CKM Unitary tests with Tau decays

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Outline

1. Introduction and Motivation

2. V_{us} from inclusive hadronic τ decays

3. V_{us} from exclusive hadronic τ decays

4. Conclusion and Outlook

1. Introduction and Motivation

1.1 Test of New Physics : V_{us}

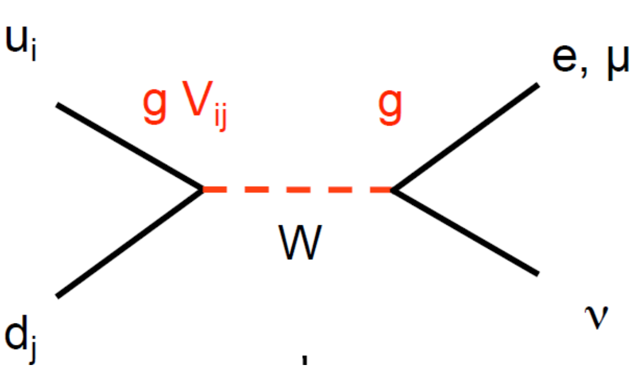
- Extraction of the Cabibbo-Kobayashi-Maskawa matrix element V_{us}
  - Fundamental parameter of the Standard Model
    Check unitarity of the first row of the CKM matrix:
    ⇒ *Cabibbo Universality*

$$\boxed{|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \stackrel{?}{=} 1}$$

Negligible
(B decays)

  - Input in UT analysis

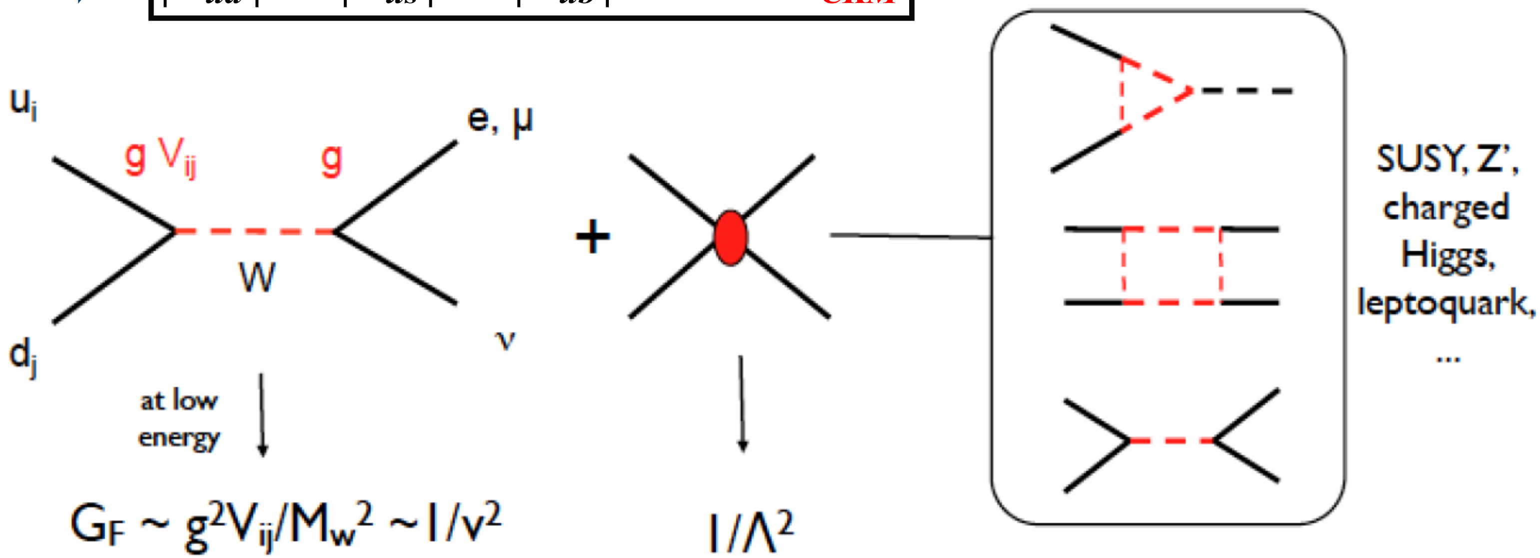
- Look for *new physics*
  - In the Standard Model : W exchange ⇒ only V-A structure



1.1 Test of New Physics : V_{us}

- BSM: sensitive to tree-level and loop effects of a large class of models

$$\boxed{|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 + \Delta_{CKM}}$$



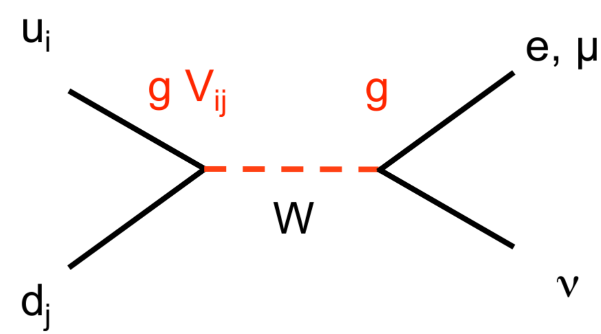
BSM effects : $\boxed{\Delta_{CKM} \sim (v/\Lambda)^2}$

- Look for new physics by comparing the extraction of V_{us} from different processes: helicity suppressed $K_{\mu 2}$, helicity allowed K_{l3}, hadronic τ decays

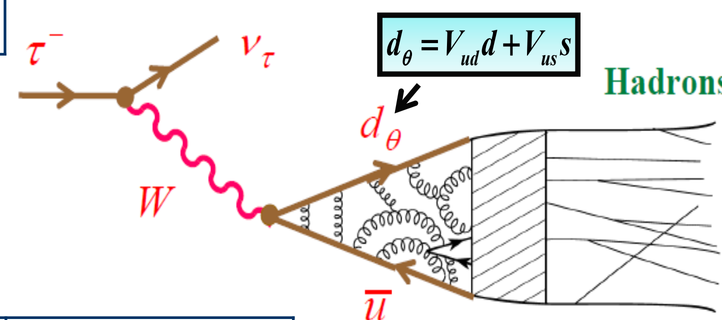
1.2 Paths to V_{ud} and V_{us}

- From kaon, pion, baryon and nuclear decays

V_{ud}	$0^+ \to 0^+$ $\pi^\pm \to \pi^0 e \nu_e$	$n \to p e \nu_e$	$\pi \to l \nu_l$
V_{us}	$K \to \pi l \nu_l$	$\Lambda \to p e \nu_e$	$K \to l \nu_l$



- From τ decays (crossed channel)



V_{ud}	$\tau \to \pi\pi\nu_\tau$		$\tau \to \pi\nu_\tau$	$\tau \to h_{NS}\nu_\tau$
V_{us}	$\tau \to K\pi\nu_\tau$		$\tau \to K\nu_\tau$	$\tau \to h_S\nu_\tau$ (inclusive)

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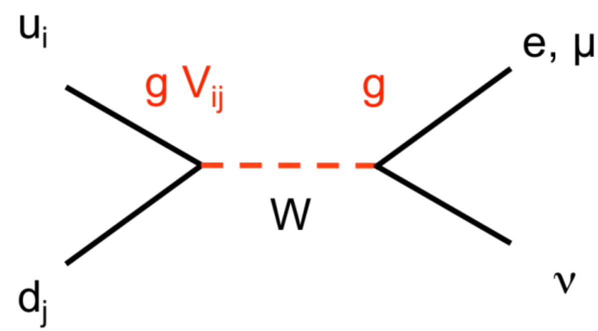

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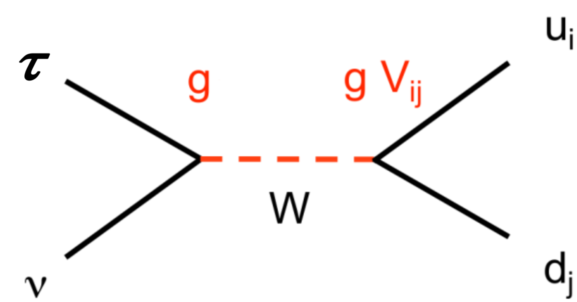

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1.2 Paths to V_{ud} and V_{us}

- These are the *golden modes* to extract V_{ud} and V_{us}
  - Only the *vector current* contributes $\langle A(p_A)|\bar{q}^i\gamma_\mu q^j|B(p_B)\rangle$
  - Normalization known in SU(2) [SU(3)] symmetry limit
  - Corrections start at 2nd order in SU(2) [SU(3)] breaking

Ademollo & Gato, Berhands & Sirlin

- Currently the most precise determination of V_{ud} and V_{us}

⇒ V_{ud} (*0.02 %*) and V_{us} (*0.5 %*)

1.2 Paths to V_{ud} and V_{us}

- From kaon, pion, baryon and nuclear decays

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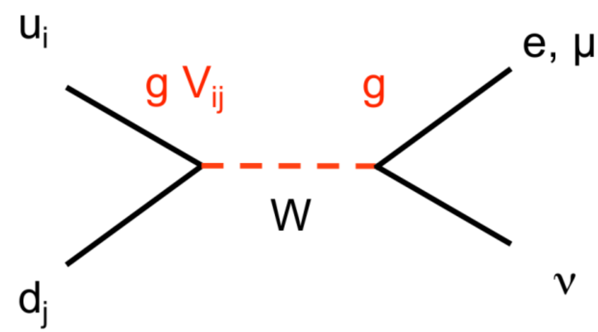

- $n \to p e \nu_e$:
  - Both *V* and *A currents* contribute ⇨ need experimental information on A (e.g. β asymmetry ($r_A = g_A/g_V$))
  - Free of nuclear uncertainties
  - Probe different combinations of BSM operators (e.g. right-handed currents, etc…)

1.2 Paths to V_{ud} and V_{us}

- From kaon, pion, baryon and nuclear decays

V_{ud}	$0^+ \to 0^+$ $\pi^\pm \to \pi^0 e \nu_e$	$n \to p e \nu_e$	$\pi \to \ell\, \nu_\ell$
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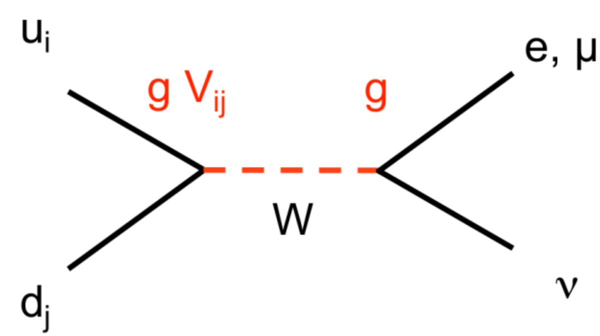

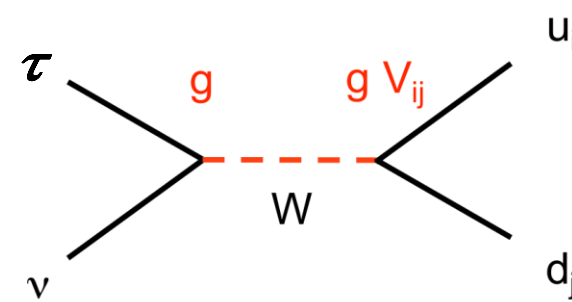

- From τ decays (crossed channel)

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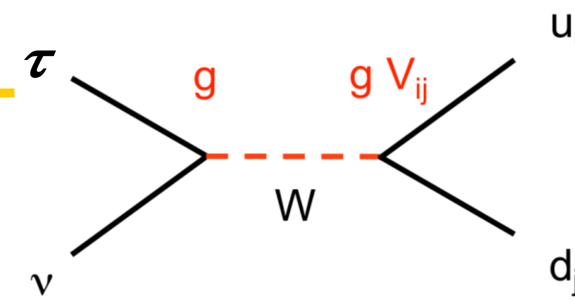
V_{us}	$\tau \to K \pi \nu_\tau$		$\tau \to K \nu_\tau$	$\tau \to h_S \nu_\tau$ (inclusive)

1.2 Paths to V_{ud} and V_{us}

- K_{l2}/π_{l2} and $\tau \to K/\pi\nu_\tau$
  - Only the *axial current* contributes
  - Need to know the decay constants F_K, F_π
    ⇨ *Lattice QCD*
  - Probe different BSM operators than from the vector case

- Input on F_K/F_π ⇨ V_{us}/V_{ud} very precisely

1.2 Paths to V_{ud} and V_{us}



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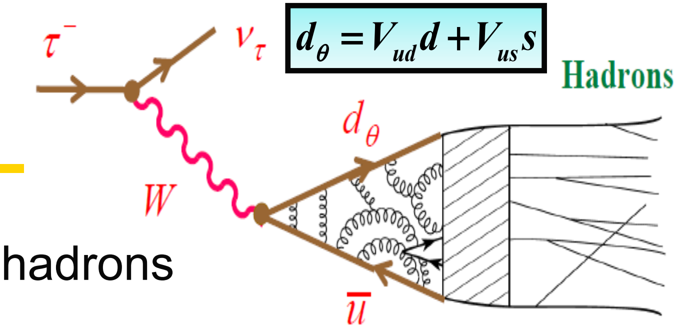
- Possibility to determine V_{ud}, V_{us} from *inclusive τ decays*
  - Use *OPE* to calculate the inclusive BRs
  - Different test of BSM operators *inclusive* vs. *exclusive*

2. V_{us} from inclusive hadronic τ decays

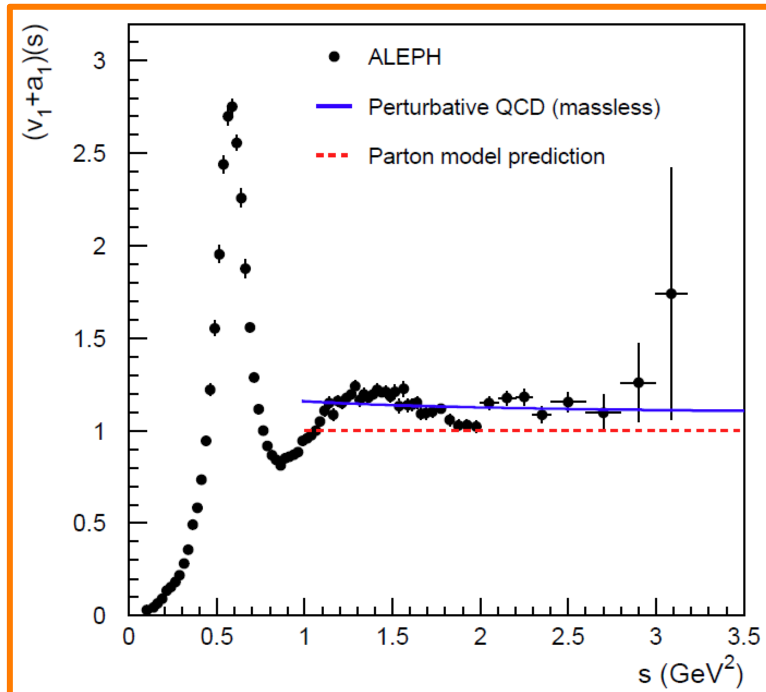
2.1 Introduction



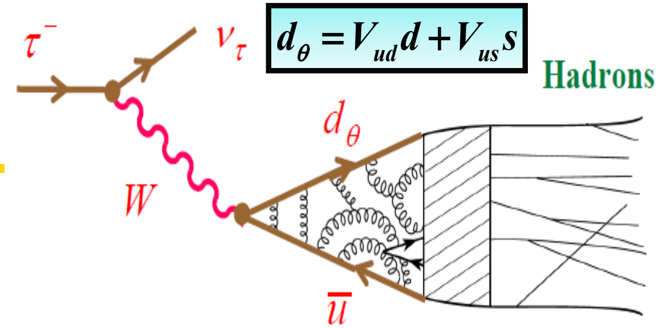
- Tau, the only lepton heavy enough to decay into hadrons
- $\boxed{m_\tau \sim 1.77\,\mathrm{GeV} > \Lambda_{QCD}}$ ⇨ use *perturbative tools: OPE…*
- Inclusive τ decays : $\boxed{\tau \to (\bar{u}d, \bar{u}s)\nu_\tau}$ ⇨ fund. SM parameters $\left(\alpha_S(m_\tau), |V_{us}|, m_s\right)$
- We consider $\boxed{\Gamma\left(\tau^- \to \nu_\tau + \mathrm{hadrons}_{S=0}\right)}$

  $\boxed{\Gamma\left(\tau^- \to \nu_\tau + \mathrm{hadrons}_{S\neq 0}\right)}$
- ALEPH and OPAL at LEP measured with precision not only the total BRs but also the energy distribution of the hadronic system ⇨ huge *QCD activity*!
- Observable studied: $\boxed{R_\tau \equiv \dfrac{\Gamma\left(\tau^- \to \nu_\tau + \mathrm{hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \bar{\nu}_e\right)}}$

Davier et al'13



2.2 Theory



- $$\boxed{R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} \approx N_C}$$ parton model prediction

- $$R_\tau = R_\tau^{NS} + R_\tau^S \approx |V_{ud}|^2 N_C + |V_{us}|^2 N_C$$

- $$\boxed{\frac{|V_{us}|^2}{|V_{ud}|^2} = \frac{R_\tau^S}{R_\tau^{NS}}} \Rightarrow |V_{us}|$$

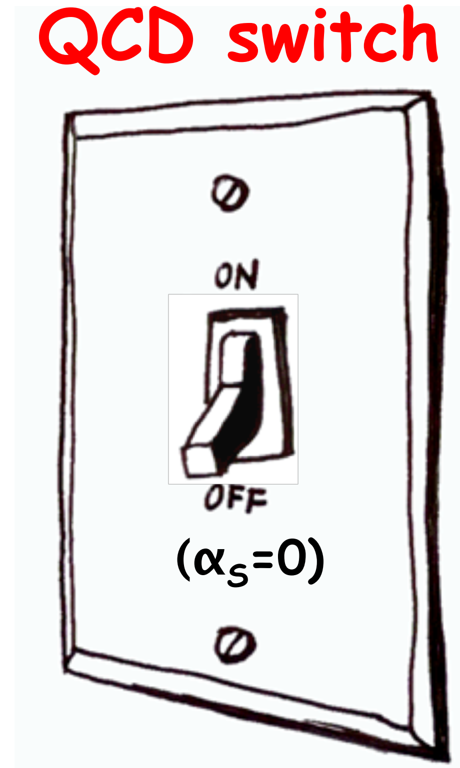
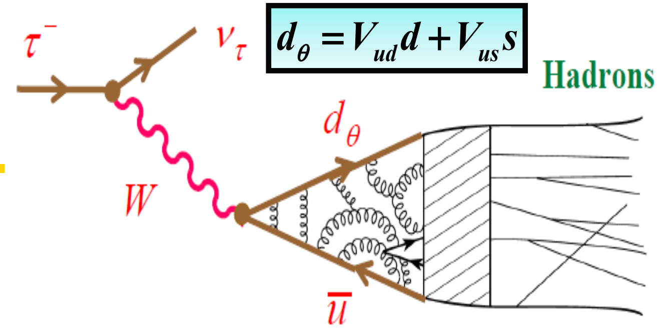


Figure from M. González Alonso'13

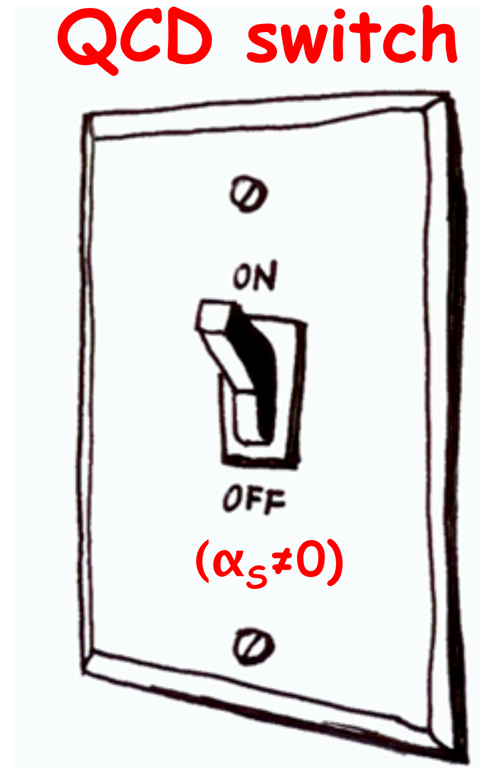
2.2 Theory



- $$R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau + \text{hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \overline{\nu}_e\right)} \approx N_C$$ parton model prediction

- $$R_\tau = R_\tau^{NS} + R_\tau^{S} \approx \left|V_{ud}\right|^2 N_C + \left|V_{us}\right|^2 N_C$$

- Experimentally: $$R_\tau = \frac{1 - B_e - B_\mu}{B_e} = 3.6291 \pm 0.0086$$



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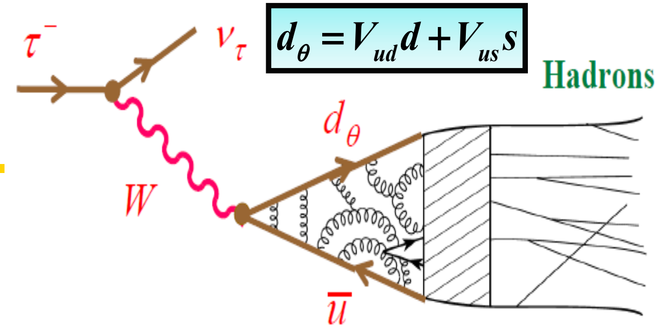

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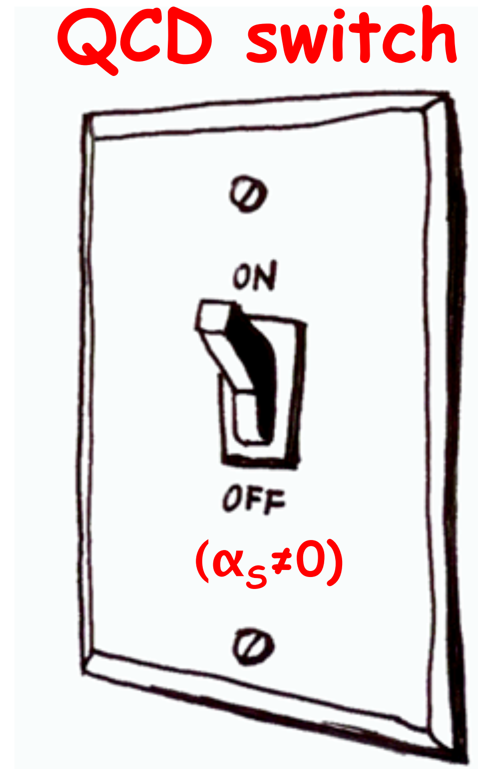
- $$R_\tau = R_\tau^{NS} + R_\tau^{S} \approx \left|V_{ud}\right|^2 N_C + \left|V_{us}\right|^2 N_C$$

- Experimentally: $$\boxed{R_\tau = \frac{1 - B_e - B_\mu}{B_e} = 3.6291 \pm 0.0086}$$

- Due to *QCD corrections*: $R_\tau = \left|V_{ud}\right|^2 N_C + \left|V_{us}\right|^2 N_C + O\left(\alpha_S\right)$



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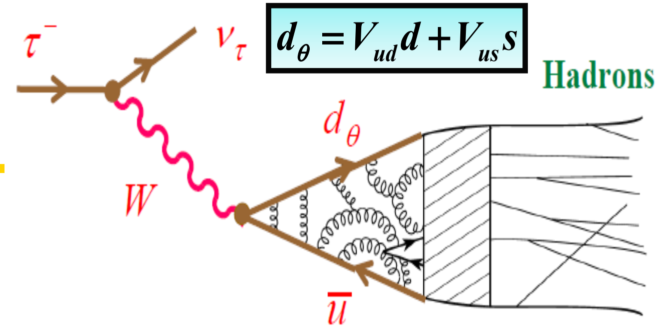
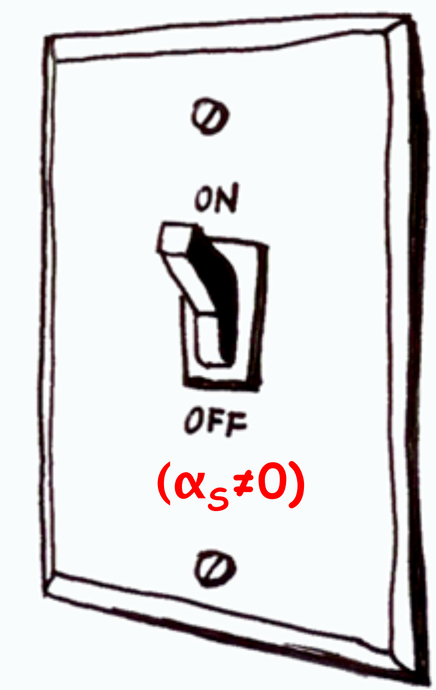


- From the measurement of the spectral functions, extraction of α_S, $|V_{us}|$

- $$\boxed{R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau + \text{hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \bar{\nu}_e\right)} \approx N_C}$$ naïve QCD prediction

QCD switch



($\alpha_S \neq 0$)

- Extraction of the strong coupling constant :

$$R_\tau^{NS} = \left|V_{ud}\right|^2 N_C + O\left(\alpha_s\right) \Rightarrow \alpha_S$$

measured calculated

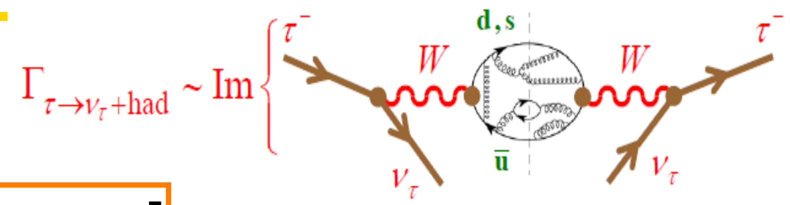
- Determination of V_{us} : $$\boxed{\frac{\left|V_{us}\right|^2}{\left|V_{ud}\right|^2} = \frac{R_\tau^S}{R_\tau^{NS}} + O\left(\alpha_s\right)}$$

- Main difficulty: compute the QCD corrections with the best accuracy

2.3 Calculation of the QCD corrections

- Calculation of R_τ:

$$R_\tau(m_\tau^2) = 12\pi S_{EW} \int_0^{m_\tau^2} \frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2 \left[\left(1+2\frac{s}{m_\tau^2}\right)\mathrm{Im}\,\Pi^{(1)}(s+i\varepsilon) + \mathrm{Im}\,\Pi^{(0)}(s+i\varepsilon)\right]$$

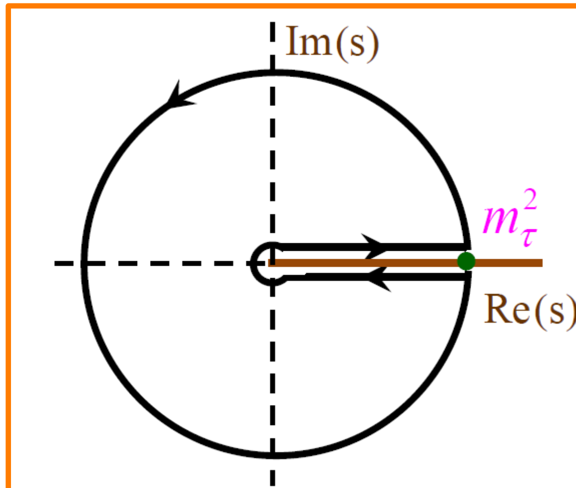


Braaten, Narison, Pich'92

- Analyticity: Π is analytic in the entire complex plane except for s real positive

⇒ Cauchy Theorem

$$R_\tau(m_\tau^2) = 6i\pi S_{EW} \oint_{|s|=m_\tau^2} \frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2 \left[\left(1+2\frac{s}{m_\tau^2}\right)\Pi^{(1)}(s) + \Pi^{(0)}(s)\right]$$



- We are now at sufficient energy to use OPE:

$$\Pi^{(J)}(s) = \sum_{D=0,2,4\ldots} \frac{1}{(-s)^{D/2}} \sum_{\dim O=D} \mathrm{C}^{(J)}(s,\mu)\,\langle O_D(\mu)\rangle$$

Wilson coefficients

Operators

μ: separation scale between short and long distances

2.3 Calculation of the QCD corrections

Braaten, Narison, Pich'92

- Calculation of R_τ:

$$\boxed{R_\tau\left(m_\tau^2\right) = N_C\ S_{EW}\left(1+\delta_P+\delta_{NP}\right)}$$

- Electroweak corrections: $S_{EW} = 1.0201(3)$ *Marciano &Sirlin'88, Braaten & Li'90, Erler'04*

- Perturbative part (D=0): $\delta_P = a_\tau + 5.20\ a_\tau^2 + 26\ a_\tau^3 + 127\ a_\tau^4 + \ldots \approx 20\%$ $\quad a_\tau = \frac{\alpha_s(m_\tau)}{\pi}$

Baikov, Chetyrkin, Kühn'08

- D=2: quark mass corrections, *neglected* for $R_\tau^{NS}\ \left(\propto m_u, m_d\right)$ but not for $R_\tau^{S}\ \left(\propto m_s\right)$

- D ≥ 4: Non perturbative part, not known, *fitted from the data*

  ⇨ Use of weighted distributions

2.3 Calculation of the QCD corrections

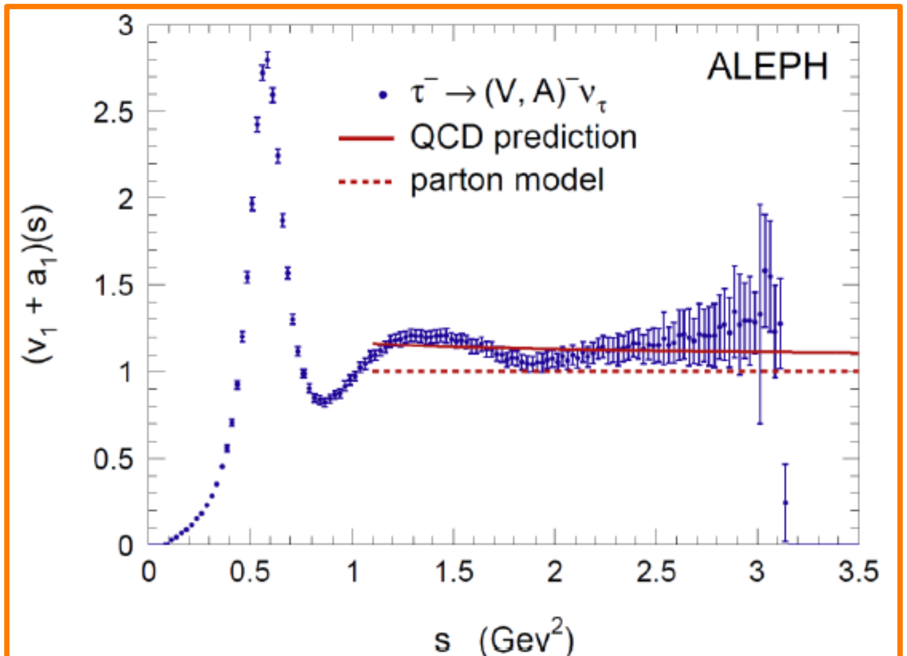
Le Diberder&Pich'92

- D ≥ 4: Non perturbative part, not known, *fitted from the data*

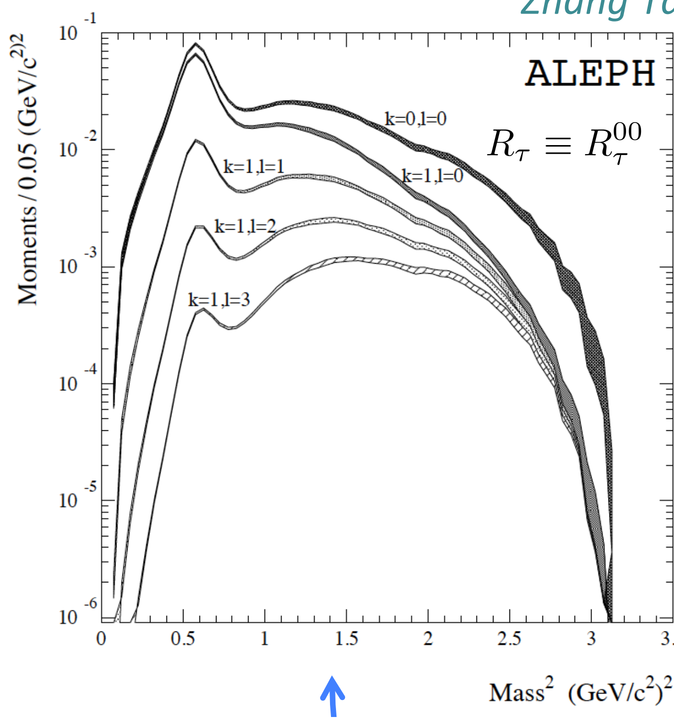
  ⇨ Use of weighted distributions

Exploit shape of the spectral functions to obtain additional experimental information

$$R_{\tau,U}^{k\ell}(s_0) = \int_0^{s_0} ds \left(1-\frac{s}{s_0}\right)^k \left(\frac{s}{s_0}\right)^\ell \frac{dR_{\tau,U}(s_0)}{ds}$$



Zhang'Tau14



2.4 Inclusive determination of V_{us}

- With QCD on: $\boxed{\dfrac{|V_{us}|^2}{|V_{ud}|^2} = \dfrac{R_\tau^S}{R_\tau^{NS}} + O(\alpha_s)}$

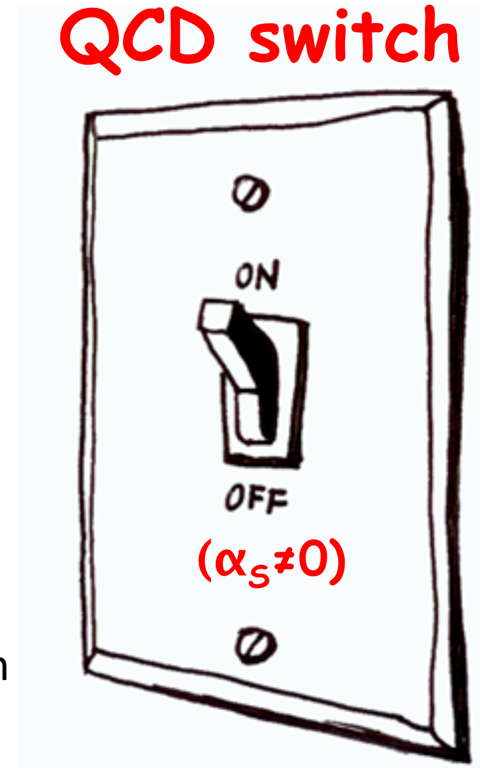
- Use OPE: $\boxed{R_\tau^{NS}(m_\tau^2) = N_C\, S_{EW} |V_{ud}|^2 \left(1+\delta_P+\delta_{NP}^{ud}\right)}$

$$\boxed{R_\tau^{S}(m_\tau^2) = N_C\, S_{EW} |V_{us}|^2 \left(1+\delta_P+\delta_{NP}^{us}\right)}$$

- $\boxed{\delta R_\tau \equiv \dfrac{R_{\tau,NS}}{|V_{ud}|^2} - \dfrac{R_{\tau,S}}{|V_{us}|^2}}$

SU(3) breaking quantity, strong dependence in m_s computed from OPE (L+T) + phenomenology

$\delta R_{\tau,th} = 0.0242(32)$ *Gamiz et al'07, Maltman'11*



$$\boxed{|V_{us}|^2 = \dfrac{R_{\tau,S}}{\dfrac{R_{\tau,NS}}{|V_{ud}|^2} - \delta R_{\tau,th}}}$$

HFAG'17

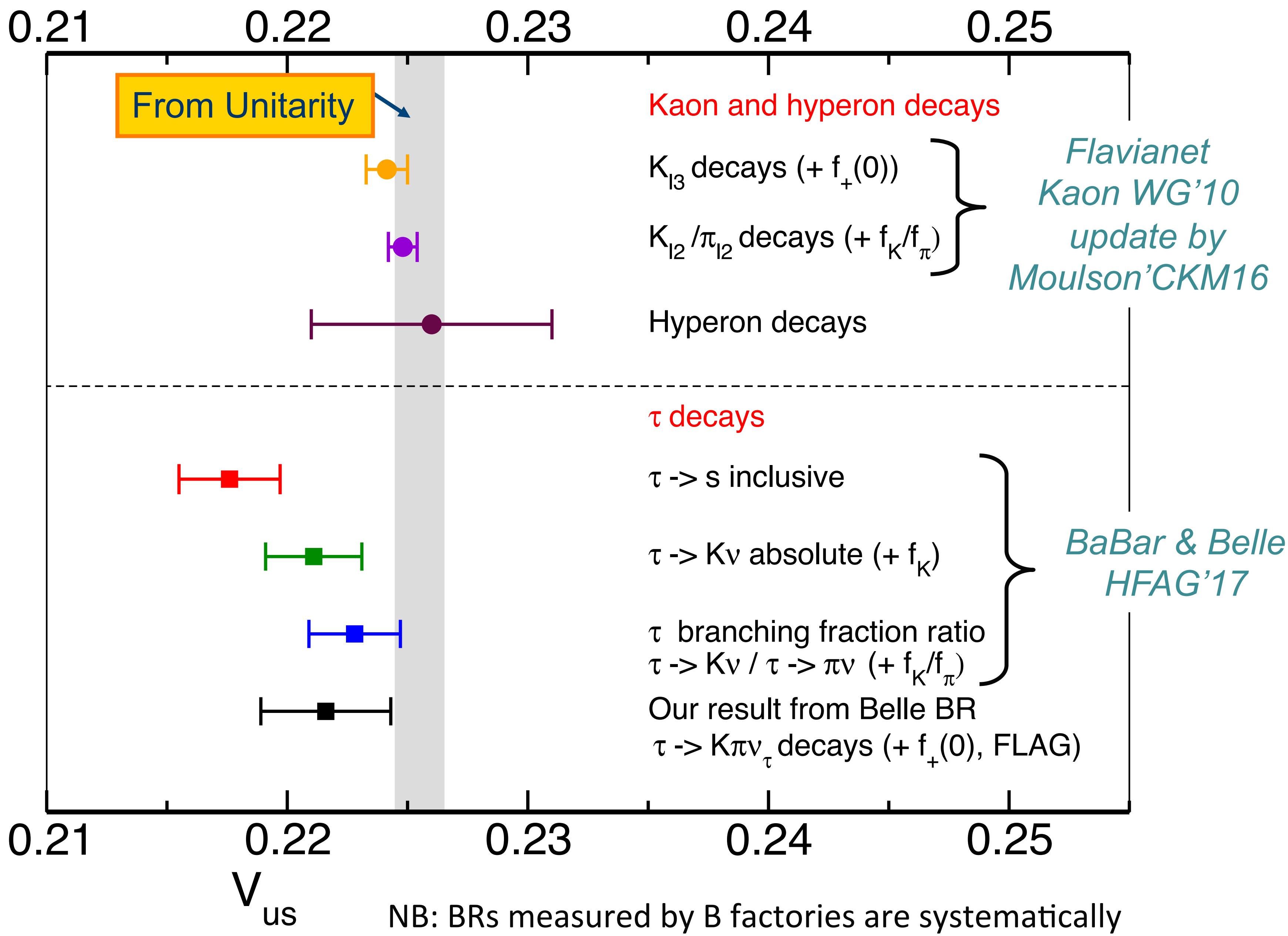
$R_{\tau,S} = 0.1633(28)$

$R_{\tau,NS} = 3.4718(84)$

$|V_{ud}| = 0.97417(21)$

$\Rightarrow$ $\boxed{|V_{us}| = 0.2186 \pm 0.0019_{\text{exp}} \pm 0.0010_{\text{th}}}$

3.1σ away from unitarity!



NB: BRs measured by B factories are systematically smaller than previous measurements

3.5 V_{us} using info on Kaon decays and $\tau \to K\pi\nu_\tau$

Antonelli, Cirigliano, Lusiani, E.P. '13

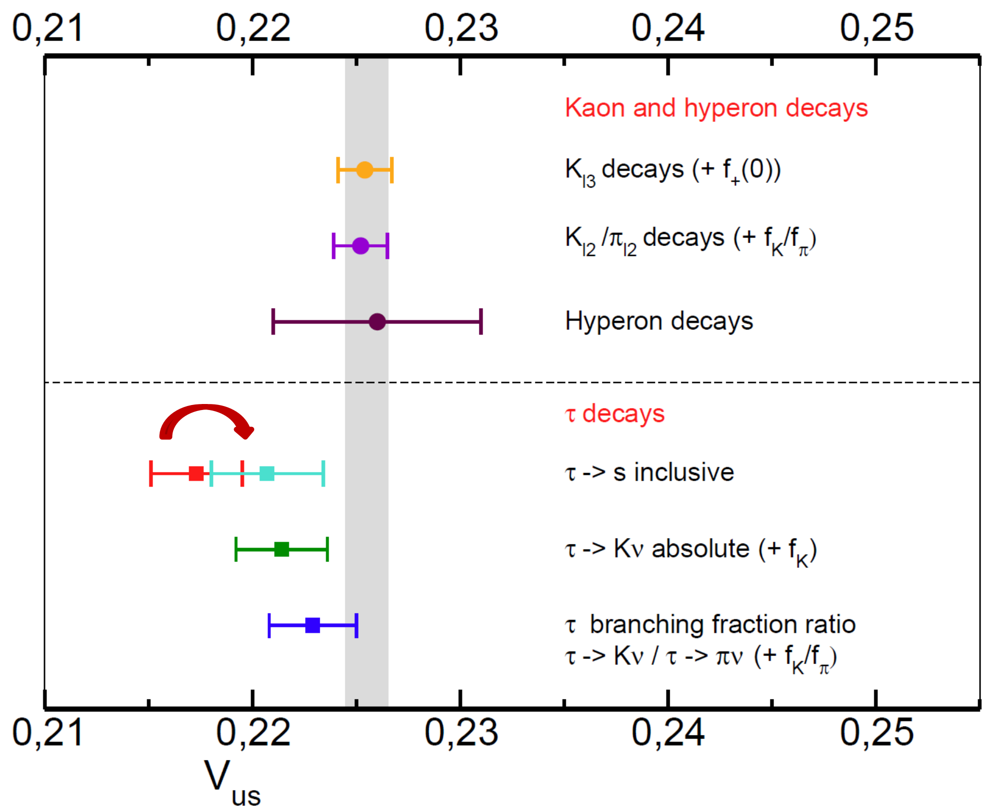
Branching fraction	HFAG Winter 2012 fit	
$\Gamma_{10} = K^-\nu_\tau$	$(0.6955 \pm 0.0096) \cdot 10^{-2}$	$\Rightarrow$ **(0.713 ± 0.003)%**
$\Gamma_{16} = K^-\pi^0\nu_\tau$	$(0.4322 \pm 0.0149) \cdot 10^{-2}$	$\Rightarrow$ **(0.471 ± 0.018)%**
$\Gamma_{23} = K^-2\pi^0\nu_\tau$ (ex. K^0)	$(0.0630 \pm 0.0222) \cdot 10^{-2}$	
$\Gamma_{28} = K^-3\pi^0\nu_\tau$ (ex. K^0, η)	$(0.0419 \pm 0.0218) \cdot 10^{-2}$	
$\Gamma_{35} = \pi^-\bar{K}^0\nu_\tau$	$(0.8206 \pm 0.0182) \cdot 10^{-2}$	$\Rightarrow$ **(0.857 ± 0.030)%**
$\Gamma_{110} = X_s^-\nu_\tau$	$(2.8746 \pm 0.0498) \cdot 10^{-2}$	$\Rightarrow$ **(2.967 ± 0.060)%**

- Longstanding inconsistencies between τ and kaon decays in extraction of V_{us} seem to have been resolved !
  R. Hudspith, R. Lewis, K. Maltman, J. Zanotti'17

- Crucial input:
  $\tau \to K\pi\nu_\tau$ Br + spectrum

$$|V_{us}| = 0.2229 \pm 0.0022_{exp} \pm 0.0004_{theo}$$

⇒ need new data



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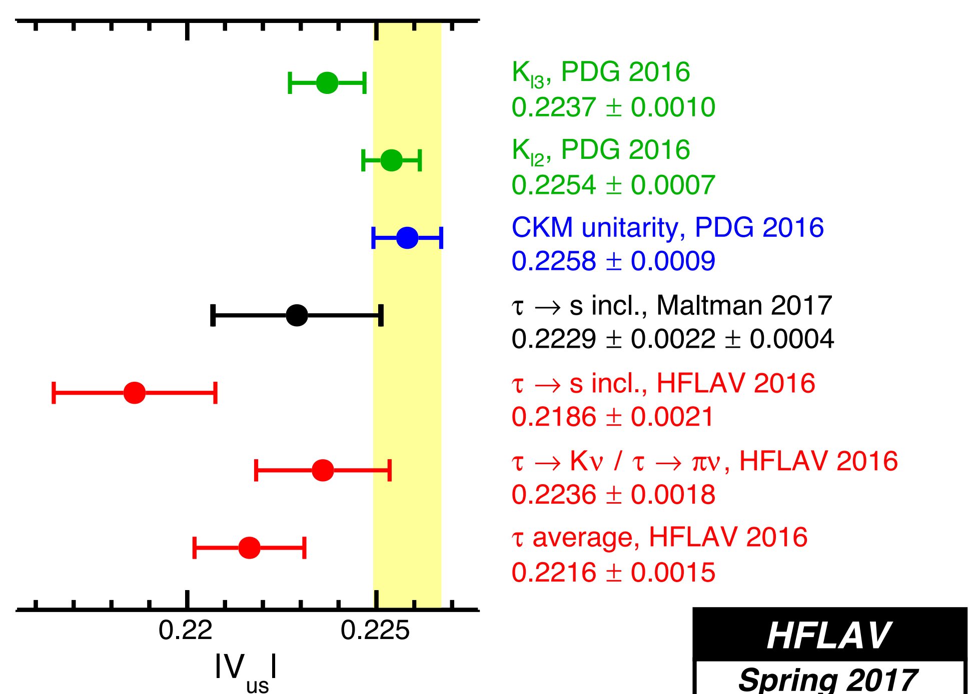


3. V_{us} from exclusive hadronic τ decays :

- $\tau \rightarrow K\pi\nu_{\tau}$ decays

3.1.1 Introduction: key ingredients

- Master formula for $\tau \to K\pi\nu_\tau$:

$$\boxed{\Gamma\left(\tau \to \overline{K}\pi\nu_\tau[\gamma]\right) = \frac{G_F^2 m_\tau^5}{96\pi^3} C_K^2 S_{EW}^\tau \left|V_{us}\right|^2 \left|f_+^{K^0\pi^-}(0)\right|^2 I_K^\tau \left(1+\delta_{\mathrm{EM}}^{K\tau}+\tilde{\delta}_{\mathrm{SU(2)}}^{K\pi}\right)^2}$$

- Experimental inputs from HFAG *Banerjee et al.'12*

3.1.2 Radiative corrections

- Master formula for $\tau \to K\pi\nu_\tau$:

$$\boxed{\Gamma\left(\tau \to \overline{K}\pi\nu_\tau\left[\gamma\right]\right) = \frac{G_F^2 m_\tau^5}{96\pi^3} C_K^2 S_{EW}^\tau \left|V_{us}\right|^2 \left|f_+^{K^0\pi^-}(0)\right|^2 I_K^\tau \left(1 + \delta_{\mathrm{EM}}^{K\tau} + \tilde{\delta}_{\mathrm{SU(2)}}^{K\pi}\right)^2}$$

- Theoretical inputs :
  - S_{ew} : Short distance electroweak correction $\Rightarrow$ $\boxed{S_{\mathrm{ew}} = 1.0201}$

Marciano &Sirlin'88, Braaten & Li'90, Erler'04

3.1.2 Radiative corrections

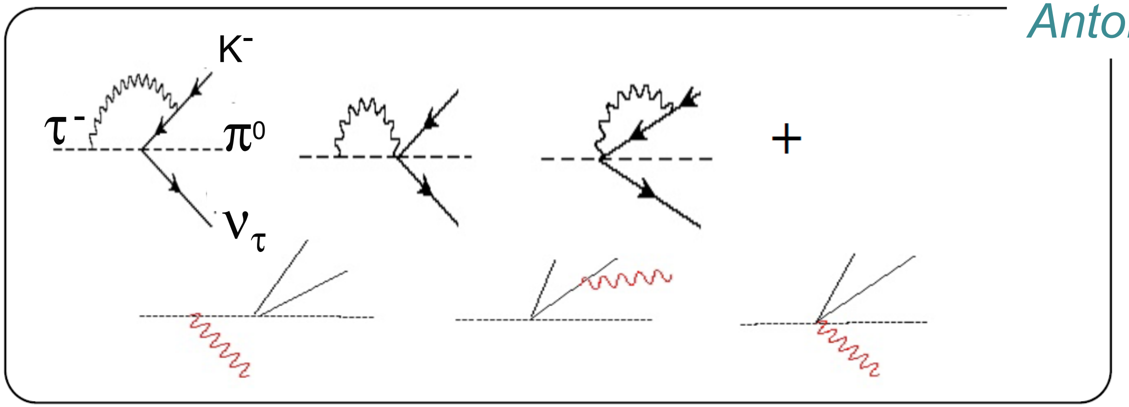
- Master formula for $\tau \to K\pi\nu_\tau$:

$$\Gamma\left(\tau \to \bar{K}\pi\nu_\tau[\gamma]\right) = \frac{G_F^2 m_\tau^5}{96\pi^3} C_K^2 S_{EW}^\tau \left|V_{us}\right|^2 \left|f_+^{K^0\pi^-}(0)\right|^2 I_K^\tau \left(1 + \delta_{\rm EM}^{K\tau} + \tilde{\delta}_{\rm SU(2)}^{K\pi}\right)^2$$

- Theoretical inputs :
  - S_{ew} : Short distance electroweak correction $\Rightarrow$ $S_{\rm ew} = 1.0201$
  - $\delta_{\rm EM}^{Kl}$: Long-distance electromagnetic corrections

F.V. Flores-Baez, J.R. Morones-Ibarra'13

Antonelli, Cirigliano, Lusiani, E.P.'13



→ ChPT to $O(p^2e^2)$

→ Counter-terms neglected

based on $\tau^- \to \pi^-\pi^0\nu_\tau$

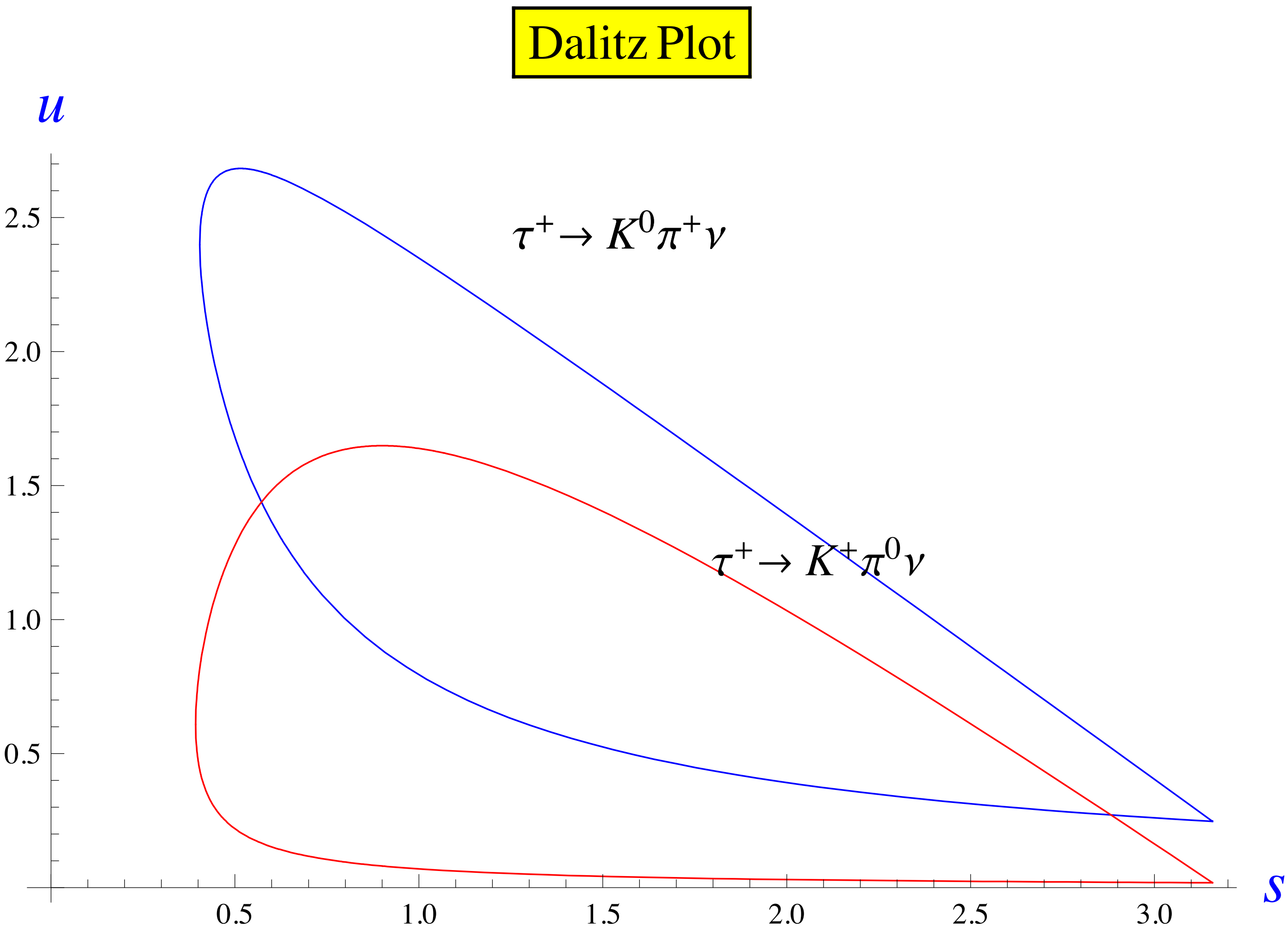
Cirigliano, Neufeld, Ecker'02

F. Flores-Baez, A. Flores-Tlalpa, G. Lopez Castro, G. Toledo Sanchez'06

$\Rightarrow$ $\delta_{\rm EM}^{\bar{K}^0\tau} = (-0.15 \pm 0.2)\%$ $\delta_{\rm EM}^{K^-\tau} = (-0.2 \pm 0.2)\%$

3.1.2 Radiative corrections

F.V. Flores-Baez, J.R. Morones-Ibarra'13



3.1.2 Radiative corrections

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$$\boxed{\Gamma\left(\tau \to \overline{K}\pi\nu_\tau[\gamma]\right) = \frac{G_F^2 m_\tau^5}{96\pi^3} C_K^2 S_{EW}^\tau \left|V_{us}\right|^2 \left|f_+^{K^0\pi^-}(0)\right|^2 I_K^\tau \left(1 + \delta_{\mathrm{EM}}^{K\tau} + \tilde{\delta}_{\mathrm{SU(2)}}^{K\pi}\right)^2}$$

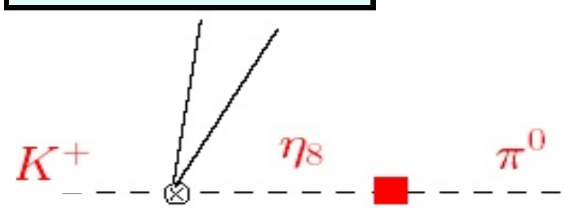
- Theoretical inputs :
  - S_{ew} : Short distance electroweak correction $\Rightarrow$ $\boxed{S_{\mathrm{ew}} = 1.0201}$
  - $\delta_{\mathrm{EM}}^{Kl}$: Long-distance electromagnetic corrections

$\Rightarrow$ $\boxed{\delta_{\mathrm{EM}}^{\bar{K}^0\tau} = (-0.15 \pm 0.2)\%}$ and $\boxed{\delta_{\mathrm{EM}}^{K^-\tau} = (-0.2 \pm 0.2)\%}$

  - $\tilde{\delta}_{\mathrm{SU(2)}}^{K\pi}$: Isospin breaking corrections

Antonelli, Cirigliano, Lusiani, E.P.'13

$$\boxed{\tilde{\delta}_{\mathrm{SU(2)}}^{K\pi} = \frac{f_+^{K^+\pi^0}(0)}{f_+^{K^0\pi^-}(0)} - 1} \Rightarrow \boxed{\tilde{\delta}_{\mathrm{SU(2)}}^{K\pi} = \left(2.9 \pm 0.4_{\mathrm{mixing}} \pm 0.5\right)\%}$$



+ IB in the K^{*-} to $K\pi$ coupling

3.1.3 Phase space integrals

- Master formula for $\tau \to K\pi\nu_\tau$:

$$\boxed{\Gamma\left(\tau \to \overline{K}\pi\nu_\tau[\gamma]\right) = \frac{G_F^2 m_\tau^5}{96\pi^3} C_K^2 S_{EW}^\tau \left|V_{us}\right|^2 \left|f_+^{K^0\pi^-}(0)\right|^2 I_K^\tau \left(1+\delta_{\mathrm{EM}}^{K\tau}+\tilde{\delta}_{\mathrm{SU(2)}}^{K\pi}\right)^2}$$

- Theoretical inputs :
  - I_K : Phase space integral ⇨ need a *parametrization* for the normalized form factors to fit the experimental distributions

$$\boxed{I_K^\tau = \int ds\, F\left(s, \overline{f}_+(s), \overline{f}_0(s)\right)}$$

Hadronic matrix element: Crossed channel from $K \to \pi l \nu_l$

$$\boxed{\langle \mathbf{K}\pi | \, \bar{\mathbf{s}}\gamma_\mu \mathbf{u} \, |0\rangle = \left[(p_K - p_\pi)_\mu + \frac{\Delta_{K\pi}}{s}(p_K + p_\pi)_\mu\right] \underbrace{f_+(s)}_{\text{vector}} - \frac{\Delta_{K\pi}}{s}(p_K + p_\pi)_\mu \underbrace{f_0(s)}_{\text{scalar}}}$$

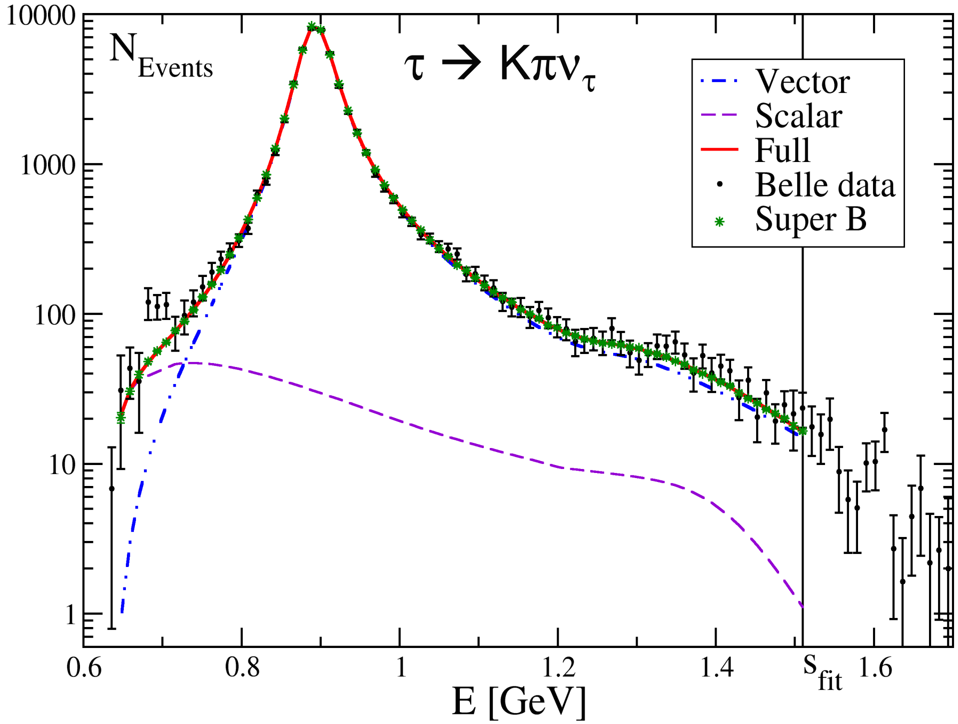
with $s = q^2 = (p_K + p_\pi)^2$, $\overline{f}_{0,+}(t) = \dfrac{f_{0,+}(t)}{f_+(0)}$

⇨ Use a *dispersive parametrization* to combine with K_{l3} analysis

Form factors

- Invariant mass spectra: constraints on FF very important for testing QCD dynamics and the SM and new physics:

$K\pi$ form factors : ⇨ V_{us}



Jamin, Pich, Portolés'08

Boito, Escribano, Jamin'09,'10

Bernard, Boito, E.P'11

Bernard'13,

Escribano, González-Solis, Jamin, Roig'14

3.1.5 Results for phase space integrals

- From the results of the fit to the Belle + K_{l3} data :

Integral	result	error	exp	theo
$I^{\tau}_{K^0}$	0.50418	0.01762	0.01689	0.00501
$I^{e}_{K^0}$	0.15472	0.00022	0.00022	0.00000
$I^{\tau}_{K^0}/I^{e}_{K^0}$	3.25864	0.11115	0.10634	0.03235
$I^{\tau}_{K^+}$	0.52387	0.01958	0.01889	0.00515
$I^{e}_{K^+}$	0.15909	0.00025	0.00025	0.00000
$I^{\tau}_{K^+}/I^{e}_{K^+}$	3.29282	0.12032	0.11589	0.03235

Precision : $I^{\tau}_{K^0}$ 3.4%, $I^{\tau}_{K^+}$ 3.7%
To be compared to the precision on I^{l}_{K} : 0.14 %
Should be improved with more *precise measurements*!

3.1.6 Extraction of V_{us}

- Result for $\tau \to K\pi\nu_\tau$:

$$\boxed{BR\left(\tau \to \overline{K^0}\pi^-\nu_\tau\right) = \left(0.416 \pm 0.008\right)\%}$$

Belle'14

$$\Rightarrow \boxed{f_+(0)|V_{us}| = 0.2141 \pm 0.0014_{I_K} \pm 0.0021_{\text{exp}}}$$

$$\Rightarrow \boxed{|V_{us}| = 0.2212 \pm 0.0026} \quad \text{with} \quad \boxed{f_+(0) = 0.9677(27)}$$

FLAG'16

3.1.6 Extraction of V_{us}

- Result for $\tau \to K\pi\nu_\tau$: $\boxed{f_+(0)|V_{us}| = 0.2141 \pm 0.0014_{I_K} \pm 0.0021_{\exp}}$

$\Rightarrow$ $\boxed{|V_{us}| = 0.2212 \pm 0.0026}$ with $\boxed{f_+(0) = 0.9677(27)}$ *FLAG'16*

- To be compared to results for K_{l3}: *FLAVIAnet Kaon WG, talk by M. Moulson @CKM16*

FLAG'16

$\boxed{f_+(0)|V_{us}| = 0.2165 \pm 0.0004}$ $\Rightarrow$ $\boxed{|V_{us}| = 0.2238 \pm 0.0004_{\exp} \pm 0.0006_{\text{theo}}}$

$\boxed{|V_{us}| = 0.2241 \pm 0.0007}$

- Not competitive yet but interesting cross check of V_{us} determination from K_{l3} and inclusive τ result

3.1.6 Extraction of V_{us}

- Result for $\tau \to K\pi\nu_\tau$: $\boxed{f_+(0)|V_{us}| = 0.2141 \pm 0.0014_{I_K} \pm 0.0021_{\exp}}$

$\Rightarrow$ $\boxed{|V_{us}| = 0.2212 \pm 0.0026}$ with $\boxed{f_+(0) = 0.9677(27)}$ *FLAG'16*

- To be compared to results for K_{l3}: *FLAVIAnet Kaon WG, talk by M. Moulson @CKM16*

FLAG'16

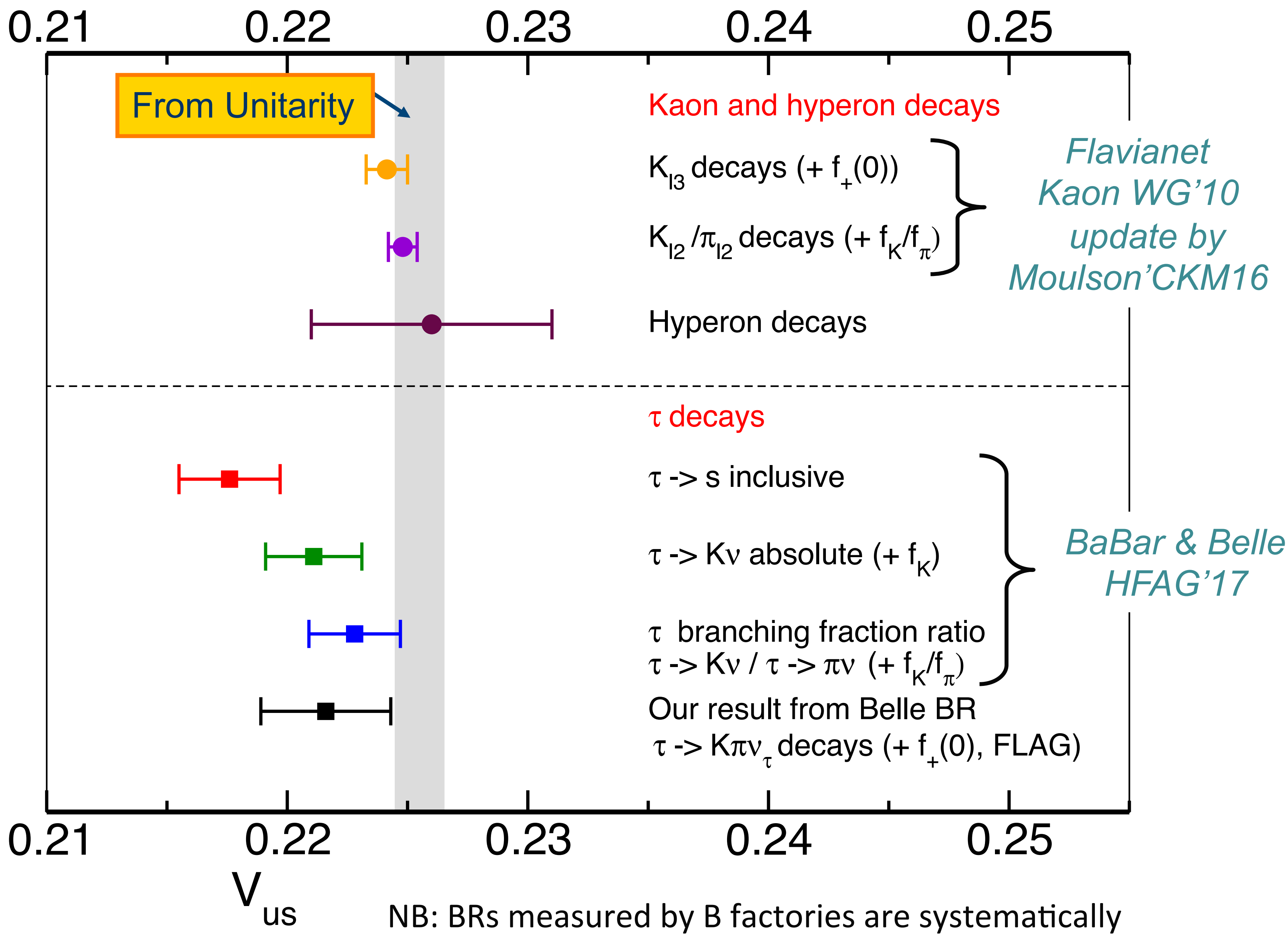
$\boxed{f_+(0)|V_{us}| = 0.2165 \pm 0.0004}$ $\Rightarrow$ $\boxed{|V_{us}| = 0.2238 \pm 0.0004_{\exp} \pm 0.0006_{\text{theo}}}$

$\boxed{|V_{us}| = 0.2241 \pm 0.0007}$

- Not competitive yet but interesting cross check of V_{us} determination from K_{l3} and inclusive τ result

Bernard'14

- Result of fit to K_{l3} + $\tau \to K\pi\nu_\tau$ and $K\pi$ scattering data including inelasticities in the dispersive FFs $\boxed{f_+(0)|V_{us}| = 0.2163 \pm 0.0014}$



NB: BRs measured by B factories are systematically smaller than previous measurements

3. V_{us} from exclusive hadronic τ decays :

- $\tau \to K\nu_\tau$ / $\tau \to \pi\nu_\tau$ decays
- $\tau \to K\nu_\tau$ decays

3.2 V_{us} from $\tau \to K\nu_\tau$ / $\tau \to \pi\nu_\tau$

- $$\boxed{\frac{\Gamma(\tau \to K\nu[\gamma])}{\Gamma(\tau \to \pi\nu[\gamma])} = \frac{\left(1 - m_{K^\pm}^2 / m_\tau^2\right)}{\left(1 - m_{\pi^\pm}^2 / m_\tau^2\right)} \frac{f_K^2}{f_\pi^2} \frac{|V_{us}|^2}{|V_{ud}|^2} \left(1 + \delta_{\mathrm{LD}}\right)}$$

  - δ_{LD} : Long-distance radiative corrections

    ⇨ $\boxed{\delta_{\mathrm{LD}} = 1.0003 \pm 0.0044}$

  - Brs from *HFAG'17 with update by A.Lusiani*
  - F_K/ F_π from lattice average: $\boxed{\frac{f_K}{f_\pi} = 1.1930 \pm 0.0030}$ *FLAG'16*
  - V_{ud} : $|V_{ud}| = 0.97417(21)$ *Towner & Hardy'14*

⇨ $\boxed{|V_{us}| = 0.2236 \pm 0.0018}$ 1.1σ away from unitarity

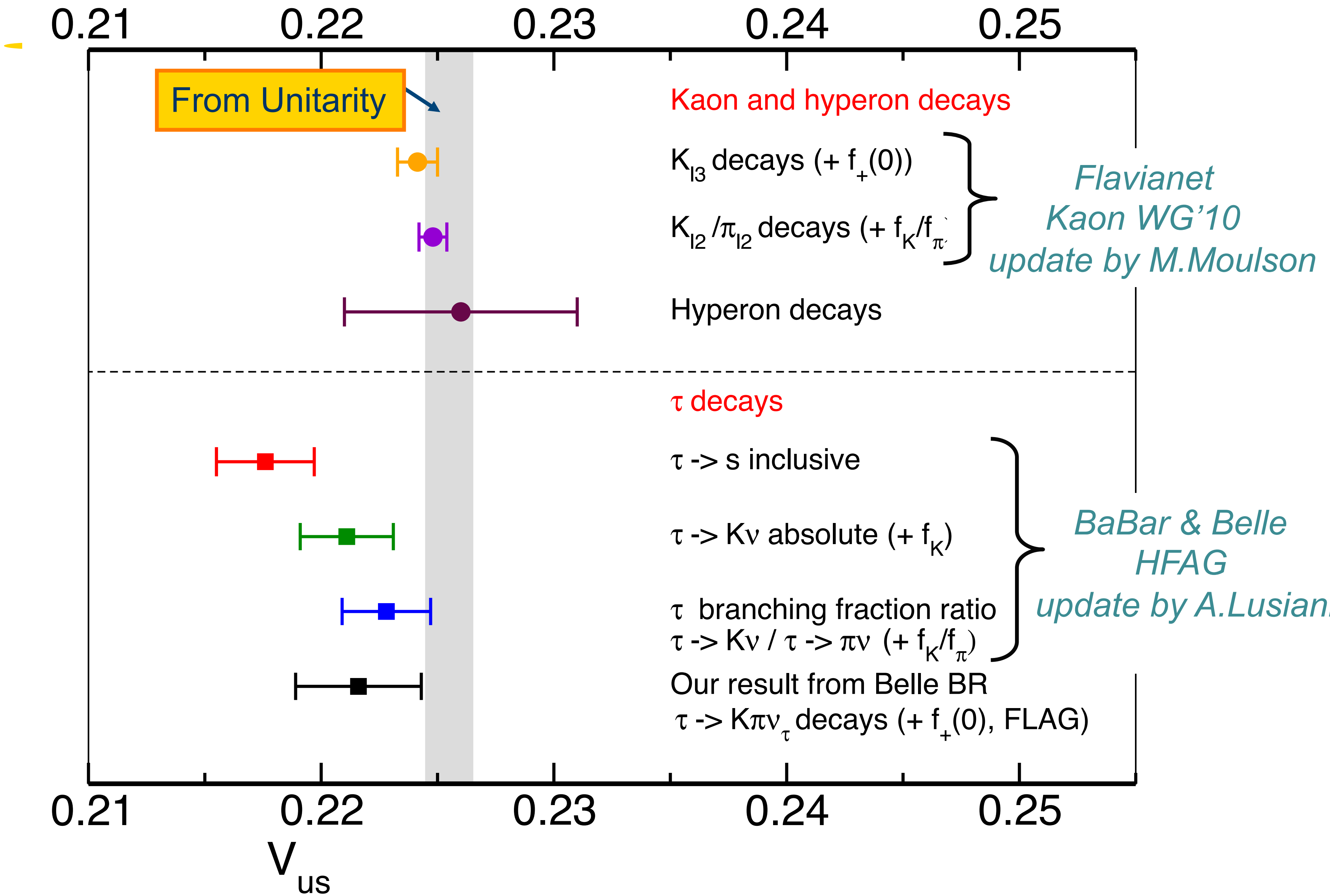
3.3 V_{us} from $\tau \to K\nu_\tau$

- $$BR\left(\tau \to K\nu[\gamma]\right) = \frac{G_F^2 m_\tau^3 S_{EW} \tau_\tau}{16\pi h}\left(1 - \frac{m_{K^\pm}^2}{m_\tau^2}\right) f_K^2 |V_{us}|^2$$

In principle less precise than ratios

- Inputs from *HFAG'17 with update by A.Lusiani*
- F_K from lattice average $\boxed{f_K = (156.3 \pm 0.9)\ \text{MeV}}$ *FLAG'16*

$\Rightarrow$ $\boxed{|V_{us}| = 0.2211 \pm 0.0020}$ 1.9σ away from unitarity


0.21
0.22
0.23
0.24
0.25
From Unitarity
Kaon and hyperon decays
K_{l3} decays (+ $f_+(0)$)
K_{l2}/π_{l2} decays (+ f_K/f_π)
Flavianet
Kaon WG'10
update by M.Moulson
Hyperon decays
τ decays
τ -> s inclusive
τ -> Kν absolute (+ f_K)
BaBar & Belle
HFAG
update by A.Lusiani
τ branching fraction ratio
τ -> Kν / τ -> πν (+ f_K/f_π)
Our result from Belle BR
τ -> $K\pi\nu_\tau$ decays (+ $f_+(0)$, FLAG)
V_{us}

4. Conclusion and Outlook

4.1 Conclusion

- Studying τ physics ⇨ very interesting tests of the Standard Model e.g. V_{us}
- Inclusive τ decays : ⇨ $\boxed{|V_{us}| = 0.2176 \pm 0.0019_{\mathrm{exp}} \pm 0.0010_{\mathrm{th}}}$

  Error dominated by experiment ⇨ Potentially the more precise extraction of V_{us}

Antonelli, Cirigliano, Lusiani, E.P. '13

- Simulated *New flavour factory* data from *Belle* data :
  Same central values but uncertainties rescaled assuming 40 ab^{-1} luminosity

Mode	BR	% err	BR(K_{e3})	τ_K	τ_τ	I_K^τ/I_K^e	Δ_{EM}	$\Delta_{\mathrm{SU(2)}}$
$\tau^- \to \bar{K}^0\pi^-\nu_\tau$	0.8427 ± 0.0122	1.45	0.22	0.41	0.34	1.24	0.46	0
$\tau^- \to K^-\pi^0\nu_\tau$	0.4631 ± 0.0079	1.71	0.06	0.12	0.34	1.25	0.47	1.00

$\boxed{|V_{us}| = 0.2176 \pm 0.0019_{\mathrm{exp}} \pm 0.0010_{\mathrm{th}}}$ ⇨ $\boxed{|V_{us}| = 0.2211 \pm 0.0006_{\mathrm{exp}} \pm 0.0010_{\mathrm{th}}}$

- *Promising!* Competitive with kaon physics!

⇨ $\boxed{|V_{us}| = 0.2255 \pm 0.0005_{\mathrm{exp}} \pm 0.0008_{\mathrm{th}}}$

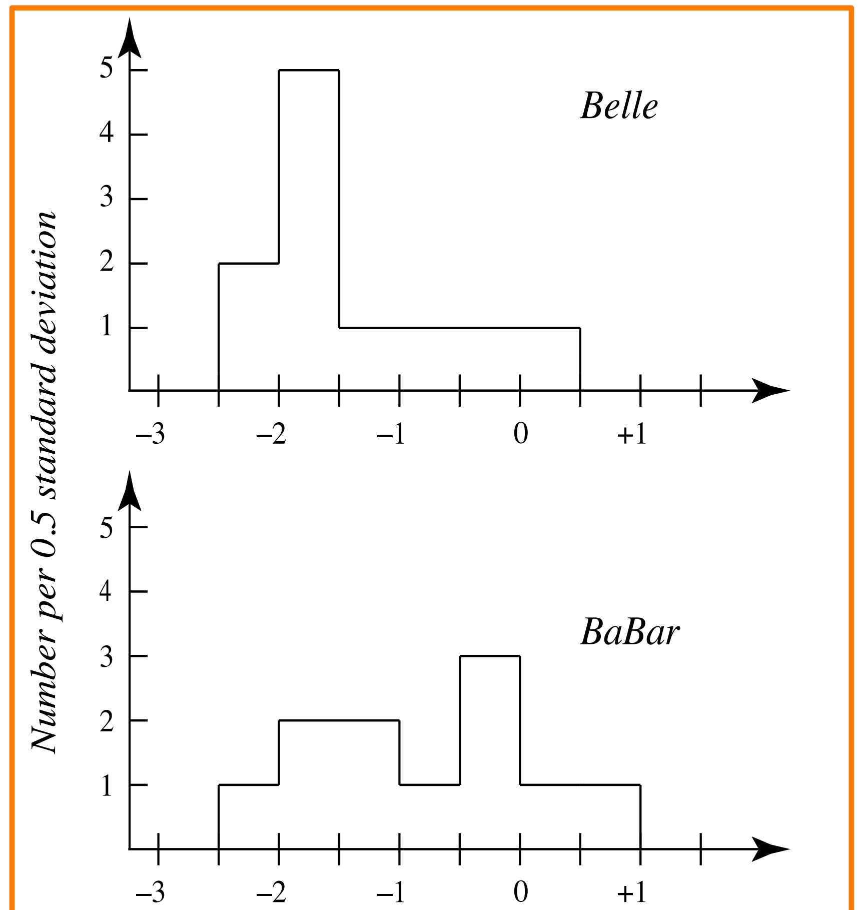
4.2 Outlook : Experimental challenges : strange τ Brs

- *PDG 2014*: « Nineteen of the 20 *B*-factory branching fraction measurements are smaller than the non-*B*-factory values. The average normalized difference between the two sets of measurements is -1.08 » (-1.41 for the 11 Belle measurements and -0.75 for the 11 BaBar measurements)

  ⇨ Supported by predictions from kaon X channel

- Measured modes by the 2 B factories:

Mode	BaBar – Belle Normalized Difference ($\#\sigma$)
$\pi^-\pi^+\pi^-\nu_\tau$ (ex. K^0)	$+1.4$
$K^-\pi^+\pi^-\nu_\tau$ (ex. K^0)	-2.9
$K^-K^+\pi^-\nu_\tau$	-2.9
$K^-K^+K^-\nu_\tau$	-5.4
$\eta\ K^-\nu_\tau$	-1.0
$\phi\ K^-\nu_\tau$	-1.3



4.2 Outlook : Experimental challenges : strange τ Brs

- *PDG 2016*: « We find that that BaBar and Belle tend to measure lower τ branching fractions and ratios than the other experiments. The average normalized difference between the two sets of measurements is -0.8σ (-0.8σ for the 16 Belle measurements and -0.9σ for the 11 BaBar measurements)»

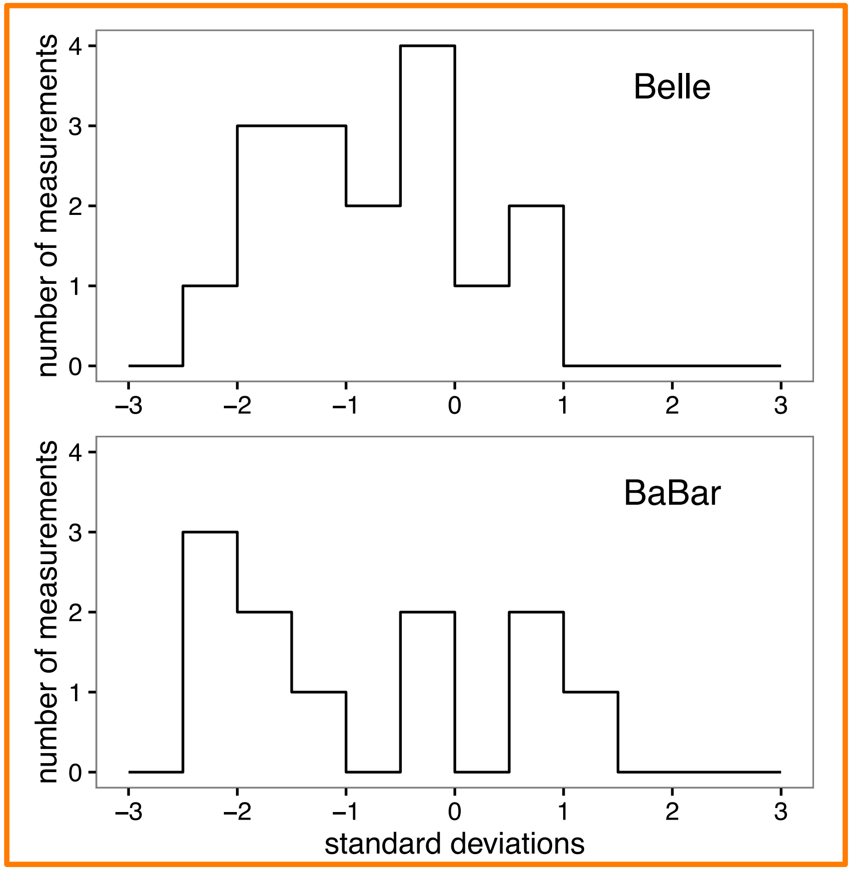


Figure 3: Distribution of the normalized difference between the 27 B-factory measurements and non-B-factory measurements. The list includes 16 measurements of branching fractions and ratios published by the Belle collaboration and 11 by the BaBar collaboration that are used in the fit and for which non-B-factory measurements exist.

4.2 Outlook

- Experimental challenges :

  strange τ BRs:
  PDG 2014: « Nineteen of the 20 *B*-factory branching fraction measurements are smaller than the non-*B*-factory values. The average normalized difference between the two sets of measurements is -1.08 »

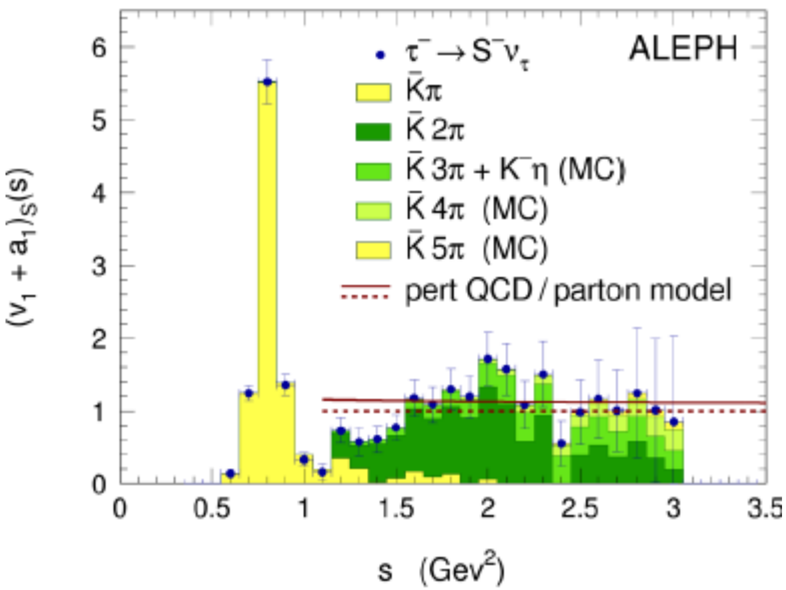
  ⇨ Supported by predictions from kaon X channel measurements

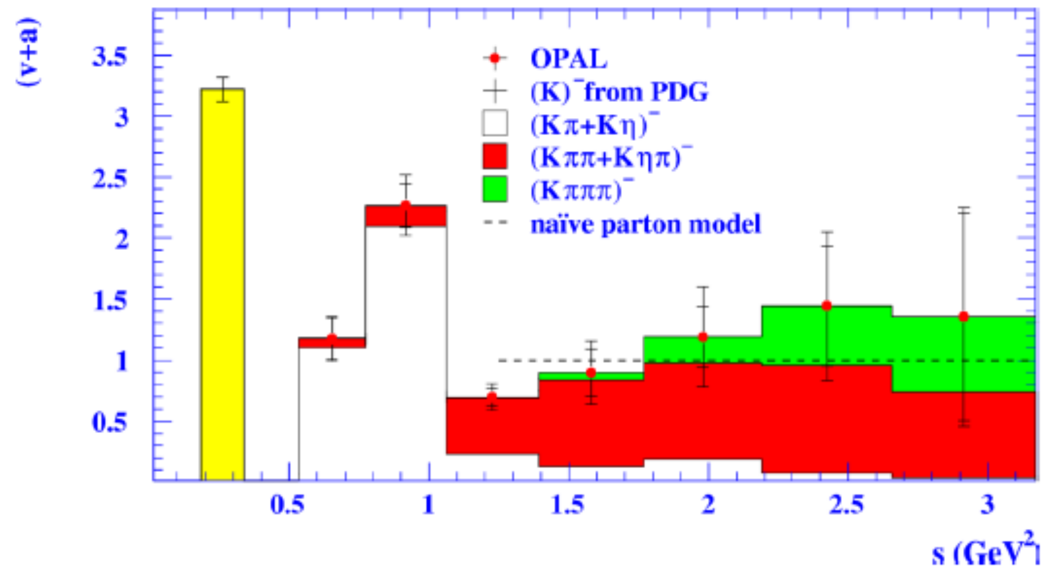
  ⇨ More *precise measurements*

- Theoretical challenges :
  - Having the hadronic uncertainties under control: OPE vs. Lattice QCD or ChPT
  - Isospin breaking
  - Electromagnetic corrections

Prospects : τ strange Brs

- Experimental measurements of the strange spectral functions not very precise





⇨ New measurements are needed !

- Before B-factories
- With B-factories new measurements :

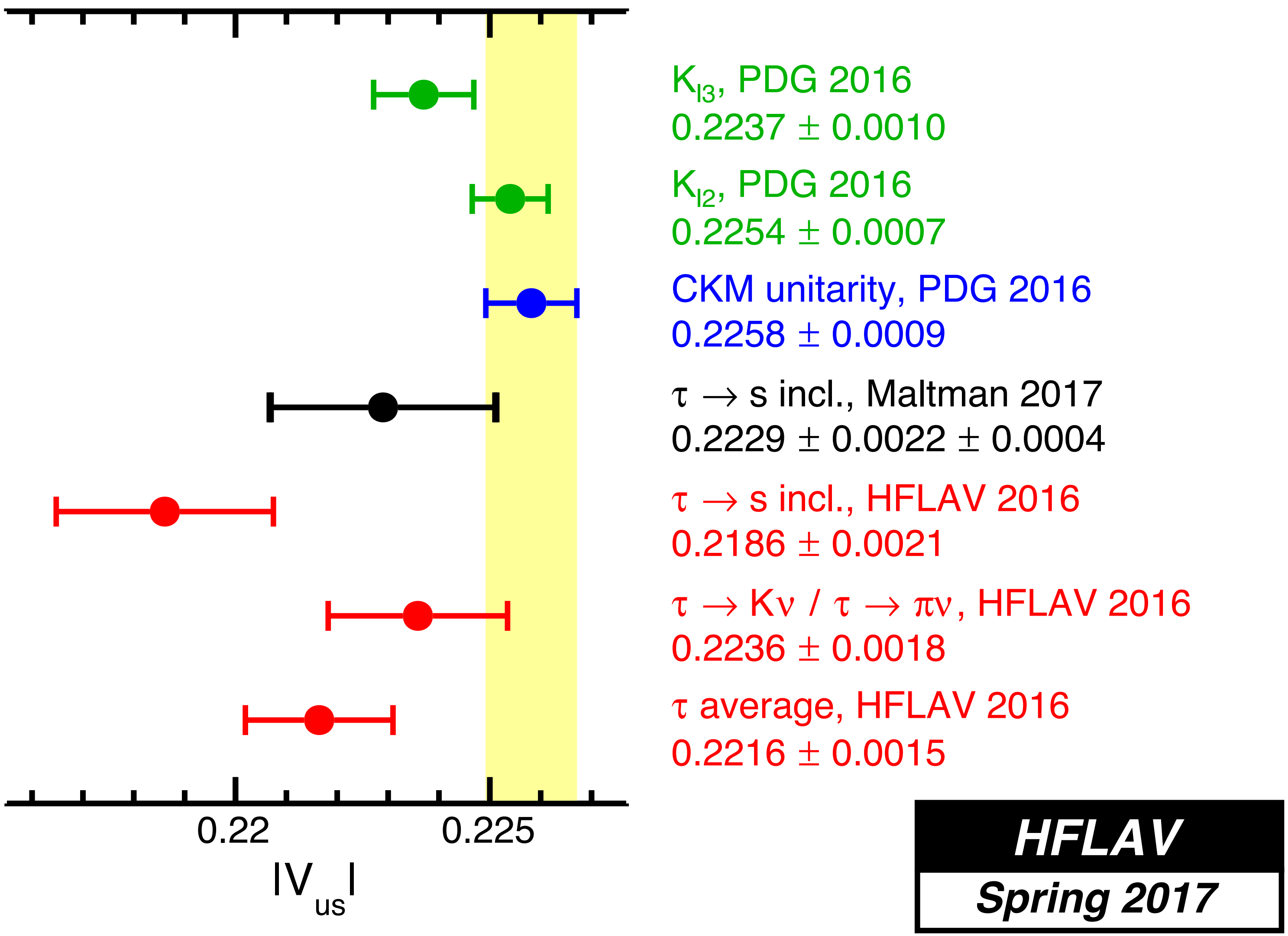
Smaller $\tau \to$ K branching ratios ⇨ smaller $R_{\tau,S}$ ⇨ smaller $\boxed{V_{us}}$

$$\boxed{R_\tau^S\Big|_{\text{old}} = 0.1686(47)} \Rightarrow \boxed{R_\tau^S\Big|_{\text{new}} = 0.1615(28)}$$

$$\boxed{|V_{us}|_{\text{old}} = 0.2214 \pm 0.0031_{\text{exp}} \pm 0.0010_{\text{th}}} \Rightarrow \boxed{|V_{us}|_{\text{new}} = 0.2186 \pm 0.0019_{\text{exp}} \pm 0.0010_{\text{th}}}$$

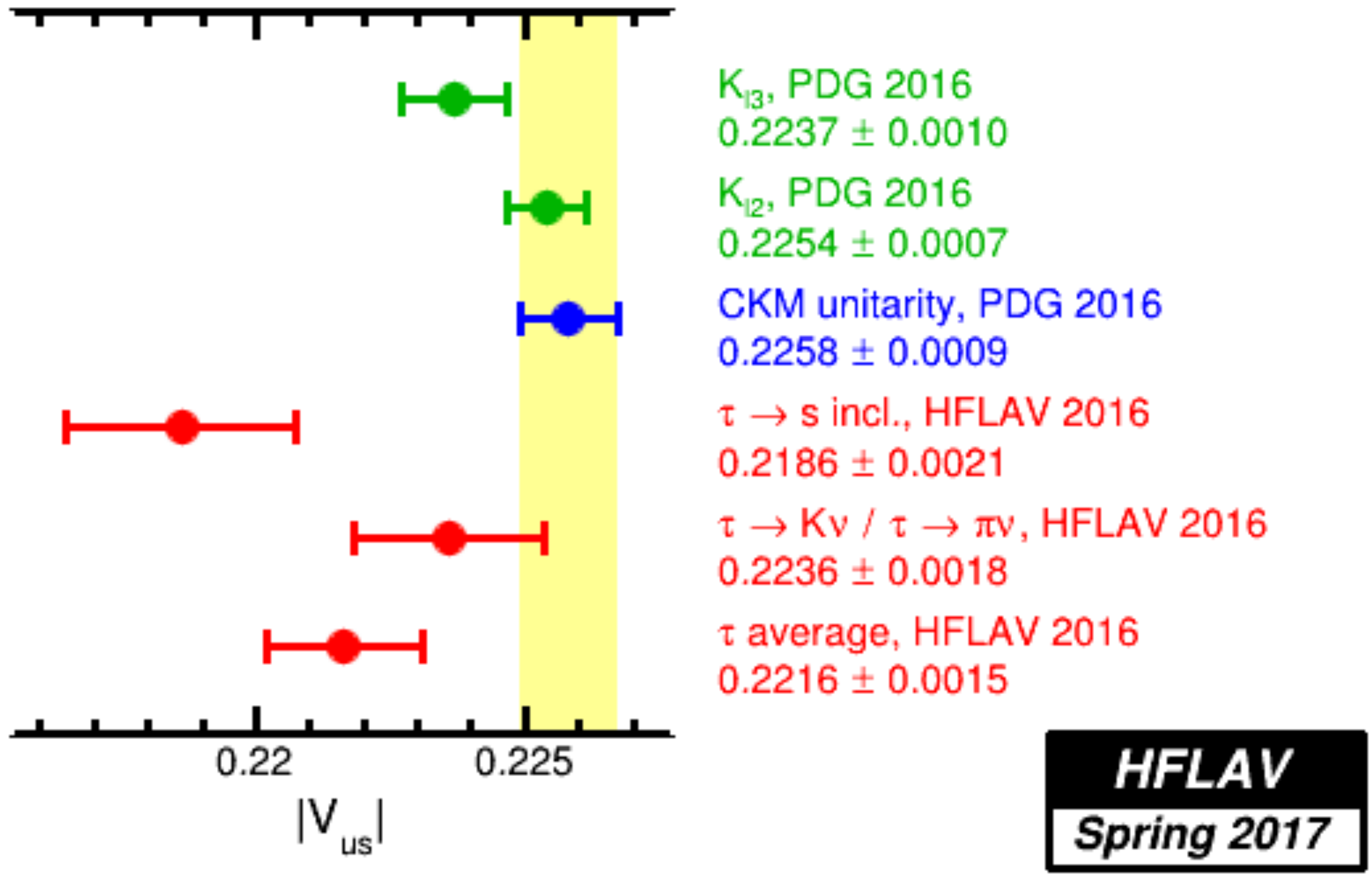
7. Back-up

V_{us} summary



0.22
0.225
$|V_{us}|$
K_{l3}, PDG 2016
0.2237 ± 0.0010
K_{l2}, PDG 2016
0.2254 ± 0.0007
CKM unitarity, PDG 2016
0.2258 ± 0.0009
$\tau \to$ s incl., Maltman 2017
0.2229 ± 0.0022 ± 0.0004
$\tau \to$ s incl., HFLAV 2016
0.2186 ± 0.0021
$\tau \to K\nu$ / $\tau \to \pi\nu$, HFLAV 2016
0.2236 ± 0.0018
τ average, HFLAV 2016
0.2216 ± 0.0015
HFLAV
Spring 2017

V_{us} summary



0.22
0.225
$|V_{us}|$
K_{l3}, PDG 2016
0.2237 ± 0.0010
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0.2258 ± 0.0009
$\tau \to$ s incl., HFLAV 2016
0.2186 ± 0.0021
$\tau \to K\nu$ / $\tau \to \pi\nu$, HFLAV 2016
0.2236 ± 0.0018
τ average, HFLAV 2016
0.2216 ± 0.0015
HFLAV
Spring 2017

2.3 Calculation of δR_τ

- $$\boxed{\delta R_\tau^{th} \equiv \frac{R_{\tau,NS}}{|V_{ud}|^2} - \frac{R_{\tau,S}}{|V_{us}|^2}} \approx N_C S_{EW} \sum_{D\geq 2} \left[\delta_{ud}^{(D)} - \delta_{us}^{(D)}\right]$$

$\Rightarrow$ $$\boxed{\delta R_\tau^{theo} \approx 24 \frac{m_s^2(m_\tau^2)}{m_\tau^2} \Delta(\alpha_s)}$$ but perturbatives series for L behave very badly!

- $\Delta_{kl}(\alpha_s)$ known to order $O(\alpha_s^3)$: *Gámiz, Jamin, Pich, Prades,Schwab'03,'05*
  - *transverse* contribution computed from *theory*
  - *longitudinal* contribution divergent $\Rightarrow$ determined from *data*

E. Gámiz, CKM'12

	$R_{us,A}^{00,L}$	$R_{us,V}^{00,L}$	$R_{ud,A}^{00,L}$
Theory:	-0.144 ± 0.024	-0.028 ± 0.021	$-(7.79 \pm 0.14) \cdot 10^{-3}$
Phenom:	-0.135 ± 0.003	-0.028 ± 0.004	$-(7.77 \pm 0.08) \cdot 10^{-3}$

2.4 Results

$$\boxed{|V_{us}|^2 = \frac{R_{\tau,S}}{\frac{R_{\tau,NS}}{|V_{ud}|^2} - \delta R_{\tau,th}}}$$

- $\delta R_{\tau,th}$ determined from OPE (L+T) + phenomenology   *E. Gámiz, CKM'12*

$$\Rightarrow \boxed{\delta R_{\tau,th} = \underbrace{(0.1544 \pm 0.0037)}_{\text{J=L}} + \underbrace{(9.3 \pm 3.4)\, m_s^2}_{\text{J=L+T, D=2}} + (0.0034 \pm 0.0028)}$$

$$\left[\delta R_{\tau,th} = 0.0239(30)\right]$$

Gamiz, Jamin, Pich, Prades, Schwab'07, Maltman'11

Input : m_s $\Rightarrow$ $m_s\left(2\ \mathrm{GeV}, \overline{\mathrm{MS}}\right) = 93.8 \pm 2.4\ \mathrm{MeV}$ lattice average *FLAG'13*

- Tau data : $R_{\tau,S} = 0.1615(28)$ and $R_{\tau,NS} = 3.4650(84)$ *HFAG'12, update by A. Lusiani*
- V_{ud} : $|V_{ud}| = 0.97425(22)$ *Towner & Hardy'08*

2.4 Results

$$|V_{us}|^2 = \frac{R_{\tau,S}}{\frac{R_{\tau,NS}}{|V_{ud}|^2} - \delta R_{\tau,th}}$$

- $\delta R_{\tau,th} = 0.239(30)$

$$\Rightarrow \boxed{|V_{us}| = 0.2176 \pm 0.0019_{\mathrm{exp}} \pm 0.0010_{\mathrm{th}}}$$

- Determination dominated by experimental uncertainties! Contrary to V_{us} from K_{l3}, dominated by uncertainties on $f_+(0)$

- 3.4σ away from unitarity!