



Lepton Flavour Violating Tau decays

Emilie Passemar Indiana University/Jefferson Laboratory

Mini Workshop on Tau physics

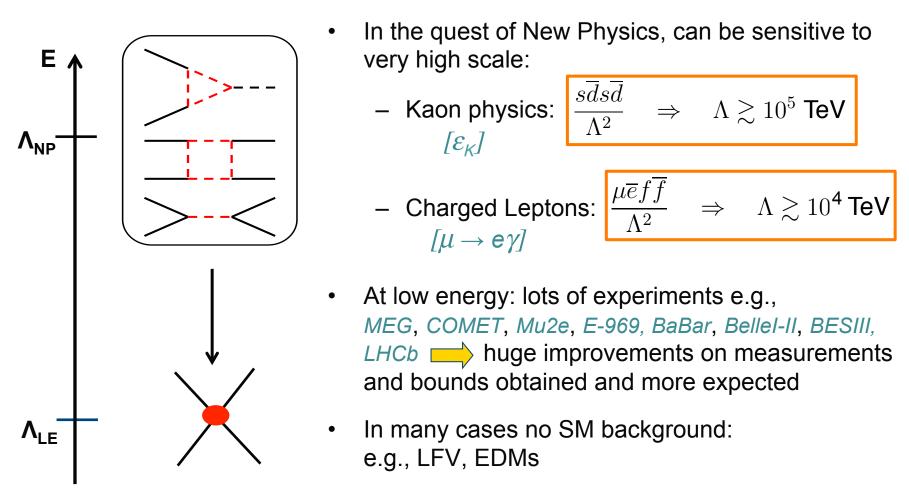
CINVESTAV, Mexico, May 23, 2017

In collaboration with A. Celis (LMU, Munich), and V. Cirigliano (LANL) PRD 89 (2014) 013008, 095014

- 1. Introduction and Motivation
- 2. Charged Lepton-Flavour Violation from tau decays
- 3. Special Role of $\tau \rightarrow \mu \pi \pi$: hadronic form factors
- 4. Results
- 5. Conclusion and Outlook

1. Introduction and Motivation

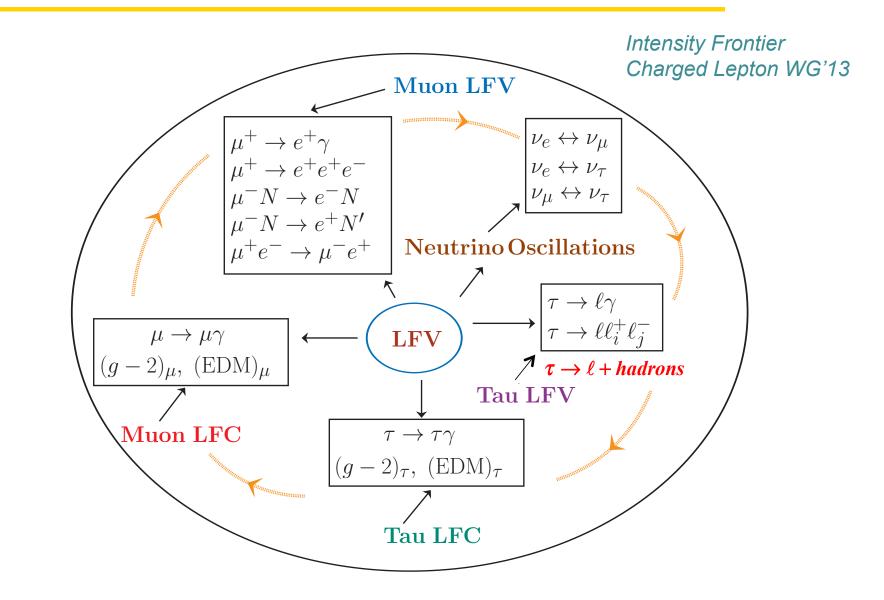
1.1 Why study charged leptons?



 For some modes accurate calculations of hadronic uncertainties essential



1.2 The Program



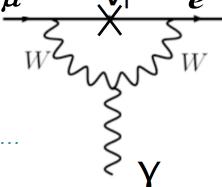
2. Charged Lepton-Flavour Violation

2.1 Introduction and Motivation

- Lepton Flavour Number is an « accidental » symmetry of the SM ($m_v=0$)
- In the SM with massive neutrinos effective CLFV vertices are tiny due to GIM suppression in unobservably small rates!

E.g.:
$$\mu \rightarrow e\gamma$$

$$Br(\mu \to e\gamma) = \frac{3\alpha}{32\pi} \left| \sum_{i=2,3} U^*_{\mu i} U_{ei} \frac{\Delta m_{1i}^2}{M_W^2} \right|^2 < 10^{-54}$$



Petcov'77, Marciano & Sanda'77, Lee & Shrock'77...

$$\left[Br\left(\tau\to\mu\gamma\right)<10^{-40}\right]$$

• Extremely *clean probe of beyond SM physics*

2.1 Introduction and Motivation

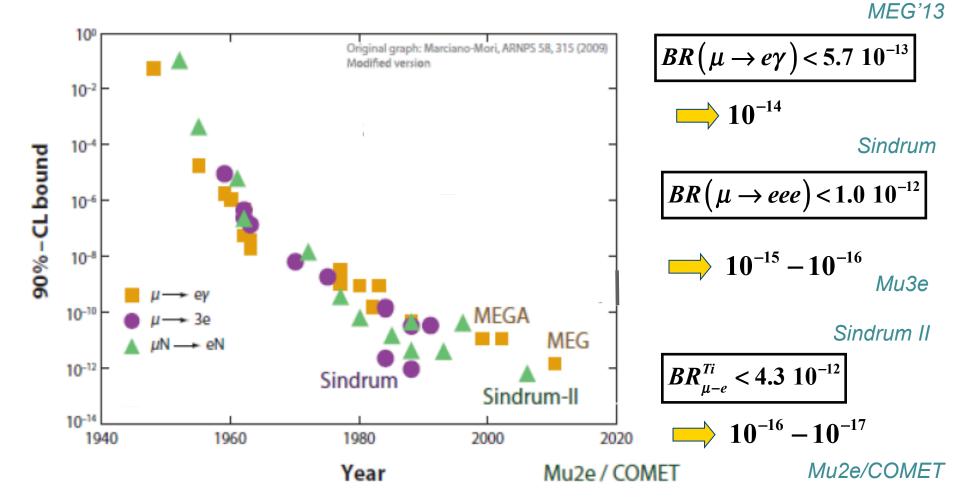
• In New Physics scenarios CLFV can reach observable levels in several channels

Talk by D. Hitlin @ CLFV2013			$\tau \rightarrow \mu \gamma \ \tau \rightarrow \ell \ell \ell$	
SM + v mixing	Lee, Shrock, PRD 16 (1977) 1444 Cheng, Li, PRD 45 (1980) 1908	Undetect	Undetectable	
SUSY Higgs	Dedes, Ellis, Raidal, PLB 549 (2002) 159 Brignole, Rossi, PLB 566 (2003) 517	10-10	10-7	
SM + heavy Maj $v_{\rm R}$	Cvetic, Dib, Kim, Kim, PRD66 (2002) 034008	10-9	10-10	
Non-universal Z'	Yue, Zhang, Liu, PLB 547 (2002) 252	10-9	10-8	
SUSY SO(10)	Masiero, Vempati, Vives, NPB 649 (2003) 189 Fukuyama, Kikuchi, Okada, PRD 68 (2003) 033012	10-8	10-10	
mSUGRA + Seesaw	JGRA + Seesaw Ellis, Gomez, Leontaris, Lola, Nanopoulos, EPJ C14 (2002) 319 Ellis, Hisano, Raidal, Shimizu, PRD 66 (2002) 115013		10 ⁻⁹	

- But the sensitivity of particular modes to CLFV couplings is model dependent
- Comparison in muonic and tauonic channels of branching ratios, conversion rates and spectra is model-diagnostic

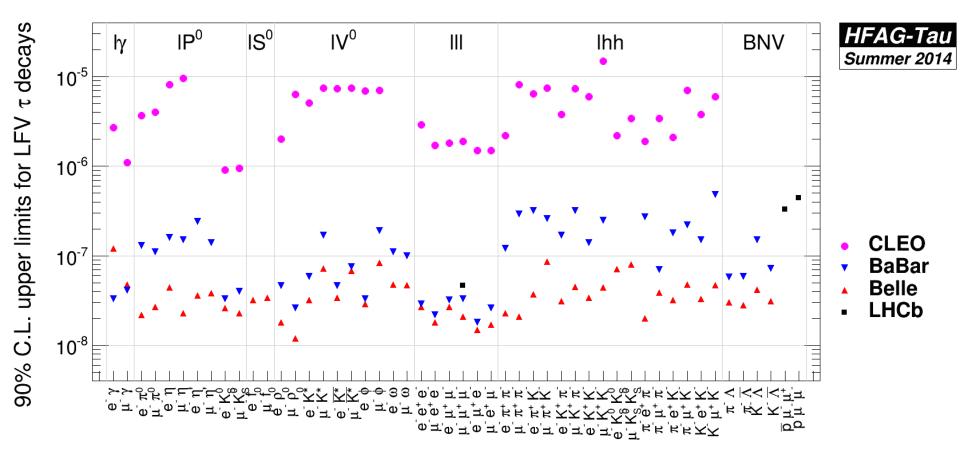
2.2 CLFV processes: muon decays

• Several processes: $\mu \to e\gamma$, $\mu \to e\overline{e}e$, $\mu(A,Z) \to e(A,Z)$



2.2 CLFV processes: tau decays

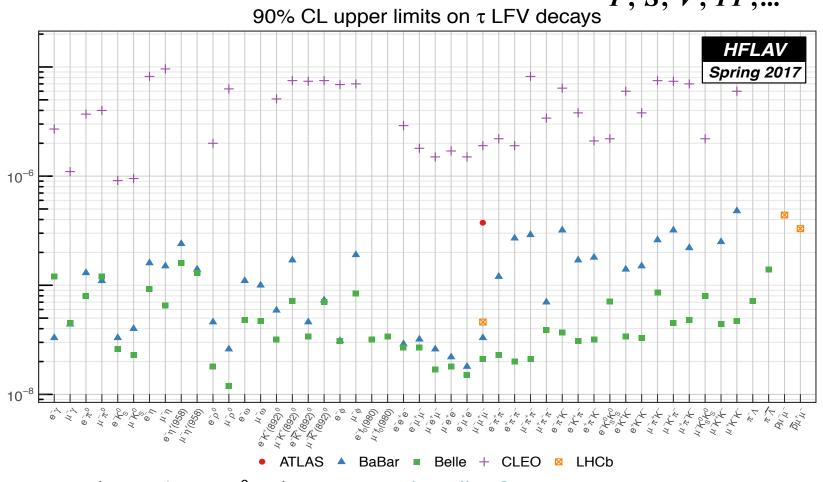
• Several processes: $\tau \to \ell \gamma, \ \tau \to \ell_{\alpha} \overline{\ell}_{\beta} \ell_{\beta}, \ \tau \to \ell Y$ $\swarrow P, S, V, P\overline{P}, ...$



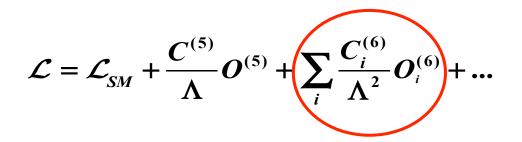
48 LFV modes studied at Belle and BaBar

2.2 CLFV processes: tau decays

• Several processes: $\tau \to \ell \gamma, \ \tau \to \ell_{\alpha} \overline{\ell}_{\beta} \ell_{\beta}, \ \tau \to \ell Y$ $P, S, V, P\overline{P}, ...$



• Expected sensitivity 10⁻⁹ or better at *LHCb, Belle II*?

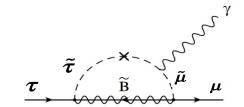


e.g.

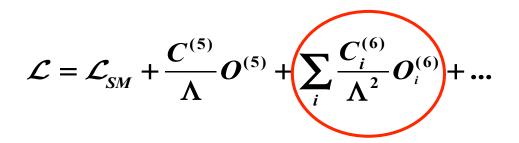
• Build all D>5 LFV operators:

> Dipole:

$$\mathcal{L}_{eff}^{D} \supset -\frac{C_{D}}{\Lambda^{2}} m_{\tau} \overline{\mu} \sigma^{\mu\nu} P_{L,R} \tau F_{\mu\nu}$$



See e.g. Black, Han, He, Sher'02 Brignole & Rossi'04 Dassinger et al.'07 Matsuzaki & Sanda'08 Giffels et al.'08 Crivellin, Najjari, Rosiek'13 Petrov & Zhuridov'14 Cirigliano, Celis, E.P.'14

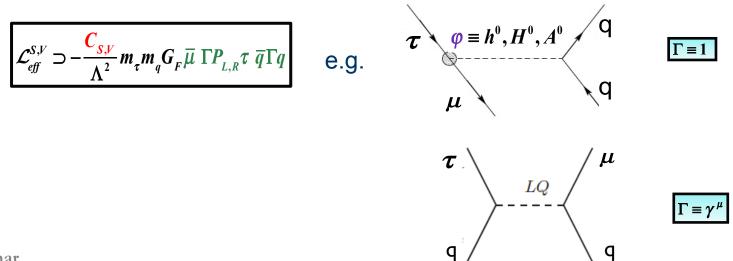


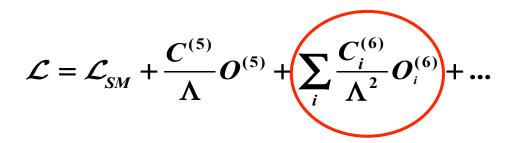
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Lepton-quark (Scalar, Pseudo-scalar, Vector, Axial-vector):





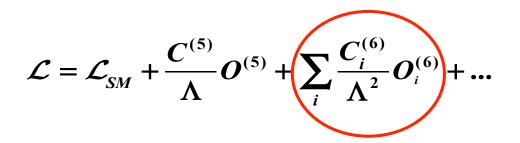
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- Lepton-quark (Scalar, Pseudo-scalar, Vector, Axial-vector):
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$$\mathcal{L}_{eff}^{S} \supset -\frac{C_{S,V}}{\Lambda^{2}} m_{\tau} m_{q} G_{F} \overline{\mu} \Gamma P_{L,R} \tau \overline{q} \Gamma q$$

Integrating out heavy quarks generates gluonic operator



• Build all D>5 LFV operators:

$$\succ \text{ Dipole: } \mathcal{L}_{eff}^{D} \supset -\frac{C_{D}}{\Lambda^{2}} m_{\tau} \overline{\mu} \sigma^{\mu\nu} P_{L,R} \tau F_{\mu\nu}$$

- Lepton-quark (Scalar, Pseudo-scalar, Vector, Axial-vector):
- $\mathcal{L}_{eff}^{S} \supset -\frac{C_{S,V}}{\Lambda^{2}} m_{\tau} m_{q} G_{F} \overline{\mu} \Gamma P_{L,R} \tau \overline{q} \Gamma q$
- > 4 leptons (Scalar, Pseudo-scalar, Vector, Axial-vector): \mathcal{L}_{e}

$${}_{eff}^{4\ell} \supset -\frac{C_{S,V}^{4\ell}}{\Lambda^2} \overline{\mu} \ \Gamma P_{L,R} \tau \ \overline{\mu} \ \Gamma P_{L,R} \mu$$

See e.g.

Black, Han, He, Sher'02

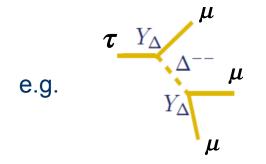
Matsuzaki & Sanda'08

Petrov & Zhuridov'14 Cirigliano, Celis, E.P.'14

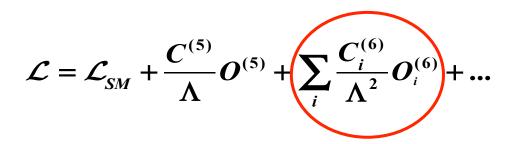
Crivellin, Najjari, Rosiek'13

Brignole & Rossi'04 Dassinger et al.'07

Giffels et al.'08



$$\Gamma \equiv 1 \,, \gamma^{\mu}$$



• Build all D>5 LFV operators:

$$\succ \text{ Dipole: } \mathcal{L}_{eff}^{D} \supset -\frac{C_{D}}{\Lambda^{2}} m_{\tau} \overline{\mu} \sigma^{\mu\nu} P_{L,R} \tau F_{\mu\nu}$$

Lepton-quark (Scalar, Pseudo-scalar, Vector, Axial-vector):

$$\mathcal{L}_{eff}^{S} \supset -\frac{C_{S,V}}{\Lambda^{2}} m_{\tau} m_{q} G_{F} \overline{\mu} \Gamma P_{L,R} \tau \overline{q} \Gamma q$$

 $\Gamma \equiv 1, \gamma^{\mu}$

$$\succ \text{ Lepton-gluon (Scalar, Pseudo-scalar): } \mathcal{L}_{eff}^G \supset -\frac{C_G}{\Lambda^2} m_{\tau} G_F \overline{\mu} P_{L,R} \tau \ G_{\mu\nu}^a G_A^{\mu\nu}$$

➤ 4 leptons (Scalar, Pseudo-scalar, Vector, Axial-vector):

$$\mathcal{L}_{eff}^{4\ell} \supset -\frac{C_{S,V}^{4\ell}}{\Lambda^2} \overline{\mu} \ \Gamma P_{L,R} \tau \ \overline{\mu} \ \Gamma P_{L,R} \mu$$

• Each UV model generates a *specific pattern* of them

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See e.g. Black, Han, He, Sher'02 Brignole & Rossi'04 Dassinger et al.'07 Matsuzaki & Sanda'08 Giffels et al.'08 Crivellin, Najjari, Rosiek'13 Petrov & Zhuridov'14 Cirigliano, Celis, E.P.'14

2.4 Model discriminating power of Tau processes

• Summary table:

Celis, Cirigliano, E.P.'14

	$\tau \to 3\mu$	$\tau \to \mu \gamma$	$\tau o \mu \pi^+ \pi^-$	$ au o \mu K \bar{K}$	$\tau \to \mu \pi$	$\tau \to \mu \eta^{(\prime)}$
$O_{S,V}^{4\ell}$	✓	—	—	_	_	_
OD	✓	✓	\checkmark	\checkmark	_	_
O_V^q	—	—	\checkmark (I=1)	$\checkmark(\mathrm{I=}0{,}1)$	—	_
$O_{\mathbf{S}}^{\mathbf{q}}$	—	—	✓ (I=0)	$\checkmark(\mathrm{I=}0{,}1)$	_	_
O _{GG}	—	—	1	\checkmark	—	_
$O^{\mathbf{q}}_{\mathbf{A}}$	—	—	—	—	\checkmark (I=1)	✓ (I=0)
O_P^q	—	—	—	—	✓ (I=1)	✓ (I=0)
$O_{G\widetilde{G}}$	—	_	_	_	_	1

- The notion of "best probe" (process with largest decay rate) is model dependent
- If observed, compare rate of processes key handle on *relative strength* between operators and hence on the *underlying mechanism*

2.4 Model discriminating power of Tau processes

• Summary table:

Celis, Cirigliano, E.P.'14

	$\tau \to 3\mu$	$\tau \to \mu \gamma$	$\tau o \mu \pi^+ \pi^-$	$\tau \to \mu K \bar{K}$	$\tau \to \mu \pi$	$\tau \to \mu \eta^{(\prime)}$
$O_{S,V}^{4\ell}$	✓	—	_	_	_	—
OD	✓	✓	\checkmark	\checkmark	_	_
O_V^q	—	—	\checkmark (I=1)	$\checkmark(\mathrm{I=}0{,}1)$	—	_
$O_{\mathbf{S}}^{\mathbf{q}}$	—	—	✓ (I=0)	$\checkmark(\mathrm{I=}0{,}1)$	—	_
O _{GG}	—	—	\checkmark	✓	_	_
O_A^q	—	—	_	_	\checkmark (I=1)	✓ (I=0)
O_P^q	—	—	—	—	✓ (I=1)	✓ (I=0)
$O_{G\widetilde{G}}$	_	_	_	_	_	1

- In addition to leptonic and radiative decays, *hadronic decays* are very important sensitive to large number of operators!
- But need reliable determinations of the hadronic part: form factors and *decay constants* (e.g. f_n, f_n['])

2.5 Ex: Non standard LFV Higgs coupling

•
$$\Delta \mathcal{L}_{Y} = -\frac{\lambda_{ij}}{\Lambda^{2}} (\overline{f}_{L}^{i} f_{R}^{j} H) H^{\dagger} H$$

$$\implies -Y_{ij} (\overline{f}_{L}^{i} f_{R}^{j}) h$$
In the SM: $Y_{ij}^{h_{SM}} = \frac{m_{i}}{V} \delta_{ij}$

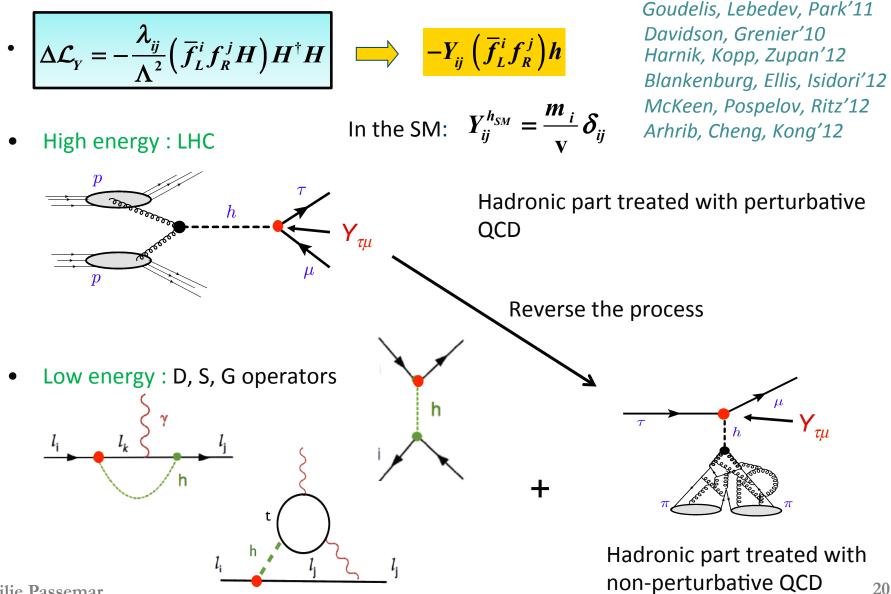
$$L_{Y} = -m_{i} \overline{f}_{L}^{i} f_{R}^{i} - h (Y_{e\mu} \overline{e}_{L} \mu_{R} + Y_{e\tau} \overline{e}_{L} \tau_{R} + Y_{\mu\tau} \overline{\mu}_{L} \tau_{R}) + \dots$$

Goudelis, Lebedev, Park'11 Davidson, Grenier'10 Harnik, Kopp, Zupan'12 Blankenburg, Ellis, Isidori'12 McKeen, Pospelov, Ritz'12 Arhrib, Cheng, Kong'12

Cheng, Sher'97

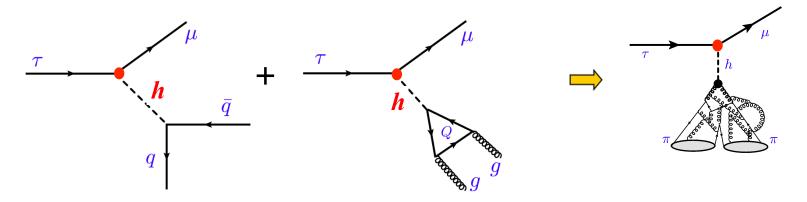
- Order of magnitude expected \longrightarrow No tuning: $|Y_{\tau\mu}Y_{\mu\tau}| \lesssim \frac{m_{\mu}m_{\tau}}{v^2}$
- In concrete models, in general further parametrically suppressed

2.5 Ex: Non standard LFV Higgs coupling



2.6 Constraints from $\tau \rightarrow \mu \pi \pi$

• Tree level Higgs exchange



• Problem : Have the hadronic part under control, ChPT not valid at these energies! $s = (p_{\pi^+} + p_{\pi^-})^2 \implies \sqrt{s} \le m_\tau - m_\mu$

Use form factors determined with dispersion relations matched at low energy to CHPT Daub, Dreiner, Hanart, Kubis, Meissner'13

Celis, Cirigliano, E.P.'14

Dispersion relations: based on unitarity, analyticity and crossing symmetry
 Take all rescattering effects into account

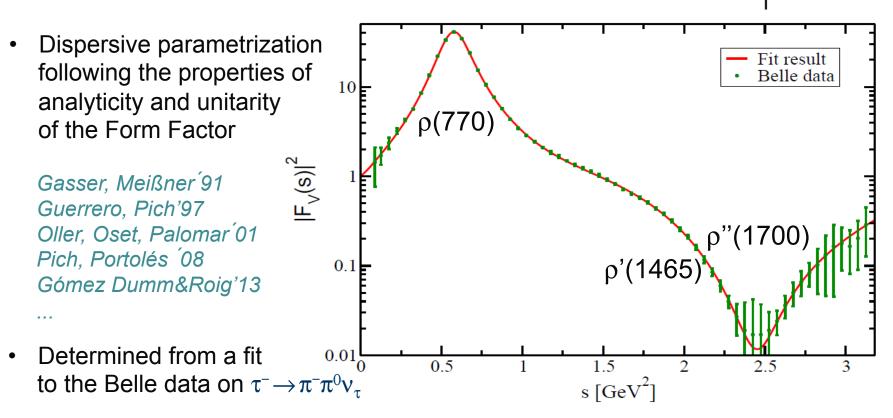
 $\pi\pi$ final state interactions important

3. Description of the hadronic form factors

3.1 Constraints from $\tau \rightarrow \mu \pi \pi$

• Photon mediated contribution requires the pion vector form factor:

$$\langle \pi^+(p_{\pi^+})\pi^-(p_{\pi^-})|\frac{1}{2}(\bar{u}\gamma^{\alpha}u-\bar{d}\gamma^{\alpha}d)|0\rangle \equiv F_V(s)(p_{\pi^+}-p_{\pi^-})^{\alpha}$$



Celis, Cirigliano, E.P.'14

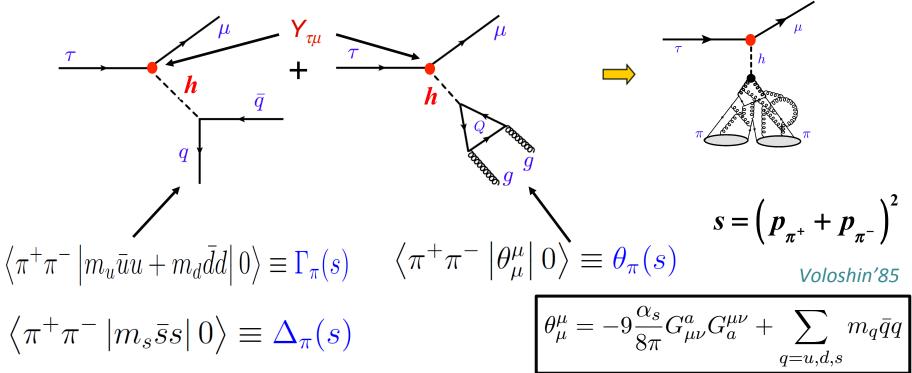
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 \bar{q}

q

3.1 Constraints from $\tau \rightarrow \mu \pi \pi$



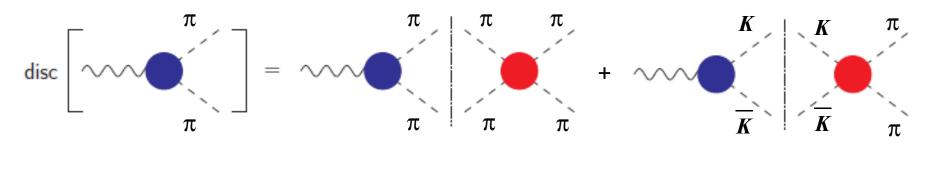


3.2 Unitarity

 Coupled channel analysis up to √s~1.4 GeV: Mushkhelishvili-Omnès approach Inputs: I=0, S-wave ππ and KK data
 Donoghue, Gasser, Leutwyler'90

See also Osset & Oller'98 Lahde & Meissner'06 Donognue, Gasser, Leutwyler 90 Moussallam'99 Daub, Dreiner, Hanart, Kubis, Meissner'13 Celis, Cirigliano, E.P.'14

Unitarity the discontinuity of the form factor is known



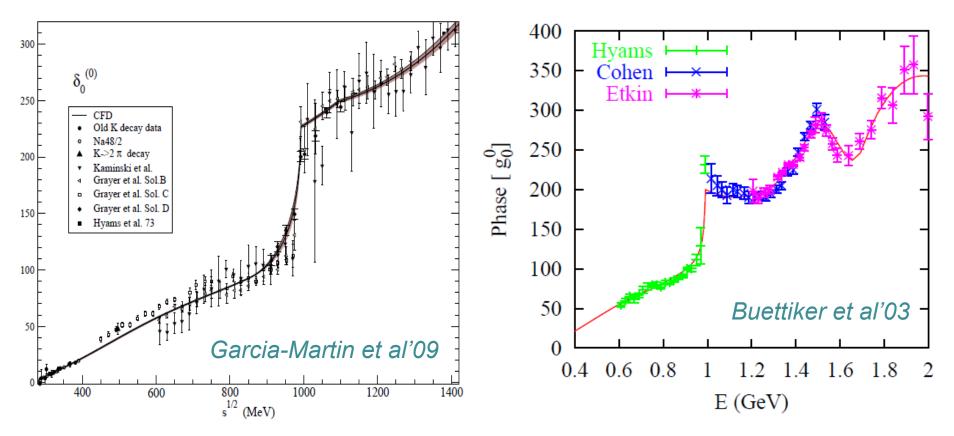
$$\operatorname{Im} F_n(s) = \sum_{m=1}^2 T^*_{nm}(s) \sigma_m(s) F_m(s)$$
$$n = \pi \pi_* K \overline{K}$$

Scattering matrix:

$$\left(\begin{array}{c} \pi\pi \to \pi\pi, \ \pi\pi \to K\overline{K} \\ K\overline{K} \to \pi\pi, \ K\overline{K} \to K\overline{K} \end{array}\right)$$

3.3 Inputs for the coupled channel analysis

• Inputs : $\pi\pi o\pi\pi, K\overline{K}$



- A large number of theoretical analyses *Descotes-Genon et al'01, Kaminsky et al'01, Buettiker et al'03, Garcia-Martin et al'09, Colangelo et al.'11* and all agree
- 3 inputs: $\delta_{\pi}(s)$, $\delta_{K}(s)$, η from *B. Moussallam* \Longrightarrow *reconstruct T matrix* Emilie Passemar

3.4 Dispersion relations

• General solution to *Mushkhelishvili-Omnès* problem:

$$\begin{pmatrix} F_{\pi}(s) \\ \frac{2}{\sqrt{3}}F_{K}(s) \end{pmatrix} = \begin{pmatrix} C_{1}(s) & D_{1}(s) \\ C_{2}(s) & D_{2}(s) \end{pmatrix} \begin{pmatrix} P_{F}(s) \\ Q_{F}(s) \end{pmatrix}$$
Canonical solution falling as 1/s for large s (obey unsubtracted dispersion relations)
Polynomial determined from a matching to ChPT + lattice

Canonical solution X(s) = C(s), D(s):

- Knowing the discontinuity of X(s) write a dispersion relation for it
- Analyticity of the FFs: X(z) is
 - real for $z < s_{th}$
 - has a branch cut for $z > s_{th}$
 - analytic for complex z
- Cauchy Theorem and Schwarz reflection principle:

$$X(s) = \frac{1}{\pi} \oint_C dz \frac{X(z)}{z-s}$$
$$= \frac{1}{2i\pi} \int_{s_{th}=4M_{\pi}^2}^{\Lambda^2} dz \frac{disc[F(z)]}{z-s-i\varepsilon} + \frac{1}{2i\pi} \int_{|z|=\Lambda^2} dz \frac{F(z)}{z-s}$$

$$\Lambda \to \infty$$

$$X(s) = \frac{1}{\pi} \int_{4M_{\pi}^2}^{\infty} dz \frac{\operatorname{Im}[X(z)]}{z - s - i\varepsilon}$$

X(s) can be reconstructed everywhere from the knowledge of ImX(s)

Im(z)

 $s_{th} \equiv 4m_{\pi}^2$

 Λ^2

 $\operatorname{Re}(z)$

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• Canonical solution found by solving the dispersive integral equations iteratively starting with Omnès functions X(s) = C(s), D(s)

$$\Omega_{\pi,K}(s) \equiv \exp\left[\frac{s}{\pi} \int_{4M_{\pi}^2}^{\infty} \frac{dt}{t} \frac{\delta_{\pi,K}(t)}{(t-s)}\right] = X(s)$$

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• Canonical solution found by solving the dispersive integral equations iteratively starting with Omnès functions X(s) = C(s), D(s)

$$\mathrm{Im}X_{n}^{(N+1)}(s) = \sum_{m=1}^{2} T_{mn}^{*}\sigma_{m}(s)X_{m}^{(N)}(s) \longrightarrow$$

$$X_n^{(N+1)}(s) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} ds' \frac{\text{Im}X_n^{(N+1)}(s')}{s'-s}$$

Determination of the polynomial

• Fix the polynomial with requiring $F_p(s) \rightarrow 1/s$ + ChPT:

Brodsky & Lepage'80

• Feynman-Hellmann theorem:

$$\Gamma_P(0) = \left(m_u \frac{\partial}{\partial m_u} + m_d \frac{\partial}{\partial m_d} \right) M_P^2$$
$$\Delta_P(0) = \left(m_s \frac{\partial}{\partial m_s} \right) M_P^2$$

• At LO in ChPT:

$$egin{aligned} M_{\pi^+}^2 &= (m_{ extsf{u}} + m_{ extsf{d}}) \, B_0 + O(m^2) \ M_{K^+}^2 &= (m_{ extsf{u}} + m_{ extsf{s}}) \, B_0 + O(m^2) \ M_{K^0}^2 &= (m_{ extsf{d}} + m_{ extsf{s}}) \, B_0 + O(m^2) \end{aligned}$$

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Brodsky & Lepage'80

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$$P_{\Gamma}(s) = \Gamma_{\pi}(0) = M_{\pi}^{2} + \cdots$$

$$Q_{\Gamma}(s) = \frac{2}{\sqrt{3}}\Gamma_{K}(0) = \frac{1}{\sqrt{3}}M_{\pi}^{2} + \cdots$$

$$P_{\Delta}(s) = \Delta_{\pi}(0) = 0 + \cdots$$

$$Q_{\Delta}(s) = \frac{2}{\sqrt{3}}\Delta_{K}(0) = \frac{2}{\sqrt{3}}\left(M_{K}^{2} - \frac{1}{2}M_{\pi}^{2}\right) + \cdots$$

Determination of the polynomial

• At LO in ChPT:

$$egin{aligned} M_{\pi^+}^2 &= (m_{ extsf{u}}+m_{ extsf{d}})\,B_0 + O(m^2)\ M_{K^+}^2 &= (m_{ extsf{u}}+m_{ extsf{s}})\,B_0 + O(m^2)\ M_{K^0}^2 &= (m_{ extsf{d}}+m_{ extsf{s}})\,B_0 + O(m^2) \end{aligned}$$

• For the scalar FFs:

$$P_{\Gamma}(s) = \Gamma_{\pi}(0) = M_{\pi}^{2} + \cdots$$

$$Q_{\Gamma}(s) = \frac{2}{\sqrt{3}}\Gamma_{K}(0) = \frac{1}{\sqrt{3}}M_{\pi}^{2} + \cdots$$

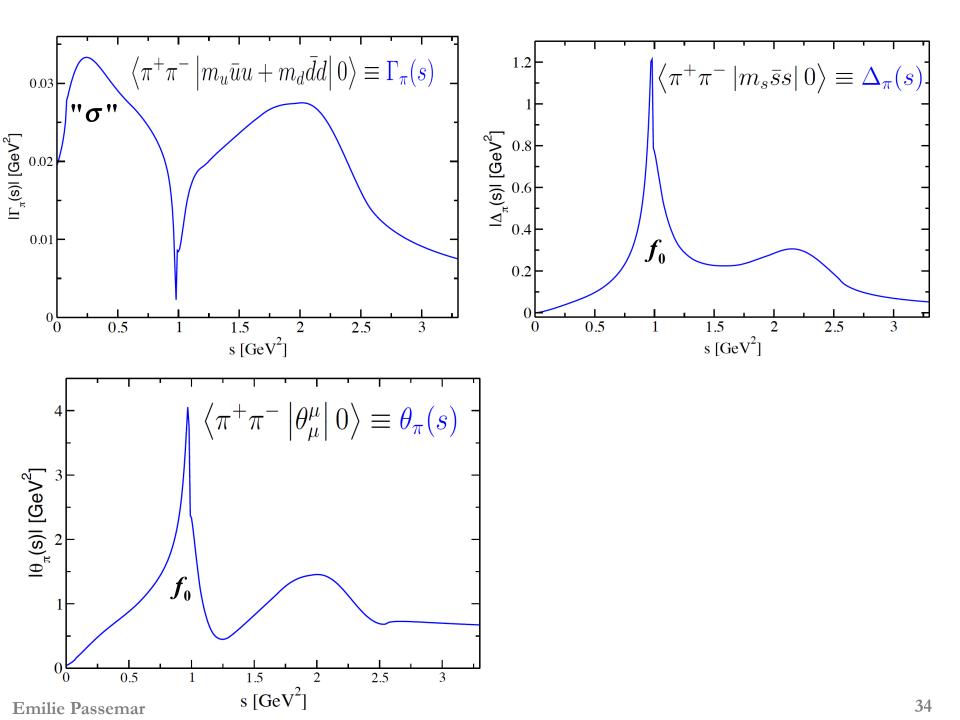
$$P_{\Delta}(s) = \Delta_{\pi}(0) = 0 + \cdots$$

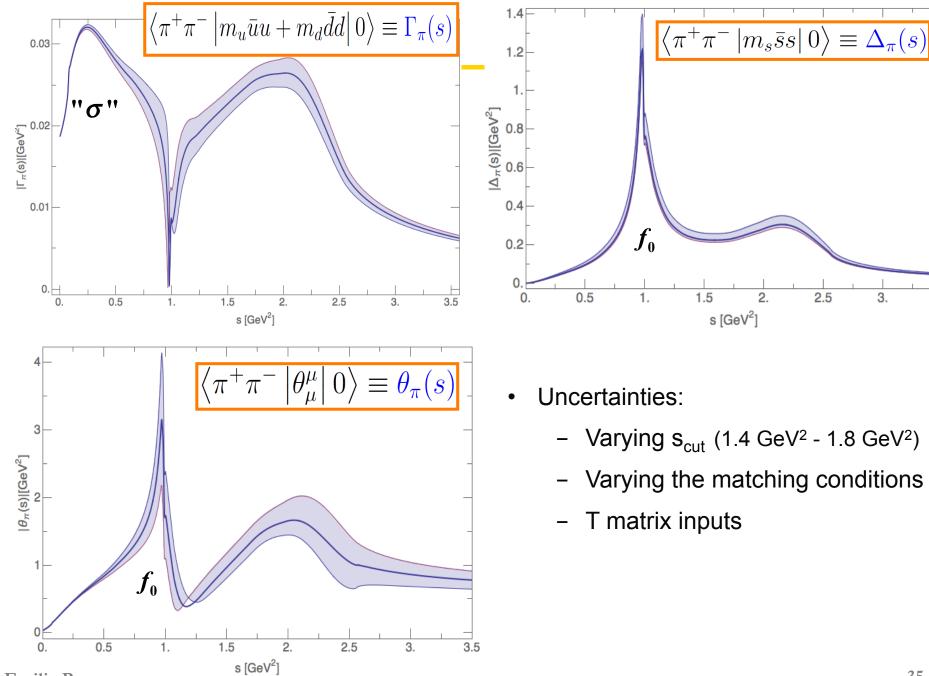
$$Q_{\Delta}(s) = \frac{2}{\sqrt{3}}\Delta_{K}(0) = \frac{2}{\sqrt{3}}\left(M_{K}^{2} - \frac{1}{2}M_{\pi}^{2}\right) + \cdots$$

Problem: large corrections in the case of the kaons!
 Use lattice QCD to determine the SU(3) LECs

 $\Gamma_K(0) = (0.5 \pm 0.1) \ M_\pi^2$ $\Delta_K(0) = 1^{+0.15}_{-0.05} \left(M_K^2 - 1/2M_\pi^2 \right)$

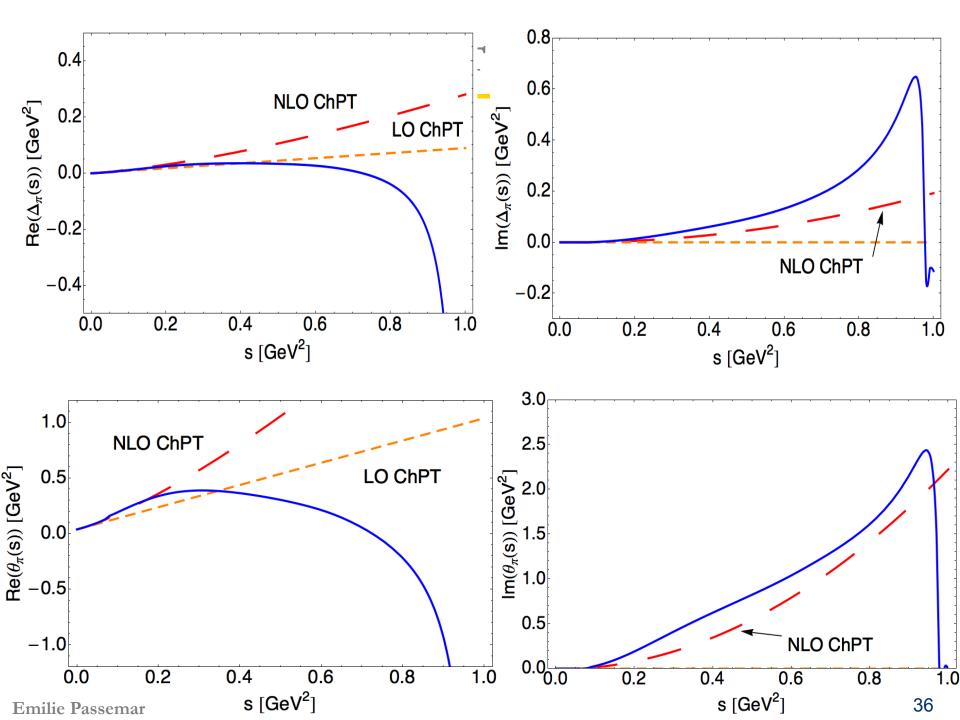
Daub, Dreiner, Hanart, Kubis, Meissner'13 Bernard, Descotes-Genon, Toucas'12





Emilie Passemar

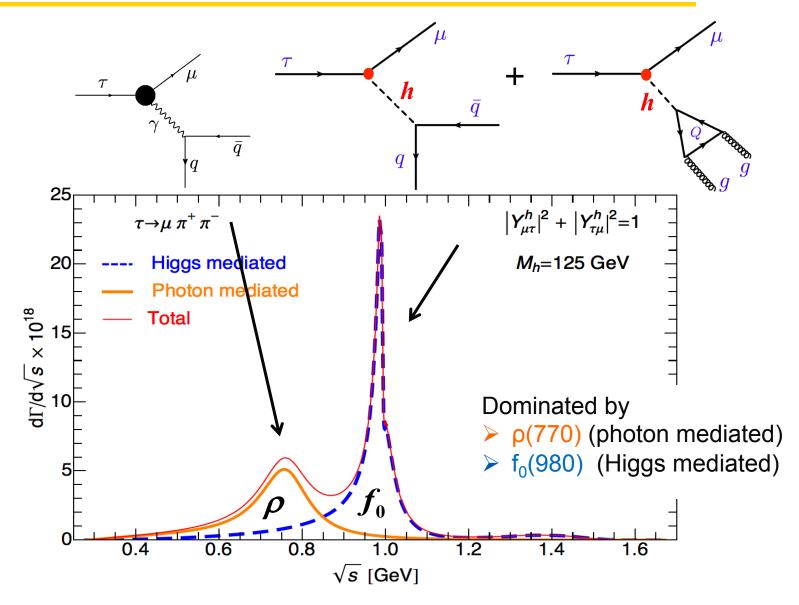
3.5



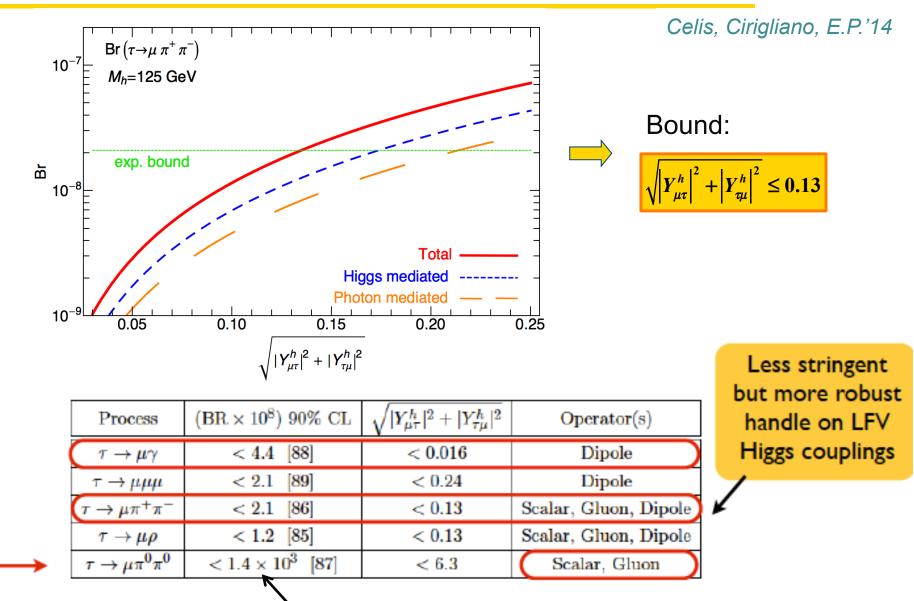
4. Results

4.1 Spectrum

Celis, Cirigliano, E.P.'14



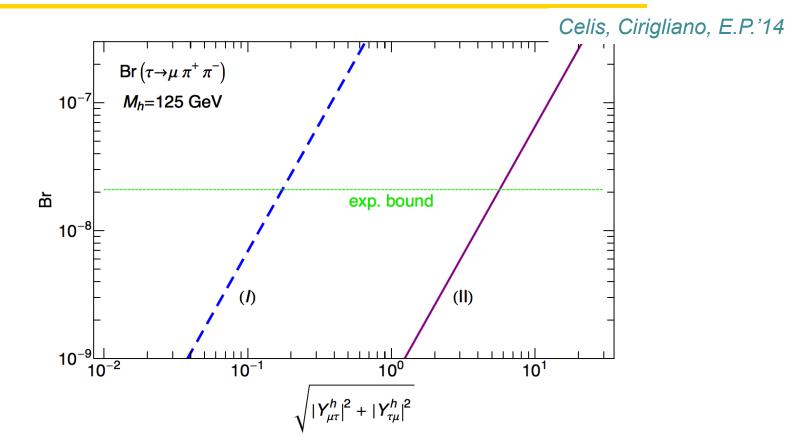
4.2 Bounds



Emilie Passemar

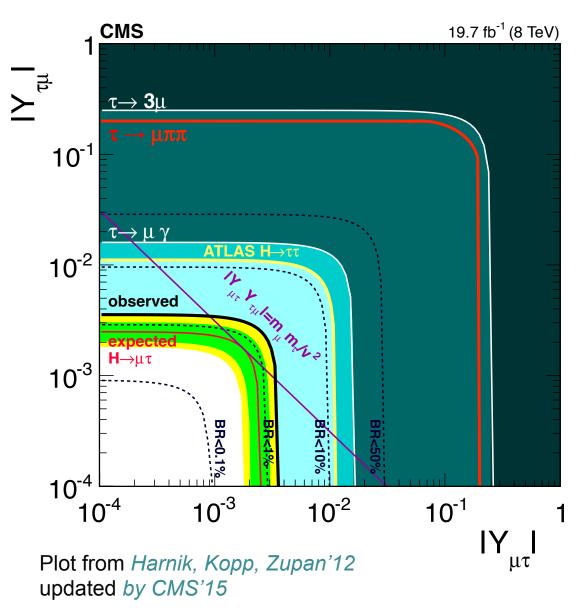
BaBar'10, Belle'10'11'13 except last from CLEO'97

4.3 Impact of our results



- Dispersive treatment of hadronic part bound reduced by one order of magnitude!
- ChPT, EFT only valid at low energy for $p \ll \Lambda = 4\pi f_{\pi} \sim 1 \text{ GeV}$ \longrightarrow not valid up to $E = (m_{\pi} - m_{\mu})!$

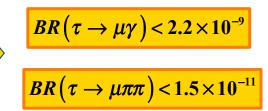
4.4 Constraints in the $\tau\mu$ sector



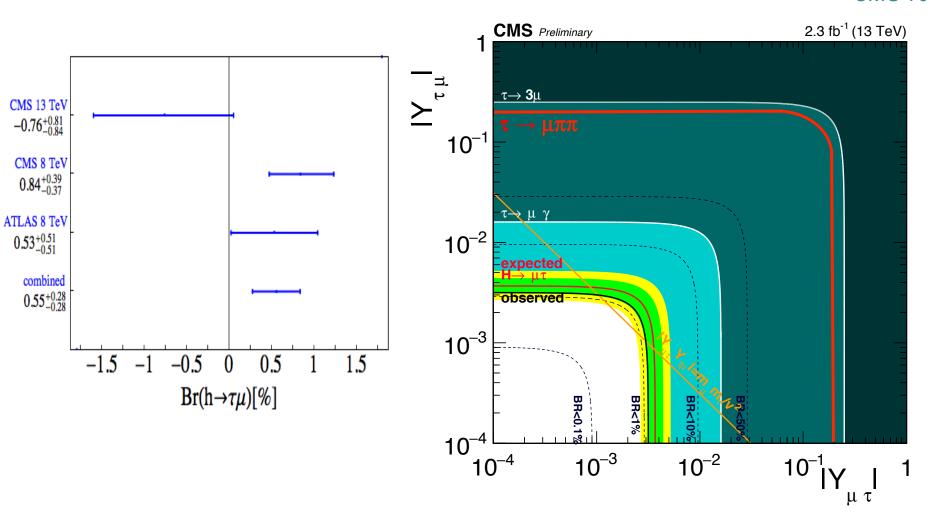
• Constraints from LE:

> $\tau \rightarrow \mu \gamma$: best constraints but loop level > sensitive to UV completion of the theory

- Constraints from HE: *LHC* wins for $\tau \mu!$
- Opposite situation for $\mu e!$
- For LFV Higgs and nothing else: LHC bound



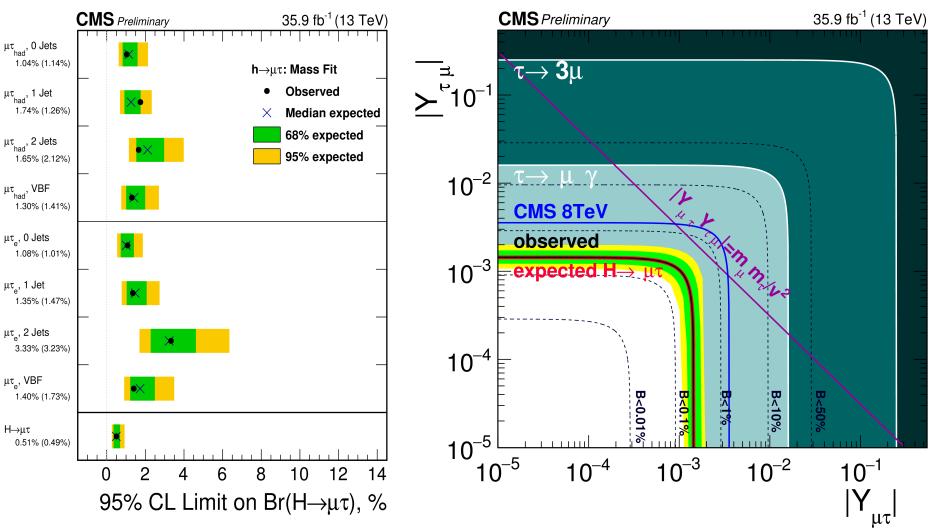
4.5 Hint of New Physics in $h \rightarrow \tau \mu$?

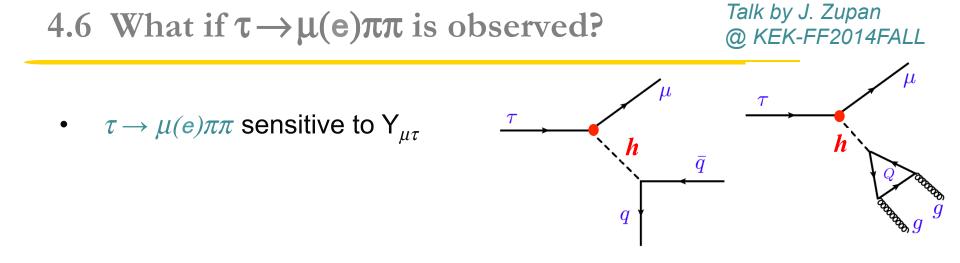


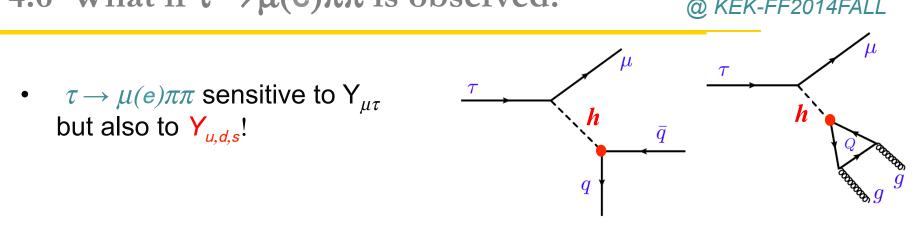
CMS'16

4.5 Hint of New Physics in $h \rightarrow \tau \mu$?

CMS'17







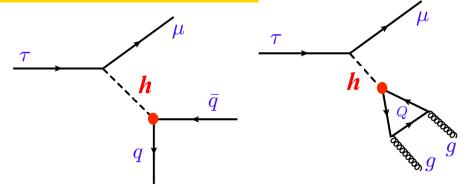
4.6 What if $\tau \rightarrow \mu(e)\pi\pi$ is observed?

Talk by J. Zupan @ KEK-FF2014FALL

4.6 What if $\tau \rightarrow \mu(e)\pi\pi$ is observed?

Talk by J. Zupan @ KEK-FF2014FALL

- $\tau \rightarrow \mu(e)\pi\pi$ sensitive to $Y_{\mu\tau}$ but also to $Y_{u,d,s}!$
- $Y_{u,d,s}$ poorly bounded



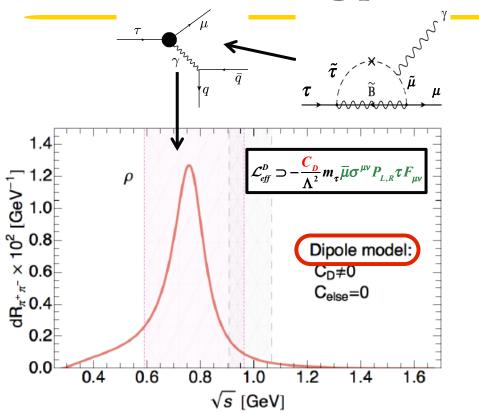
• For
$$Y_{u,d,s}$$
 at their SM values :
 $Br(\tau \to \mu \pi^+ \pi^-) < 1.6 \times 10^{-11}, Br(\tau \to \mu \pi^0 \pi^0) < 4.6 \times 10^{-12}$
 $Br(\tau \to e \pi^+ \pi^-) < 2.3 \times 10^{-10}, Br(\tau \to e \pi^0 \pi^0) < 6.9 \times 10^{-11}$

• But for $Y_{u,d,s}$ at their upper bound: $Br(\tau \to \mu \pi^+ \pi^-) < 3.0 \times 10^{-8}, Br(\tau \to \mu \pi^0 \pi^0) < 1.5 \times 10^{-8}$ $Br(\tau \to e \pi^+ \pi^-) < 4.3 \times 10^{-7}, Br(\tau \to e \pi^0 \pi^0) < 2.1 \times 10^{-7}$

below present experimental limits!

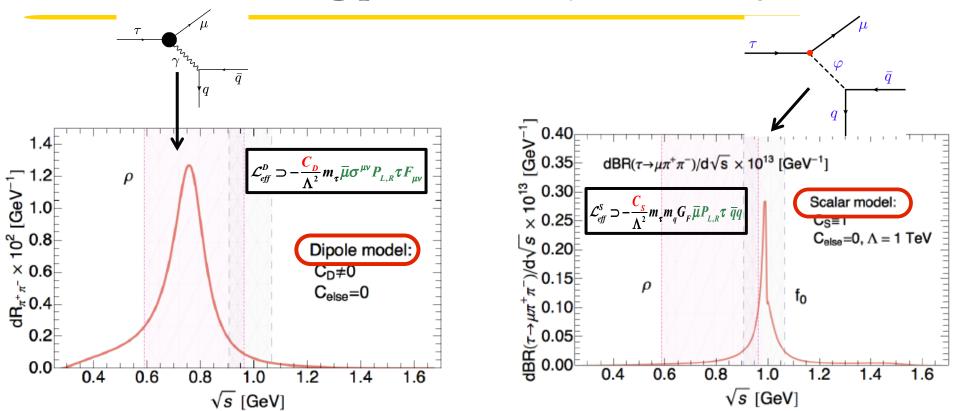
If discovered upper limit on Y_{u,d,s}!
 Interplay between high-energy and low-energy constraints!

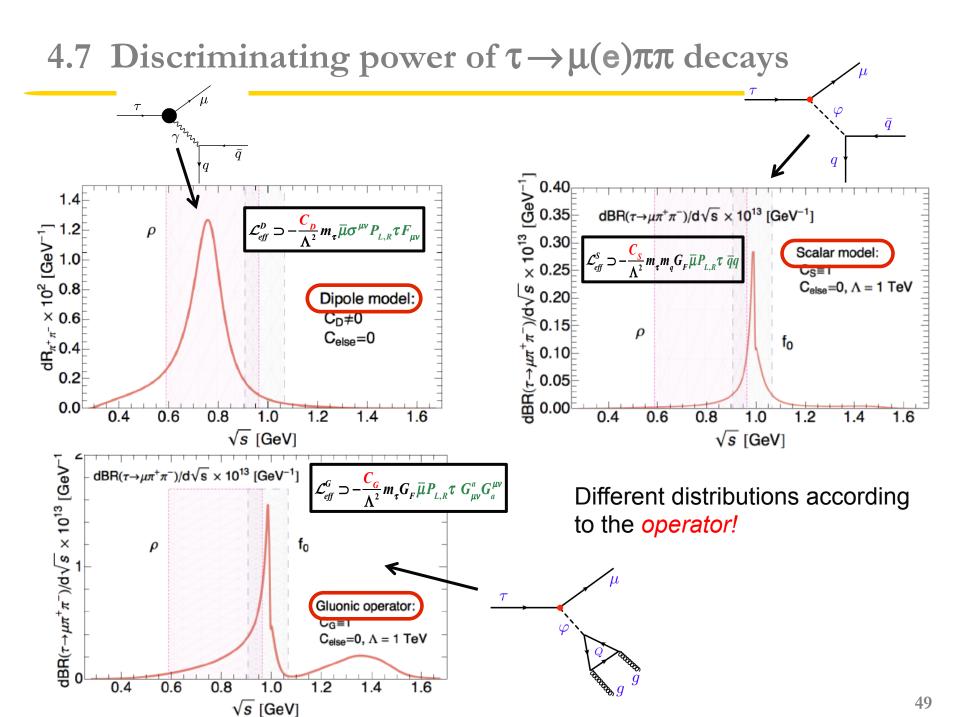
4.7 Discriminating power of $\tau \rightarrow \mu(e)\pi\pi$ decays



Celis, Cirigliano, E.P.'14

4.7 Discriminating power of $\tau \rightarrow \mu(e)\pi\pi$ decays





5. Conclusion and Outlook

- Direct searches for new physics at the TeV-scale at LHC by ATLAS and CMS penergy frontier
- Probing new physics orders of magnitude beyond that scale and helping to decipher possible TeV-scale new physics requires to work hard on the intensity and precision frontiers
- Charged LFV are a very important probe of new physics
 - Extremely small SM rates
 - Experimental results at low energy are very precise

very high scale sensitivity

• CLFV decays excellent model discriminating tools especially τ decays *Hadronic decays* such as $\tau \rightarrow \mu(e)\pi\pi$ important!

Summary

- To consider hadronic decays, need to control the hadronic uncertainties: need to know hadronic matrix elements, form factors etc.
- For $\tau \to \mu(e)\pi\pi$: need to know the $\pi\pi$ form factors



- Dispersion relations rely on analyticity, unitarity and crossing symmetry
 Rigorous treatment of two and three hadronic final state
- $\tau \rightarrow \mu(e)\pi\pi$ gives interesting constraints on LFV new physics operators involving quarks
- Interplay low energy and collider physics: LFV of the Higgs boson
- Complementarity with LFC sector: EDMs, g-2 and colliders:
 New physics models usually strongly correlate these sectors

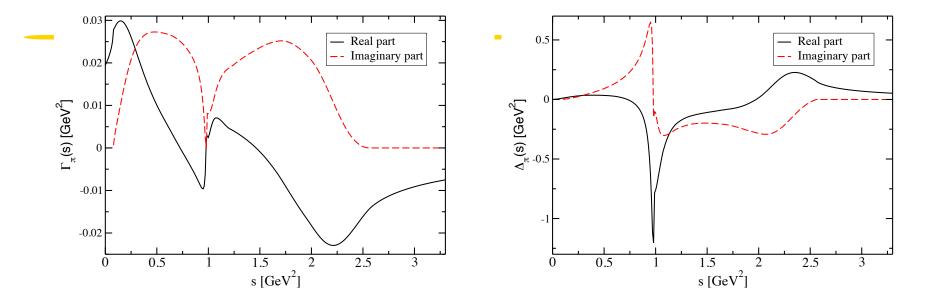
6. Back-up

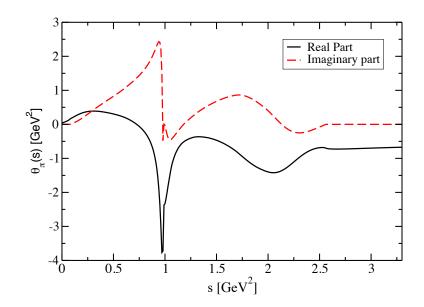
$$S_{mn} = \delta_{mn} + 2i \sqrt{\sigma_m \sigma_n} T_{mn}$$

$$S = \begin{pmatrix} \cos\gamma \ e^{2i\delta_{\pi}} & i \sin\gamma \ e^{i(\delta_{\pi} + \delta_{K})} \\ i \sin\gamma \ e^{i(\delta_{\pi} + \delta_{K})} & \cos\gamma \ e^{2i\delta_{K}} \end{pmatrix}$$

• Inelasticity:
$$\eta_0^0 \equiv \cos \gamma$$

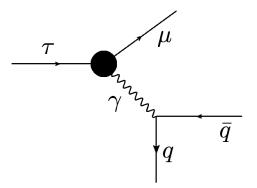
- + $\delta_{\pi}(s)$: $\pi\pi$ S wave phase shift
- + $\delta_K(s)$: KK S wave phase shift





3.1 Constraints from $\tau \rightarrow \mu \pi \pi$

• Contribution from dipole diagrams



$$L_{eff} = c_L Q_{L\gamma} + c_R Q_{R\gamma} + h.c.$$

with the dim-5 EM penguin operators :

$$Q_{L\gamma,R\gamma} = \frac{e}{8\pi^2} m_{\tau} \left(\mu \sigma^{\alpha\beta} P_{L,R} \tau\right) F_{\alpha\beta}$$

•
$$\frac{d\Gamma(\tau \to \ell \pi^+ \pi^-)}{d\sqrt{s}} = \frac{\alpha^2 |F_V(s)|^2 (|c_L|^2 + |c_R|^2)}{768\pi^5 m_\tau} \frac{(s - 4m_\pi^2)^{3/2} (m_\tau^2 - s)^2 (s + 2m_\tau^2)}{s^2}$$

with the vector form factor :

$$C_{L,R} = f\left(\boldsymbol{Y}_{\tau \mu}\right)$$

$$\left\langle \pi^{+}(p_{\pi^{+}})\pi^{-}(p_{\pi^{-}}) \right| \frac{1}{2} (\bar{u}\gamma^{\alpha}u - \bar{d}\gamma^{\alpha}d) \left| 0 \right\rangle \equiv F_{V}(s)(p_{\pi^{+}} - p_{\pi^{-}})^{\alpha}$$

• Diagram only there in the case of $\tau^- \to \mu^- \pi^+ \pi^-$ absent for $\tau^- \to \mu^- \pi^0 \pi^0$ neutral mode more model independent

Determination of F_V(s)

Vector form factor

Precisely known from experimental measurements

$$e^+e^- \rightarrow \pi^+\pi^-$$
 and $\tau^- \rightarrow \pi^0\pi^-\nu_{\tau}$ (isospin rotation)

> Theoretically: Dispersive parametrization for $F_V(s)$

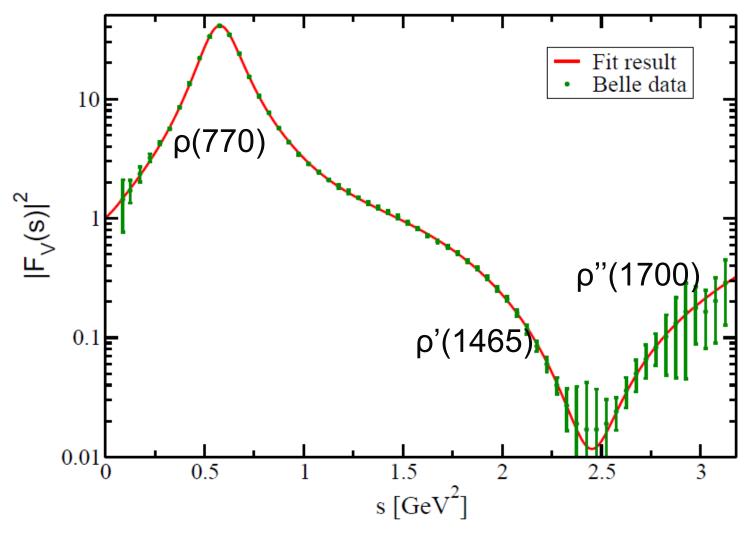
Guerrero, Pich'98, Pich, Portolés'08 Gomez, Roig'13

$$F_{V}(s) = \exp\left[\lambda_{V}^{\prime}\frac{s}{m_{\pi}^{2}} + \frac{1}{2}\left(\lambda_{V}^{\prime\prime} - \lambda_{V}^{\prime2}\right)\left(\frac{s}{m_{\pi}^{2}}\right)^{2} + \frac{s^{3}}{\pi}\int_{4m_{\pi}^{2}}^{\infty}\frac{ds^{\prime}}{s^{\prime3}}\frac{\phi_{V}(s^{\prime})}{\left(s^{\prime}+s-i\varepsilon\right)}\right]$$

Extracted from a model including 3 resonances $\rho(770)$, $\rho'(1465)$ and $\rho''(1700)$ fitted to the data

> Subtraction polynomial + phase determined from a *fit* to the Belle data $\tau^- \rightarrow \pi^0 \pi^- v_{\tau}$

Determination of $F_V(s)$



Determination of $F_V(s)$ thanks to precise measurements from Belle!

CPV AND FV HIGGS COUPLINGS TO SM FERMIONS

• if SM an EFT, the Yukawas get corrected by higher dim. ops

$$\mathcal{L}_{SM} = -\left[\lambda_{ij}(\bar{f}_L^i f_R^j)H + h.c.
ight]$$

$$\Delta \mathcal{L}_Y = -\frac{\lambda'_{ij}}{\Lambda^2} (\bar{f}^i_L f^j_R) H(H^{\dagger} H) + h.c. + \cdots$$

decouples mass terms from yukawas

$$\mathcal{L}_Y = -m_i \bar{f}_L^i f_R^i - Y_{ij} (\bar{f}_L^i f_R^j) h + h.c. + \cdots,$$

- can lead to flavor violating Higgs decays
- can lead to CPV Higgs decays
- different models lead to different patterns of flavor diagonal and flavor violating Yukawas

5

J. Zupan CP and flavor violation in Higgs...

A GENERAL BENCHMARK

- what is a reasonable aim for precision on Y_{ij} ?
 - if off-diagonals are large ⇒ spectrum in general not hierarchical
 - no tuning, if

$$|Y_{\tau\mu}Y_{\mu\tau}| \lesssim \frac{m_{\mu}m_{\tau}}{v^2}$$

Cheng, Sher, 1987

• in concrete models it will be typically further suppressed parametrically

see e.g, Dery, Efrati, Nir, Soreq, Susic, 1408.1371; Dery, Efrati, Hochberg, Nir, 1302.3229; Arhrib, Cheng, Kong, 1208.4669 KEK-FF2014FALL, Oct 29 2014, Tsukuba

J. Zupan CP and flavor violation in Higgs...

8

SUMMARY OF MODELS

an example: higgs couplings to 2nd&3rd gen. charged leptons

adapted from Dery, Efrati, Hochberg, Nir, 1302.3229 and extended

(
Model	$\hat{\mu}_{ au au}$	$(\hat{\mu}_{\mu\mu}/\hat{\mu}_{ au au})/(m_{\mu}^2/m_{ au}^2)$	$\hat{\mu}_{\mu au}/\hat{\mu}_{ au au}$				
SM	1	1	0				
NFC	$(V_{h\ell}^*v/v_\ell)^2$	1	0				
MSSM	$(\sin \alpha / \cos \beta)^2$	1	0				
MFV	$1+2av^2/\Lambda^2$	$1-4bm_{ au}^2/\Lambda^2$	0				
\mathbf{FN}	$1+{\cal O}(v^2/\Lambda^2)$	$1 + \mathcal{O}(v^2/\Lambda^2)$	$\mathcal{O}(U_{23} ^2 v^4/\Lambda^4)$				
GL	9	25/9	${\cal O}(\hat{\mu}_{\mu\mu}/\hat{\mu}_{ au au})$				
$\mathrm{RS}~(i)$	$1 + \mathcal{O}(ar{Y}^2 v^2 / m_{KK}^2)$	$1 + \mathcal{O}(ar{Y}^2 v^2 / m_{KK}^2)$	$\mathcal{O}(\bar{Y}^2 v^2 / m_{KK}^2) \sqrt{m_\tau / m_\mu}$				
RS(ii)	$1 + O(\bar{Y}^2 v^2 / m_{KK}^2)$	$1 + \mathcal{O}(\bar{Y}^2 v^2 / m_{KK}^2)$	$\mathcal{O}(\bar{Y}^2 v^2 / m_{KK}^2)$				
PGB (1 rep.)	$1 - v^2/f^2$	1	0				

6

3.3 Handles

- Two handles:
 - Branching ratios:

$$R_{F,M} \equiv \frac{\Gamma(\tau \to F)}{\Gamma(\tau \to F_M)}$$

with ${\rm F}_{\rm M}$ dominant LFV mode for

model M

> Spectra for > 2 bodies in the final state:

$$\frac{dBR(\tau \to \mu \pi^+ \pi^-)}{d\sqrt{s}} \text{ and } dR_{\pi^+ \pi^-} \equiv \frac{1}{\Gamma(\tau \to \mu \gamma)} \frac{d\Gamma(\tau \to \mu \pi^+ \pi^-)}{d\sqrt{s}}$$

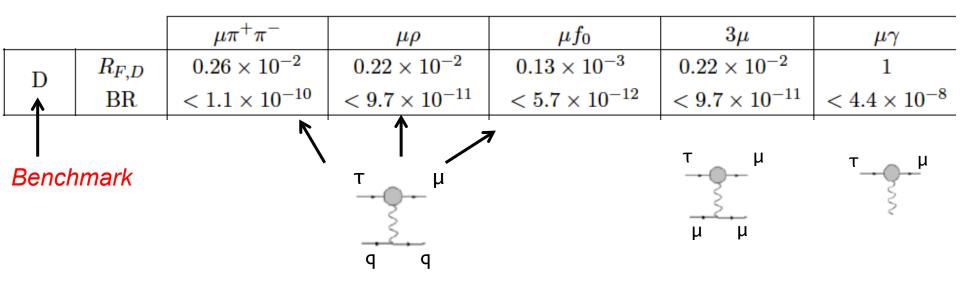
- Benchmarks:
 - ➤ Dipole model: $C_D \neq 0$, $C_{else} = 0$
 - > Scalar model: $C_S \neq 0$, $C_{else} = 0$
 - > Vector (gamma, Z) model: $C_V \neq 0$, $C_{else} = 0$
 - > Gluonic model: $C_{GG} ≠ 0$, $C_{else} = 0$

3.3 Branching ratios

- Two handles:
 - Branching ratios:

$$R_{F,M} \equiv \frac{\Gamma(\tau \to F)}{\Gamma(\tau \to F_M)} \,,$$

with F_M dominant LFV mode for model M



- $\rho(770)$ resonance (J^{PC}=1⁻⁻): cut in the $\pi^+\pi^-$ invariant mass: 587 MeV $\leq \sqrt{s} \leq 962$ MeV
- $f_0(980)$ resonance $(J^{PC}=0^{++})$: cut in the $\pi^+\pi^-$ invariant mass: 906 MeV $\leq \sqrt{s} \leq 1065$ MeV

3.3 Branching ratios

- Two handles: ٠

> Branching ratios: $R_{F,M} \equiv \frac{\Gamma(\tau \to F)}{\Gamma(\tau \to F_M)}$ with F_M dominant LFV mode for model M

		$\mu\pi^+\pi^-$	μho	μf_0	3μ	$\mu\gamma$
D	$R_{F,D}$	$0.26 imes10^{-2}$	$0.22 imes 10^{-2}$	$0.13 imes10^{-3}$	$0.22 imes 10^{-2}$	1
	\mathbf{BR}	$<1.1\times10^{-10}$	$<9.7\times10^{-11}$	$< 5.7 \times 10^{-12}$	$<9.7\times10^{-11}$	$<4.4\times10^{-8}$
S	$R_{F,S}$	1	0.28	0.7	-	-
	BR	$<~2.1\times10^{-8}$	$<~5.9\times10^{-9}$	$<~1.47\times10^{-8}$	-	-
$\mathrm{V}^{(\gamma)}$	$R_{F,V^{(\gamma)}}$	1	0.86	0.1	-	-
	BR	$<~1.4\times10^{-8}$	$<~1.2\times10^{-8}$	$<~1.4\times10^{-9}$	-	-
Z	$R_{F,Z}$	1	0.86	0.1	-	-
	BR	$<~1.4\times10^{-8}$	$<~1.2\times10^{-8}$	$<~1.4\times10^{-9}$	-	-
G	$R_{F,G}$	1	0.41	0.41	-	-
	\mathbf{BR}	$<~2.1\times10^{-8}$	$< 8.6 imes 10^{-9}$	$<~8.6\times10^{-9}$	-	-
				7		
Bench	nmark	x	τ μ		τµ →Q≁	τ _→ ,μ
			The second se			~
			q q		μμ	56

4.1 Constraints from $\tau \rightarrow lP$

Tree level Higgs exchange
 ≻ η, η'

$$\Gamma\left(\tau \to \ell\eta^{(\prime)}\right) = \frac{\bar{\beta}\left(m_{\tau}^2 - m_{\eta}^2\right)\left(|Y_{\mu\tau}^A|^2 + |Y_{\tau\mu}^A|^2\right)}{256\,\pi\,M_A^4\,v^2\,m_{\tau}} \Big[(y_u^A + y_d^A)h_{\eta'}^q + \sqrt{2}y_s^Ah_{\eta'}^s - \sqrt{2}a_{\eta'}\sum_{q=c,b,t}\,y_q^A\Big]^2$$

with the decay constants :

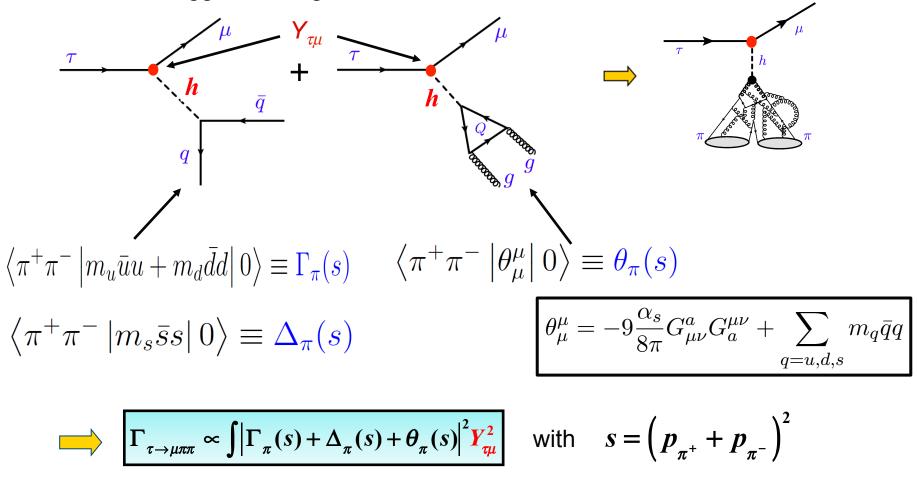
$$\langle \eta^{(\prime)}(p) | \bar{q} \gamma_5 q | 0 \rangle = -\frac{i}{2\sqrt{2}m_q} h^q_{\eta^{(\prime)}} \qquad \langle \eta^{(\prime)}(p) | \bar{s} \gamma_5 s | 0 \rangle = -\frac{i}{2m_s} h^s_{\eta^{(\prime)}}$$

$$\langle \eta^{(\prime)}(p) | \frac{\alpha_s}{4\pi} G^{\mu\nu}_a \widetilde{G}^a_{\mu\nu} | 0 \rangle = a_{\eta^{(\prime)}}$$

$$\geqslant \pi : \Gamma(\tau \to \ell \pi^0) = \frac{f^2_\pi m^4_\pi m_\tau}{256\pi M^4_A v^2} \left(|Y^A_{\tau\mu}|^2 + |Y^A_{\mu\tau}|^2 \right) \left(y^A_u - y^A_d \right)^2$$

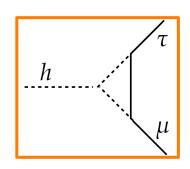
3.1 Constraints from $\tau \rightarrow \mu \pi \pi$

• Tree level Higgs exchange

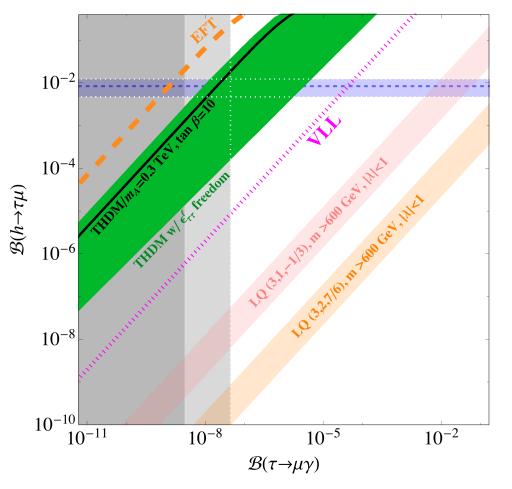


4.5 Interplay between LHC & Low Energy

- If real what type of NP?
- If $h \rightarrow \tau \mu$ due to loop corrections:
 - extra charged particles necessary
 - $\tau \rightarrow \mu \gamma$ too large



- h → τ μ possible to explain if extra scalar doublet:
 ⇒ 2HDM of type III
- Constraints from $\tau \rightarrow \mu \gamma$ important! \Rightarrow Belle II

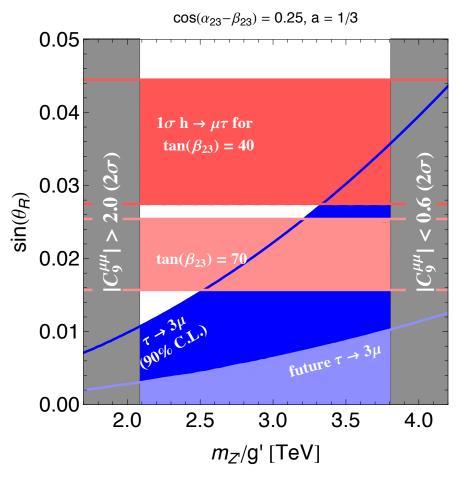


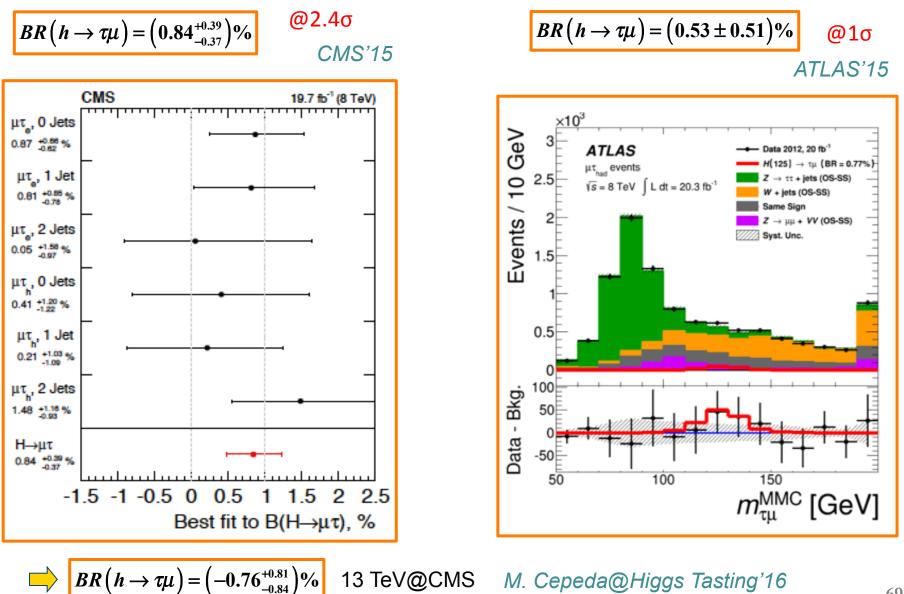
Dorsner et al.'15

4.5 Interplay between LHC & Low Energy

- 2HDMs with gauged L_μ − L_τ
 Z', explain anomalies for
 - $\ h \to \tau \mu$
 - $\ B \to K^* \mu \mu$
 - $R_K = B \rightarrow K \mu \mu / B \rightarrow K e e$
- Constraints from $\tau \rightarrow 3\mu$ crucial \Rightarrow Belle II, LHCb
- See also: Aristizabal-Sierra & Vicente'14, Lima et al'15, Omhura, Senaha, Tobe '15

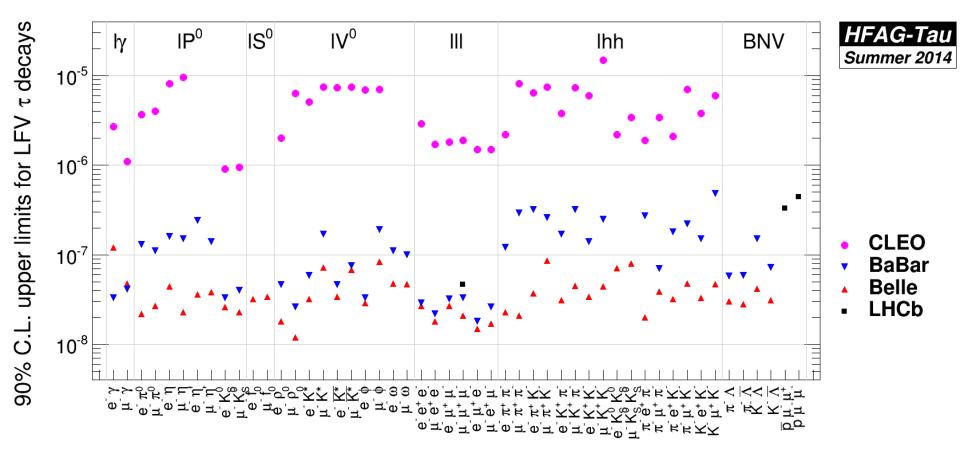
Altmannshofer & Straub'14, Crivellin et al'15 Crivellin, D'Ambrosio, Heeck.'15





2.2 CLFV processes: tau decays

• Several processes: $\tau \to \ell \gamma, \ \tau \to \ell_{\alpha} \overline{\ell}_{\beta} \ell_{\beta}, \ \tau \to \ell Y$ $\swarrow P, S, V, P\overline{P}, ...$



Expected sensitivity 10⁻⁹ or better at *LHCb, Belle II*?

Determination of the polynomial

• For θ_P enforcing the asymptotic constraint is not consistent with ChPT The unsubtracted DR is not saturated by the 2 states

Relax the constraints and match to ChPT

$$\begin{array}{lll} P_{\theta}(s) &=& 2M_{\pi}^2 + \left(\dot{\theta}_{\pi} - 2M_{\pi}^2 \dot{C}_1 - \frac{4M_K^2}{\sqrt{3}} \dot{D}_1\right) s \\ Q_{\theta}(s) &=& \frac{4}{\sqrt{3}} M_K^2 + \frac{2}{\sqrt{3}} \left(\dot{\theta}_K - \sqrt{3}M_{\pi}^2 \dot{C}_2 - 2M_K^2 \dot{D}_2\right) s \end{array}$$

with
$$\dot{f} = \left(\frac{df}{ds}\right)_{s=0}$$

• At LO ChPT:
$$\dot{\theta}_{\pi,K} = 1$$

• Higher orders $\implies \dot{\theta}_{K} = 1.15 \pm 0.1$