_{(m)} f^{(m)}(\bar{x}) A_{\bar{\mu}}^{(m)a}(x)$$

$$\alpha^a(x, \bar{x}) = \sum_{(m)} f^{(m)}(\bar{x}) \alpha^{(m)a}(x)$$

$\mathcal{L}_{\text{eff}}^{(0)} = \mathcal{L}_{\text{SM}}^{(0)} + \mathcal{L}_{\text{d>4}}^{(0)} \rightarrow$  Sólo modos cero; contiene al ME

$\mathcal{L}_{\text{eff}}^{(0)(m)} = \mathcal{L}_{\text{d=4}}^{(0)(m)} + \mathcal{L}_{\text{d>4}}^{(0)(m)} \rightarrow$  Modos cero y excitados; primeras contribuciones de DEs a observables del ME

$\mathcal{L}_{\text{eff}}^{(m)} = \mathcal{L}_{\text{d=4}}^{(m)} + \mathcal{L}_{\text{d>4}}^{(m)} \rightarrow$  Sólo modos excitados

- ♦ **Renormalización de las teorías de Kaluza-Klein en un sentido amplio**
- ♦ **Fenomenología: estudio de contribuciones de Kaluza-Klein a observables**

# PROBLEMAS Y AVANCES EN LA DESCRIPCION DE ESPINES SUPERIORES



$$\mathcal{L} : [S_{\mu\nu}, S_{\rho\sigma}] = i(g_{\mu\rho}S_{\nu\sigma} - g_{\nu\rho}S_{\mu\sigma} + g_{\mu\sigma}S_{\rho\nu} - g_{\nu\sigma}S_{\rho\mu}),$$

$$\mathcal{T}_4 : [P_\mu, P_\nu] = 0, \quad F_{AB} = \frac{1}{4}[S^{\mu\nu}]_{AD}[S_{\mu\nu}]_{DB},$$

$$[S_{\mu\nu}, P_\lambda] = 0, \quad G_{AB} = \frac{1}{8}\epsilon_{\mu\nu\alpha\beta}[S^{\mu\nu}]_{AC}[S_{\alpha\beta}]_{CB}, \quad A, B, C, D, \dots = 1, \dots, d.$$

$$F D^{(j_1, j_2) \oplus (j_2, j_1)} = c_{(j_1, j_2)} D^{(j_1, j_2) \oplus (j_2, j_1)},$$

$$c_{(j_1, j_2)} = \frac{1}{2} (K(K+2) + M^2), \quad K = j_1 + j_2, \quad M = |j_1 - j_2|, \quad \Psi^{(n)}_{a_1 a_2 \dots a_n}, \quad a_i = 1, 2, 3, 4,$$

$$D^{(n/2, 0) \oplus (0, n/2)} \text{ irrep.} \quad \mathcal{P}_F^{(n/2, 0)} = \Pi_{kl} \times \left( \frac{F - c_{(j_k, j_l)}}{c_{(n/2, 0)} - c_{(j_k, j_l)}} \right),$$

$$\left( \partial_\mu \partial^\mu \left[ \mathcal{P}_F^{(n/2, 0)} \right]_{a_1 a_2 \dots a_n}^{b_1 b_2 \dots b_n} + m^2 \right) \Psi_{b_1 b_2 \dots b_n}^{(n)} = 0, \quad n = 2j.$$

$$\Psi_{a_1 a_2}^{(2)} \simeq D^{(1/2, 0) \oplus (0, 1/2)} \otimes D^{(1/2, 0) \oplus (0, 1/2)} = D^{(1, 0) \oplus (0, 1)} \oplus 2D^{(0, 0)} \oplus 2D^{(1/2, 1/2)},$$

$$\mathcal{P}_F^{(1, 0)} = \frac{1}{2} (2F^2 - 3F). \quad \left[ \mathcal{P}_F^{(1, 0)} \right]_{b_1 b_2}^{a_1 a_2} = \frac{1}{4} (\sigma_{\mu\nu})^{a_1}_{b'_1} (\sigma_{\mu\nu})^{a_2}_{b'_2} F^{b'_1 b'_2}_{b_1 b_2}$$

First calculation of the Compton scattering off  $(3/2, 0) \oplus (0, 3/2)$   
**g factor = 2/3**

$c_{(0,0)}$	=	0	for	$(0, 0)$ ,
$c_{(1,0)}$	=	2	for	$(1, 0) \oplus (0, 1)$ ,
$c_{(1/2, 1/2)}$	=	$\frac{3}{2}$	for	$(1/2, 1/2)$ .



# TEORIA DE LA RELATIVIDAD PSEUDO COMPLEJA

$$X = X_1 + IX_2 \quad , \quad I^2 = 1$$

Alternativa:  $\sigma_{\pm} = \frac{1}{2}(1 \pm I) \rightarrow X = X_+ \sigma_+ + X_- \sigma_-$

$$\sigma_{\pm}^2 = \sigma_{\pm} \quad \sigma_+ \sigma_- = 0 \quad !$$

Solo una extensión pseudo-compleja no produce estados no-fisicos, como fantasmas y tachyones.

$$F(X)G(X) = F(X_+)G(X_+)\sigma_+ + F(X_-)G(X_-)\sigma_-$$

$$X^\mu = x^\mu + I \frac{\ell}{c} \frac{dx^\mu}{d\tau} \quad \frac{D}{DX^\mu} = \frac{1}{2}(\partial_{1,\mu} + I\partial_{2,\mu})$$

$$\delta S \in P^0$$

$$g_{\mu\nu} = g_{\mu\nu}^+ \sigma_+ + g_{\mu\nu}^- \sigma_-$$

$$D_{+,\mu} \left( \frac{D_+ \mathcal{L}_+}{D_+(D_{+,\mu} \Phi_{+,r})} \right) - \frac{D_+ \mathcal{L}_+}{D_+ \Phi_{+,r}} = 0$$

$$D_{-,\mu} \left( \frac{D_- \mathcal{L}_-}{D_-(D_{-,\mu} \Phi_{-,r})} \right) - \frac{D_- \mathcal{L}_-}{D_- \Phi_{-,r}} - A_-^r \sigma_- = 0$$

$$\mathcal{R}^{\mu\nu} - \frac{1}{2}g^{\mu\nu}\mathcal{R} = -\frac{8\pi\kappa}{c^2}T_{-}^{\mu\nu}\sigma_-$$

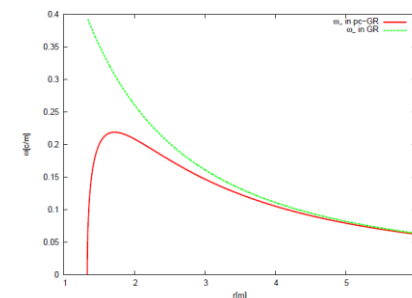
- Al final uno debe proyectar reemplazando todas las variables pseudocomplejas por su parte pseudoreal y tomar  $\ell = 0$

$$\Omega_- = B / r^2$$

- Schwarzschild  $d\omega^2 = \left(1 - \frac{2m}{r} + \frac{\Omega_-}{2r}\right) e^{\frac{r}{2}} (dx^0)^2 - \left(1 - \frac{2m}{r} + \frac{\Omega_-}{2r}\right)^{-1} (dr)^2 - r^2 ((d\theta)^2 + \sin^2\theta(d\phi)^2)$

$$\omega_{GR} = c \sqrt{\frac{m}{r^3}}$$

$$\omega_{pc-GR} = \sqrt{\frac{m}{r^3} - \frac{3B}{4r^5}}$$





# AN OVERVIEW OF QUANTUM FIELD THEORY IN CURVED SPACETIMES



- **Ecuación de Klein Gordon**  $(\square - m^2 + \xi R)\Phi = 0$
- **Campos**  $A = \Phi(f)$  **forman un algebra**  $\mathcal{A}$ ,
  1.  $f \mapsto \Phi(f)$  is a linear map (linearity),
  2.  $\Phi(f)^* = \Phi(f)$  (hermicity),
  3.  $\Phi((\square - m^2 + \xi R)f) = 0$  (field equation),
  4.  $[\Phi(f), \Phi(g)] = -iE(f, g)\mathbb{1}$ ,  $g \in C_0^\infty(M)$  (covariant CCR).
- **Los estados  $\omega$  son funcionales**
  1.  $\omega : A \mapsto z \in \mathbb{C}$  is linear (linearity)
  2.  $\omega(\mathbb{1}) = 1$  (normalisation),
  3.  $\omega(A^*A) \geq 0$  (positivity).
- **Método de Gelfand-Neimark y Siegel**

GNS: Given  $\mathcal{A}$  and  $\omega$ , construct the Hilbert space of the theory and have algebra elements represented on it.
- **Produce una representación de los campos como operadores lineales en el espacio de Hilbert y un producto interno.**

# EXTRACTING GEOMETRY FROM QUANTUM SPACETIME: OBSTACLES DOWN THE ROAD



Extended classical probe

Effective trajectory

CM in GR

W. Beigböck, *Commun. Math. Phys.* **5**, 106 (1967)

W.G. Dixon, *Il Nuovo Cimento* **34**, 317 (1961)

$$g_{ab}(x_0)U_0^a \Xi_{(i)}^b(x_0, U_0) = 0.$$

$$\exp_{x_0} [\Xi_{(i)}^a(x_0, U_0)] = y_{(i)}(x_0, U_0).$$

$$S(x_0) = \exp_{x_0} \left[ \frac{\sum_{(i)=1}^N \Xi_{(i)}^a(x_0) E_{(i)}(x_0)}{\sum_{j=1}^N E_{(j)}(x_0)} \right]$$

$$\sum_{(i)=1}^N \Xi_{(i)}^a(Z) E_{(i)}(Z) = 0$$

• Relativistic center of mass (Pryce, 1948)

Requirements:

1. Part of four-vector (worldline frame independent)
2. At rest in COMom frame
3. Move in straight line (no external forces)
4.  $X_i$ 's commute (???)

Definitions:

1.  $X_i$ 's weighted by rest mass (Eddington, 1946)
2. Apply (1) in COMom, LT to other frames
3.  $X_i$ 's weighted by energies (*centroid*)
4. Apply (3) in COMom, LT to other frames (*CM*)

Score

	1	2	3	4
1	-	-	-	x
2	x	-	-	x
3	-	x	x	-
4	x	x	x	-

3.  $[q_i, q_j] = -\epsilon_{ijk} H^{-2} S_k$
4.  $[X_i, X_j] = \epsilon_{ijk} M^{-2} S_k$

Is it an atom or is it a molecule?

Not associative (never mentioned):

(AB)C ≠ A(BC)

CoM worldline not geodesic

Effective geometry?

No effective metric

# QUANTUM COSMOLOGY OF QUADRATIC F(R) THEORIES WITH A FRW METRIC



$$A = \frac{1}{2\kappa^2} \int \sqrt{-g} (R + \alpha R^2) d^4x \quad , \quad A = \frac{1}{2\kappa^2} \int \sqrt{-g} [R + \phi(\beta\phi + R)] d^4x$$

$$3k^2 + a^2 (6\dot{a}\ddot{a} - 3\ddot{a}^2 - 2k\beta) + \dot{a}^2 (6a\ddot{a} - 9\dot{a}^2 - 2\beta a^2 - 6k) = 0$$

$$\pi_a = -\frac{6a}{N} [2(\phi + 1)\dot{a} + a\dot{\phi}] \quad \text{and} \quad \pi_\phi = -\frac{6a^2\dot{a}}{N}$$

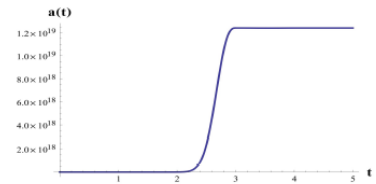


Figure:  $a(t)$  with  $\beta = -200$  ( $k = 0$ )

$$\left[ (\phi + 1) \frac{\partial^2}{\partial \phi^2} - a \frac{\partial^2}{\partial \phi \partial a} + 2 \frac{\partial}{\partial \phi} + 6\beta\phi^2 a^6 + 36k(\phi + 1) a^4 \right] \psi(a, \phi) = 0$$

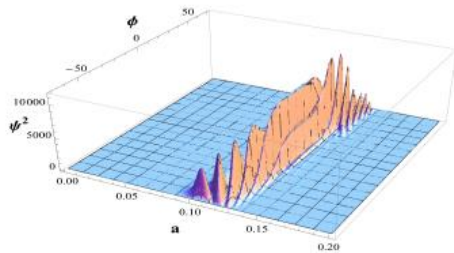


Figure:  $\psi(a, \phi)$ ,  $\beta = -200$ .

$$\Psi(a, \tau) = \frac{1}{\sqrt{\int_0^\infty da |\psi(a, \phi)|^2}} \psi(a, \phi) \Big|_{\phi=\tau}$$

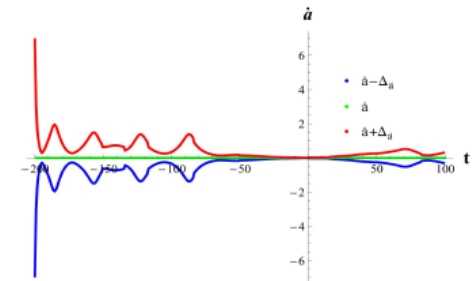


Figure:  $\dot{a}(t)$  with fluctuations

# GEOMETRY OF THE QUANTUM SPACE



$$\mathcal{F}(\lambda, \lambda + \delta\lambda) = |\langle \psi(\lambda + \delta\lambda) | \psi(\lambda) \rangle|, \quad \mathcal{F}(\lambda, \lambda + \delta\lambda) = 1 + \frac{1}{2} G_{ab} \delta\lambda^a \delta\lambda^b + \dots,$$

$$G_{ab} = \langle \partial_a \psi | \partial_b \psi \rangle - \langle \partial_a \psi | \psi \rangle \langle \psi | \partial_b \psi \rangle. \quad \langle \psi | \partial_a \psi \rangle \equiv -iA_a$$

$$\frac{1}{2} F_{ab} \equiv \mathbf{Im} G_{ab} = \frac{1}{2i} (\langle \partial_a \psi | \partial_b \psi \rangle - \langle \partial_b \psi | \partial_a \psi \rangle) = \frac{1}{2} \{ \partial_a A_b - \partial_b A_a \}$$

## A Lagrangian approach to the quantum information metric and Berry curvature

$$-\infty < t < 0 \quad \mathcal{L}_0(\lambda^a) \quad |\Omega_0\rangle \quad 0 < t < \infty \quad \mathcal{L}_1 = \mathcal{L}_0 + \delta\lambda^a \mathcal{O}_a \quad |\Omega_1\rangle$$

$$\langle \Omega_1 | \Omega_0 \rangle = \frac{\langle \exp\left(-\int_0^\infty d\tau \int d^d x \delta\lambda^a \mathcal{O}_a\right) \rangle}{\langle \exp\left(-\int_{-\infty}^\infty d\tau \int d^d x \delta\lambda^a \mathcal{O}_a\right) \rangle^{1/2}},$$

$$g_{ab} = \int_{-\infty}^0 d\tau_1 \int_0^\infty d\tau_2 \left( \frac{1}{2} \langle \{ \mathcal{O}_a(\tau_1), \mathcal{O}_b(\tau_2) \} \rangle - \langle \mathcal{O}_a(\tau_1) \rangle \langle \mathcal{O}_b(\tau_2) \rangle \right)$$

$$F_{ab} = \frac{1}{i} \int_{-\infty}^0 d\tau_1 \int_0^\infty d\tau_2 \langle [ \mathcal{O}_a(\tau_1), \mathcal{O}_b(\tau_2) ] \rangle.$$

### EJEMPLO

$$H = Zp^2 + Y \{p, q\} + Xq^2,$$

$$\{X, Y, Z\} \rightarrow \{X + \delta X, Y + \delta Y, Z + \delta Z\}$$

# ASPECTOS MODERNOS DE LAS AMPLITUDES DE DISPERSION



- Base de los resultados experimentales  $\frac{d\sigma}{d\Omega} \propto |\mathcal{A}|^2$
- La idea básica es emplear la representación espinorial

$$p_{a\dot{a}} = p_\mu \sigma_{a\dot{a}}^\mu \quad \gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix} \quad \sigma_{a\dot{a}}^\mu = (I, \vec{\sigma}), \quad \bar{\sigma}^{\mu\dot{a}a} = (I, -\vec{\sigma}) \quad u_-(\vec{p}) \bar{u}_-(\vec{p}) = \frac{1}{2}(1 - \gamma_5)(-\not{p}) = \begin{pmatrix} 0 & -p_{a\dot{a}} \\ 0 & 0 \end{pmatrix}$$

$$u_-(\vec{p}) = \begin{pmatrix} \phi_a \\ 0 \end{pmatrix} \quad \bar{u}_-(\vec{p}) = (0, \phi_a^*), \quad p_{a\dot{a}} = -\phi_a \phi_{\dot{a}}^* \quad |p] = u_-(\mathbf{p}) = v_+(\mathbf{p}),$$

$$[pk] = \phi^a \kappa_a = \bar{u}_+(\vec{p}) u_-(\vec{k}) = -[kp] \quad \langle pk \rangle = \phi_a^* \kappa^{a\dot{a}} = \bar{u}_-(\vec{p}) u_+(\vec{k}) = -\langle kp \rangle \quad |p\rangle = u_+(\mathbf{p}) = v_-(\mathbf{p}),$$

$$[p| = \bar{u}_+(\mathbf{p}) = \bar{v}_-(\mathbf{p}), \quad \langle p| = \bar{u}_-(\mathbf{p}) = \bar{v}_+(\mathbf{p}).$$

Some of the most important formulas that are needed to compute scattering amplitudes :

$[ij] = -[ji],$

$\langle i|\gamma_\mu|j\rangle = [j|\gamma_\mu|i],$

$\langle ij\rangle = [ji]^*,$

$\langle i|\gamma_\mu|j\rangle \langle k|\gamma^\mu|l\rangle = 2\langle ik\rangle[lj],$

$\langle ij\rangle[ji] = \langle ij\rangle\langle ij\rangle^* = |\langle ij\rangle|^2,$

$\langle ab\rangle\langle cd\rangle = \langle ac\rangle\langle bd\rangle + \langle ad\rangle\langle cb\rangle,$

$\langle ij\rangle[ji] = -2k_i \cdot k_j = s_{ij},$

$\sum_{k=1}^n \langle ik\rangle[kj] = 0,$

$\langle pk\rangle = [pk] = 0$

**160 amplitudes, 132 son cero**  
**Quedan 28, sólo 14 por simetría.**

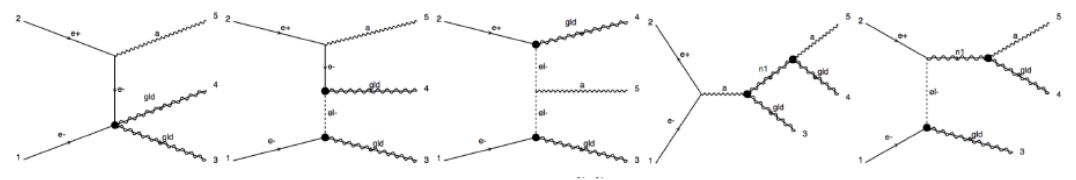


Figure: Feynman Diagrams for  $e^+e^- \rightarrow \gamma \tilde{G} \tilde{G}$

**MUCHAS GRACIAS**