## Lepton Flavour Violating Tau decays

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## Outline

1. Introduction and Motivation
2. Charged Lepton-Flavour Violation from tau decays
3. Special Role of $\tau \rightarrow \mu \pi \pi$ : hadronic form factors
4. Results
5. Conclusion and Outlook
6. Introduction and Motivation

### 1.1 Why study charged leptons?



- In the quest of New Physics, can be sensitive to very high scale:
$-\underset{\left[\varepsilon_{K}\right]}{\text { Kaon physics: }} \frac{s \bar{d} s \bar{d}}{\Lambda^{2}} \Rightarrow \Lambda \gtrsim 10^{5} \mathrm{TeV}$
- Charged Leptons: $\left.\frac{\mu \bar{e} f \bar{f}}{\Lambda^{2}} \Rightarrow \Lambda \rightarrow \mathrm{e} \gamma\right] \mathrm{L} \gtrsim 10^{4} \mathrm{TeV}$
- At low energy: lots of experiments e.g., MEG, COMET, Mu2e, E-969, BaBar, Bellel-II, BESIII, $\mathrm{LHCb} \square$ huge improvements on measurements and bounds obtained and more expected
- In many cases no SM background: e.g., LFV, EDMs
- For some modes accurate calculations of hadronic uncertainties essential
$\square$ Charged leptons very important to look for New Physics!


### 1.2 The Program



## 2. Charged Lepton-Flavour Violation

### 2.1 Introduction and Motivation

- Lepton Flavour Number is an « accidental » symmetry of the $\mathrm{SM}\left(\mathrm{m}_{\mathrm{v}}=0\right)$
- In the $S M$ with massive neutrinos effective CLFV vertices are tiny due to GIM suppression $\Rightarrow$ unobservably small rates!
E.g.: $\mu \rightarrow \boldsymbol{e} \boldsymbol{\gamma}$
$\operatorname{Br}(\mu \rightarrow e \gamma)=\frac{3 \alpha}{32 \pi}\left|\sum_{i=2,3} U_{\mu i}^{*} U_{e i} \frac{\Delta m_{1 i}^{2}}{M_{W}^{2}}\right|^{2}<10^{-54}$
Petcov'77, Marciano \& Sanda'77, Lee \& Shrock'77...

$$
\left[\operatorname{Br}(\tau \rightarrow \mu \gamma)<10^{-40}\right]
$$



- Extremely clean probe of beyond SM physics


### 2.1 Introduction and Motivation

- In New Physics scenarios CLFV can reach observable levels in several channels

| Talk by D.Hitlin@ CLFV2013 |  | $\tau \rightarrow \mu \gamma \tau \rightarrow \ell \ell \ell$ |  |
| :---: | :---: | :---: | :---: |
| $S M+v$ mixing | Lee, Shrock, PRD 16 (1977) 1444 Cheng, Li, PRD 45 (1980) 1908 | Unde |  |
| SUSY Higgs | Dedes, Ellis, Raidal, PLB 549 (2002) 159 Brignole, Rossi, PLB 566 (2003) 517 | $10^{-10}$ | $10^{-7}$ |
| $S M+$ heavy Maj $v_{R}$ | Cvetic, Dib, Kim, Kim , PRD66 (2002) 034008 | $10^{-9}$ | 10-10 |
| Non-universal $Z^{\prime}$ | Yue, Zhang, Liu, PLB 547 (2002) 252 | $10^{-9}$ | $10^{-8}$ |
| SUSY SO(10) | Masiero, Vempati, Vives, NPB 649 (2003) 189 Fukuyama, Kikuchi, Okada, PRD 68 (2003) 033012 | $10^{-8}$ | 10-10 |
| mSUGRA + Seesaw | Ellis, Gomez, Leontaris, Lola, Nanopoulos, EPJ C14 (2002) 319 Ellis, Hisano, Raidal, Shimizu, PRD 66 (2002) 115013 | $10^{-7}$ | $10^{-9}$ |

- But the sensitivity of particular modes to CLFV couplings is model dependent
- Comparison in muonic and tauonic channels of branching ratios, conversion rates and spectra is model-diagnostic


### 2.2 CLFV processes: muon decays

- Several processes: $\mu \rightarrow \boldsymbol{e} \gamma, \mu \rightarrow \boldsymbol{e} \overline{\boldsymbol{e}}, \mu(A, Z) \rightarrow \boldsymbol{e}(A, Z)$

MEG'13


### 2.2 CLFV processes: tau decays

- Several processes: $\tau \rightarrow \ell \gamma, \tau \rightarrow \ell_{\alpha} \bar{\ell}_{\beta} \ell_{\beta}, \tau \rightarrow \ell \boldsymbol{Y}_{k}$

$$
\nwarrow_{P}, S, V, P \bar{P}, . .
$$



| HFAG-Tau |
| :--- |
| Summer 2014 |

- CLEO
- BaBar
- Belle
- LHCb
- 48 LFV modes studied at Belle and BaBar


### 2.2 CLFV processes: tau decays

- Several processes: $\tau \rightarrow \ell \gamma, \tau \rightarrow \ell_{\alpha} \bar{\ell}_{\beta} \ell_{\beta}, \tau \rightarrow \ell Y_{\kappa}$ $90 \%$ CL upper limits on $\tau$ LFV decays $\boldsymbol{P}, \boldsymbol{S}, \boldsymbol{V}, \boldsymbol{P} \overline{\boldsymbol{P}}, \ldots$

- Expected sensitivity $10^{-9}$ or better at LHCb , Belle II?


### 2.3 Effective Field Theory approach

$$
\mathcal{L}=\mathcal{L}_{S M}+\frac{C^{(5)}}{\Lambda} O^{(5)}+\sum_{i} \frac{C_{i}^{(6)}}{\Lambda^{2}} O_{i}^{(6)}+\ldots
$$

- Build all D>5 LFV operators:

See e.g.
Black, Han, He, Sher'02
Brignole \& Rossi'04
Dassinger et al.'07
Matsuzaki \& Sanda'08
Giffels et al.'08
Crivellin, Najjari, Rosiek'13
Petrov \& Zhuridov'14
Cirigliano, Celis, E.P.'14
> Dipole:

$$
\mathcal{L}_{\text {eff }}^{D} \supset-\frac{C_{D}}{\Lambda^{2}} \boldsymbol{m}_{\tau} \bar{\mu} \sigma^{\mu \nu} \boldsymbol{P}_{L, R} \tau F_{\mu \nu}
$$

e.g.


### 2.3 Effective Field Theory approach



- Build all D>5 LFV operators:
$>$ Dipole: $\mathcal{L}_{e f f}^{D} \supset-\frac{C_{D}}{\Lambda^{2}} \boldsymbol{m}_{\tau} \bar{\mu} \sigma^{\mu \nu} P_{L, R} \tau F_{\mu \nu}$

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Crivellin, Najjari, Rosiek'13
Petrov \& Zhuridov'14
Cirigliano, Celis, E.P.' 14
> Lepton-quark (Scalar, Pseudo-scalar, Vector, Axial-vector):

$$
\mathcal{L}_{e f f}^{S, V} \supset-\frac{C_{S, V}}{\Lambda^{2}} \boldsymbol{m}_{\tau} \boldsymbol{m}_{q} \boldsymbol{G}_{F} \bar{\mu} \Gamma \boldsymbol{P}_{L, R} \tau \bar{q} \Gamma q
$$

e.g.



### 2.3 Effective Field Theory approach



- Build all D>5 LFV operators:
$>$ Dipole: $\mathcal{L}_{\text {eff }}^{D} \supset-\frac{C_{D}}{\Lambda^{2}} \boldsymbol{m}_{\tau} \bar{\mu} \sigma^{\mu \nu} P_{L, R} \tau F_{\mu \nu}$

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Cirigliano, Celis, E.P.' 14
$>$ Lepton-quark (Scalar, Pseudo-scalar, Vector, Axial-vector): $\mathcal{L}_{\text {fff }}^{s} \supset-\frac{C_{S, V}}{\Lambda^{2}} \boldsymbol{m}_{\tau} \boldsymbol{m}_{q} \boldsymbol{G}_{F} \bar{\mu} \Gamma P_{L, R} \tau \bar{q} \Gamma q$
> Integrating out heavy quarks generates gluonic operator

### 2.3 Effective Field Theory approach



- Build all D>5 LFV operators:
$>$ Dipole: $\mathcal{L}_{e f f}^{D} \supset-\frac{C_{D}}{\Lambda^{2}} \boldsymbol{m}_{\tau} \bar{\mu} \sigma^{\mu \nu} P_{L, R} \tau F_{\mu \nu}$

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$>$ Lepton-quark (Scalar, Pseudo-scalar, Vector, Axial-vector): $\mathcal{L}_{f f f}^{s} \supset-\frac{C_{S, V}}{\Lambda^{2}} \boldsymbol{m}_{\tau} \boldsymbol{m}_{q} \boldsymbol{G}_{F} \bar{\mu} \Gamma P_{L, R} \tau \bar{q} \Gamma q$
$>4$ leptons (Scalar, Pseudo-scalar, Vector, Axial-vector): $\mathcal{L}_{\epsilon f f}^{4 \ell} \supset-\frac{C_{S, V}^{4 \ell}}{\Lambda^{2}} \bar{\mu} \Gamma P_{L, R} \tau \bar{\mu} \Gamma P_{L, R} \mu$

$\Gamma \equiv 1, \gamma^{\mu}$

### 2.3 Effective Field Theory approach



- Build all D>5 LFV operators:
$>$ Dipole: $\mathcal{L}_{\text {eff }}^{D} \supset-\frac{C_{D}}{\Lambda^{2}} \boldsymbol{m}_{\tau} \bar{\mu} \sigma^{\mu \nu} P_{L, R} \tau F_{\mu \nu}$

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Cirigliano, Celis, E.P.' 14
$>$ Lepton-quark (Scalar, Pseudo-scalar, Vector, Axial-vector): $\mathcal{L}_{\text {eff }}^{S} \supset-\frac{C_{S, V}}{\Lambda^{2}} \boldsymbol{m}_{\tau} \boldsymbol{m}_{q} \boldsymbol{G}_{F} \bar{\mu} \Gamma P_{L, R} \tau \bar{q} \Gamma q$
$>$ Lepton-gluon (Scalar, Pseudo-scalar): $\mathcal{L}_{e f f}^{G} \supset-\frac{C_{G}}{\Lambda^{2}} \boldsymbol{m}_{\tau} \boldsymbol{G}_{\boldsymbol{F}} \bar{\mu} P_{L, R} \tau G_{\mu \nu}^{a} G_{a}^{\mu \nu}$
$>4$ leptons (Scalar, Pseudo-scalar, Vector, Axial-vector): $\mathcal{L}_{\epsilon f f}^{4 \ell} \supset-\frac{C_{S, V}^{4 \ell}}{\Lambda^{2}} \bar{\mu} \Gamma P_{L, R} \tau \bar{\mu} \Gamma P_{L, R} \mu$

- Each UV model generates a specific pattern of them
$\Gamma \equiv 1, \gamma^{\mu}$


### 2.4 Model discriminating power of Tau processes

- Summary table:

|  | $\tau \rightarrow 3 \mu$ | $\tau \rightarrow \mu \gamma$ | $\tau \rightarrow \mu \pi^{+} \pi^{-}$ | $\tau \rightarrow \mu K \bar{K}$ | $\tau \rightarrow \mu \pi$ | $\tau \rightarrow \mu \eta^{(\prime)}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{O}_{\mathrm{S}, \mathrm{V}}^{4 \ell}$ | $\checkmark$ | - | - | - | - | - |
| $\mathrm{O}_{\mathrm{D}}$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | - | - |
| $\mathrm{O}_{\mathrm{V}}^{\mathrm{q}}$ | - | - | $\checkmark(\mathrm{I}=1)$ | $\checkmark(\mathrm{I}=0,1)$ | - | - |
| $\mathrm{O}_{\mathrm{S}}^{\mathrm{q}}$ | - | - | $\checkmark(\mathrm{I}=0)$ | $\boldsymbol{\checkmark}(\mathrm{I}=0,1)$ | - | - |
| $\mathrm{O}_{\mathrm{GG}}$ | - | - | $\checkmark$ | $\checkmark$ | - | - |
| $\mathrm{O}_{\mathrm{A}}^{\mathrm{q}}$ | - | - | - | - | $\checkmark(\mathrm{I}=1)$ | $\checkmark(\mathrm{I}=0)$ |
| $\mathrm{O}_{\mathrm{P}}^{\mathrm{q}}$ | - | - | - | - | $\checkmark(\mathrm{I}=1)$ | $\boldsymbol{\checkmark}$ |
| $\mathrm{O}_{\mathrm{G} \widetilde{\mathrm{G}}}$ | - | - | - | - | - | $\checkmark$ |

- The notion of "best probe" (process with largest decay rate) is model dependent
- If observed, compare rate of processes $\Rightarrow$ key handle on relative strength between operators and hence on the underlying mechanism


### 2.4 Model discriminating power of Tau processes

- Summary table:

|  | $\tau \rightarrow 3 \mu$ | $\tau \rightarrow \mu \gamma$ | $\tau \rightarrow \mu \pi^{+} \pi^{-}$ | $\tau \rightarrow \mu K \bar{K}$ | $\tau \rightarrow \mu \pi$ | $\tau \rightarrow \mu \eta^{(\prime)}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{O}_{\mathrm{S}, \mathrm{V}}^{4 \ell}$ | $\checkmark$ | - | - | - | - | - |
| $\mathrm{O}_{\mathrm{D}}$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | - | - |
| $\mathrm{O}_{\mathrm{V}}^{\mathrm{q}}$ | - | - | $\checkmark(\mathrm{I}=1)$ | $\checkmark(\mathrm{I}=0,1)$ | - | - |
| $\mathrm{O}_{\mathrm{S}}^{\mathrm{q}}$ | - | - | $\checkmark(\mathrm{I}=0)$ | $\checkmark(\mathrm{I}=0,1)$ | - | - |
| $\mathrm{O}_{\mathrm{GG}}$ | - | - | $\checkmark$ | $\checkmark$ | - | - |
| $\mathrm{O}_{\mathrm{A}}^{\mathrm{q}}$ | - | - | - | - | $\checkmark(\mathrm{I}=1)$ | $\checkmark(\mathrm{I}=0)$ |
| $\mathrm{O}_{\mathrm{P}}^{\mathrm{q}}$ | - | - | - | - | $\checkmark(\mathrm{I}=1)$ | $\checkmark(\mathrm{I}=0)$ |
| $\mathrm{O}_{\mathrm{G} \widetilde{ }}$ | - | - | - | - | - | $\checkmark$ |

- In addition to leptonic and radiative decays, hadronic decays are very important sensitive to large number of operators!
- But need reliable determinations of the hadronic part: form factors and decay constants (e.g. $f_{\eta}, f_{\eta}$, )


### 2.5 Ex: Non standard LFV Higgs coupling


In the SM: $\quad \boldsymbol{Y}_{i j}^{\boldsymbol{h}_{S M}}=\frac{\boldsymbol{m}_{\boldsymbol{i}}}{\mathbf{v}} \boldsymbol{\delta}_{i j}$

$$
L_{Y}=-m_{i} \bar{f}_{L}^{i} f_{R}^{i}-h\left(Y_{e \mu} \bar{e}_{L} \mu_{R}+Y_{e \tau} \bar{e}_{L} \tau_{R}+Y_{\mu \tau} \bar{\mu}_{L} \tau_{R}\right)+\ldots
$$

Goudelis, Lebedev, Park'11 Davidson, Grenier'10 Harnik, Kopp, Zupan'12
Blankenburg, Ellis, Isidori'12 McKeen, Pospelov, Ritz'12
Arhrib, Cheng, Kong'12

- Arise in several models Cheng, Sher'97, Goudelis, Lebedev,Park'11 Davidson, Grenier'10

Cheng, Sher'97

- Order of magnitude expected $\square$ No tuning:

$$
\left|Y_{\tau \mu} Y_{\mu \tau}\right| \lesssim \frac{m_{\mu} m_{\tau}}{v^{2}}
$$

- In concrete models, in general further parametrically suppressed


### 2.5 Ex: Non standard LFV Higgs coupling

$\Delta \mathcal{C}_{Y}=-\frac{\lambda_{i j}}{\Lambda^{2}}\left(\bar{f}_{L}^{i} f_{R}^{j} H\right) H^{\dagger} H$

- High energy : LHC


In the SM: $\quad \boldsymbol{Y}_{i j}^{\boldsymbol{h}_{S M}}=\frac{\boldsymbol{m}_{\boldsymbol{i}}}{\mathbf{v}} \boldsymbol{\delta}_{i j}$ QCD

Goudelis, Lebedev, Park'11
Davidson, Grenier'10
Harnik, Kopp, Zupan'12
Blankenburg, Ellis, Isidori'12
McKeen, Pospelov, Ritz'12
Arhrib, Cheng, Kong'12

Hadronic part treated with perturbative

- Low energy : D, S, G operators


Hadronic part treated with non-perturbative QCD

### 2.6 Constraints from $\tau \rightarrow \mu \pi \pi$

- Tree level Higgs exchange

- Problem : Have the hadronic part under control, ChPT not valid at these energies! $s=\left(p_{\pi^{+}}+p_{\pi^{-}}\right)^{2} \Rightarrow \sqrt{s} \leq m_{\tau}-m_{\mu}$
$\square$ Use form factors determined with dispersion relations matched at low energy to CHPT

Daub, Dreiner, Hanart, Kubis, Meissner'13
Celis, Cirigliano, E.P.' 14

- Dispersion relations: based on unitarity, analyticity and crossing symmetry $\square$ Take all rescattering effects into account
$\pi \pi$ final state interactions important


## 3. Description of the hadronic form factors

### 3.1 Constraints from $\tau \rightarrow \mu \pi \pi$

- Photon mediated contribution requires the pion vector form factor:

$$
\left\langle\pi^{+}\left(p_{\pi^{+}}\right) \pi^{-}\left(p_{\pi^{-}}\right)\right| \frac{1}{2}\left(\bar{u} \gamma^{\alpha} u-\bar{d} \gamma^{\alpha} d\right)|0\rangle \equiv F_{V}(s)\left(p_{\pi^{+}}-p_{\pi^{-}}\right)^{\alpha}
$$



- Dispersive parametrization following the properties of analyticity and unitarity of the Form Factor

Gasser, Meißner'91
Guerrero, Pich'97
Oller, Oset, Palomar'01 Pich, Portolés '08
Gómez Dumm\&Roig'13

- Determined from a fit


Celis, Cirigliano, E.P.'14

### 3.1 Constraints from $\tau \rightarrow \mu \pi \pi$

- Tree level Hags exchange

$\left\langle\pi^{+} \pi^{-}\right| m_{u} \bar{u} u+m_{d} \bar{d} d|0\rangle \equiv \Gamma_{\pi}(s) \quad\left\langle\pi^{+} \pi^{-}\right| \theta_{\mu}^{\mu}|0\rangle \equiv \theta_{\pi}(s)$
$\left\langle\pi^{+} \pi^{-}\right| m_{s} \bar{s} s|0\rangle \equiv \Delta_{\pi}(s)$

$$
\theta_{\mu}^{\mu}=-9 \frac{\alpha_{s}}{8 \pi} G_{\mu \nu}^{a} G_{a}^{\mu \nu}+\sum_{q=u, d, s} m_{q} \bar{q} q
$$

$$
\frac{d \Gamma\left(\tau \rightarrow \mu \pi^{+} \pi^{-}\right)}{d \sqrt{s}}=\frac{\left(m_{\tau}^{2}-s\right)^{2} \sqrt{s-4 m_{\pi}^{2}}}{256 \pi^{3} m_{\tau}^{3}} \frac{\left(\left|Y_{\tau \mu}^{h}\right|^{2}+\left|Y_{\mu \tau}^{h}\right|^{2}\right)}{M_{h}^{4} v^{2}}\left|\mathcal{K}_{\Delta} \Delta_{\pi}(s)+\mathcal{K}_{\Gamma} \Gamma_{\pi}(s)+\mathcal{K}_{\theta} \theta_{\pi}(s)\right|^{2}
$$

### 3.2 Unitarity

- Coupled channel analysis up to $\sqrt{ } \mathbf{s} \sim 1.4 \mathrm{GeV}$ : Mushkhelishvili-Omnès approach Inputs: I=0, S-wave $\pi$ m and KK data

Donoghue, Gasser, Leutwyler'90 Moussallam'99
See also Osset \& Oller'98 Lahde \& Meissner'06

Daub, Dreiner, Hanart, Kubis, Meissner'13
Celis, Cirigliano, E.P.'14

- Unitarity $\square$ the discontinuity of the form factor is known


$$
\Rightarrow \quad \operatorname{Im} F_{n}(s)=\sum_{m=1}^{2} T_{n m}^{*}(s) \sigma_{m}(s) F_{m}(s)
$$

$$
n=\pi \pi, K \bar{K}
$$

Scattering matrix:

$$
\binom{\pi \pi \rightarrow \pi \pi, \pi \pi \rightarrow \boldsymbol{K} \overline{\boldsymbol{K}}}{\boldsymbol{K} \overline{\boldsymbol{K}} \rightarrow \pi \pi, \boldsymbol{K} \overline{\boldsymbol{K}} \rightarrow \boldsymbol{K} \overline{\boldsymbol{K}}}
$$

### 3.3 Inputs for the coupled channel analysis

- Inputs : $\pi \pi \rightarrow \pi \pi, \boldsymbol{K} \overline{\boldsymbol{K}}$

- A large number of theoretical analyses Descotes-Genon et al'01, Kaminsky et al'01, Buettiker et al'03, Garcia-Martin et al'09, Colangelo et al.' 11 and all agree
- 3 inputs: $\delta_{\pi}(\mathrm{s}), \delta_{K}(\mathrm{~s}), \eta$ from $B$. Moussallam $\longrightarrow$ reconstruct $T$ matrix


### 3.4 Dispersion relations

- General solution to Mushkhelishvili-Omnès problem:

$$
\binom{F_{\pi}(s)}{\frac{2}{\sqrt{3}} F_{K}(s)}=\left(\begin{array}{cc}
C_{1}(s) & D_{1}(s) \\
C_{2}(s) & D_{2}(s)
\end{array}\right)\binom{P_{F}(s)}{Q_{F}(s)}
$$

Canonical solution falling as $1 / \mathrm{s}$ for large s (obey unsubtracted dispersion relations)

Polynomial determined from a matching to ChPT + lattice

## Canonical solution $X(s)=C(s), D(s)$ :

- Knowing the discontinuity of $X(s) \Rightarrow$ write a dispersion relation for it
- Analyticity of the FFs: $\mathrm{X}(\mathrm{z})$ is
- real for $z<S_{\text {th }}$
- has a branch cut for $z>s_{\text {th }}$
- analytic for complex z
- Cauchy Theorem and Schwarz reflection principle:

$$
\begin{aligned}
X(s) & =\frac{1}{\pi} \oint_{C} d z \frac{X(z)}{z-s} \\
& =\frac{1}{2 i \pi} \int_{s_{s_{h}}=4 M_{\pi}^{2}}^{\Lambda^{2}} d z \frac{\operatorname{disc}[F(z)]}{z-s-i \varepsilon}+\frac{1}{2 i \pi} \int_{|z|=\Lambda^{2}} d z \frac{F(z)}{z-s}
\end{aligned}
$$

$$
\Lambda \rightarrow \infty
$$

$$
X(s)=\frac{1}{\pi} \int_{4 M_{\pi}^{2}}^{\infty} d z \frac{\operatorname{Im}[X(z)]}{z-s-i \varepsilon}
$$

X(s) can be reconstructed everywhere from the knowledge of $\operatorname{ImX}(\mathrm{s})$

### 3.4 Dispersion relations

- General solution to Mushkhelishvili-Omnès problem:

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C_{1}(s) & D_{1}(s) \\
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\end{array}\right)\binom{P_{F}(s)}{Q_{F}(s)}
$$

Canonical solution falling as $1 / \mathrm{s}$ for large s (obey unsubtracted dispersion relations)

- Canonical solution found by solving the dispersive integral equations iteratively starting with Omnès functions

$$
X(s)=C(s), D(s)
$$

$$
\Omega_{\pi, K}(s) \equiv \exp \left[\frac{s}{\pi} \int_{4 M_{\pi}^{2}}^{\infty} \frac{d t}{t} \frac{\delta_{\pi, K}(t)}{(t-s)}\right]=\boldsymbol{X}(\boldsymbol{s})
$$

### 3.4 Dispersion relations

- General solution to Mushkhelishvili-Omnès problem:

$$
\binom{F_{\pi}(s)}{\frac{2}{\sqrt{3}} F_{K}(s)}=\left(\begin{array}{cc}
C_{1}(s) & D_{1}(s) \\
C_{2}(s) & D_{2}(s)
\end{array}\right)\binom{P_{F}(s)}{Q_{F}(s)}
$$

Canonical solution falling as $1 / \mathrm{s}$ for large s (obey unsubtracted dispersion relations)

- Canonical solution found by solving the dispersive integral equations iteratively starting with Omnès functions

Polynomial determined from a matching to ChPT + lattice

$$
X(s)=C(s), D(s)
$$



## Determination of the polynomial

- Fix the polynomial with requiring

$$
F_{P}(s) \rightarrow 1 / s+\mathrm{ChPT}:
$$

Brodsky \& Lepage'80

- Feynman-Hellmann theorem:

$$
\Gamma_{P}(0)=\left(m_{u} \frac{\partial}{\partial m_{u}}+m_{d} \frac{\partial}{\partial m_{d}}\right) M_{P}^{2}
$$

$$
\Delta_{P}(0)=\left(m_{s} \frac{\partial}{\partial m_{s}}\right) M_{P}^{2}
$$

- At LO in ChPT:

$$
\begin{aligned}
& M_{\pi^{+}}^{2}=\left(m_{\mathrm{u}}+m_{\mathrm{d}}\right) B_{0}+O\left(m^{2}\right) \\
& M_{K^{+}}^{2}=\left(m_{\mathrm{u}}+m_{\mathrm{s}}\right) B_{0}+O\left(m^{2}\right) \\
& M_{K^{0}}^{2}=\left(m_{\mathrm{d}}+m_{\mathrm{s}}\right) B_{0}+O\left(m^{2}\right)
\end{aligned}
$$

## Determination of the polynomial

- Fix the polynomial with requiring

$$
F_{P}(s) \rightarrow 1 / s+\text { ChPT: }
$$

Brodsky \& Lepage'80

- Feynman-Hellmann theorem:

$$
\Gamma_{P}(0)=\left(m_{u} \frac{\partial}{\partial m_{u}}+m_{d} \frac{\partial}{\partial m_{d}}\right) M_{P}^{2}
$$

$$
\Delta_{P}(0)=\left(m_{s} \frac{\partial}{\partial m_{s}}\right) M_{P}^{2}
$$

- At LO in ChPT:

$$
\begin{aligned}
& M_{\pi^{+}}^{2}=\left(m_{\mathrm{u}}+m_{\mathrm{d}}\right) B_{0}+O\left(m^{2}\right) \\
& M_{K^{+}}^{2}=\left(m_{\mathrm{u}}+m_{\mathrm{s}}\right) B_{0}+O\left(m^{2}\right) \\
& M_{K^{0}}^{2}=\left(m_{\mathrm{d}}+m_{\mathrm{s}}\right) B_{0}+O\left(m^{2}\right)
\end{aligned}
$$

$$
\begin{aligned}
P_{\Gamma}(s) & =\Gamma_{\pi}(0)=M_{\pi}^{2}+\cdots \\
Q_{\Gamma}(s) & =\frac{2}{\sqrt{3}} \Gamma_{K}(0)=\frac{1}{\sqrt{3}} M_{\pi}^{2}+\cdots \\
P_{\Delta}(s) & =\Delta_{\pi}(0)=0+\cdots \\
Q_{\Delta}(s) & =\frac{2}{\sqrt{3}} \Delta_{K}(0)=\frac{2}{\sqrt{3}}\left(M_{K}^{2}-\frac{1}{2} M_{\pi}^{2}\right)+\cdots
\end{aligned}
$$

## Determination of the polynomial

- At LO in ChPT:

$$
\begin{aligned}
& M_{\pi^{+}}^{2}=\left(m_{\mathrm{u}}+m_{\mathrm{d}}\right) B_{0}+O\left(m^{2}\right) \\
& M_{K^{+}}^{2}=\left(m_{\mathrm{u}}+m_{\mathrm{s}}\right) B_{0}+O\left(m^{2}\right) \\
& M_{K^{0}}^{2}=\left(m_{\mathrm{d}}+m_{\mathrm{s}}\right) B_{0}+O\left(m^{2}\right)
\end{aligned}
$$

- For the scalar FFs:

$$
\begin{aligned}
P_{\Gamma}(s) & =\Gamma_{\pi}(0)=M_{\pi}^{2}+\cdots \\
Q_{\Gamma}(s) & =\frac{2}{\sqrt{3}} \Gamma_{K}(0)=\frac{1}{\sqrt{3}} M_{\pi}^{2}+\cdots \\
P_{\Delta}(s) & =\Delta_{\pi}(0)=0+\cdots \\
Q_{\Delta}(s) & =\frac{2}{\sqrt{3}} \Delta_{K}(0)=\frac{2}{\sqrt{3}}\left(M_{K}^{2}-\frac{1}{2} M_{\pi}^{2}\right)+\cdots
\end{aligned}
$$

- Problem: large corrections in the case of the kaons! $\square$ Use lattice QCD to determine the SU(3) LECs

$$
\begin{aligned}
& \Gamma_{K}(0)=(0.5 \pm 0.1) M_{\pi}^{2} \\
& \Delta_{K}(0)=1_{-0.05}^{+0.15}\left(M_{K}^{2}-1 / 2 M_{\pi}^{2}\right)
\end{aligned}
$$







## 4. Results



### 4.2 Bounds



Celis, Cirigliano, E.P.'14

## Bound:

$$
\sqrt{\left|Y_{\mu \tau}^{h}\right|^{2}+\left|Y_{\tau \mu}^{h}\right|^{2}} \leq 0.13
$$

| Process | $\left(\mathrm{BR} \times 10^{8}\right) 90 \% \mathrm{CL}$ | $\sqrt{\left\|Y_{\mu \tau}^{h}\right\|^{2}+\left\|Y_{\tau \mu}^{h}\right\|^{2}}$ | Operator(s) |
| :---: | :---: | :---: | :---: |
| $\tau \rightarrow \mu \gamma$ | $<4.4[88]$ | $<0.016$ | Dipole |
| $\tau \rightarrow \mu \mu \mu$ | $<2.1[89]$ | $<0.24$ | Dipole |
| $\tau \rightarrow \mu \pi^{+} \pi^{-}$ | $<2.1[86]$ | $<0.13$ | Scalar, Gluon, Dipole |
| $\tau \rightarrow \mu \rho$ | $<1.2[85]$ | $<0.13$ | Scalar, Gluon, Dipole |
| $\tau \rightarrow \mu \pi^{0} \pi^{0}$ | $<1.4 \times 10^{3}[87]$ | $<6.3$ | Scalar, Gluon |
| $\tau$ |  |  |  |

Less stringent but more robust handle on LFV Higgs couplings

### 4.3 Impact of our results



- Dispersive treatment of hadronic part $\square$ bound reduced by one order of magnitude!
- ChPT, EFT only valid at low energy for $\mathrm{p} \ll \Lambda=4 \pi f_{\pi} \sim \mathbf{1} \mathbf{~ G e V}$ $\rightleftarrows$ not valid up to $E=\left(m_{\tau}-m_{\mu}\right)$ !


### 4.4 Constraints in the $\tau \mu$ sector



- Constraints from LE:
$>\tau \rightarrow \mu \gamma$ : best constraints but loop level $\Rightarrow$ sensitive to UV completion of the theory
$>\tau \rightarrow \mu \pi \pi$ : tree level diagrams $\Rightarrow$ robust handle on LFV
- Constraints from HE:

LHC wins for $\tau \mu$ !

- Opposite situation for $\mu \mathrm{e}$ !
- For LFV Higgs and nothing else: LHC bound

$$
\begin{aligned}
& B R(\tau \rightarrow \mu \gamma)<2.2 \times 10^{-9} \\
& B R(\tau \rightarrow \mu \pi \pi)<1.5 \times 10^{-11}
\end{aligned}
$$

### 4.5 Hint of New Physics in $h \rightarrow \tau \mu$ ?

CMS'16


### 4.5 Hint of New Physics in $h \rightarrow \tau \mu$ ?

CMS'17


### 4.6 What if $\tau \rightarrow \mu(\mathrm{e}) \pi \pi$ is observed?

Talk by J. Zupan
@ KEK-FF2014FALL


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### 4.6 What if $\tau \rightarrow \mu(\mathrm{e}) \pi \pi$ is observed?

- $\quad \tau \rightarrow \mu(e) \pi \pi$ sensitive to $\mathrm{Y}_{\mu \tau}$ but also to $Y_{u, d, s}$ !
- $Y_{u, d, s}$ poorly bounded
- For $Y_{\mu, d, s}$ at their $S M$ values :

$$
\begin{aligned}
& \operatorname{Br}\left(\tau \rightarrow \mu \pi^{+} \pi^{-}\right)<1.6 \times 10^{-11}, \operatorname{Br}\left(\tau \rightarrow \mu \pi^{0} \pi^{0}\right)<4.6 \times 10^{-12} \\
& \operatorname{Br}\left(\tau \rightarrow e \pi^{+} \pi^{-}\right)<2.3 \times 10^{-10}, \operatorname{Br}\left(\tau \rightarrow e \pi^{0} \pi^{0}\right)<6.9 \times 10^{-11}
\end{aligned}
$$

- But for $Y_{\mu, d, s}$ at their upper bound:

$$
\begin{aligned}
& \operatorname{Br}\left(\tau \rightarrow \mu \pi^{+} \pi^{-}\right)<3.0 \times 10^{-8}, \operatorname{Br}\left(\tau \rightarrow \mu \pi^{0} \pi^{0}\right)<1.5 \times 10^{-8} \\
& \operatorname{Br}\left(\tau \rightarrow e \pi^{+} \pi^{-}\right)<4.3 \times 10^{-7}, \operatorname{Br}\left(\tau \rightarrow e \pi^{0} \pi^{0}\right)<2.1 \times 10^{-7}
\end{aligned}
$$

below present experimental limits!

- If discovered $\Rightarrow$ upper limit on $Y_{u, d, s}$ !
$\Rightarrow$ Interplay between high-energy and low-energy constraints!


### 4.7 Discriminating power of $\tau \rightarrow \mu(\mathrm{e}) \pi \pi$ decays



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### 4.7 Discriminating power of $\tau \rightarrow \mu(\mathrm{e}) \pi \pi$ decays





Different distributions according to the operator!

## 5. Conclusion and Outlook

- Direct searches for new physics at the TeV-scale at LHC by ATLAS and CMS $\square$ energy frontier
- Probing new physics orders of magnitude beyond that scale and helping to decipher possible TeV-scale new physics requires to work hard on the intensity and precision frontiers
- Charged LFV are a very important probe of new physics
> Extremely small SM rates
> Experimental results at low energy are very precise
$\Rightarrow$ very high scale sensitivity
- CLFV decays excellent model discriminating tools especially $\tau$ decays Hadronic decays such as $\tau \rightarrow \mu(e) \pi \tau$ important!
- To consider hadronic decays, need to control the hadronic uncertainties: need to know hadronic matrix elements, form factors etc.
- For $\tau \rightarrow \mu(e) \pi \pi$ : need to know the $\pi \pi$ form factors
$\square$ Use dispersion relations
- Dispersion relations rely on analyticity, unitarity and crossing symmetry $\Rightarrow$ Rigorous treatment of two and three hadronic final state
- $\tau \rightarrow \mu(e) \pi \pi$ gives interesting constraints on LFV new physics operators involving quarks
- Interplay low energy and collider physics: LFV of the Higgs boson
- Complementarity with LFC sector: EDMs, g-2 and colliders:
$\Rightarrow$ New physics models usually strongly correlate these sectors

6. Back-up

## T matrix parametrization

$$
S_{m n}=\delta_{m n}+2 i \sqrt{\sigma_{m} \sigma_{n}} T_{m n}
$$

$S=\left(\begin{array}{cc}\cos \gamma e^{2 i \delta_{\pi}} & i \sin \gamma e^{i\left(\delta_{\pi}+\delta_{K}\right)} \\ i \sin \gamma e^{i\left(\delta_{\pi}+\delta_{K}\right)} & \cos \gamma e^{2 i \delta_{K}}\end{array}\right)$

- Inelasticity: $\eta_{0}^{0} \equiv \cos \gamma$.
- $\delta_{\pi}(s): \pi \pi S$ wave phase shift
- $\delta_{K}(s)$ : KK $S$ wave phase shift




### 3.1 Constraints from $\tau \rightarrow \mu \pi \pi$

- Contribution from dipole diagrams


$$
L_{e f f}=c_{L} Q_{L \gamma}+c_{R} Q_{R \gamma}+h . c .
$$

with the dim-5 EM penguin operators :

$$
Q_{L \gamma, R \gamma}=\frac{e}{8 \pi^{2}} m_{\tau}\left(\mu \sigma^{\alpha \beta} P_{L, R} \tau\right) F_{\alpha \beta}
$$

- $\frac{d \Gamma\left(\tau \rightarrow \ell \pi^{+} \pi^{-}\right)}{d \sqrt{s}}=\frac{\alpha^{2}\left|F_{V}(s)\right|^{2}\left(\left|c_{L}\right|^{2}+\left|c_{R}\right|^{2}\right)}{768 \pi^{5} m_{\tau}} \frac{\left(s-4 m_{\pi}^{2}\right)^{3 / 2}\left(m_{\tau}^{2}-s\right)^{2}\left(s+2 m_{\tau}^{2}\right)}{s^{2}}$
with the vector form factor :

$$
C_{L, R}=f\left(Y_{q u}\right)
$$

$$
\left\langle\pi^{+}\left(p_{\pi^{+}}\right) \pi^{-}\left(p_{\pi^{-}}\right)\right| \frac{1}{2}\left(\bar{u} \gamma^{\alpha} u-\bar{d} \gamma^{\alpha} d\right)|0\rangle \equiv F_{V}(s)\left(p_{\pi^{+}}-p_{\pi^{-}}\right)^{\alpha}
$$

- Diagram only there in the case of $\tau^{-} \rightarrow \mu^{-} \pi^{+} \pi^{-}$absent for $\tau^{-} \rightarrow \mu^{-} \pi^{0} \pi^{0}$ $\Rightarrow$ neutral mode more model independent


## Determination of $\mathrm{F}_{\mathrm{V}}(\mathrm{s})$

- Vector form factor
> Precisely known from experimental measurements

$$
\boldsymbol{e}^{+} e^{-} \rightarrow \pi^{+} \pi^{-} \text {and } \tau^{-} \rightarrow \pi^{0} \pi^{-} \boldsymbol{v}_{\tau} \text { (isospin rotation) }
$$

> Theoretically: Dispersive parametrization for $\mathrm{F}_{\mathrm{V}}(\mathrm{s})$
Guerrero, Pich'98, Pich, Portolés'08

$$
\left.F_{V}(s)=\exp \left[\lambda_{V}^{\prime} \frac{s}{m_{\pi}^{2}}+\frac{1}{2}\left(\lambda_{V}^{\prime \prime}-\lambda_{V}^{\prime 2}\right)\left(\frac{s}{m_{\pi}^{2}}\right)^{2}+\frac{s^{3}}{\pi} \int_{4 m_{\pi}^{2}}^{\infty} \frac{d s^{\prime}}{s^{\prime 3}} \frac{\phi_{V}\left(s^{\prime}\right)}{\left(s^{\prime} \mathcal{F}-i \varepsilon\right)}\right]\right]
$$

Extracted from a model including 3 resonances $\rho(770)$, $\rho^{\prime}(1465)$ and $\rho "(1700)$ fitted to the data
>Subtraction polynomial + phase determined from a fit to the Belle data $\boldsymbol{\tau}^{-} \rightarrow \boldsymbol{\pi}^{0} \boldsymbol{\pi}^{-} \boldsymbol{v}_{\boldsymbol{\tau}}$

## Determination of $\mathrm{F}_{\mathrm{V}}(\mathrm{s})$



Determination of $\mathrm{F}_{\mathrm{V}}(\mathrm{s})$ thanks to precise measurements from Belle!

## CPV AND FV HIGGS

## COUPLINGS TO SM FERMIONS

- if SM an EFT, the Yukawas get corrected by higher dim. ops

$$
\mathcal{L}_{S M}=-\left[\lambda_{i j}\left(\bar{f}_{L}^{i} f_{R}^{j}\right) H+\text { h.c. }\right]
$$

$$
\Delta \mathcal{L}_{Y}=-\frac{\lambda_{i j}^{\prime}}{\Lambda^{2}}\left(\bar{f}_{L}^{i} f_{R}^{j}\right) H\left(H^{\dagger} H\right)+\text { h.c. }+\cdots
$$

- decouples mass terms from yukawas

$$
\mathcal{L}_{Y}=-m_{i} \bar{f}_{L}^{i} f_{R}^{i}-Y_{i j}\left(\bar{f}_{L}^{i} f_{R}^{j}\right) h+h . c .+\cdots
$$

- can lead to flavor violating Higgs decays
- can lead to CPV Higgs decays
- different models lead to different patterns of flavor diagonal and flavor violating Yukawas


## A GENERAL BENCHMARK

- what is a reasonable aim for precision on $Y_{i j}$ ?
- if off-diagonals are large $\Rightarrow$ spectrum in general not hierarchical
- no tuning, if

$$
\left|Y_{\tau \mu} Y_{\mu \tau}\right| \lesssim \frac{m_{\mu} m_{\tau}}{v^{2}}
$$

- in concrete models it will be typically further suppressed parametrically


## SUMMARY OF MODELS

- an example: higgs couplings to 2nd\&3rd gen. charged leptons
adapted from Dery, Efrati, Hochberg, Nir, 1302.3229 and extended

| Model | $\hat{\mu}_{\tau \tau}$ | $\left(\hat{\mu}_{\mu \mu} / \hat{\mu}_{\tau \tau}\right) /\left(m_{\mu}^{2} / m_{\tau}^{2}\right)$ | $\hat{\mu}_{\mu \tau} / \hat{\mu}_{\tau \tau}$ |
| :---: | :---: | :---: | :---: |
| SM | 1 | 1 | 0 |
| NFC | $\left(V_{h \ell}^{*} v / v_{\ell}\right)^{2}$ | 1 | 0 |
| MSSM | $(\sin \alpha / \cos \beta)^{2}$ | 1 | 0 |
| MFV | $1+2 a v^{2} / \Lambda^{2}$ | $1-4 b m_{\tau}^{2} / \Lambda^{2}$ | 0 |
| FN | $1+\mathcal{O}\left(v^{2} / \Lambda^{2}\right)$ | $1+\mathcal{O}\left(v^{2} / \Lambda^{2}\right)$ | $\mathcal{O}\left(\left\|U_{23}\right\|^{2} v^{4} / \Lambda^{4}\right)$ |
| GL | 9 | $25 / 9$ | $\mathcal{O}\left(\hat{\mu}_{\mu \mu} / \hat{\mu}_{\tau \tau}\right)$ |
| RS $(i)$ | $1+\mathcal{O}\left(\bar{Y}^{2} v^{2} / m_{K K}^{2}\right)$ | $1+\mathcal{O}\left(\bar{Y}^{2} v^{2} / m_{K K}^{2}\right)$ | $\mathcal{O}\left(\bar{Y}^{2} v^{2} / m_{K K}^{2}\right) \sqrt{m_{\tau} / m_{\mu}}$ |
| RS (ii) | $1+\mathcal{O}\left(\bar{Y}^{2} v^{2} / m_{K K}^{2}\right)$ | $1+\mathcal{O}\left(\bar{Y}^{2} v^{2} / m_{K K}^{2}\right)$ | $\mathcal{O}\left(\bar{Y}^{2} v^{2} / m_{K K}^{2}\right)$ |
| PGB (1 rep.) | $1-v^{2} / f^{2}$ | 1 | 0 |

### 3.3 Handles

- Two handles:
$>$ Branching ratios: $\boldsymbol{R}_{F, M} \equiv \frac{\Gamma(\tau \rightarrow \boldsymbol{F})}{\Gamma\left(\tau \rightarrow F_{M}\right)}$ with $\mathrm{F}_{\mathrm{M}}$ dominant LFV mode for model M
$>$ Spectra for $>2$ bodies in the final state:

$$
\frac{d B R\left(\tau \rightarrow \mu \pi^{+} \pi^{-}\right)}{d \sqrt{s}} \text { and } d R_{\pi^{+} \pi^{-}} \equiv \frac{1}{\Gamma(\tau \rightarrow \mu \gamma)} \frac{d \Gamma\left(\tau \rightarrow \mu \pi^{+} \pi^{-}\right)}{d \sqrt{s}}
$$

- Benchmarks:
$>$ Dipole model: $\mathrm{C}_{\mathrm{D}} \neq 0, \mathrm{C}_{\text {else }}=0$
$\Rightarrow$ Scalar model: $\mathrm{C}_{\mathrm{S}} \neq 0, \mathrm{C}_{\text {else }}=0$
$>$ Vector (gamma, $Z$ ) model: $C_{V} \neq 0, C_{\text {else }}=0$
$>$ Gluonic model: $\mathrm{C}_{\mathrm{GG}} \neq 0, \mathrm{C}_{\text {else }}=0$


### 3.3 Branching ratios

- Two handles:

Wo handles:
$>$ Branching ratios: $\boldsymbol{R}_{F, M} \equiv \frac{\Gamma(\tau \rightarrow \boldsymbol{F})}{\Gamma\left(\tau \rightarrow F_{M}\right)}$ with $\mathrm{F}_{\mathrm{M}}$ dominant LFV mode for model M

|  |  | $\mu \pi^{+} \pi^{-}$ | $\mu \rho$ | $\mu f_{0}$ | $3 \mu$ | $\mu \gamma$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| D | $R_{F, D}$ | $0.26 \times 10^{-2}$ | $0.22 \times 10^{-2}$ | $0.13 \times 10^{-3}$ | $0.22 \times 10^{-2}$ | 1 |
| $\uparrow$ | BR | $<1.1 \times 10^{-10}$ | $<9.7 \times 10^{-11}$ | $<5.7 \times 10^{-12}$ | $<9.7 \times 10^{-11}$ | $<4.4 \times 10^{-8}$ |

- $\quad \rho(770)$ resonance ( $\mathrm{JPC}^{\mathrm{PC}}=1-$ ): cut in the $\pi^{+} \pi^{-}$invariant mass: $587 \mathrm{MeV} \leq \sqrt{s} \leq 962 \mathrm{MeV}$
- $\mathrm{f}_{0}(980)$ resonance $\left(\mathrm{JPC}^{\mathrm{P}}=0^{++}\right)$: cut in the $\pi^{+} \pi^{-}$invariant mass: $906 \mathrm{MeV} \leq \sqrt{s} \leq 1065 \mathrm{MeV}$


### 3.3 Branching ratios

- Two handles:
$>$ Branching ratios: $\boldsymbol{R}_{F, M} \equiv \frac{\Gamma(\tau \rightarrow \boldsymbol{F})}{\Gamma\left(\tau \rightarrow F_{M}\right)}$ with $\mathrm{F}_{\mathrm{M}}$ dominant LFV mode for model M

|  |  | $\mu \pi^{+} \pi^{-}$ | $\mu \rho$ | $\mu f_{0}$ | $3 \mu$ | $\mu \gamma$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| D | $\begin{gathered} \hline R_{F, D} \\ \mathrm{BR} \end{gathered}$ | $\begin{gathered} 0.26 \times 10^{-2} \\ <1.1 \times 10^{-10} \end{gathered}$ | $\begin{gathered} 0.22 \times 10^{-2} \\ <9.7 \times 10^{-11} \end{gathered}$ | $\begin{gathered} 0.13 \times 10^{-3} \\ <5.7 \times 10^{-12} \end{gathered}$ | $\begin{gathered} 0.22 \times 10^{-2} \\ <9.7 \times 10^{-11} \end{gathered}$ | $\begin{gathered} 1 \\ <4.4 \times 10^{-8} \end{gathered}$ |
| S | $\begin{gathered} R_{F, S} \\ \mathrm{BR} \end{gathered}$ | $\begin{gathered} 1 \\ <2.1 \times 10^{-8} \end{gathered}$ | $\begin{gathered} 0.28 \\ <\quad 5.9 \times 10^{-9} \end{gathered}$ | $\begin{gathered} 0.7 \\ <1.47 \times 10^{-8} \\ \hline \end{gathered}$ |  |  |
| $\mathrm{V}^{(\gamma)}$ | $\begin{gathered} R_{F, V(\gamma)} \\ \text { BR } \end{gathered}$ | $\begin{gathered} 1 \\ <1.4 \times 10^{-8} \end{gathered}$ | $\begin{gathered} 0.86 \\ <1.2 \times 10^{-8} \end{gathered}$ | $\begin{gathered} 0.1 \\ <1.4 \times 10^{-9} \end{gathered}$ |  |  |
| Z | $\begin{gathered} R_{F, Z} \\ \mathrm{BR} \end{gathered}$ | $\begin{gathered} 1 \\ <\quad 1.4 \times 10^{-8} \end{gathered}$ | $\begin{gathered} 0.86 \\ <1.2 \times 10^{-8} \end{gathered}$ | $\begin{gathered} 0.1 \\ <1.4 \times 10^{-9} \end{gathered}$ |  |  |
| G | $\begin{gathered} R_{F, G} \\ \text { BR } \end{gathered}$ | $\begin{gathered} 1 \\ <2.1 \times 10^{-8} \end{gathered}$ | $\begin{gathered} 0.41 \\ <8.6 \times 10^{-9} \end{gathered}$ | $\begin{gathered} 0.41 \\ <8.6 \times 10^{-9} \end{gathered}$ |  |  |
|  | ark | $7$ |  |  |  |  <br> 56 |

### 4.1 Constraints from $\tau \rightarrow 1 \mathrm{P}$

- Tree level Higgs exchange
$>\eta, \eta$ '

$$
\Gamma\left(\tau \rightarrow \ell \eta^{(\prime)}\right)=\frac{\bar{\beta}\left(m_{\tau}^{2}-m_{\eta}^{2}\right)\left(\left|Y_{\mu \tau}^{A}\right|^{2}+\left|Y_{\tau \mu}^{A}\right|^{2}\right)}{256 \pi M_{A}^{4} v^{2} m_{\tau}}\left[\left(y_{u}^{A}+y_{d}^{A}\right) h_{\eta^{\prime}}^{q}+\sqrt{2} y_{s}^{A} h_{\eta^{\prime}}^{s}-\sqrt{2} a_{\eta^{\prime}} \sum_{q=c, b, t} y_{q}^{A}\right]^{2}
$$

with the decay constants :

$$
\begin{aligned}
& \left\langle\eta^{(\prime)}(p)\right| \bar{q} \gamma_{5} q|0\rangle=-\frac{i}{2 \sqrt{2} m_{q}} h_{\eta^{(\prime)}}^{q} \quad\left\langle\eta^{(\prime)}(p)\right| \bar{s} \gamma_{5} s|0\rangle=-\frac{i}{2 m_{s}} h_{\eta^{(\prime)}}^{s} \\
& \left\langle\eta^{(\prime)}(p)\right| \frac{\alpha_{s}}{4 \pi} G_{a}^{\mu \nu} \widetilde{G}_{\mu \nu}^{a}|0\rangle=a_{\eta^{(\prime)}}
\end{aligned}
$$

$$
>\pi: \quad \Gamma\left(\tau \rightarrow \ell \pi^{0}\right)=\frac{f_{\pi}^{2} m_{\pi}^{4} m_{\tau}}{256 \pi M_{A}^{4} v^{2}}\left(\left|Y_{\tau \mu}^{A}\right|^{2}+\left|Y_{\mu \tau}^{A}\right|^{2}\right)\left(y_{u}^{A}-y_{d}^{A}\right)^{2}
$$

### 3.1 Constraints from $\tau \rightarrow \mu \pi \pi$

- Tree level Higgs exchange

$$
\begin{aligned}
& \left\langle\pi^{+} \pi^{-}\right| m_{s} \bar{s} s|0\rangle \equiv \Delta_{\pi}(s) \\
& \theta_{\mu}^{\mu}=-9 \frac{\alpha_{s}}{8 \pi} G_{\mu \nu}^{a} G_{a}^{\mu \nu}+\sum_{q=u, d, s} m_{q} \bar{q} q \\
& \square \Gamma_{\tau \rightarrow \mu \pi \pi} \propto \int\left|\Gamma_{\pi}(s)+\Delta_{\pi}(s)+\theta_{\pi}(s)\right|^{2} Y_{\tau u}^{2} \quad \text { with } \quad s=\left(p_{\pi^{+}}+p_{\pi^{-}}\right)^{2}
\end{aligned}
$$

### 4.5 Interplay between LHC \& Low Energy

- If real what type of NP?
- If $h \rightarrow \tau \mu$ due to loop corrections:
- extra charged particles necessary
$-\tau \rightarrow \mu \gamma$ too large

- $h \rightarrow \tau \mu$ possible to explain if extra scalar doublet:
$\Rightarrow 2 H D M$ of type III

- Constraints from $\tau \rightarrow \mu \nu$ important! $\Rightarrow$ Belle II


### 4.5 Interplay between LHC \& Low Energy

- 2HDMs with gauged $L_{\mu}-L_{\tau}$ $\Rightarrow Z^{\prime}$, explain anomalies for
$-\mathrm{h} \rightarrow \tau \mu$
$-B \rightarrow K^{*} \mu \mu$
- $\mathrm{R}_{\mathrm{K}}=\mathrm{B} \rightarrow \mathrm{K} \mu \mu / \mathrm{B} \rightarrow \mathrm{Kee}$
- Constraints from $\tau \rightarrow 3 \mu$ crucial $\Rightarrow$ Belle II, LHCb
- See also:

Aristizabal-Sierra \& Vicente'14, Lima et al'15,
Omhura, Senaha, Tobe '15

Altmannshofer \& Straub'14, Crivellin et al'15
Crivellin, D'Ambrosio, Heeck.'15


### 4.5 Hint of New Physics in $h \rightarrow \tau \mu$ ? See talk by $A$. Crivellin

$$
B R(h \rightarrow \tau \mu)=\left(0.84_{-0.37}^{+0.39}\right) \% \quad @ 2.4 \sigma
$$

CMS'15


$$
B R(h \rightarrow \tau \mu)=(0.53 \pm 0.51) \% \text { @1 } \sigma
$$

ATLAS'15

$\square$

$$
B R(h \rightarrow \tau \mu)=\left(-0.76_{-0.84}^{+0.81}\right) \%
$$

### 2.2 CLFV processes: tau decays

- Several processes: $\tau \rightarrow \ell \gamma, \tau \rightarrow \ell_{\alpha} \bar{\ell}_{\beta} \ell_{\beta}, \tau \rightarrow \ell \boldsymbol{K}_{\boldsymbol{R}}, S, V, P \bar{P}, \ldots$

- Expected sensitivity $10^{-9}$ or better at LHCb , Belle II?


## Determination of the polynomial

- For $\theta_{p}$ enforcing the asymptotic constraint is not consistent with ChPT The unsubtracted DR is not saturated by the 2 states

Relax the constraints and match to ChPT

$$
\begin{aligned}
P_{\theta}(s) & =2 M_{\pi}^{2}+\left(\dot{\theta}_{\pi}-2 M_{\pi}^{2} \dot{C}_{1}-\frac{4 M_{K}^{2}}{\sqrt{3}} \dot{D}_{1}\right) s \\
Q_{\theta}(s) & =\frac{4}{\sqrt{3}} M_{K}^{2}+\frac{2}{\sqrt{3}}\left(\dot{\theta}_{K}-\sqrt{3} M_{\pi}^{2} \dot{C}_{2}-2 M_{K}^{2} \dot{D}_{2}\right) s
\end{aligned}
$$

with $\dot{f}=\left(\frac{d f}{d s}\right)_{s=0}$

- At LO ChPT: $\dot{\theta}_{\pi, K}=\mathbf{1}$
- Higher orders $\Rightarrow \dot{\theta}_{K}=1.15 \pm 0.1$

