Hadronic tau decays and the strong coupling

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Mini Workshop on Tau Physics, Cinvestav, Mexico City
May 22-23, 2017

Values for the strong coupling from tau decays (ALEPH data):

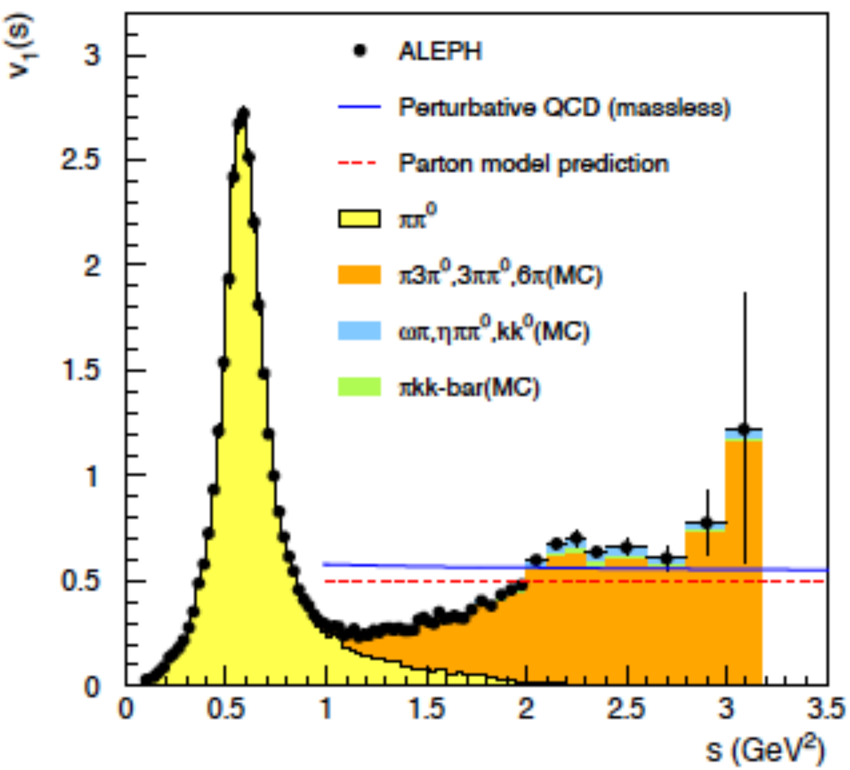
- Pich, Rodríguez-Sánchez (PRD94 (‘16) 034027): $\alpha_s(m_\tau^2) = 0.328(12)$
  Davier *et al.* (EPJC74 (‘14) 2803): $\alpha_s(m_\tau^2) = 0.332(12)$
  Boito *et al.* (PRD91 (‘15) 034003, 95 (‘17) 034024): $\alpha_s(m_\tau^2) = 0.301(10)$
  (including OPAL data: $\alpha_s(m_\tau^2) = 0.309(9)$)

- PDG (2016):
  At the Z mass (from tau): $\alpha_s(m_Z^2) = 0.1192(18)$ $\alpha_s(m_\tau^2) = 0.325(15)$
  lattice: $\alpha_s(m_Z^2) = 0.1184(11)$ $\alpha_s(m_\tau^2) = 0.315(9)$
  world average w/o tau: $\alpha_s(m_Z^2) = 0.1179(11)$ $\alpha_s(m_\tau^2) = 0.314(9)$

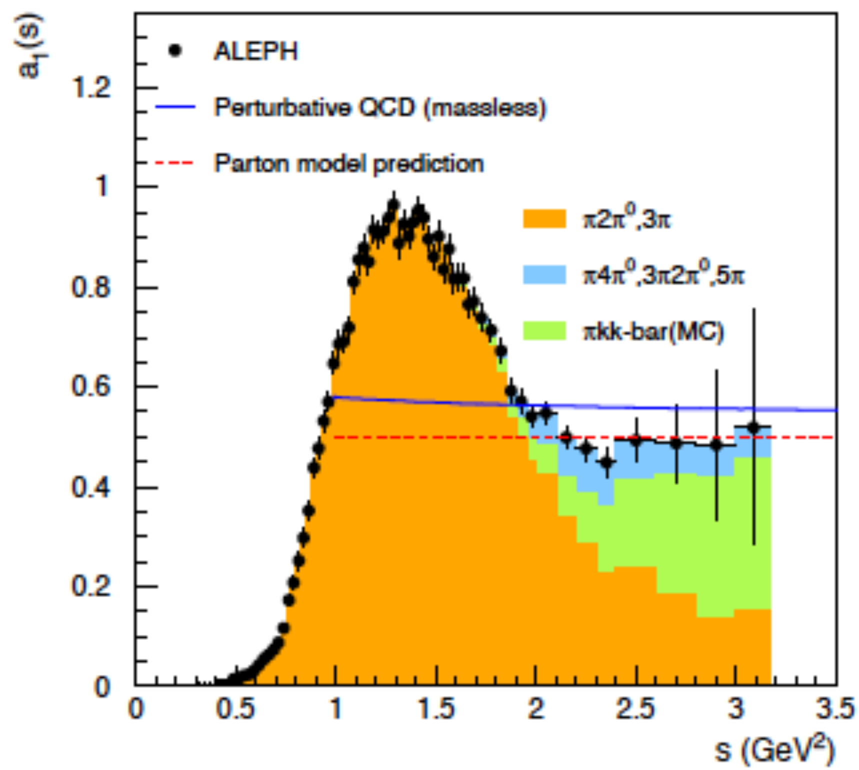
- Can we do better? Note disagreements well outside errors!

- *Technical note:* these are averages between CIPT and FOPT values.

Experimental data



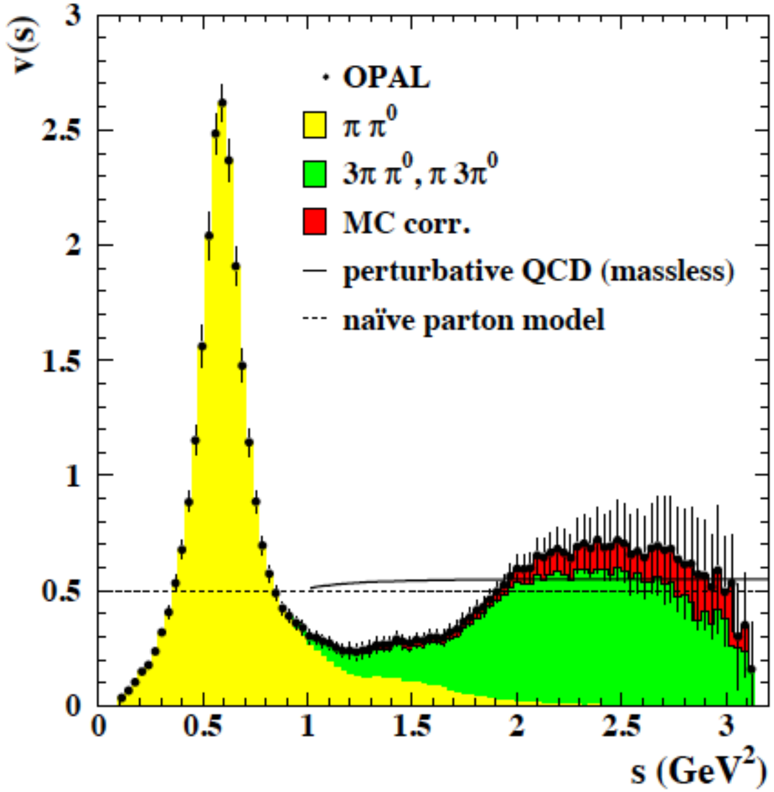
vector non-strange



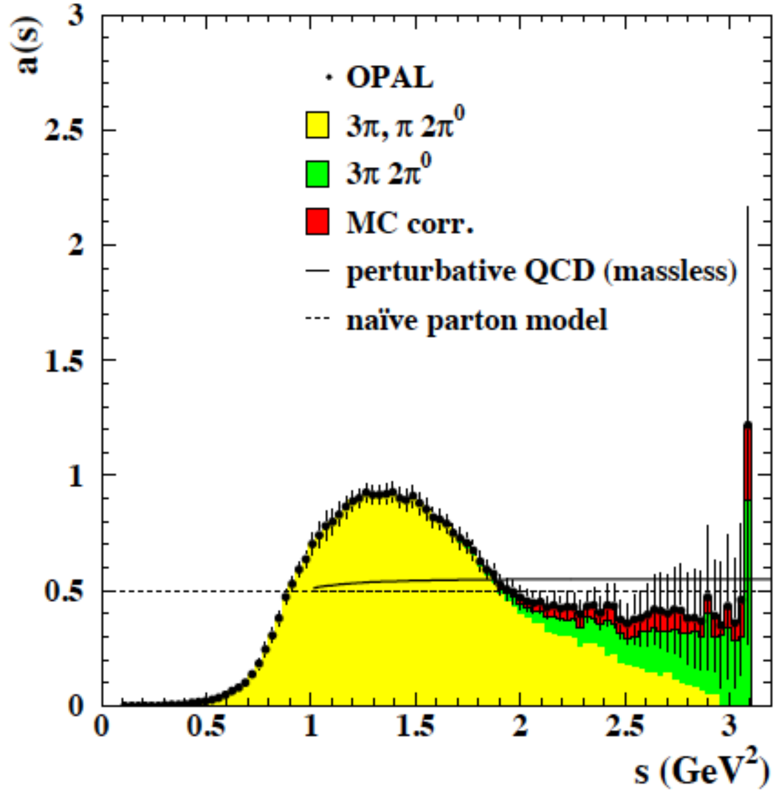
axial non-strange

ALEPH, Davier *et al.* (EPJC74 (2014) 2803)

Experimental data



vector non-strange

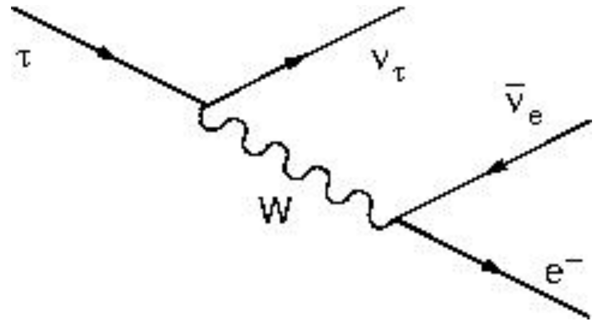


axial non-strange

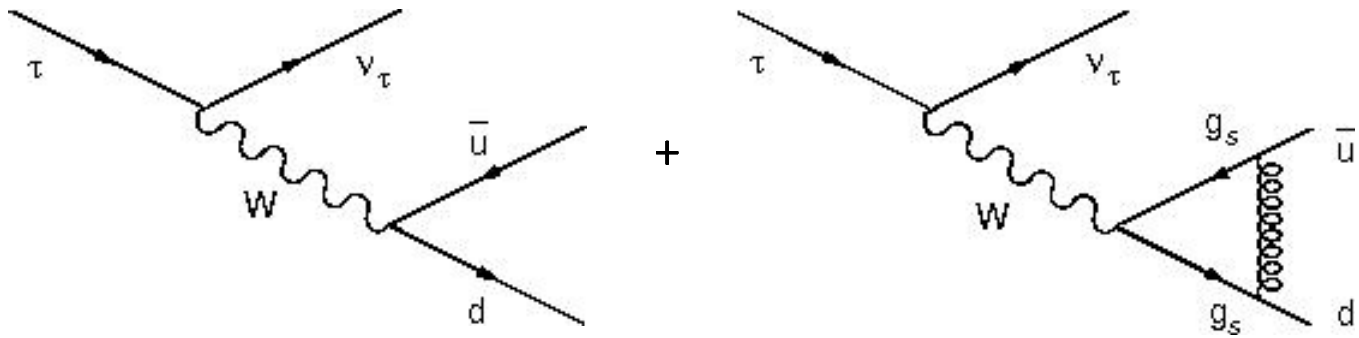
OPAL, Ackerstaff *et al.* (EPJC7 (1999) 571)

τ decays

leptonic:



hadronic:

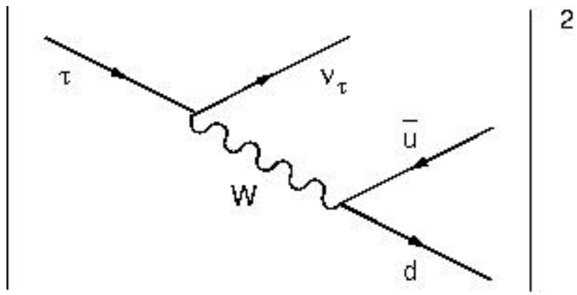


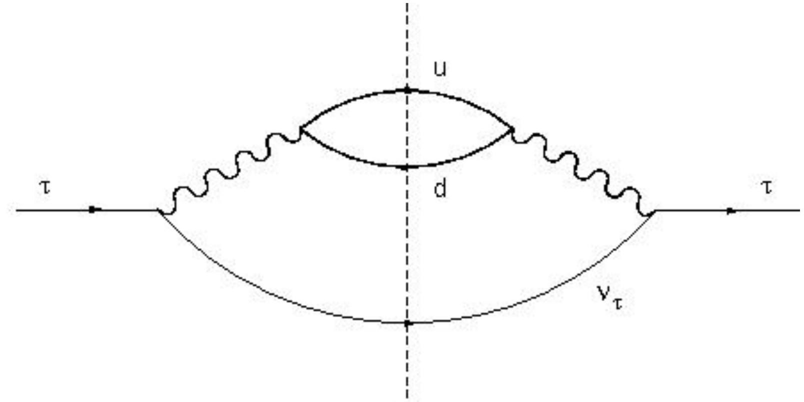
$$R_\tau^{ud} = \frac{\Gamma \to \nu_\tau \mathrm{hadrons}_{ud}}{\Gamma \to \nu_\tau e \overline{\nu}_e} = 3 S_{\mathrm{EW}} |V_{ud}|^2 \left[1 + \frac{\alpha_s}{\pi} + 5.2 \left(\frac{\alpha_s}{\pi} \right)^2 + \ldots \right]$$

- use this to determine $\alpha_s(m_\tau)$ from non-strange decays
- see $\tau \to \nu_\tau \rho^* \to \nu_\tau \,\mathrm{pions}$, not $\tau \to \nu_\tau \,\mathrm{jets}$
  $\rho^* = \rho(770),\ \rho(1450),\ \rho(1700)$ (and others: incl. axial, kaons, …)

$\longrightarrow$ relation with perturbative regime?

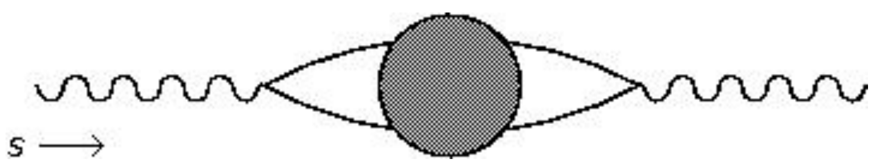
$\Gamma(\tau \to \nu_\tau \mathrm{hadrons}_{ud})$

$\propto$ 

$\propto$ Im 

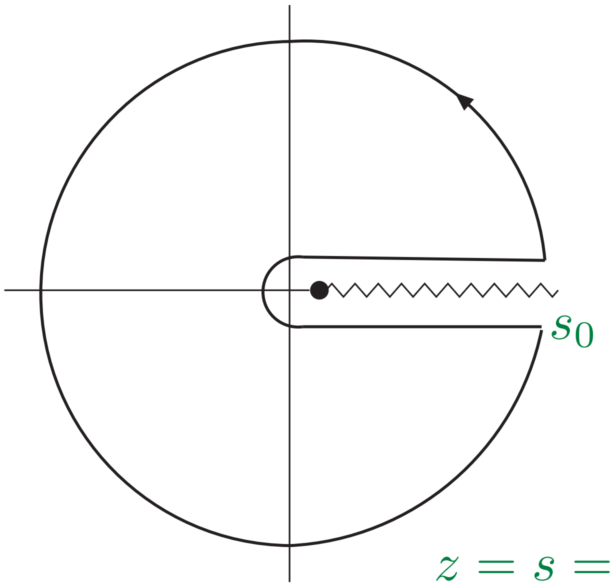
(optical theorem)

$$R_\tau^{ud}(s_0) = 12\pi S_{EW}|V_{ud}|^2 \int_0^{s_0} \frac{ds}{s_0}\left(1-\frac{s}{s_0}\right)^2\left(1+\frac{2s}{s_0}\right)\underbrace{\mathrm{Im}\,\Pi(s)}_{=\pi\rho(s)}$$

$\Pi(s) =$ 

$s = (q_W)^2$

$0 \le s \le s_0 = m_\tau^2$ depending on how much momentum ν_τ carries away



$z = s = q^2$ plane

$$\rho(s) = \frac{1}{\pi} \mathrm{Im}\,\Pi(s)$$

inclusive non-strange spectral function, measured in tau decays (ALEPH, OPAL)

$\Pi(z = q^2 = s)$ current two point function, gives access to strong coupling

polynomial weight

Cauchy:

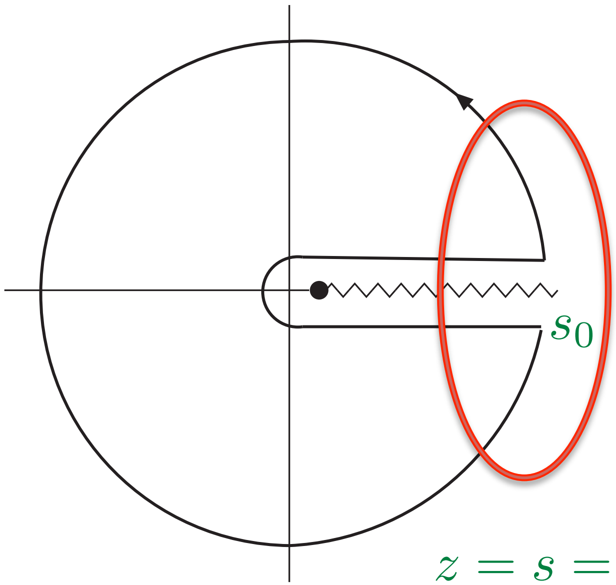
$$\int_0^{s_0} ds\, w(s)\, \rho(s) = -\frac{1}{2\pi i} \oint_{|z|=s_0} dz\, w(z)\, \Pi(z)$$

$$\Pi(z) = \Pi^{\mathrm{pert}}(z) + \Pi^{\mathrm{nonpert}}_{\mathrm{OPE}}(z) + \Pi^{\mathrm{nonpert}}_{\mathrm{DV}}(z)$$

α_s

resonances

$$\Pi_{\mathrm{OPE}}(q^2) = \frac{C_4}{q^4} - \frac{C_6}{q^6} + \frac{C_8}{q^8} - \dots$$



$z = s = q^2$ plane

$$\rho(s) = \frac{1}{\pi}\,\mathrm{Im}\,\Pi(s)$$

OPE not valid!!

$\mathrm{D}_{\mathrm{e}}\mathrm{V}_{\mathrm{il}}$ of Duality Violations:

resonances

polynomial weight

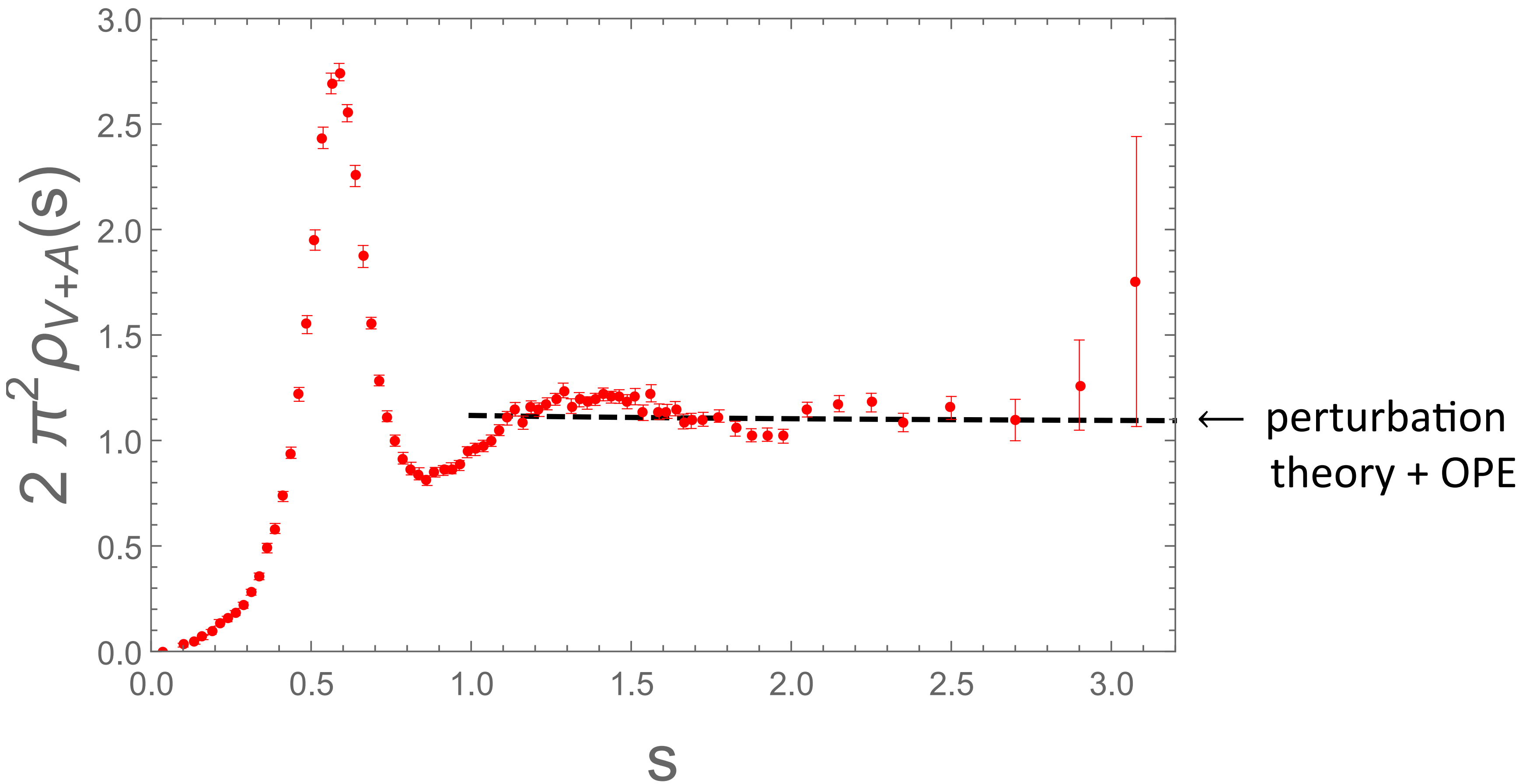
Cauchy: $$\int_0^{s_0} ds\, w(s)\, \rho(s) = -\frac{1}{2\pi i}\oint_{|z|=s_0} dz\, w(z)\, \Pi(z)$$

$$\Pi(z) = \Pi^{\mathrm{pert}}(z) + \Pi^{\mathrm{nonpert}}_{\mathrm{OPE}}(z) + \Pi^{\mathrm{nonpert}}_{\mathrm{DV}}(z)$$

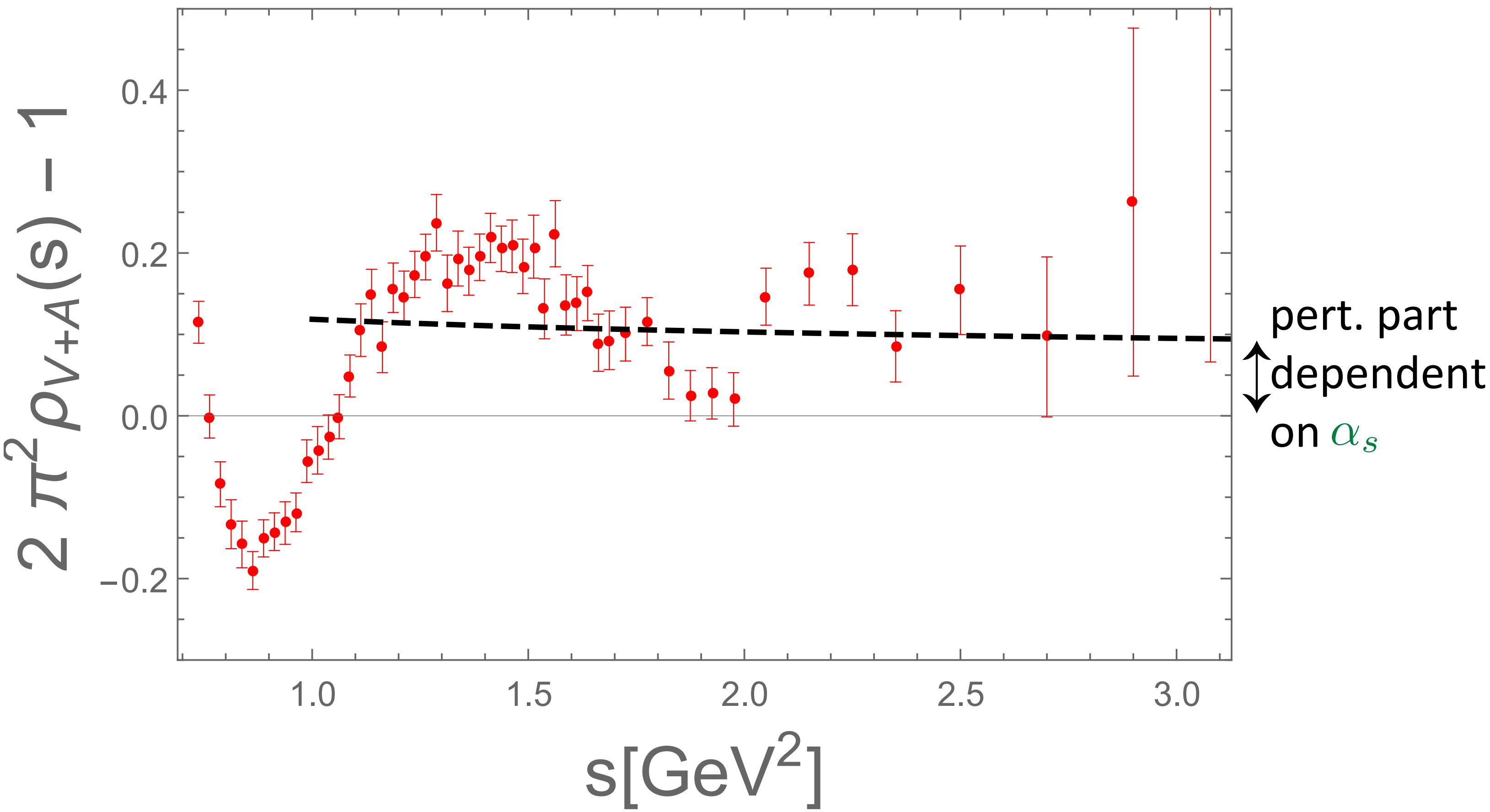
α_s resonances

$$\Pi_{\mathrm{OPE}}(q^2) = \frac{C_4}{q^4} - \frac{C_6}{q^6} + \frac{C_8}{q^8} - \dots$$

V+A non-strange spectral function (Davier *et al.*, 2014, ALEPH)



Blow up of large *s* region:



Duality violations – resonance effects – are **not** small!

Two approaches to non-perturbative "contamination"

- ALEPH (Davier *et al.*), OPAL, Pich & Rodriguez ("Truncated-OPE approach"):
  Ignore Duality Violations, but suppress dangerous region by "pinching":
  choose only higher-order polynomials with multiple zeroes at $s = s_0 = m_\tau^2$
  Fit $\alpha_s(m_\tau^2)$ and C_4 , C_6 , C_8 , set higher orders and DVs to zero
  Problem: inconsistent treatment of the OPE

- Boito *et al.* ("DV-model approach"):
  Treat OPE consistently, keep only low orders: choose simple polynomials
  Model DVs with *ansatz* $\rho_{\rm DV}(s) = e^{-\gamma s - \delta} \sin{(\alpha + \beta s)}$
  Vary s_0 between $s_{min} \approx 1.55 \text{ GeV}^2$ and m_τ^2
  Fit $\alpha_s(m_\tau^2)$ and C_6 , C_8 , α , β , γ , δ
  Problem: need to model DVs

Truncated-OPE approach: take $s_0 = m_\tau^2$ and weights ($x \equiv z/s_0$)

$$w_{00}(x) = (1-x)^2(1+2x)$$
$$w_{10}(x) = (1-x)^3(1+2x)$$
$$w_{11}(x) = (1-x)^3(1+2x)x$$ (ALEPH)
$$w_{12}(x) = (1-x)^3(1+2x)x^2$$
$$w_{13}(x) = (1-x)^3(1+2x)x^3$$

Assume: $C_{10} = C_{12} = C_{14} = C_{16} = 0$ and Duality Violations negligible

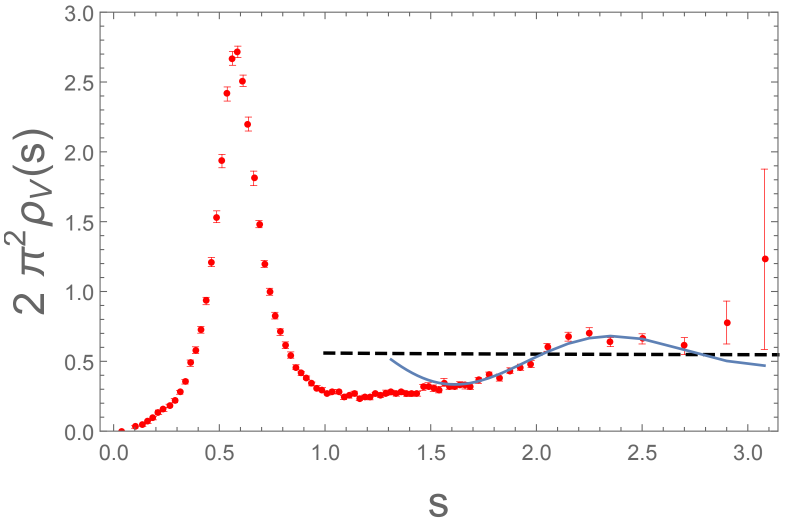
$\Rightarrow$ fit four parameters (α_s , C_4 , C_6 , C_8) to five data spectral integrals

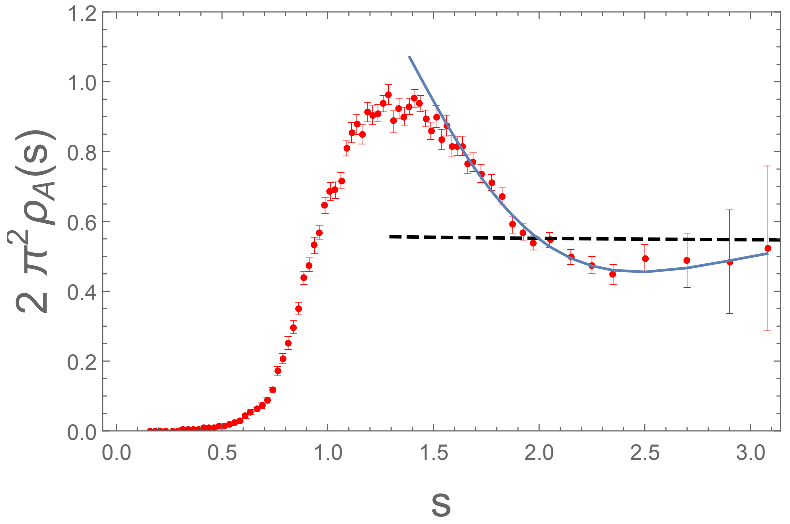
However: $$\frac{1}{2\pi i}\oint dz\, z^n \frac{C_{2k}}{z^k} = C_{2(n+1)}\delta_{k,n+1}$$

- need OPE coefficients up to C_{16}
- resonance oscillations around OPE clearly visible in V+A spectral function!

This approach can be tested: fake data

- Start from model with, by construction, lower $\alpha_s(m_\tau^2) = 0.312$ (CIPT) and non-negligible DVs, compatible with the experimental spectral function. Test Truncated-OPE approach

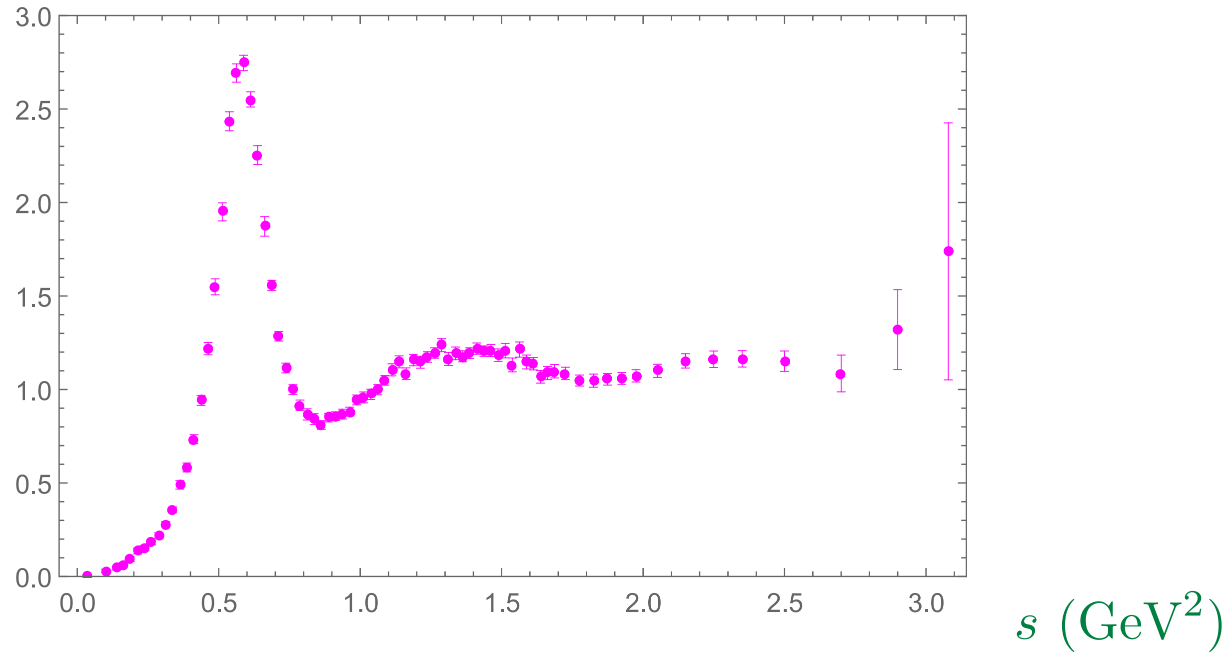




- Generate fake data from this model (using real-data covariances).
- Perform Truncated-OPE type fits on these fake data.
- Compare the parameter values ($\alpha_s(m_\tau^2)$) obtained from these fits to the input value, $\alpha_s(m_\tau^2) = 0.312$.

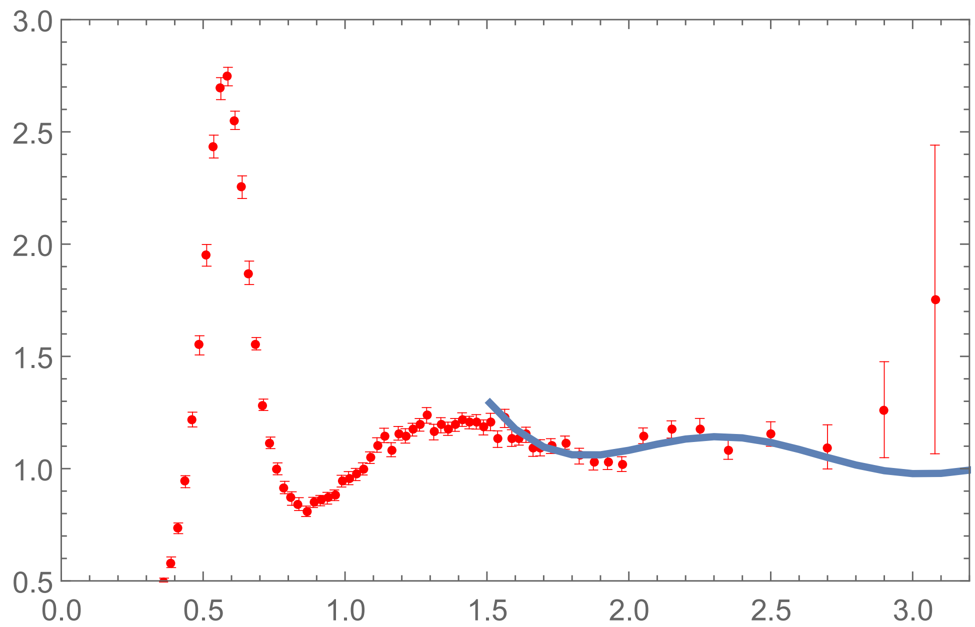
Fake data:

(fake above $s = 1.55\ \mathrm{GeV}^2$)



Real data:

(V+A spectral functions)



Results of this test:

ALEPH	$\alpha_s(m_\tau^2)$	$C_{4,V+A}$ (GeV^4)	$C_{6,V+A}$ (GeV^6)	$C_{8,V+A}$ (GeV^8)	χ^2/dof
true values	0.312	0.0027	-0.013	0.035	
fake data fit	0.334(3)	-0.0024(4)	0.0007(3)	-0.0008(4)	0.95/1

optimal	$\alpha_s(m_\tau^2)$	$C_{6,V+A}$ (GeV^6)	$C_{8,V+A}$ (GeV^8)	$C_{10,V+A}$ (GeV^{10})	χ^2/dof
true values	0.312	-0.013	0.035	-0.083	
fake data fit	0.334(4)	0.0008(4)	-0.0008(5)	0.0001(3)	0.92/1

(CIPT, statistical errors only)

- Truncated-OPE approach gets it wrong (similar conclusion for FOPT), despite good χ^2 : systematic overestimation of α_s
- big difference in behavior of OPE

DV-model approach (Boito *et al.*):

- Use simple weights:

$$w_0(x) = 1\,, \quad w_2(x) = 1 - x^2\,, \quad w_3(x) = (1-x)^2(1+2x)$$

  hence $C_6\,,\ C_8$ only OPE coefficients needed

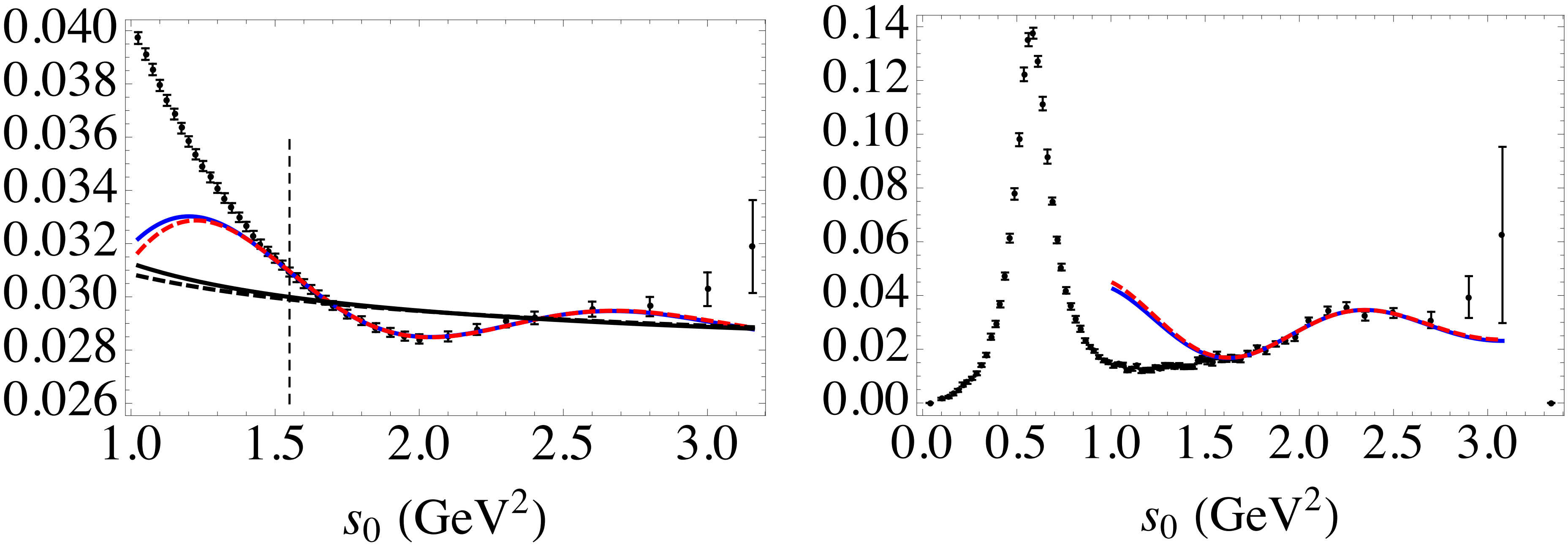
- No attempt to suppress DVs, hence use *ansatz* $\rho_{\rm DV}(s) = e^{-\gamma s-\delta}\sin\left(\alpha+\beta s\right)$

  for the DV part of the spectral function, and fit $\alpha\,,\ \beta\,,\ \gamma\,,\ \delta$

- To do this, vary $s_{min} \le s_0 \le m_\tau^2$, make use of the data!

  Fit determines $s_{min} \approx 1.55\ {\rm GeV}^2$

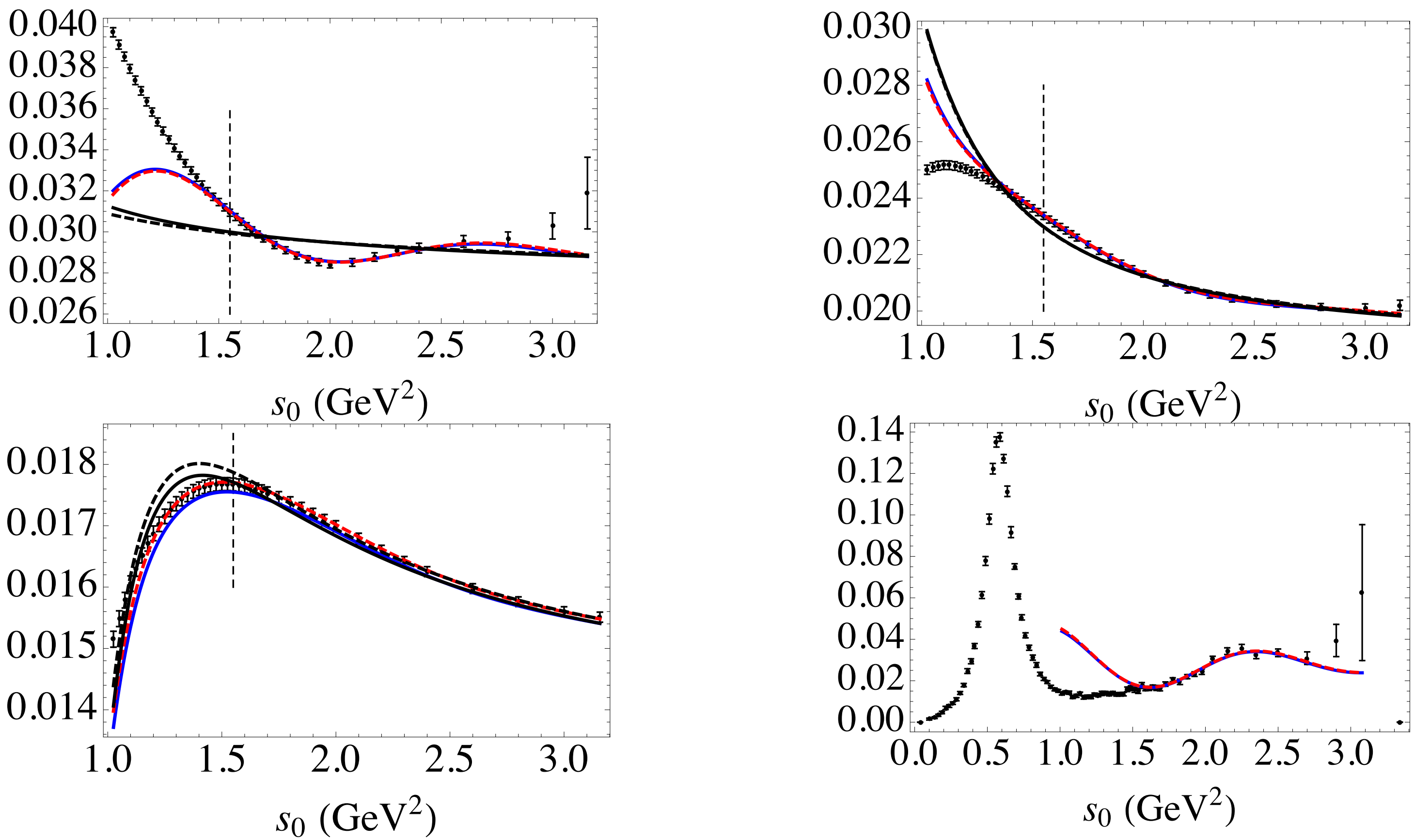
Example: simplest fit, vector channel $w_0(x) = 1$



Blue: FOPT $\alpha_s(m_\tau^2) = 0.296(11)$ Red: CIPT $\alpha_s(m_\tau^2) = 0.310(14)$

Black: OPE contribution only

Example: 3-weight $w_{0,2,3}(x)$, vector channel fit



Blue: FOPT $\alpha_s(m_\tau^2) = 0.296(10)$ Red: CIPT $\alpha_s(m_\tau^2) = 0.310(14)$

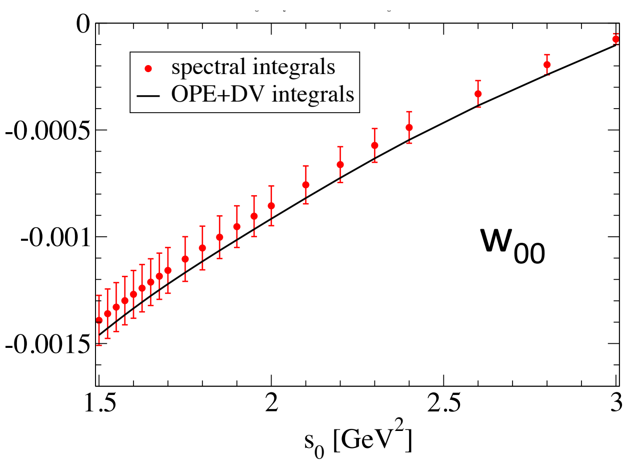
Black: OPE contribution only

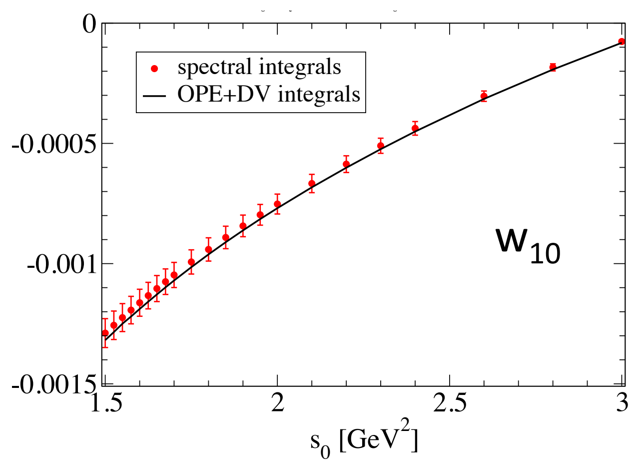
Vector as good as Vector+Axial!

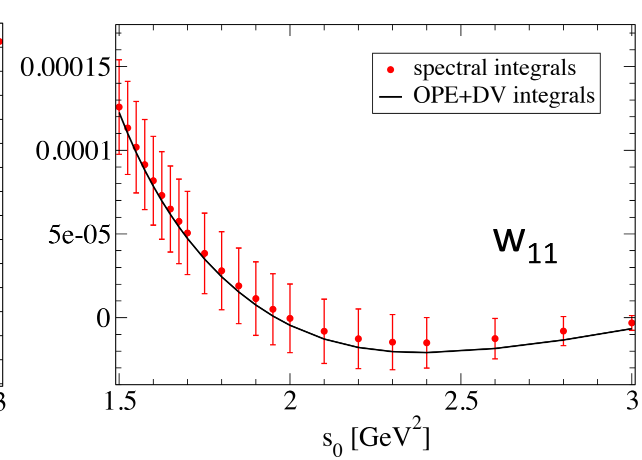
This approach passes many tests (here V+A):

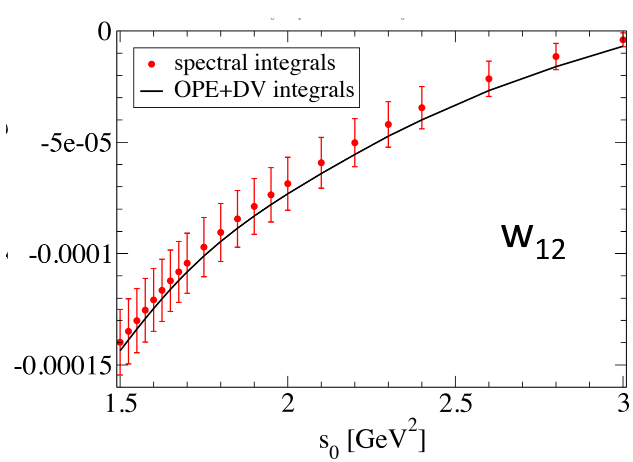
Check s_0 dependence, should work above $\sim 1.5\ \mathrm{GeV}^2$

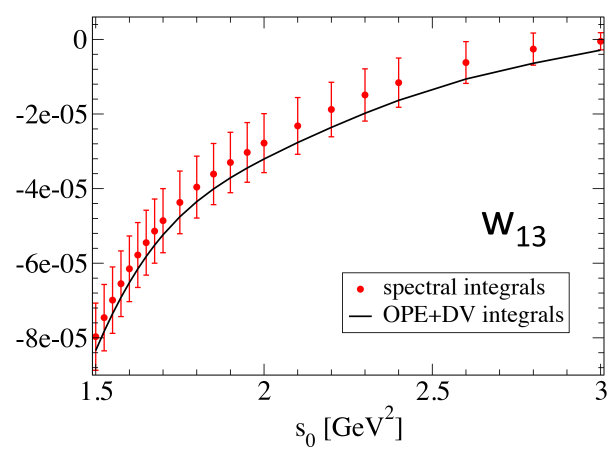
ALEPH moments for Boito *et al.* (only w_{00} used in fits; $s_0 = m_\tau^2$ minus s_0 diffs.):







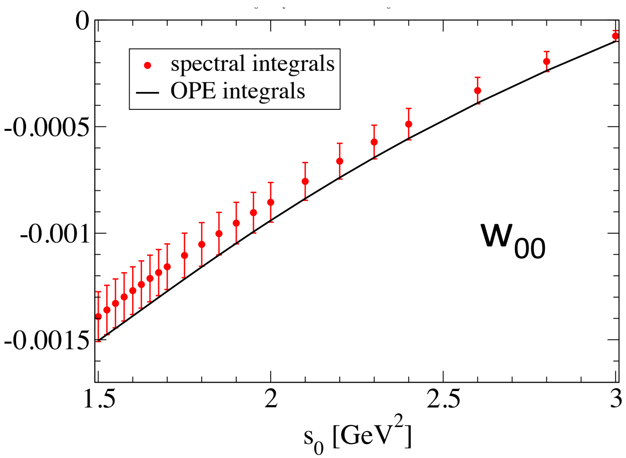


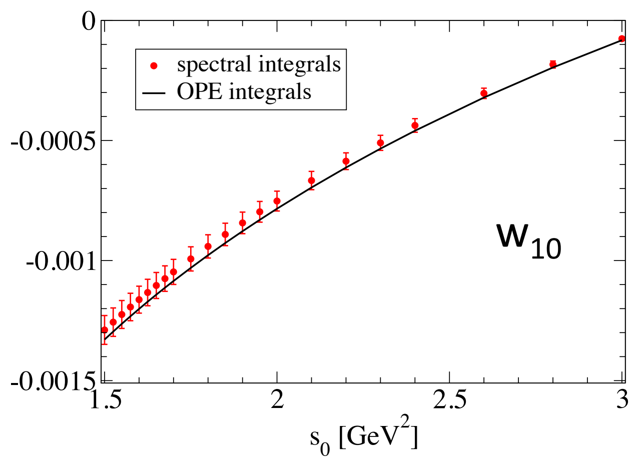


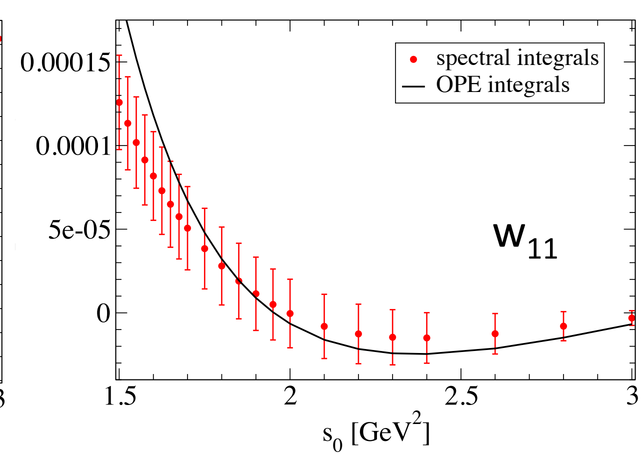
Same tests for truncated-OPE approach (again, V+A):

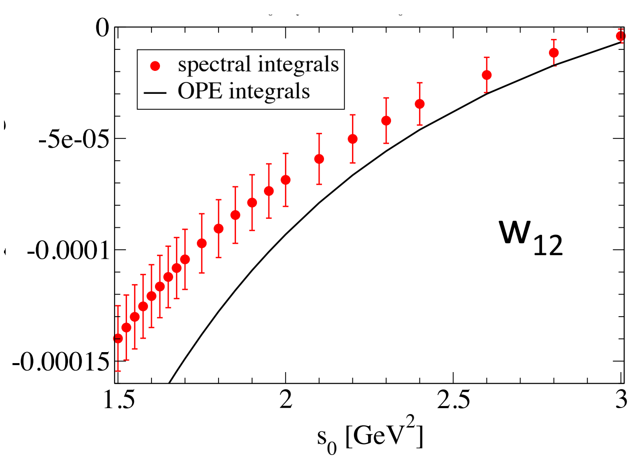
Check s_0 dependence, should work above $\sim 2\ \mathrm{GeV}^2$

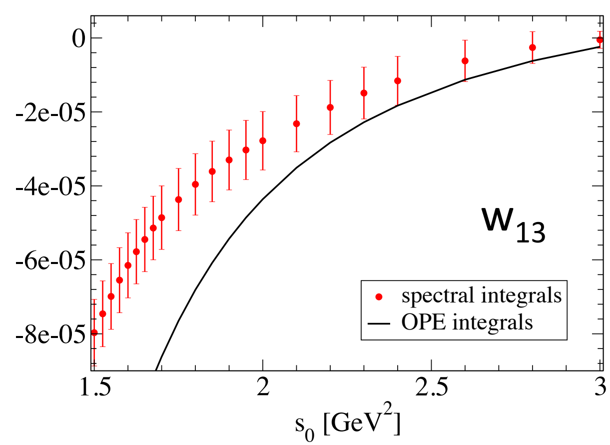
ALEPH moments for P&R (all used in fit):











Model dependence of DV-model approach:

- Depends on a model for the effect of Duality Violations = resonance effects
- Ingredients: Regge behavior of spectrum for $s \geq s_{min}$:

  $$M^2(n) = M^2(0) + \sigma n \ , \quad n = 0, 1, 2, \dots \ , \quad \sigma \sim \Lambda^2_{\rm QCD} \sim 1 \ {\rm GeV}^2$$

  Large N_c : $\Gamma(n) \propto M(n)/N_c$

- Model satisfying these constraints and analyticity
  (Blok, Shifman & Zhang '98, Bigi, Shifman, Uraltsev & Vainshtein '99
  Catà, Golterman & Peris '05,'08)
- More general arguments: in preparation (Boito *et al.*)
- Important to test this with data!

FOPT and CIPT

FOPT = fixed order perturbation theory

CIPT = contour improved perturbation theory
(Pivovarov '92, Le Diberder & Pich '92)

- Different ways of partially resumming perturbation theory on the theory side of the sum rules.

- Differences: $0.017\ (5.1\%)$ (Davier *et al.*)
  $0.016\ (4.9\%)$ (Pich & Rodriguez)
  $0.014\ (4.7\%)$ (Boito *et al.*)

- Renormalon analysis somewhat favors FOPT
  (Beneke & Jamin '08, Beneke, Boito & Jamin '12)

Outlook

- Recent theory advances may help extract $\alpha_s(m_\tau^2)$ from non-perturbative hadronic τ-decay data to higher precision, but need to understand the physics of resonances better

  $\Rightarrow$ precision tests of the DV *ansatz*? Important!

- Belle and Belle-II will have thousands times more τ pairs than ALEPH/OPAL!

- Focus on vector channel – less clear that $s \leq m_\tau^2$ is already asymptotic in axial channel; reduce errors especially for $s > 2.5\ \mathrm{GeV}^2$

  Will improving the $\tau \to \nu_\tau 4\pi$ non-strange vector decay already help? (Large errors in the region $s > 1.5\ \mathrm{GeV}^2$ dominated by this decay)

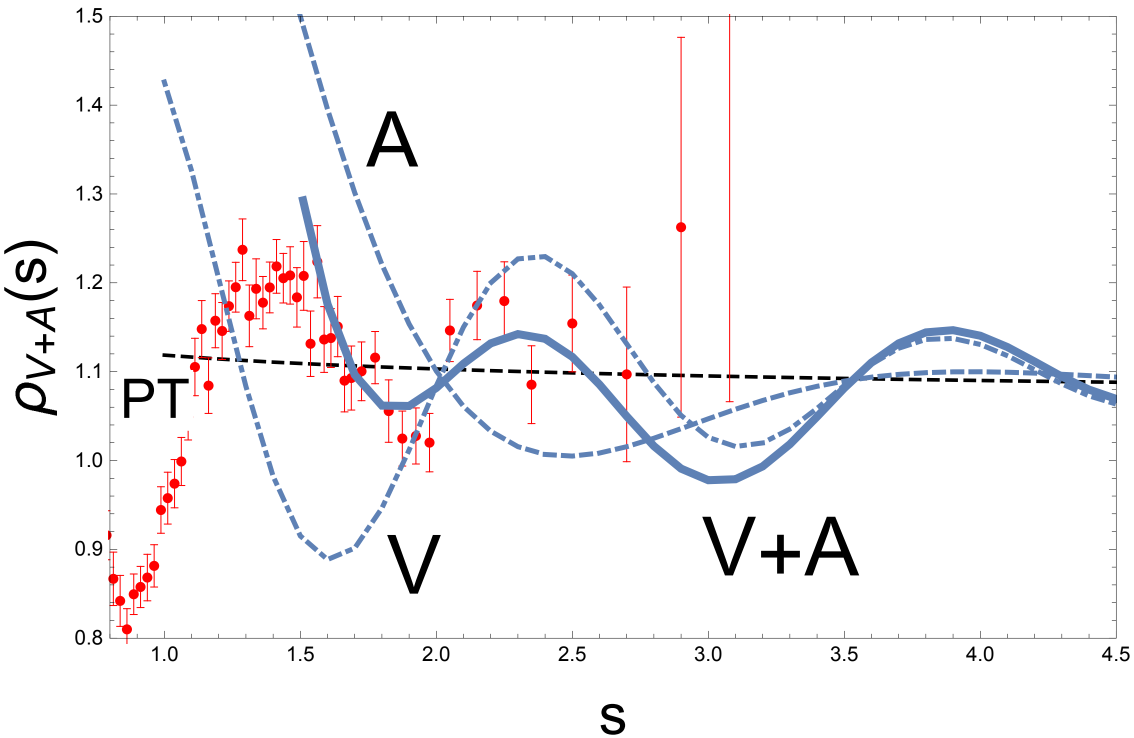
- Open theory question: CIPT vs. FOPT? (cf. Boito, Beneke & Jamin, '12)

  Comparison with value from other high-precision value, at common scale?

BACKUP SLIDES

Why does the Truncated-OPE approach get it wrong?

- Rely on uncontrolled assumption about the OPE in higher orders.
- Assume that duality violations (resonance effects) can be neglected, at least in V+A, *without testing this*.



Why does the Truncated-OPE approach get it wrong?

- Rely on uncontrolled assumption about the OPE in higher orders.
- Assume that duality violations (resonance effects) can be neglected, at least in V+A, *without testing this*.
- Potentially large effect at $s_0 = m_\tau^2$! **Not excluded by data.**