Study of Michel parameters in τ decays at e^+e^- B factories

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Mini Workshop on Tau Physics
CINVESTAV, Mexico, 22-23 May 2017

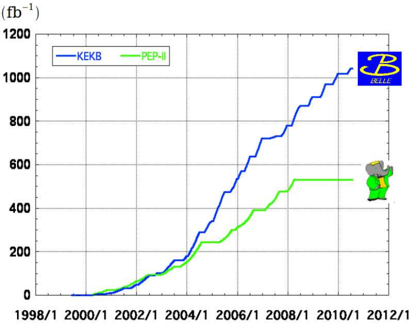
Outline:

Introduction

- The world largest statistics of τ leptons collected by e^+e^- B factories (Belle and *BABAR*) opens new era in the precision tests of the Standard Model (SM).
- Basic tau properties, like: lifetime, mass, couplings, electric dipole moment, anomalous magnetic dipole moment and other appear as free parameters in the SM, which should be measured experimentally as precise as possible, or provide unique possibility to test SM and search for the effects of New Physics.
- In the SM τ decays due to the charged weak interaction described by the exchange of $W^\pm$ with a pure vector coupling to only left-handed fermions. There are two main classes of tau decays:
  - Decays with leptons, like: $\tau^- \to \ell^-\bar{\nu}_\ell\nu_\tau$, $\tau^- \to \ell^-\bar{\nu}_\ell\nu_\tau\gamma$, $\tau^- \to \ell^-\ell'^+\ell'^-\bar{\nu}_\ell\nu_\tau$; $\ell, \ell' = e, \mu$. They provide very clean laboratory to probe electroweak couplings, which is complementary/competitive to precision studies with muon (in experiments with muon beam). Plenty of New Physics models can be tested/constrained in the precision studies of the dynamics of decays with leptons.
  - Hadronic decays of τ offer unique tools for the precision study of low energy QCD.

Introduction: e^+e^- B factories

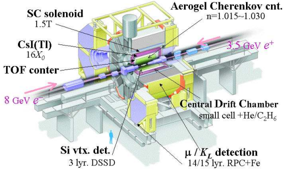
Integrated luminosity of B factories



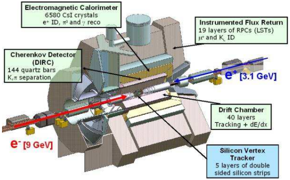
> 1 ab^{-1}
On resonance:
$\Upsilon(5S)$: 121 fb^{-1}
$\Upsilon(4S)$: 711 fb^{-1}
$\Upsilon(3S)$: 3 fb^{-1}
$\Upsilon(2S)$: 25 fb^{-1}
$\Upsilon(1S)$: 6 fb^{-1}
Off reson./scan:
~ 100 fb^{-1}

~ 550 fb^{-1}
On resonance:
$\Upsilon(4S)$: 433 fb^{-1}
$\Upsilon(3S)$: 30 fb^{-1}
$\Upsilon(2S)$: 14 fb^{-1}
Off resonance:
~ 54 fb^{-1}

Belle Detector



BABAR detector



$$\sigma(b\bar{b}) = 1.05 \text{ nb } N_{b\bar{b}} = 1.2 \times 10^9$$
$$\sigma(c\bar{c}) = 1.3 \text{ nb } N_{c\bar{c}} = 2.0 \times 10^9$$
$$\sigma(\tau\tau) = 0.9 \text{ nb } N_{\tau\tau} = 1.4 \times 10^9$$

B-factories are also charm- and τ-factories !

B-factory experimental strategy is proved to be fruitful to search for New Physics.

Precision studies of τ at e^+e^- B factories

- **Michel parameters in $\tau \to \ell\nu\nu(\gamma)$ (ρ, η, ξ, δ, $\bar{\eta}$, $\xi\kappa$):**

  Belle(prelim.): $\bar{\eta} = -1.3 \pm 1.5 \pm 0.8$, $\xi\kappa = 0.5 \pm 0.4 \pm 0.2$; arXiv:1609.08280

  Belle: Measurement of ρ, η, ξ, $\xi\delta$ is going on; Belle-CONF-1402: arXiv:1409.4969

- Tau lifetime:
  Belle: $\tau_\tau = (290.17 \pm 0.53(\mathrm{stat}) \pm 0.33(\mathrm{syst}))$ fs; PRL 112, 031801 (2014)
  BABAR(prelim.): $\tau_\tau = (289.40 \pm 0.91(\mathrm{stat}) \pm 0.90(\mathrm{syst}))$ fs; Nucl. Phys. B 144, 105 (2005)

- Tau mass:
  Belle: $m_\tau = (1776.61 \pm 0.13(\mathrm{stat}) \pm 0.35(\mathrm{syst}))$ MeV/c^2; PRL 99, 011801 (2007)
  BABAR: $m_\tau = (1776.68 \pm 0.12(\mathrm{stat}) \pm 0.41(\mathrm{syst}))$ MeV/c^2; PRD 80, 092005 (2009)
  Accuracy comparable with the most precision measurements done by **KEDR** and **BES** at the $\tau^+\tau^-$ production threshold.

- Tau electric dipole moment (EDM):
  Belle: $\mathrm{Re}(d_\tau) = (1.15 \pm 1.70) \times 10^{-17}$ e·cm, $\mathrm{Im}(d_\tau) = (-0.83 \pm 0.86) \times 10^{-17}$ e·cm;
  PLB 551, 16 (2003) ($\int Ldt = 29.5$ fb^{-1}) We are working on EDM with full statistics

- Hadronic contribution to a_μ ($\tau^- \to \pi^-\pi^0\nu_\tau$):
  Belle: $a_\mu^{\pi\pi} = (523.5 \pm 1.1(\mathrm{stat}) \pm 3.7(\mathrm{syst})) \times 10^{-10}$; PRD 78, 072006 (2008)

- Lepton universality:
  BABAR: $\left(\frac{g_\mu}{g_e}\right)_\tau = 1.0036 \pm 0.0020$, $\left(\frac{g_\tau}{g_\mu}\right)_\mathrm{h} = 0.9850 \pm 0.0054$, h=$\pi$, K;
  PRL 105, 051602 (2010)

- Anomalous magnetic moment of τ (a_τ):
  - Not promising in $\tau^- \to \ell^-\bar{\nu}_\ell\nu_\tau\gamma$ at e^+e^- B factories: JHEP 1603, 140 (2016)
  - **Technique to measure a_τ in $e^+e^- \to \tau^+\tau^-$ process is under discussion.**

Michel parameters

In the SM charged weak interaction is described by the exchange of $W^{\pm}$ with a pure vector coupling to only left-handed fermions ("V-A" Lorentz structure). Deviations from "V-A" indicate New Physics. $\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau$ ($\ell = e, \mu$) decays provide clean laboratory to probe electroweak couplings.

The most general, Lorentz invariant four-lepton interaction matrix element:

$$\mathcal{M} = \frac{4G}{\sqrt{2}} \sum_{\substack{N=S,V,T \\ i,j=L,R}} g_{ij}^N \left[\bar{u}_i(l^-)\Gamma^N v_n(\bar{\nu}_l)\right] \left[\bar{u}_m(\nu_\tau)\Gamma_N u_j(\tau^-)\right],$$

$$\Gamma^S = 1,\ \Gamma^V = \gamma^\mu,\ \Gamma^T = \frac{i}{2\sqrt{2}}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$$

Ten couplings g_{ij}^N, in the SM the only non-zero constant is $g_{LL}^V = 1$

Four bilinear combinations of g_{ij}^N, which are called as Michel parameters (MP): ρ, η, ξ and δ appear in the energy spectrum of the outgoing lepton:

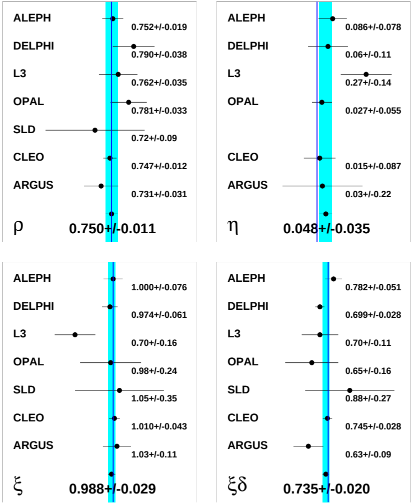
$$\frac{d\Gamma(\tau^\mp)}{d\Omega dx} = \frac{4G_F^2 M_\tau E_{\max}^4}{(2\pi)^4}\sqrt{x^2 - x_0^2}\left(x(1-x) + \frac{2}{9}\rho(4x^2 - 3x - x_0^2) + \eta x_0(1-x)\right.$$

$$\left.\mp\frac{1}{3}P_\tau cos\theta_\ell \xi\sqrt{x^2 - x_0^2}\left[1 - x + \frac{2}{3}\delta\left(4x - 4 + \sqrt{1 - x_0^2}\right)\right]\right),\ x = \frac{E_\ell}{E_{\max}},\ x_0 = \frac{m_\ell}{E_{\max}}$$

In the SM: $\rho = \frac{3}{4}$, $\eta = 0$, $\xi = 1$, $\delta = \frac{3}{4}$

Status of Michel parameters in τ decays

Michel par.	Measured value	Experiment	SM value
ρ (e or μ)	$0.747 \pm 0.010 \pm 0.006$ 1.2%	CLEO-97	3/4
η (e or μ)	$0.012 \pm 0.026 \pm 0.004$ 2.6%	ALEPH-01	0
ξ (e or μ)	$1.007 \pm 0.040 \pm 0.015$ 4.3%	CLEO-97	1
$\xi\delta$ (e or μ)	$0.745 \pm 0.026 \pm 0.009$ 2.8%	CLEO-97	3/4
ξ_{h} (all hadr.)	$0.992 \pm 0.007 \pm 0.008$ 1.1%	ALEPH-01	1



Status of Michel parameters in τ decays

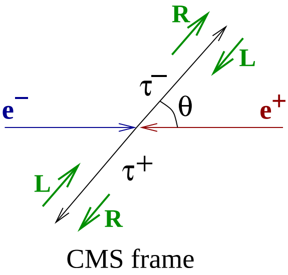
With Belle statistics, which is about 300 times larger than the previous experimental $\tau\tau$ data samples, we can improve MP uncertainties by one order of magnitude.
In BSM models the couplings to τ are expected to be larger than those to μ. Contribution from New Physics in τ decays can be enhanced by a factor of $(\frac{m_\tau}{m_\mu})^2$.

- **Type II 2HDM:** $\eta_\mu(\tau) = \frac{m_\mu M_\tau}{2}\left(\frac{\tan^2\beta}{M^2_{H\pm}}\right)^2$; $\frac{\eta_\mu(\tau)}{\eta_e(\mu)} = \frac{M_\tau}{m_e} \approx 3500$
- **Tensor interaction:**
  $\mathcal{L} = \frac{g}{2\sqrt{2}}W^\mu\left\{\bar\nu\gamma_\mu(1-\gamma^5)\tau + \frac{\kappa^W_\tau}{2m_\tau}\partial^\nu\left(\bar\nu\sigma_{\mu\ nu}(1-\gamma^5)\tau\right)\right\}$,
  $-0.096 < \kappa^W_\tau < 0.037$: DELPHI Abreu EPJ C16 (2000) 229.
- **Unparticles:** Moyotl PRD 84 (2011) 073010, Choudhury PLB 658 (2008) 148.
- **Lorentz and CPTV:** Hollenberg PLB **701** (2011) 89
- **Heavy Majorana neutrino:** M. Doi *et al.*, Prog. Theor. Phys. **118** (2007) 1069.

Method, spin-spin correlation in $\tau^+\tau^-$

To measure ξ and δ MP we have to know τ spin direction. Effect of τ spin-spin correlation in $e^+e^- \to \tau^+(\vec{\zeta}^+)\tau^-(\vec{\zeta}^-)$ can be used:

$$\frac{d\sigma(\vec{\zeta}^-,\vec{\zeta}^+)}{d\Omega} = \frac{\alpha^2}{64E_\tau^2}\beta_\tau(D_0+D_{ij}\zeta_i^-\zeta_j^+)$$



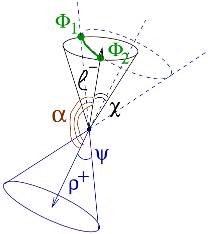
$$D_0 = 1 + cos^2\theta + \frac{1}{\gamma_\tau^2}\sin^2\theta$$

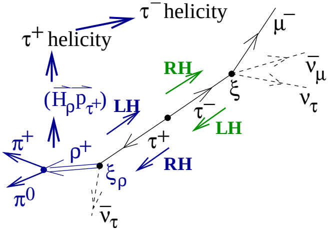
$$D_{ij} = \begin{pmatrix} (1+\frac{1}{\gamma_\tau^2})\sin^2\theta & 0 & \frac{1}{\gamma_\tau}\sin 2\theta \\ 0 & -\beta_\tau^2\sin^2\theta & 0 \\ \frac{1}{\gamma_\tau}\sin 2\theta & 0 & 1+\cos^2\theta - \frac{1}{\gamma_\tau^2}\sin^2\theta \end{pmatrix}$$

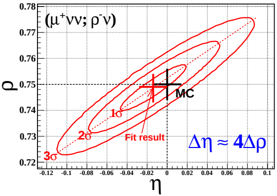
τ^- and τ^+ helicities are 95% anti-correlated

Method, study of $(\ell\nu\nu;\ \rho\nu)$ and $(\rho\nu;\ \rho\nu)$ events

Effect of τ spin-spin correlation is used to measure ξ and δ MP.
Events of the $(\tau^{\mp} \to \ell^{\mp}\nu\nu;\ \tau^{\pm} \to \rho^{\pm}\nu)$ topology are used to measure: ρ, η, $\xi_\rho\xi$ and $\xi_\rho\xi\delta$, while $(\tau^{\mp} \to \rho^{\mp}\nu;\ \tau^{\pm} \to \rho^{\pm}\nu)$ events are used to extract ξ_ρ^2.







$$\frac{d\sigma(\ell^{\mp}\nu\nu,\rho^{\pm}\nu)}{dE^*_\ell\, d\Omega^*_\ell\, d\Omega^*_\rho\, dm^2_{\pi\pi}\, d\tilde{\Omega}_\pi\, d\Omega_\tau} = A_0 + \rho A_1 + \eta A_2 + \xi_\rho\xi A_3 + \xi_\rho\xi\delta A_4 = \sum_{i=0}^{4} A_i\Theta_i$$

$$\mathcal{F}(\vec{z}) = \frac{d\sigma(\ell^{\mp}\nu\nu,\rho^{\pm}\nu)}{dp_\ell\, d\Omega_\ell\, dp_\rho\, d\Omega_\rho\, dm^2_{\pi\pi}\, d\tilde{\Omega}_\pi} = \int\limits_{\Phi_1}^{\Phi_2} \frac{d\sigma(\ell^{\mp}\nu\nu,\rho^{\pm}\nu)}{dE^*_\ell\, d\Omega^*_\ell\, d\Omega^*_\rho\, dm^2_{\pi\pi}\, d\tilde{\Omega}_\pi\, d\Omega_\tau} \left|\frac{\partial(E^*_\ell,\Omega^*_\ell,\Omega^*_\rho,\Omega_\tau)}{\partial(p_\ell,\Omega_\ell,p_\rho,\Omega_\rho,\Phi_\tau)}\right| d\Phi_\tau$$

$$L = \prod_{k=1}^{N} \mathcal{P}^{(k)},\ \mathcal{P}^{(k)} = \mathcal{F}(\vec{z}^{(k)})/\mathcal{N}(\vec{\Theta}),\ \mathcal{N}(\vec{\Theta}) = \int \mathcal{F}(\vec{z})d\vec{z},\ \vec{\Theta} = (1,\rho,\eta,\xi_\rho\xi_\ell,\xi_\rho\xi_\ell\delta_\ell)$$

MP are extracted in the unbinned maximum likelihood fit of $(\ell\nu\nu;\ \rho\nu)$ events in the 9D phase space $\vec{z} = (p_\ell,\ \cos\theta_\ell,\ \phi_\ell,\ p_\rho,\ \cos\theta_\rho,\ \phi_\rho,\ m^2_{\pi\pi},\ \cos\tilde{\theta}_\pi,\ \tilde{\phi}_\pi)$ in CMS.

$$J^\mu =< h|\bar{d}\gamma^\mu(c_V + c_A\gamma^5)u|0 >$$

Michel formalism for the $\tau^- \to h^-\nu_\tau$ includes
$\xi_h = -\frac{2Re(c_V^* c_A)}{|c_V|^2+|c_A|^2} = -h_{\nu_\tau}$(=1 in SM):

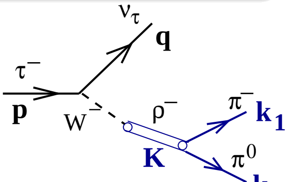
$$\frac{d\Gamma(\tau^\mp \to \pi^\mp\nu)}{d\Omega_\pi} = C\left(1 \pm \xi_\pi P_\tau \cos\theta_\pi\right)$$

$$\frac{d\Gamma(\tau^\mp \to \rho^\mp\nu)}{dm^2_{\pi\pi} d\Omega_\rho d\Omega^*_\pi} = f(\vec{k}_1, \vec{k}_2) \pm \xi_\rho \vec{P}_\tau \vec{g}(\vec{k}_1, \vec{k}_2) = f(\vec{k}_1, \vec{k}_2)(1 \pm \xi_\rho \vec{P}_\tau \vec{H}_\rho)$$

$\vec{H}_\rho = M_\tau \frac{2(q,Q)\vec{Q}+Q^2\vec{K}}{2(p,Q)(q,Q)-Q^2(p,q)}$

$\vec{H}_\rho$- polarimeter vector

$\tau^\pm \to \rho^\pm(\to \pi^\pm\pi^0)\nu$ can be used as a spin analyzer



$\mathbf{Q} = \mathbf{k_1} - \mathbf{k_2}$

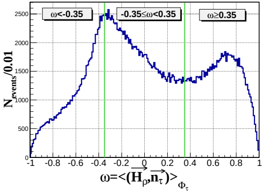
$\mathbf{K} = \mathbf{k_1} + \mathbf{k_2}$

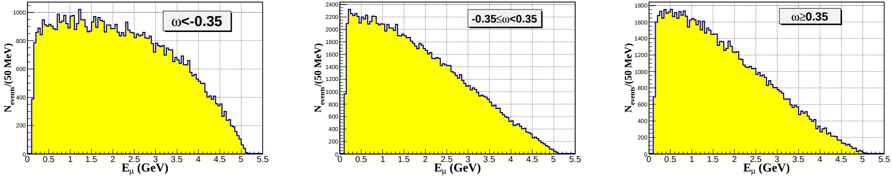
Method, helicity sensitive variable ω

M. Davier *et. al* Phys. Lett. B **306** (1993) 411.

Helicity sensitive variable ω is introduced as:

$$\omega = \frac{1}{\Phi_2 - \Phi_1} \int_{\Phi_1}^{\Phi_2} (\vec{H}_{\rho^\pm}, \vec{n}_{\tau^\pm}) d\Phi = < (\vec{H}_{\rho^\pm}, \vec{n}_{\tau^\pm}) >_{\Phi_\tau}$$





Spin-spin correlation manifests itself through momentum-momentum correlations of final lepton and pions.

Theoretical framework

- W. Fetscher, Phys. Rev. D **42** (1990) 1544. $\ell_1^{\mp} - \ell_2^{\pm}$, $\ell^{\mp} - h^{\pm}$, $\ell = e,\ \mu;\ h = \pi,\ K$.
- K. Tamai, Nucl. Phys. B **668** (2003) 385. (KEK Preprint 2003-14, Belle note 471) $\ell^{\mp} - \rho^{\pm}(\to \pi^{\pm}\pi^{0})$ + feasibility study.

$$\frac{d\sigma(\vec{\zeta},\vec{\zeta}')}{d\Omega} = \frac{\alpha^2}{64E_\tau^2}\beta_\tau(D_0 + D_{ij}\zeta_i\zeta_j')$$

$$\frac{d\Gamma(\tau^{\mp}(\vec{\zeta}^*) \to \ell^{\mp}\nu\nu)}{dx^* d\Omega_\ell^*} = \kappa_\ell(A(x^*) \mp \xi\vec{n}_\ell^*\vec{\zeta}^* B(x^*)),\ x^* = E_\ell^*/E_{\ell max}^*$$

$$A(x^*) = A_0(x^*) + \rho A_1(x^*) + \eta A_2(x^*),\ B(x^*) = B_1(x^*) + \delta B_2(x^*)$$

$$\frac{d\Gamma(\tau^{\pm}(\vec{\zeta'}^*) \to \rho^{\pm}\nu)}{dm_{\pi\pi}^2 d\Omega_\rho^* d\tilde{\Omega}_\pi} = \kappa_\rho(A' \mp \xi_\rho\vec{B'}\vec{\zeta'}^*)W(m_{\pi\pi}^2)$$

$$A' = 2(q,Q)Q_0^* - Q^2q_0^*,\ \vec{B'} = Q^2\vec{K}^* + 2(q,Q)\vec{Q}^*,\ W = |F_\pi(m_{\pi\pi}^2)|^2\frac{p_\rho(m_{\pi\pi}^2)\tilde{p}_\pi(m_{\pi\pi}^2)}{M_\tau m_{\pi\pi}}$$

$$\frac{d\sigma(\ell^{\mp},\rho^{\pm})}{dE_\ell^* d\Omega_\ell^* d\Omega_\rho^* dm_{\pi\pi}^2 d\tilde{\Omega}_\pi d\Omega_\tau} = \kappa_\ell\kappa_\rho\frac{\alpha^2\beta_\tau}{64E_\tau^2}(D_0A'A(E_\ell^*) + \xi_\rho\xi_\ell D_{ij}n_{\ell i}^*B_j'B(E_\ell^*))W(m_{\pi\pi}^2)$$

$$\frac{d\sigma(\ell^{\mp},\rho^{\pm})}{dp_\ell d\Omega_\ell dp_\rho d\Omega_\rho dm_{\pi\pi}^2 d\tilde{\Omega}_\pi} = \int_{\Phi_1}^{\Phi_2}\frac{d\sigma(\ell^{\mp},\rho^{\pm})}{dE_\ell^* d\Omega_\ell^* d\Omega_\rho^* dm_{\pi\pi}^2 d\tilde{\Omega}_\pi d\Omega_\tau}\left|\frac{\partial(E_\ell^*,\Omega_\ell^*,\Omega_\rho^*,\Omega_\tau)}{\partial(p_\ell,\Omega_\ell,p_\rho,\Omega_\rho,\Phi_\tau)}\right|d\Phi_\tau$$

Multidimensional unbinned maximum likelihood fit

4 Michel parameters ($\vec{\Theta} = (1,\ \rho,\ \eta,\ \xi_\rho\xi_\ell,\ \xi_\rho\xi_\ell\delta_\ell)$) are extracted in the unbinned maximum likelihood fit of $(\ell\nu\nu;\ \rho\nu)$ events in the 9D phase space in CMS,
$\vec{z} = (p_\ell,\ \cos\theta_\ell,\ \phi_\ell,\ p_\rho,\ \cos\theta_\rho,\ \phi_\rho,\ m_{\pi\pi},\ \cos\tilde{\theta}_\pi,\ \tilde{\phi}_\pi)$. The PDF for individual k-th event is written in the form:

$$\mathcal{P}^{(k)} = \frac{\mathcal{F}(\vec{z}^{(k)})}{\mathcal{N}(\vec{\Theta})},\ \mathcal{N}(\vec{\Theta}) = \int \mathcal{F}(\vec{z})d\vec{z}$$

Likelihood function for N events:

$$L = \prod_{k=1}^{N}\mathcal{P}^{(k)},\ \mathcal{L} = -\ln L = N\ln\mathcal{N}(\vec{\Theta}) - \sum_{k=1}^{N}\ln\mathcal{F}^{(k)},\ \mathcal{F}^{(k)} = \mathcal{F}(\vec{z}^{(k)})$$

$$\mathcal{F}^{(k)} = A_0^{(k)}\Theta_0 + A_1^{(k)}\Theta_1 + A_2^{(k)}\Theta_2 + A_3^{(k)}\Theta_3 + A_4^{(k)}\Theta_4 = \sum_{i=0}^{4}A_i^{(k)}\Theta_i$$

$$\mathcal{N} = C_0\Theta_0 + C_1\Theta_1 + C_2\Theta_2 + C_3\Theta_3 + C_4\Theta_4,\ C_j = \frac{1}{N}\sum_{k=1}^{N}C_j^{(k)},\ C_j^{(k)} = \frac{A_j^{(k)}}{\sum_{i=0}^{4}A_i^{(k)}\Theta_i^{MC}}$$

$$\vec{\Theta}^{MC} = (1,\ 0.75,\ 0,\ 1,\ 0.75),\ \mathcal{L} = N\ln\left(\sum_{j=0}^{4}C_j\Theta_j\right) - \sum_{k=1}^{N}\ln\left(\sum_{i=0}^{4}A_i^{(k)}\Theta_i\right)$$

As a result fitted statistics is represented by a set of $5\times N$ values of $A_i^{(k)}$ $(k = 1 \div N,\ i = 0 \div 4)$, which is calculated only once.

C_i $(i = 0 \div 4)$ are calculated using MC simulation.

In ideal case (no rad. corr., $\varepsilon = 100\%$): $C_0 = 1$, $C_2 = 4m_\ell/m_\tau$, $C_{1,3,4} = 0$

Suppose we have $N_{\rm MC}$ MC events, which were simulated with particular set $\vec{\Theta}^{\rm MC}$. By reweighting each event we can calculate normalization for arbitrary set $\vec{\Theta}$:

$$\mathcal{N}(\vec{\Theta}) \approx \frac{1}{N_{MC}} \sum_{k=1}^{N_{MC}} w^{(k)},\ w^{(k)} = \frac{A_i^{(k)}\Theta_i}{A_j^{(k)}\Theta_j^{MC}} = B_m^{(k)}\Theta_m,\ B_m^{(k)} = \frac{A_m^{(k)}}{A_j^{(k)}\Theta_j^{MC}}$$

$$\mathcal{N}(\vec{\Theta}) = C_i\Theta_i,\ C_i = \frac{1}{N_{MC}} \sum_{k=1}^{N_{MC}} B_i^{(k)}$$

This algorithm can be easily extended to take into account selection efficiency:

$$\mathcal{F}(\vec{z}) \to \mathcal{F}'(\vec{z}) = \mathcal{F}(\vec{z})\epsilon(\vec{z}),\ \mathcal{N}'(\vec{\Theta}) = \int \mathcal{F}(\vec{z})\epsilon(\vec{z})d\vec{z}$$

$$\mathcal{L} = N_{\rm sel} \ln \mathcal{N}'(\vec{\Theta}) - \sum_{k=1}^{N_{\rm sel}} \ln(\mathcal{F}^{(k)}\epsilon(\vec{z})) = N_{\rm sel} \ln(C_i'\Theta_i) - \sum_{k=1}^{N_{\rm sel}} \ln(A_i^{(k)}\Theta_i) - \cancel{\sum_{k=1}^{N_{\rm sel}} \ln\epsilon(\vec{z})}$$

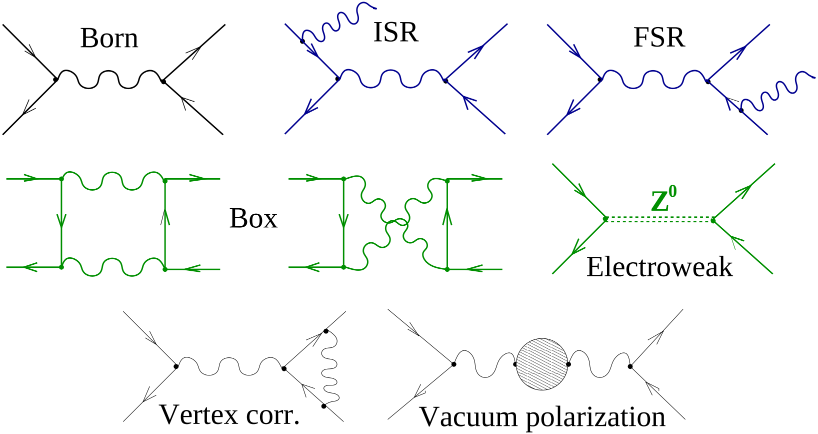
$$C_i' = \frac{1}{N_{MC}} \sum_{k=1}^{N_{MC}^{\rm sel}} B_i^{(k)}$$

Accuracy of the evaluation of the C_i' coefficients is crucial in the precision measurement of Michel parameters.

Physical corrections

- **Radiative corrections to $e^+e^- \to \tau^+\tau^-$**
  - All $\mathcal{O}(\alpha^3)$ QED and electroweak higher order corrections to $e^+e^- \to \tau^+\tau^-(\gamma)$ are included:
    S. Jadach and Z. Was, Acta Phys. Polon. B **15** (1984) 1151 [Erratum-ibid. B **16** (1985) 483].
    A. B. Arbuzov *et al* JHEP **9710** (1997) 001.
  - KKMC based approach:
    We generate table of ISR photons and then use it to calculate visible differential cross section in CMS.
- **Radiative leptonic decays $\tau^- \to \ell^-\bar{\nu}_\ell\nu_\tau\gamma$**
  - Analytical approach based on:
    A. B. Arbuzov, Phys. Lett. B **524** (2002) 99. $\mathcal{O}(\alpha)$.
    A. Arbuzov, A. Czarnecki and A. Gaponenko, Phys. Rev. D **65** (2002) 113006. $\mathcal{O}(\alpha^2 \ln^2(\frac{m_\mu}{m_e}))$.
    A. Arbuzov and K. Melnikov, Phys. Rev. D **66** (2002) 093003. $\mathcal{O}(\alpha^2 \ln(\frac{m_\mu}{m_e}))$.
  - TAUOLA based approach:
    M. Jezabek, Comput. Phys. Commum. **70** (1992) 69.
    A. Czarnecki, M. Jezabek and J. H. Kuhn, Nucl. Phys. B **351** (1991) 70.
- **Radiative corrections to $\tau^- \to \pi^-\pi^0\nu_\tau$**
  - Analytical approach based on:
    F. Flores-Baez *et al*, Phys. Rev. Lett. D **74** (2006) 071301(R).
    A. Flores-Tlalpa *et al*, Nucl. Phys. B (Proc. Suppl.) **169** (2007) 250.
  - PHOTOS based approach

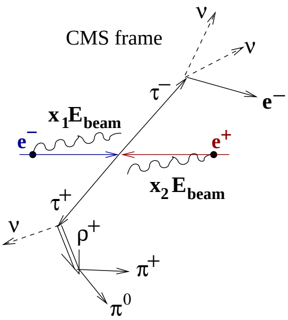
$\mathcal{O}(\alpha^3)$ corrections to $e^+e^- \to \tau^+\tau^-(\gamma)$

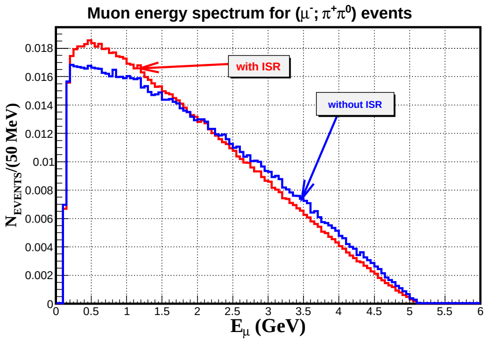


S. Jadach and Z. Was, Acta Phys. Polon. B **15** (1984) 1151 [Erratum-ibid. B **16** (1985) 483].
A. B. Arbuzov *et al* JHEP **9710** (1997) 001.

Charge-odd part of the cross section comes from the interference of the ISR and FSR diagrams as well as box and Born diagrams, and Z^0-exchange and Born diagrams.

Initial state radiation (ISR)





$$\frac{d\sigma_{\text{vis}}(s)}{dp_\ell d\Omega_\ell dp_\rho d\Omega_\rho dm^2_{\pi\pi} d\tilde{\Omega}_\pi} = \iint_0^1 dx_1 dx_2 D(x_1) D(x_2) \frac{d\sigma(s(1-x_1)(1-x_2))}{dp'_\ell d\Omega'_\ell dp'_\rho d\Omega'_\rho dm^2_{\pi\pi} d\tilde{\Omega}_\pi} \left|\frac{\partial(p'_\ell, \Omega'_\ell)}{\partial(p_\ell, \Omega_\ell)}\right| \left|\frac{\partial(p'_\rho, \Omega'_\rho)}{\partial(p_\rho, \Omega_\rho)}\right|$$

- $D(x) = x^{\beta/2-1} h(x)$ - probability function for initial $e^\mp$ to emit a γ-quantum jet carrying $x_{1,2}$ part of $e^\mp$ energy $E_{\text{beam}} = \sqrt{s}/2$. $\beta = \frac{2\alpha}{\pi}(\ln\frac{s}{m^2} - 1)$, $h(x)$ - smooth limited function.
- $\left|\frac{\partial(p'_i, \Omega'_i)}{\partial(p_i, \Omega_i)}\right|$ $(i = \ell,\ \rho)$ - Jacobian of transformation from the $\tau^+\tau^-$ rest frame to the Belle CMS.

Detector effects

In our fit procedure the probability density function for individual event after all physical corrections applied is:

$$\mathcal{P}^{vis}(\vec{z}|\vec{\Theta}) = \frac{\mathcal{F}^{vis}(\vec{z}|\vec{\Theta})}{\mathcal{N}(\vec{\Theta})},\ \mathcal{F}^{vis}(\vec{z}|\vec{\Theta}) = \frac{d\sigma_{vis}(\ell^{\mp},\rho^{\pm})}{dp_\ell d\Omega_\ell dp_\rho d\Omega_\rho dm^2_{\pi\pi} d\tilde{\Omega}_\pi} = A_i^{vis}(\vec{z})\Theta_i,$$

$$\mathcal{N}(\vec{\Theta}) = \int \mathcal{F}^{vis}(\vec{z})d\vec{z} = C_i^{vis}\Theta_i,\ C_i^{vis} = \int A_i^{vis}(\vec{z})d\vec{z}$$

$$\vec{z} = (p_\ell,\ \cos\theta_\ell,\ \phi_\ell,\ p_\rho,\ \cos\theta_\rho,\ \phi_\rho,\ m_{\pi\pi},\ \cos\tilde{\theta}_\pi,\ \tilde{\phi}_\pi)$$

For the reconstructed values $\vec{\tilde{z}}$:

$$\mathcal{P}_R^{vis}(\vec{\tilde{z}}|\vec{\Theta}) = \frac{1}{\tilde{\mathcal{N}}(\vec{\Theta})}\int \mathcal{F}^{vis}(\vec{z}|\vec{\Theta})\cdot\mathcal{R}(\vec{\tilde{z}},\vec{z})d\vec{z} = \frac{1}{\tilde{\mathcal{N}}(\vec{\Theta})}\tilde{A}_i^{vis}(\vec{\tilde{z}})\Theta_i,\ \tilde{\mathcal{N}}(\vec{\Theta}) = \tilde{C}_i^{vis}\Theta_i$$

$\mathcal{R}(\vec{\tilde{z}},\vec{z})$ includes:

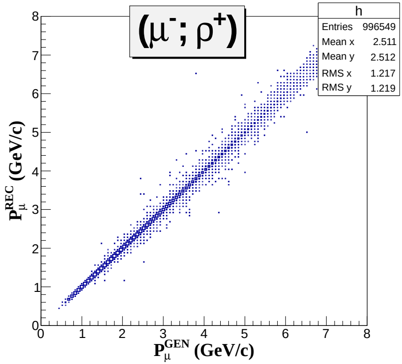
- Track momentum resolution ($\ell^{\mp},\ \pi^{\pm}$)
- γ energy and angular resolution (π^0)
- Effect of external bremsstrahlung for $e-\rho$ events
- Beam energy spread

Track momentum resolution

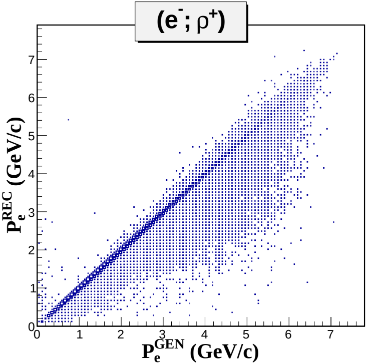
Uncertainties for the track helix parameters $(d_\rho,\ \phi_0,\ \kappa,\ d_z,\ tan\lambda)$ are propagated to the uncertainties of $(p_x,\ p_y,\ p_z)$ providing the covariance matrix $\hat{D}$. The resolution function for 1 track:

$$\mathcal{R}(\Delta p = \tilde{\vec{p}} - \vec{p}) = \frac{1}{(2\pi)^{3/2}\sqrt{det(\hat{D})}} e^{-\frac{1}{2}\Delta p^T \hat{D}^{-1} \Delta p}$$

Muon

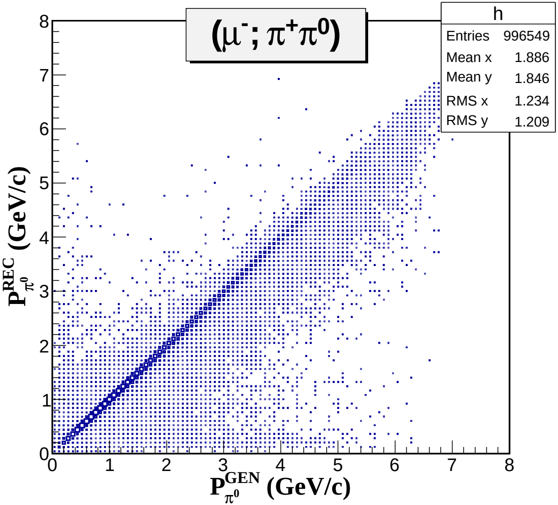


Electron



γ energy and angular resolution

Angular ($\theta,\ \varphi$) resolution is described by gaussian function. Energy resolution is described by logarithmic gaussian function.



Bremsstrahlung for $e-\rho$ events

Bremsstrahlung photon spectrum per unit of length ($E_e >> \frac{m}{\alpha Z^{1/3}}$): $\frac{dN_\gamma}{dxdE_\gamma} \simeq \frac{1}{X_0 E_\gamma}$. As a result the probability density function to emit bremsstrahlung photon:

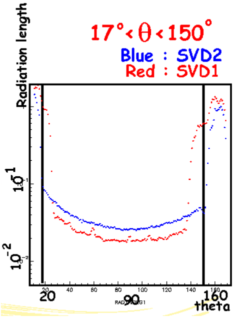
$$f(\varepsilon, \theta_{\rm electron}) = (1-p)\delta(\varepsilon) + pH(\varepsilon-\varepsilon_{\rm min})\frac{1}{\varepsilon ln(\frac{1}{\varepsilon_{\rm min}})}$$

$$\varepsilon = \frac{E_\gamma}{E_e},\ \varepsilon_{\rm min} = \frac{E_{\gamma min}}{E_e} = 10^{-4}$$

$$p = \frac{L}{1-\varepsilon_{\rm min}} ln(\frac{1}{\varepsilon_{\rm min}}),\ L = \frac{d_{\rm SVD} + d_{\rm vac.chamber}}{\sin\theta_{\rm electron}}$$

Belle SVD1 sample : $d_{\rm SVD} + d_{\rm vac.chamber} = 1.9\% X_0$

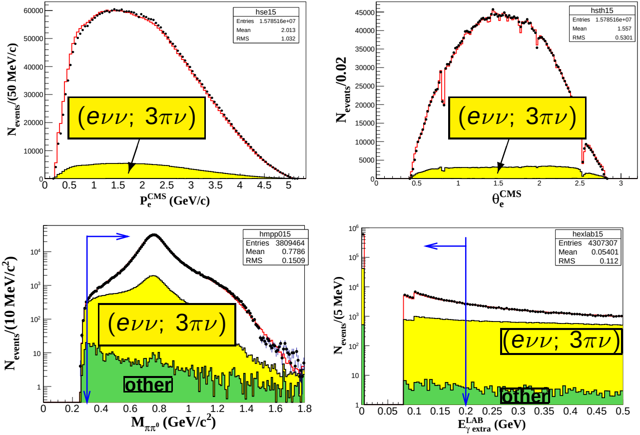
Belle SVD2 sample : $d_{\rm SVD} + d_{\rm vac.chamber} = 2.7\% X_0$



Selection criteria

- After the standard preselections we take events with two oppositely charged tracks, one of them is identified as lepton (eID, $\mu ID > 0.9$) and the other one as pion ($PID(\pi/K) > 0.4$).
- π^0 candidate is reconstructed from the pair of gammas ($E_\gamma^{\rm LAB} > 80$ MeV) satisfying 115 MeV/$c^2 < M_{\gamma\gamma} <$ 150 MeV/c^2, $P_{\pi^0}^{\rm CMS} > 0.3$ GeV/c.
- $\cos(\vec{P}_{\rm lep}, \vec{P}_\pi) < 0$, $\cos(\vec{P}_{\rm lep}, \vec{P}_{\pi^0}) < 0$, 0.3 GeV/$c^2 < M_{\pi\pi^0} <$ 1.8 GeV/c^2.
- $E_{{\rm rest}\gamma}^{\rm LAB} < 0.2$ GeV

Detection efficiency $\varepsilon_{\rm det} \simeq 12\%$



Background

The main background comes from $(\ell\nu\nu;\ \pi\pi^0\pi^0\nu)$($\sim$10%), $(\pi\nu;\ \pi\pi^0\nu)$($\sim$1.5%) and $(\rho^+\nu;\ \rho^-\nu)$($\sim$0.5%) events, it is included in the PDF analytically. The remaining background ($\sim$2.0%) is taken into account using MC-based approach. Background from the non-$\tau\tau$ events is $\lesssim$ 0.1%.

Remaining background for $(\mu;\ \rho)$ events

#	$(\ell/h_1\nu(\nu);\ \ell/h_2\nu(\nu))$	$E_{\gamma\ rest}^{LAB} < 0.3$ GeV
1	$(\mu\nu\nu;\ \text{other})$	9.3%
2	$(\mu\nu\nu;\ e\nu\nu)$	0.6%
3	$(\mu\nu\nu;\ \mu\nu\nu)$	0.4%
4	$(\mu\nu\nu;\ \pi\nu)$	6.3%
5	$(\mu\nu\nu;\ 3\pi\nu)$	1.5%
6	$(\mu\nu\nu;\ \pi K_S\nu)$	4.9%
7	$(\mu\nu\nu;\ \pi K_L\nu)$	1.1%
8	$(\mu\nu\nu;\ K\pi^0\nu)$	6.2%
9	$(\mu\nu\nu;\ 3\pi\pi^0\nu)$	7.0%
10	$(\mu\nu\nu;\ \pi 3\pi^0\nu)$	10.6%
11	$(\mu\nu\nu;\ \pi K_S K_L\nu)$	0.7%
12	$(\mu\nu\nu;\ KK_L\pi^0\nu)$	0.7%
13	$(\mu\nu\nu;\ K2\pi^0\nu)$	0.3%
14	$(\mu\nu\nu;\ \pi K_S\pi^0\nu)$	2.0%
15	$(\mu\nu\nu;\ \pi K_L\pi^0\nu)$	11.5%
16	$(\pi\nu;\ \pi 2\pi^0\nu)$	5.5%
17	$(\rho\nu;\ \pi 2\pi^0\nu)$	1.8%
18	$(3\pi\nu;\ \rho\nu)$	0.4%
19	$(\pi 2\pi^0\nu;\ \rho\nu)$	1.1%
20	$(K\nu;\ \rho\nu)$	4.9%
21	$(K\nu;\ \pi 2\pi^0\nu)$	0.6%
22	$(\pi K_L\nu;\ \rho\nu)$	1.1%
23	$(K\pi^0\nu;\ \rho\nu)$	0.5%
24	$(KK_L\nu;\ \rho\nu)$	0.3%
	sum	95.3%
	rest	4.7%

Remaining background for $(e;\ \rho)$ events

#	$(e/h_1\nu(\nu);\ h_2\nu)$	$E_{\gamma\ rest}^{LAB} < 0.3$ GeV
1	$(e\nu\nu;\ \text{other})$	13.2%
2	$(e\nu\nu;\ \pi\nu)$	7.6%
3	$(e\nu\nu;\ 3\pi\nu)$	2.1%
4	$(e\nu\nu;\ \pi K_S\nu)$	7.0%
5	$(e\nu\nu;\ \pi K_L\nu)$	1.3%
6	$(e\nu\nu;\ K\pi^0\nu)$	8.5%
7	$(e\nu\nu;\ 3\pi\pi^0\nu)$	8.5%
8	$(e\nu\nu;\ \pi 3\pi^0\nu)$	15.9%
9	$(e\nu\nu;\ \pi K_S K_L\nu)$	1.0%
10	$(e\nu\nu;\ K\pi^0 K_S\nu)$	0.3%
11	$(e\nu\nu;\ K\pi^0 K_L\nu)$	1.2%
12	$(e\nu\nu;\ K2\pi^0\nu)$	0.5%
13	$(e\nu\nu;\ \pi\pi^0 K_S\nu)$	3.2%
14	$(e\nu\nu;\ \pi\pi^0 K_L\nu)$	17.1%
15	$(e\nu\nu;\ \pi 4\pi^0\nu)$	0.4%
16	$(e\nu\nu;\ \pi\omega\pi^0\nu)$	0.3%
17	$(\pi\nu;\ \pi 2\pi^0\nu)$	0.7%
18	$(\rho\nu;\ \rho\nu)$	2.8%
	sum	96.7%
	rest	3.3%

Description of background

Total PDF

$$\mathcal{P}(x) = \frac{\overline{\varepsilon(x)}}{\bar{\varepsilon}}\Bigg((1-\sum_i \lambda_i)\frac{S(x)}{\int \frac{\overline{\varepsilon(x)}}{\bar{\varepsilon}} S(x)dx} + \lambda_{3\pi}\frac{\tilde{B}_{3\pi}(x)}{\int \frac{\overline{\varepsilon(x)}}{\bar{\varepsilon}} \tilde{B}_{3\pi}(x)dx} + \lambda_{\pi}\frac{\tilde{B}_{\pi}(x)}{\int \frac{\overline{\varepsilon(x)}}{\bar{\varepsilon}} \tilde{B}_{\pi}(x)dx} + \lambda_{\rho}\frac{\tilde{B}_{\rho}(x)}{\int \frac{\overline{\varepsilon(x)}}{\bar{\varepsilon}} \tilde{B}_{\rho}(x)dx} +$$

$$+(1-\sum_i \lambda_i)\frac{N^{\rm sel}_{\rm rest}(x)}{N^{\rm sel}_{\rm sig}(x)} S_{\rm SM}(x)\Bigg)$$

$$\tilde{B}_{3\pi}(x) = \int 2(1-\varepsilon_{\pi^0}(y))\varepsilon_{\rm add}(y) B_{3\pi}(x,y)dy,\ \tilde{B}_{\pi}(x) = \frac{\varepsilon^{\mu ID}_{\pi\to\mu}(p_\ell,\ \Omega_\ell)}{\varepsilon^{\mu ID}_{\mu\to\mu}(p_\ell,\ \Omega_\ell)} B_{\pi}(x)$$

$$\tilde{B}_{\rho}(x) = \frac{\varepsilon^{\mu ID}_{\pi\to\mu}(p_\ell,\ \Omega_\ell)}{\varepsilon^{\mu ID}_{\mu\to\mu}(p_\ell,\ \Omega_\ell)} \int (1-\varepsilon_{\pi^0}(y))\varepsilon_{\rm add}(y) B_{\rho}(x,y)dy,\ \overline{\varepsilon(x)} = \epsilon^{\rm EXP}_{\rm corr}(x)\varepsilon(x)$$

- $x = (p_\ell,\ \Omega_\ell,\ p_\rho,\ \Omega_\rho,\ m^2_{\pi\pi},\ \tilde{\Omega}_\pi)$; $y = (p_{\pi^0},\ \Omega_{\pi^0})$;
- $S(x)$ - theoretical density of signal $(\ell^\mp\nu\nu,\ \rho^\pm\nu)$ events;
- $B_{3\pi}(x,y)$ - theoretical density of background $(\ell^\mp\nu\nu,\ \pi^\pm 2\pi^0\nu)$ events;
- $B_\pi(x)$ - theoretical density of background $(\pi^\mp\nu,\ \rho^\pm\nu)$ events;
- $B_\rho(x)$ - theoretical density of background $(\rho^\mp\nu,\ \rho^\pm\nu)$ events;
- $\varepsilon(x)$ - detection efficiency for signal events (**common multiplier**);
- $N^{\rm sel}_{\rm rest}(x)/N^{\rm sel}_{\rm sig}(x)$ - number of the selected (remaining/signal) MC events in the multidimensional cell around "x". Admixture of the remaining background is $(1 \div 2)$%.
- λ_i - i-th background fraction (from MC)
- $\varepsilon_{\pi^0}(y)$ - π^0 detection efficiency (tabulated from MC);
- $\varepsilon_{\rm add}(y) = \varepsilon^{3\pi}_{\rm add}(y)/\varepsilon^{\rm sig}_{\rm add}$ - ratio of the $E^{\rm LAB}_{\gamma\rm rest}$ cut efficiencies (tabulated from MC);
- $\varepsilon^{\mu ID}_{\pi\to\mu}(p_\ell,\ \Omega_\ell)/\varepsilon^{\mu ID}_{\mu\to\mu}(p_\ell,\ \Omega_\ell)$ is tabulated from MC;
- $\epsilon^{\rm EXP}_{\rm corr}(x)$ - EXP/MC efficiency correction.

Background: $\tau \to \pi^{\pm}\pi^0\pi^0\nu$

D. M. Asner *et al.* [CLEO] Phys. Rev. D **61** (1999) 012002.

$$J^\mu = \beta_1 j_1^\mu(\rho\pi^0)_{S-wave} + \beta_2 j_2^\mu(\rho'\pi^0)_{S-wave} + \beta_3 j_3^\mu(\rho\pi^0)_{D-wave} + \beta_4 j_4^\mu(\rho'\pi^0)_{D-wave} +$$
$$+\beta_5 j_5^\mu(f_2(1270)\pi)_{P-wave} + \beta_6 j_6^\mu(f_0(500)\pi)_{P-wave} + \beta_7 j_7^\mu(f_0(1370)\pi)_{P-wave}$$

$$\frac{d\Gamma(\tau^\mp \to \pi^\mp 2\pi^0\nu)}{d\Omega^*_{3\pi} dm^2_{3\pi} d\tilde{\Omega}_\rho dm^2_{\pi\pi} d\tilde{\Omega}_\pi} = \kappa_{3\pi}(A' \pm \vec{B'}\vec{\zeta'}^*)W$$

$$A' = H_1 + \xi_{a_1} H_2,\ \vec{B'} = \xi_{a_1}\vec{G_1} + \vec{G_2},\ \xi_{a_1} = 1$$

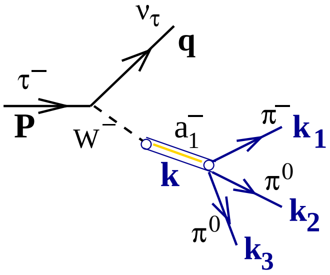
$$W = \frac{p_\pi(m^2_{\pi\pi})}{m_{\pi\pi}} \frac{p_\rho(m^2_{\pi\pi}, m^2_{3\pi})}{m_{3\pi}} \frac{p_{a_1}(m^2_{3\pi})}{M_\tau}$$

$$H_1 = (P, J^*)(q, J) + (P, J)(q, J^*) - (J, J^*)(P, q)$$

$$H_2 = ie^{\mu\nu\sigma\delta} J_\mu J^*_\nu P_\sigma q_\delta$$

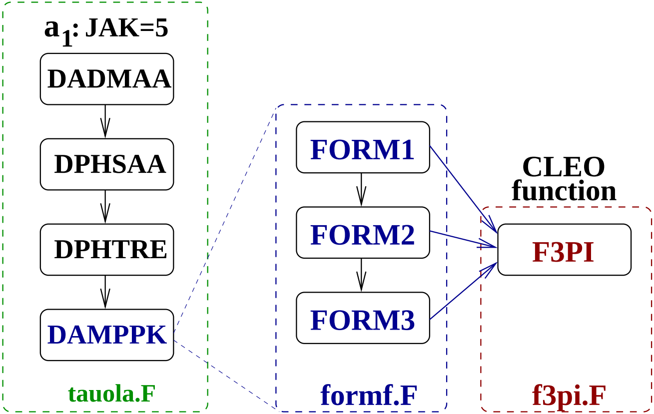
$$G_1^\lambda = M_\tau((q, J)J^{*\lambda} + (q, J^*)J^\lambda - (J, J^*)q^\lambda)$$

$$G_2^\lambda = iM_\tau e^{\lambda\mu\nu\sigma} J_\mu J^*_\nu q_\sigma$$



$\tau^- \to \pi^-\pi^0\pi^0\nu_\tau$ in TAUOLA

D. M. Asner *et al.* [CLEO] Phys. Rev. D **61** (1999) 012002.



$$J^\mu = \beta_1 j_1^\mu(\rho\pi^0)_{S-wave} + \beta_2 j_2^\mu(\rho'\pi^0)_{S-wave} + \beta_3 j_3^\mu(\rho\pi^0)_{D-wave} + \beta_4 j_4^\mu(\rho'\pi^0)_{D-wave} +$$

$$+\beta_5 j_5^\mu(f_2(1270)\pi)_{P-wave} + \beta_6 j_6^\mu(f_0(500)\pi)_{P-wave} + \beta_7 j_7^\mu(f_0(1370)\pi)_{P-wave}$$

Calculation of $\tilde{B}_{3\pi}(x)$

$$\frac{d\sigma(\vec{\zeta},\vec{\zeta}')}{d\Omega} = \frac{\alpha^2}{64E_\tau^2}\beta_\tau(D_0 + D_{ij}\zeta_i\zeta_j')$$

$$\frac{d\Gamma(\tau^\mp(\vec{\zeta}^*) \to \ell^\mp\nu\nu)}{dx^* d\Omega_\ell^*} = \kappa_\ell(A(x^*) \mp \xi\vec{n}_\ell^*\vec{\zeta}^* B(x^*)),\ x^* = E_\ell^*/E_{\ell max}^*$$

$$A(x^*) = A_0(x^*) + \rho A_1(x^*) + \eta A_2(x^*),\ B(x^*) = B_1(x^*) + \delta B_2(x^*)$$

$$\frac{d\Gamma(\tau^\mp \to \pi^\mp 2\pi^0\nu)}{d\Omega_{3\pi}^* dm_{3\pi}^2 d\tilde{\Omega}_\rho dm_{\pi\pi}^2 d\tilde{\Omega}_\pi} = \kappa_{3\pi}(H_1 + \xi_{a_1}H_2 \pm (\xi_{a_1}\vec{G_1} + \vec{G_2}, \vec{\zeta'}^*))W(m_{\pi\pi}^2, m_{3\pi}^2)$$

$$\frac{d\sigma(\ell^\mp, \pi^\pm 2\pi^0)}{dE_\ell^* d\Omega_\ell^* d\Omega_{3\pi}^* dm_{3\pi}^2 d\tilde{\Omega}_\rho dm_{\pi\pi}^2 d\tilde{\Omega}_\pi d\Omega_\tau} = \kappa_\ell\kappa_{3\pi}\frac{\alpha^2\beta_\tau}{64E_\tau^2}(D_0 A(E_\ell^*)(H_1 + \xi_{a_1}H_2) +$$

$$+ B(E_\ell^*)D_{ij}n_{\ell i}^*(\xi_{a_1}G_{1j} + G_{2j}))W(m_{\pi\pi}^2, m_{3\pi}^2)$$

$$\frac{d\sigma(\ell^\mp, \pi^\pm 2\pi^0)}{dp_\ell d\Omega_\ell dp_\rho d\Omega_\rho dm_{\pi\pi}^2 d\tilde{\Omega}_\pi dp_{\pi^0} d\Omega_{\pi^0}} = \int_{\Phi_1}^{\Phi_2} \frac{d\sigma(\ell^\mp, \pi^\pm 2\pi^0)}{dE_\ell^* d\Omega_\ell^* d\Omega_{3\pi}^* dm_{3\pi}^2 d\tilde{\Omega}_\rho dm_{\pi\pi}^2 d\tilde{\Omega}_\pi d\Omega_\tau} \times$$

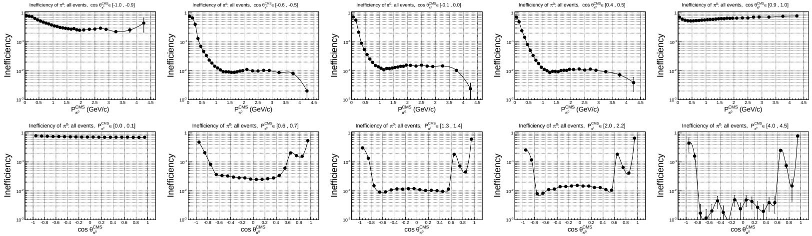
$$\times \left|\frac{\partial(E_\ell^*, \Omega_\ell^*)}{\partial(p_\ell, \Omega_\ell)}\right| \cdot \left|\frac{\partial(\Omega_{3\pi}^*, \Omega_\tau)}{\partial(p_{3\pi}, \Omega_{3\pi}, \Phi_\tau)}\right| \cdot \left|\frac{\partial(m_{3\pi}^2, \tilde{\Omega}_\rho, p_{3\pi}, \Omega_{3\pi})}{\partial(p_\rho, \Omega_\rho, p_{\pi^0}, \Omega_{\pi^0})}\right| d\Phi_\tau$$

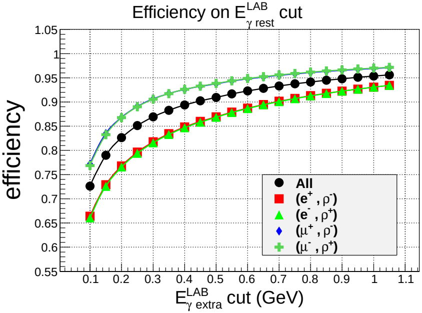
$$\tilde{B}_{3\pi}(x) = \frac{d\sigma_{obs}(\ell^\mp, \pi^\pm 2\pi^0)}{dp_\ell d\Omega_\ell dp_\rho d\Omega_\rho dm_{\pi\pi}^2 d\tilde{\Omega}_\pi} = \int 2(1 - \varepsilon_{\pi^0}(p_{\pi^0}, \Omega_{\pi^0}))\varepsilon_{\mathrm{add}}(p_{\pi^0}, \Omega_{\pi^0}) \times$$

$$\times \frac{d\sigma(\ell^\mp, \pi^\pm 2\pi^0)}{dp_\ell d\Omega_\ell dp_\rho d\Omega_\rho dm_{\pi\pi}^2 d\tilde{\Omega}_\pi dp_{\pi^0} d\Omega_{\pi^0}} dp_{\pi^0} d\Omega_{\pi^0}$$

Tabulated $(1 - \varepsilon_{\pi^0})\varepsilon^{3\pi}_{\rm add}$ and $\varepsilon^{\rm sig}_{\rm add}$

The inefficiency was tabulated in 28 bins in π^0 momentum, $P^{\rm CMS}_{\pi^0}$, and 20 bins in $\cos\theta^{\rm CMS}_{\pi^0}$.





Formulation of the total PDF

Likelihood per event, $\ell - \rho$ + $\ell - 3\pi$

$$S^{(k)} = A_0^{(k)} + A_1^{(k)}\rho + A_2^{(k)}\eta + A_3^{(k)}\xi_\rho\xi + A_4^{(k)}\xi_\rho\xi\delta$$

$$\tilde{B}_{3\pi}^{(k)} = \tilde{B}_{3\pi,0}^{(k)} + \tilde{B}_{3\pi,1}^{(k)}\rho + \tilde{B}_{3\pi,2}^{(k)}\eta + \tilde{B}_{3\pi,3}^{(k)}\xi_\rho\xi + \tilde{B}_{3\pi,4}^{(k)}\xi_\rho\xi\delta$$

$$\mathcal{P}^{(k)} = \frac{\overline{\varepsilon(x)}}{\bar{\varepsilon}}\Bigg((1-\sum_i \lambda_i)\mathbf{Z_{sig}}\frac{A_0^{(k)} + A_1^{(k)}\rho + A_2^{(k)}\eta + A_3^{(k)}\xi_\rho\xi + A_4^{(k)}\xi_\rho\xi\delta}{\frac{1}{N_{sel}^{MC}}\sum\limits_{i=1}^{N_{sel}^{MC}}\frac{A_0^{(i)}+A_1^{(i)}\rho+A_2^{(i)}\eta+A_3^{(i)}\xi_\rho\xi+A_4^{(i)}\xi_\rho\xi\delta}{A_0^{(i)}+A_1^{(i)}\cdot 0.75+A_2^{(i)}\cdot 0.0+A_3^{(i)}\cdot 1.0+A_4^{(i)}\cdot 0.75}} +$$

$$+\lambda_{3\pi}\mathbf{Z_{3\pi}}\frac{\tilde{B}_{3\pi,0}^{(k)} + \tilde{B}_{3\pi,1}^{(k)}\rho + \tilde{B}_{3\pi,2}^{(k)}\eta + \tilde{B}_{3\pi,3}^{(k)}\xi_\rho\xi + \tilde{B}_{3\pi,4}^{(k)}\xi_\rho\xi\delta}{\frac{1}{N_{sel}^{MC}}\sum\limits_{i=1}^{N_{sel}^{MC}}\frac{\tilde{B}_{3\pi,0}^{(i)}+\tilde{B}_{3\pi,1}^{(i)}\rho+\tilde{B}_{3\pi,2}^{(i)}\eta+\tilde{B}_{3\pi,3}^{(i)}\xi_\rho\xi+\tilde{B}_{3\pi,4}^{(i)}\xi_\rho\xi\delta}{\tilde{B}_{3\pi,0}^{(i)}+\tilde{B}_{3\pi,1}^{(i)}0.75+\tilde{B}_{3\pi,2}^{(i)}0.0+\tilde{B}_{3\pi,3}^{(i)}1.0+\tilde{B}_{3\pi,4}^{(i)}0.75}} + \ldots\Bigg)$$

Absolute normalizations, Z_{sig} and $Z_{3\pi}$, are calculated using huge samples of the MC generated events.

Background: $(\pi^{\mp},\ \pi^{\pm}\pi^{0})$ events

In the $(\pi^{\mp},\ \pi^{\pm}\pi^{0})$ events the direction of τ axis is determined with the two-fold ambiguity.

$$\frac{d\sigma(\vec{\zeta},\vec{\zeta}')}{d\Omega} = \frac{\alpha^2}{64E_\tau^2}\beta_\tau(D_0 + D_{ij}\zeta_i\zeta_j')$$

$$\frac{d\Gamma(\tau^{\mp}(\vec{\zeta}^*) \to \pi^{\mp}\nu)}{d\Omega_\pi^*} = \kappa_\pi(1 \pm \xi_\pi \vec{n}_\pi^* \vec{\zeta}^*)$$

$$\frac{d\Gamma(\tau^{\pm}(\vec{\zeta'}^*) \to \rho^{\pm}\nu)}{dm_{\pi\pi}^2 d\Omega_\rho^* d\tilde{\Omega}_\pi} = \kappa_\rho(A' \mp \xi_\rho \vec{B'} \vec{\zeta'}^*)W(m_{\pi\pi}^2)$$

$$A' = 2(q,Q)Q_0^* - Q^2 q_0^*,\ \vec{B'} = Q^2\vec{K}^* + 2(q,Q)\vec{Q}^*,\ W = |F_\pi(m_{\pi\pi}^2)|^2\frac{p_\rho(m_{\pi\pi}^2)\tilde{p}_\pi(m_{\pi\pi}^2)}{M_\tau m_{\pi\pi}}$$

$$\frac{d\sigma(\pi^{\mp},\rho^{\pm})}{d\Omega_\pi^* d\Omega_\rho^* dm_{\pi\pi}^2 d\tilde{\Omega}_\pi d\Omega_\tau} = \kappa_\pi\kappa_\rho\frac{\alpha^2\beta_\tau}{64E_\tau^2}(D_0 A' - \xi_\rho\xi_\pi D_{ij} n_{\pi i}^* B_j'),\ \xi_\rho\xi_\pi = 1$$

$$B_\pi(x) = \frac{d\sigma(\pi^{\mp},\rho^{\pm})}{dp_\pi d\Omega_\pi dp_\rho d\Omega_\rho dm_{\pi\pi}^2 d\tilde{\Omega}_\pi} = \sum_{\Phi_1,\Phi_2}\frac{d\sigma(\pi^{\mp},\rho^{\pm})}{d\Omega_\pi^* d\Omega_\rho^* dm_{\pi\pi}^2 d\tilde{\Omega}_\pi d\Omega_\tau}\left|\frac{\partial(\Omega_\pi^*,\Omega_\rho^*,\Omega_\tau)}{\partial(p_\pi,\Omega_\pi,p_\rho,\Omega_\rho)}\right|$$

Efficiency ratio $\varepsilon_\pi(x)/\varepsilon(x) = \varepsilon_{\pi\to\mu}(p_\ell^{\rm LAB},\theta_\ell^{\rm LAB})/\varepsilon_{\mu\to\mu}(p_\ell^{\rm LAB},\theta_\ell^{\rm LAB})$ was tabulated from MC as a function of $(p_\ell^{\rm LAB},\theta_\ell^{\rm LAB})$ and used to get for each event:

$$\tilde{B}_\pi(x) = \frac{\varepsilon_{\pi\to\mu}(p_\ell^{\rm LAB},\theta_\ell^{\rm LAB})}{\varepsilon_{\mu\to\mu}(p_\ell^{\rm LAB},\theta_\ell^{\rm LAB})}B_\pi(x)$$

Description of the remaining background

D. Schmidt et al., Nucl. Instr. and Meth. A328 (1993) 547.

PDF for the remaining background

$$\mathcal{P}_{other}(x_i) = \frac{N^{sel}_{other}(x_i)/V_i}{N^{sel\ TOT}_{other}}, \quad \mathcal{P}_{sig}(x_i) = \frac{N^{sel}_{sig}(x_i)/V_i}{N^{sel\ TOT}_{sig}}, \quad S(x_i) = \frac{N^{gen}_{sig}(x_i)/V_i}{N^{gen\ TOT}_{sig}}$$

$$\frac{\bar{\varepsilon}}{\varepsilon(x_i)} = \frac{N^{sel\ TOT}_{sig}}{N^{sel}_{sig}(x_i)} \frac{N^{gen}_{sig}(x_i)}{N^{gen\ TOT}_{sig}}, \quad \frac{\bar{\varepsilon}}{\varepsilon(x_i)} \mathcal{P}_{other}(x_i) = \frac{1 - \sum \lambda_i}{\lambda_{other}} \frac{N^{sel}_{other}(x_i)}{N^{sel}_{sig}(x_i)} S(x_i)$$

MC statistics we have is not enough to tabulate PDF for the remaining background in 9D with sufficient accuracy. Therefore we determine PDF in the reduced phase space in such a way that the main correlations are still taken into account.

$$(\, p_l,\ \cos\theta_l,\ \varphi_l \,)\ ;\ (\, p_\rho,\ \cos\theta_\rho,\ \varphi_\rho \,)\ ;\ (\, m^2_{\pi\pi},\ \cos\tilde{\theta}_\pi,\ \tilde{\varphi}_\pi \,)$$

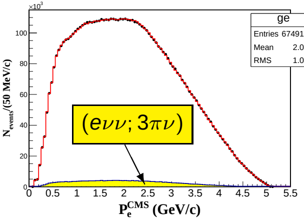
$$(P_\ell, \cos\psi_{\ell\rho}, P_\rho) \otimes \frac{(P_\ell, \cos\theta_\rho)}{\mathcal{P}(P_\ell)} \otimes \frac{(P_\rho, \cos\tilde{\theta_\pi})}{\mathcal{P}(P_\rho)} \otimes \frac{(P_\rho, m^2_{\pi\pi})}{\mathcal{P}(P_\rho)} \otimes (\varphi_\rho, \tilde{\varphi}_\pi) \otimes \frac{(\cos\psi_{\ell\rho}, \Delta\varphi_{\ell\rho})}{\mathcal{P}(\cos\psi_{\ell\rho})}$$

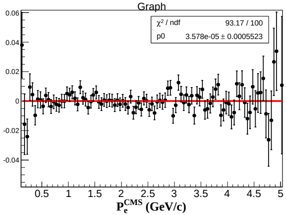
Validation of the fitter (dominant background)

For each configuration 5M MC sample is fitted. The other, statistically independent, 5M MC sample was used to calculate normalization.

$(e^+\nu\nu;\rho^-\nu)$

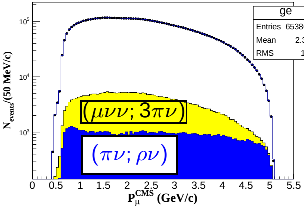
ρ	=	0.7517	$\pm$	0.0010
η	=	0	–	fixed
ξ	=	1.0092	$\pm$	0.0043
$\xi\delta$	=	0.7538	$\pm$	0.0027

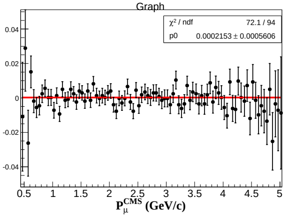




$(\mu^+\nu\nu;\rho^-\nu)$

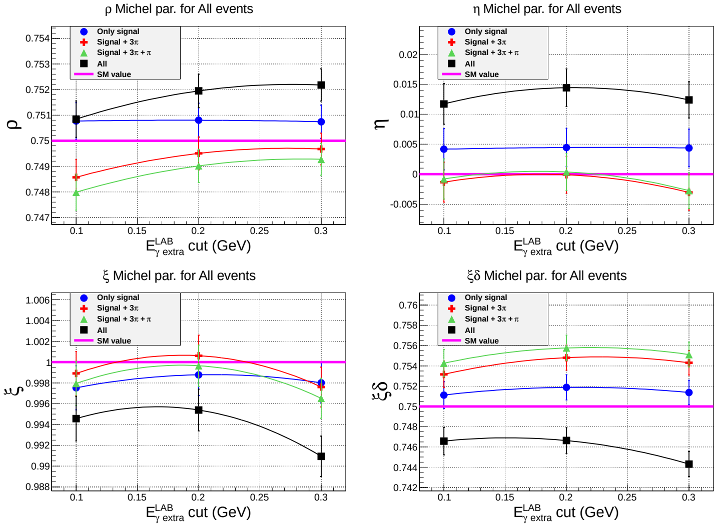
ρ	=	0.7494	$\pm$	0.0027
η	=	0.0052	$\pm$	0.0101
ξ	=	0.9995	$\pm$	0.0050
$\xi\delta$	=	0.7519	$\pm$	0.0033





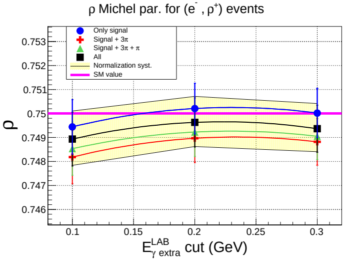
Validation of the fitter (full sample)

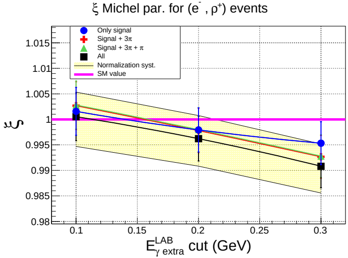
Simultaneous fit of MC $(e^{\pm}\nu\nu;\rho^{\mp}\nu)$, $(\mu^{\pm}\nu\nu;\rho^{\mp}\nu)$ events

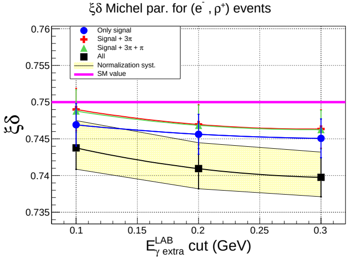


All parameters agree with their SM expectations within systematic uncertainties (ρ(0.3%), η(0.8%), ξ(0.5%), $\xi\delta$(0.4%))

Impact of the accuracy of the normalization (I)

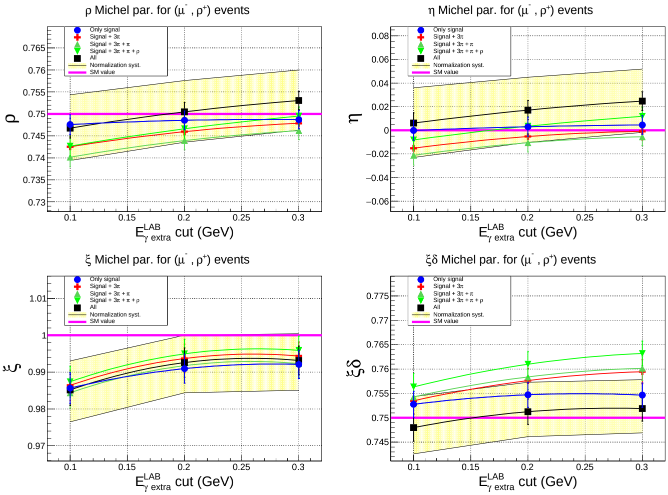






In the fit of ($e^{\pm}\nu\nu$; $\rho^{\mp}\nu$) events the systematic uncertainties due to the normalization are: ρ(**0.11%**), ξ(**0.53%**), $\xi\delta$(**0.33%**).

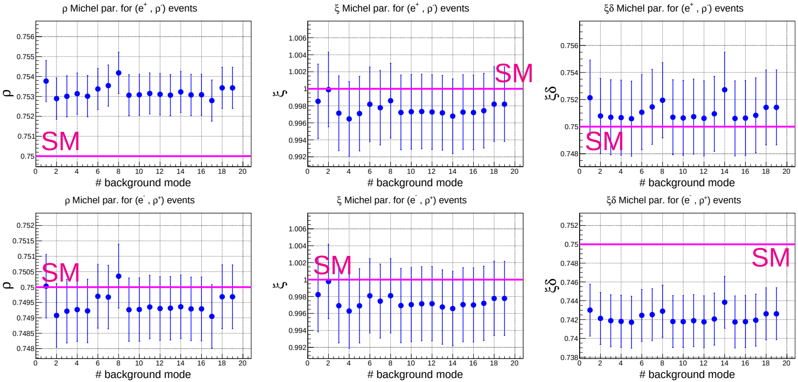
Impact of the accuracy of the normalization (II)



In the fit of $(\mu^{\pm}\nu\nu;\ \rho^{\mp}\nu)$ events the systematic uncertainties due to the normalization are: ρ(**0.76%**), η(**3.0%**), ξ(**0.84%**), $\xi\delta$(**0.59%**) ! Only half of all available MC statistics was used to evaluate normalization coefficients.

Remaining background in $(e\nu\nu;\ \rho\nu)$

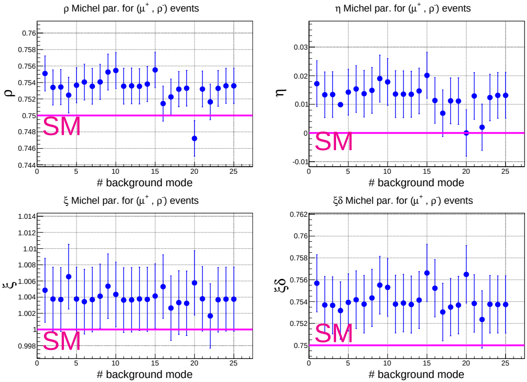
The remaining background in the $(e\nu\nu;\ \rho\nu)$ events is populated mainly by 18 different τ decay modes (see backup slides)



Switching off each mode, one by one, in the fitted MC sample we (conservatively) tested the impact of the MC description of all modes on Michel parameters ($\lesssim 0.2\%$).

Remaining background in $(\mu\nu\nu;\ \rho\nu)$

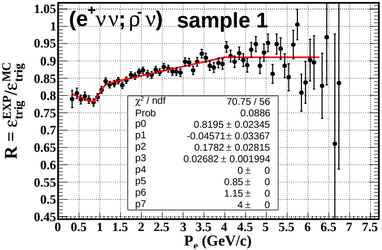
The remaining background in the $(\mu\nu\nu;\ \rho\nu)$ events is populated mainly by 24 different τ decay modes (see backup slides)

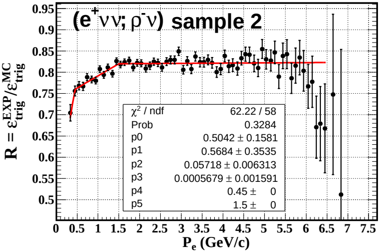


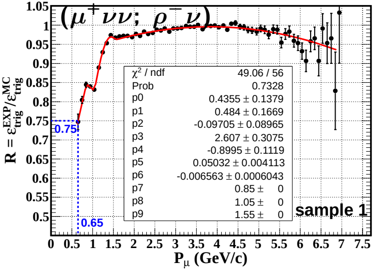
Switching off each mode, one by one, in the fitted MC sample we (conservatively) tested the impact of the MC description of all modes on Michel parameters ($\lesssim$ 0.5%).

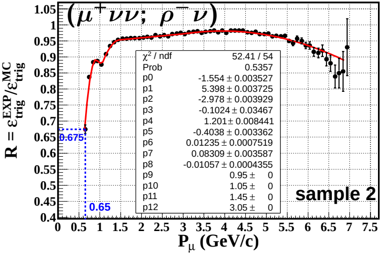
Trigger EXP/MC efficiency corrections (I)

We found that the EXP/MC trigger efficiency correction, $\mathcal{R}_{\rm trg}$, is the dominant one. Two independent subtriggers (energy trigger and track trigger) are used to evaluate it.



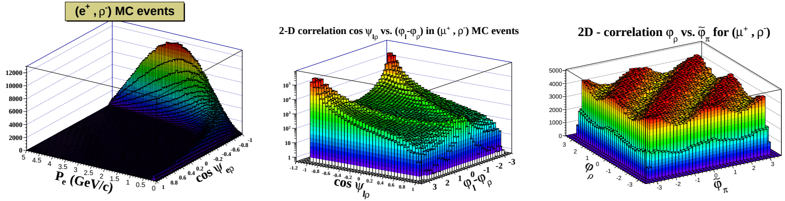






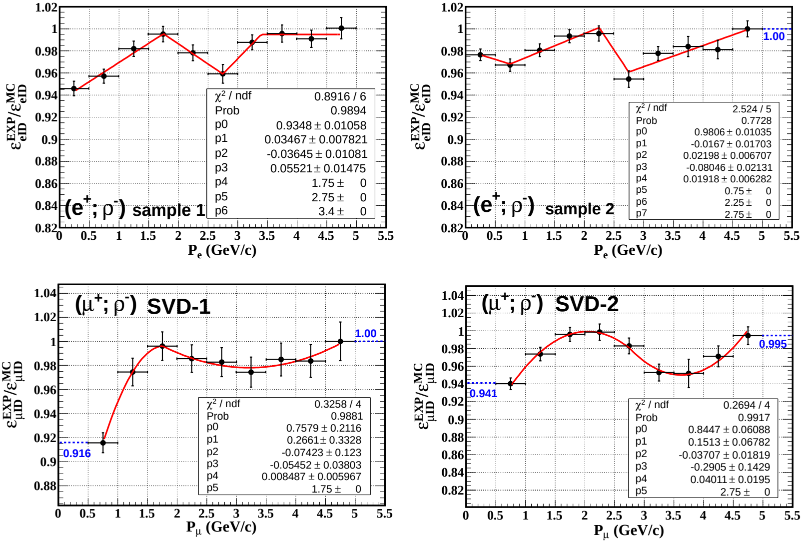
Trigger EXP/MC efficiency corrections (II)

$\mathcal{R}_{\rm trg}$ varies in the 9D phase space, appropriate parametrization is needed. We use a set of 2D-maps to approximate $\mathcal{R}_{\rm trg}$.



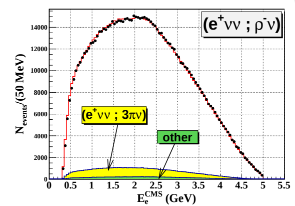
Still, we have notable $\mathcal{R}_{\rm trg}$-related systematic uncertainty:
ξ, $\xi\delta$ ($\sim$ 3%), η ($\sim$ 2%), ρ ($\sim$ 1%).

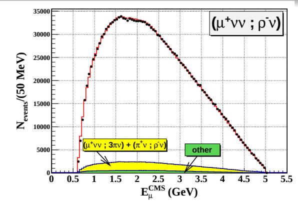
ℓID efficiency corrections

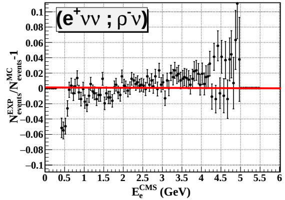


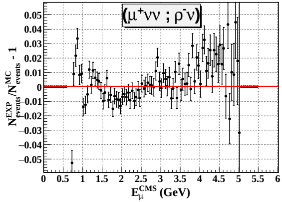
In the extraction of the experimental efficiency corrections it is important to use appropriate approximation near the kinematical boundaries, where the statistics is small. And the accuracy of the correction induces notable systematic uncertainty of Michel parameters.

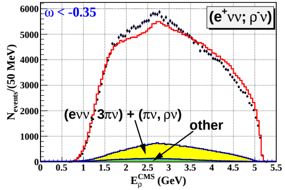
Fit of the experimental data, $(\ell^+\nu\nu;\rho^-\nu)$

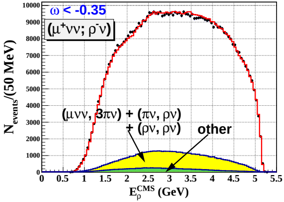












Experimental data sample of $485\ \mathrm{fb}^{-1}$ (446 $\times 10^6$ $\tau^+\tau^-$) was analyzed

Systematic uncertainties

Source	$\Delta(\rho)$, %	$\Delta(\eta)$, %	$\Delta(\xi_\rho\xi)$, %	$\Delta(\xi_\rho\xi\delta)$, %
Physical corrections				
ISR+$\mathcal{O}(\alpha^3)$	0.10	0.30	0.20	0.15
$\tau \to \ell\nu\nu\gamma$	0.03	0.10	0.09	0.08
$\tau \to \rho\nu\gamma$	0.06	0.16	0.11	0.02
Background	0.20	0.60	0.20	0.20
Apparatus corrections				
Resolution $\oplus$ brems.	0.10	0.33	0.11	0.19
$\sigma(E_{\mathrm{beam}})$	0.07	0.25	0.03	0.15
Normalization				
$\Delta\mathcal{N}$	0.11	0.50	0.17	0.13
without EXP/MC corr.	**0.29**	**0.95**	**0.38**	**0.38**
$\mathcal{R}_{\mathrm{trg}}$	~ 1	~ 2	~ 3	~ 3

We are working on the various EXP/MC efficiency corrections (trigger, ℓID, track rec., π^0 rec.).

Summary

- The procedure to measure 4 Michel parameters (MP) (ρ, η, ξ, $\xi\delta$) in leptonic τ decays at B factory has been developed and tested. It is based on the analysis of the $(\ell^{\mp}\nu\nu;\ \rho^{\pm}\nu)$, $\ell = e,\ \mu$ events and utilizes spin-spin correlation of tau leptons.
- We confirmed that with the whole Belle data sample the statistical accuracy of MP is by one order of magnitude better than in the previous best measurements (CLEO, ALEPH).
- The main background components $((\ell\nu\nu;\ \pi 2\pi^0\nu)$, $(\pi\nu;\ \rho\nu)$, $(\rho\nu;\ \rho\nu))$ are described analytically in the fitter, the remaining background (with the fraction of about 2.0%) is described with help of the MC-based method. We reached acceptable description of the backgrounds in the PDF.
- Various EXP/MC efficiency corrections provide the dominant contribution to the systemtic uncertainties of MP now. We are working on the trigger, ℓID, track rec., π^0 rec. efficiency corrections.
- The analysis is going on.