SKETCHING $\eta, \eta' \rightarrow \gamma\gamma^*$ TRANSITION FORM FACTORS

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Motivation

- Understanding strong interactions is still being a challenge for physicists, although scientists have developed the fundamental theory of quarks and gluons, namely, Quantum Chromodynamics (QCD).

- The fundamental degrees of freedom of QCD, quarks and gluons, are not found free in nature, but instead in color singlet composite particles known as hadrons. Quarks inside hadrons acquire mass dynamically; the strong interactions of QCD are responsible for 98% of the mass of the visible matter. These emerging phenomena of hadron matter are dubbed as confinement and dynamical chiral symmetry breaking (DCSB), respectively.

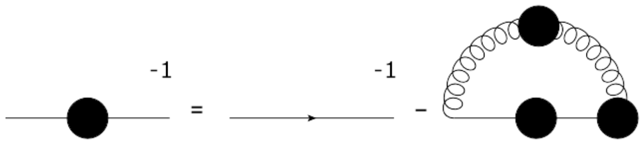
- Due to the non perturbative nature of QCD, unraveling the hadron structure from first principles is an outstanding problem. However, Dyson-Schwinger equations (DSEs), the equations of motion of QCD, combine the IR and UV behavior of the theory at once. Therefore, DSEs are an ideal platform to study quarks and hadrons.

- DSE community has done several efforts on the subject. Nowadays, people have successfully computed the hadron spectra (masses and decay constants), elastic and transition form factors, parton distribution amplitudes (PDAs), parton distribution functions (PDFs), etc.

Outline

1. The basics (DSEs)
   - Quark propagator and Bethe-Salpeter equation
   - Perturbation theory integral representations (PTIRs)
2. Parton distribution amplitudes
3. Transition form factors ($\gamma\gamma^* \rightarrow$ PS)
4. Conclusions and scope

Quark propagator



- The renormalised DSE for the quark propagator (gap equation) is:

$$S^{-1}(p,\zeta) = [\mathcal{Z}_{2F} S_0^{-1}(p)] + \mathcal{Z}_{1F}\int_q^\Lambda g^2 D_{\mu\nu}(p-q,\zeta)\frac{\lambda^a}{2}\gamma_\mu S(q)\Gamma^a_\nu(p,q;\zeta)$$

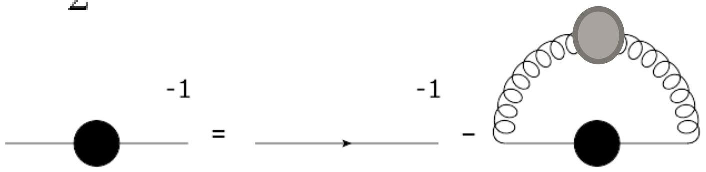
- A general solution is written as:

$$S(p,\zeta) = Z(p^2;\zeta^2)(i\gamma\cdot p + M(p^2))^{-1} = (i\gamma\cdot p\, A(p^2;\zeta^2) + B(p^2;\zeta^2))^{-1}$$

- The simplest, yet symmetry preserving truncation, is the rainbow truncation. With $k = p - q$ and $G(k^2)$ being an effective coupling, we have:

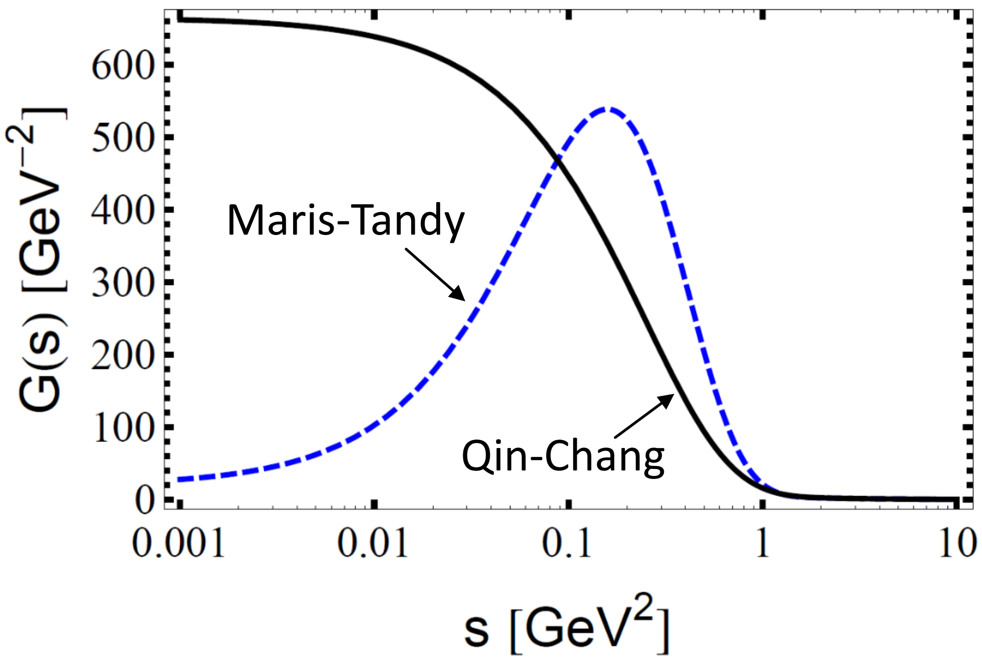
$$S^{-1}(p,\zeta) = [\mathcal{Z}_{2F} S_0^{-1}(p)] + \int_q^\Lambda G((p-q)^2) D^0_{\mu\nu}(p-q,\zeta)\frac{\lambda^a}{2}\gamma_\mu S(q)\frac{\lambda^a}{2}\gamma_\nu,$$

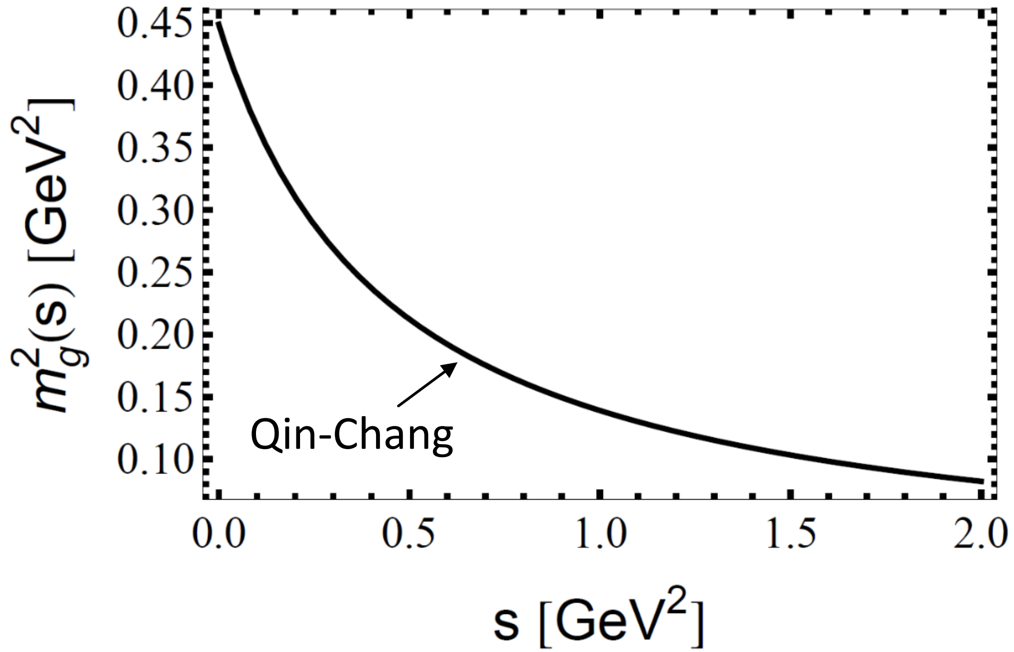
$$\mathcal{Z}_{1F} g^2 D_{\mu\nu}(k)\Gamma^a_\nu(q,p) \to k^2 G(k^2) D^2_{\mu\nu}(k)\frac{\lambda^a}{2}\gamma_\nu\ ,$$



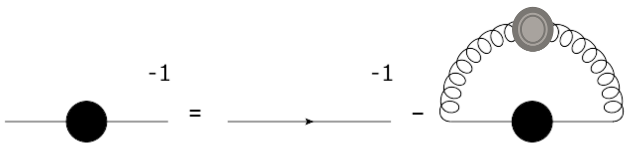
Effective coupling

- G(s) is modeled as in **Phys.Rev. C84, 042202(R) (2011)** by S.-x. Qin, L. Chang et al.
- This coupling, unlike previous DSEs ansätze (**Phys.Rev. C60 (1999) 055214** by P. Maris and P. Tandy, for example), induces a massive non-vanishing gluon in the IR.

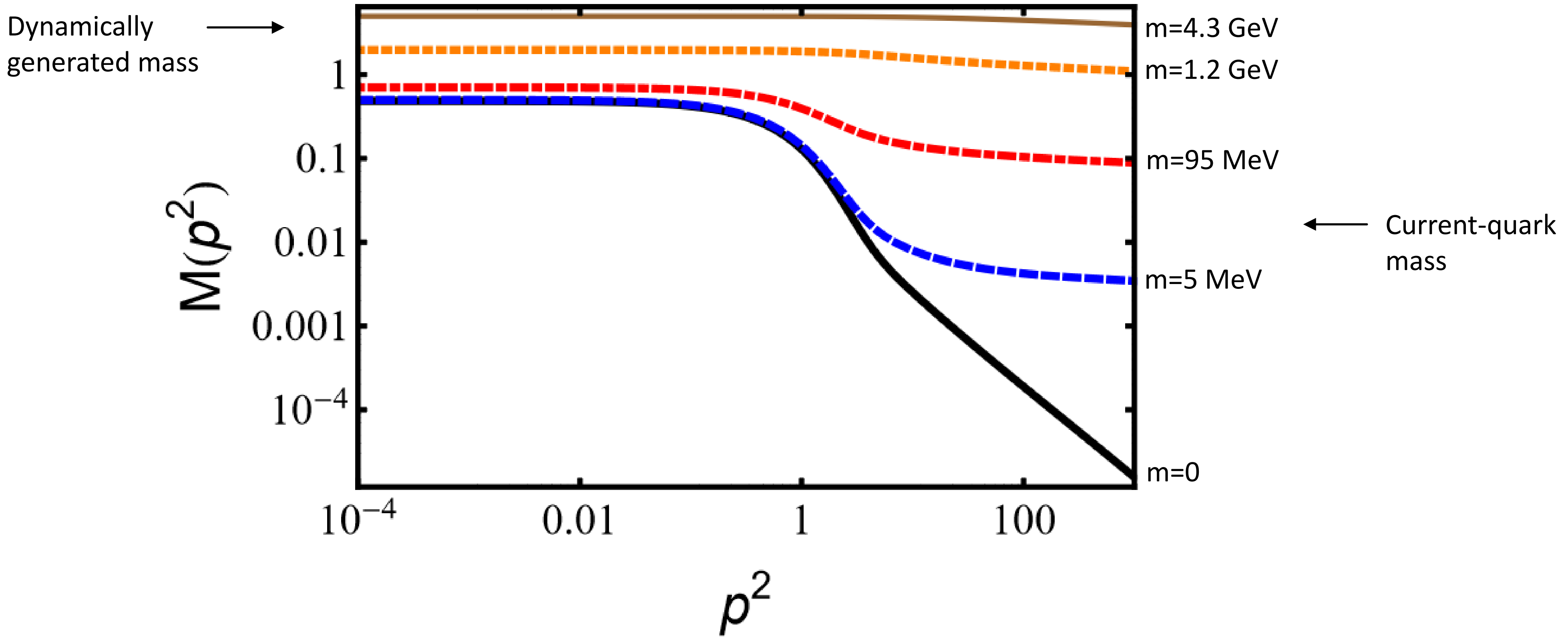


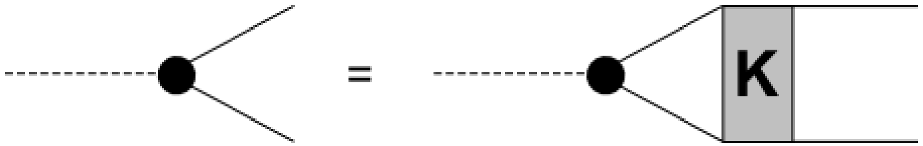


Quark propagator



- **Mass function** for different current-quark masses: The lighter the quark is, the stronger the effect of DCSB is. Even when m=0, a dynamically generated mass appears (this is DCSB).

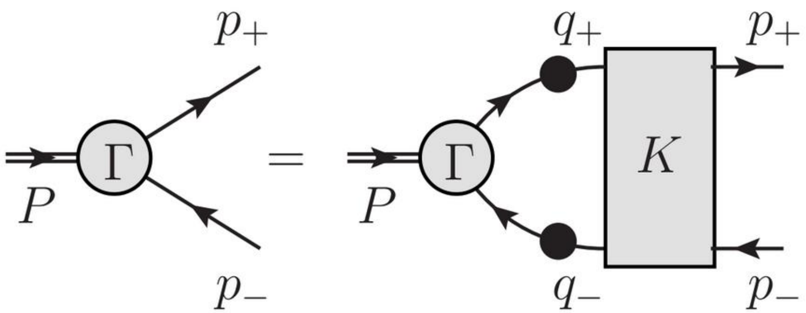




Bethe-Salpeter equation

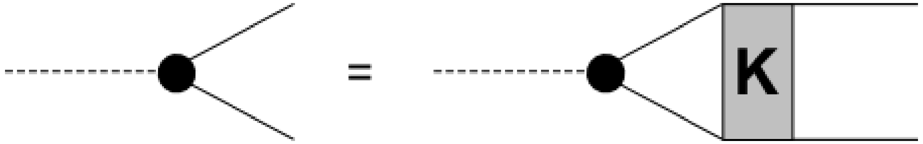
- Quarks and gluons are not found free in nature, they form hadrons. Baryons are color-singlet bound states of three quarks and mesons are color singlet-bound states of quark-antiquark pairs.

- The Bethe-Salpeter equation (BSE), the relativistic equation of a two bound-state particle is:



- $\Gamma(p;P)$ is the Bethe-Salpeter amplitude (BSA), its structure depends on the meson's nature (spin, parity, etc.). $\mathrm{K}(q,p;P)$ is the scattering kernel, which should be determined and is related to the truncation of the gap equation.

Bethe-Salpeter equation



- The Bethe-Salpeter equation (BSE) is is written as:

$$\Gamma_M(p;P) = \int_q^{\Lambda} K(q,p;P)S(q^+)\Gamma_M(q;P)S(q^-) \ , \ q^{\pm} = q \pm P/2$$

- The Interaction kernel, K(q,p;P), is related to the truncation of the gap equation via the axial vector Ward-Takahashi identity (**Phys.Lett. B733 (2014) 202-208**, Qin et al.):

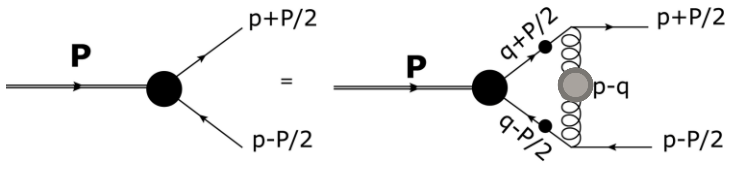
$$[\Sigma(p^+)\gamma_5 + \gamma_5\Sigma(p^-)] = \int_q^{\Lambda} K(q,p;P)[\gamma_5 S(q^-) + S(q^+)\gamma_5]$$

- It implies:

$$K(p,q;P) = -G((p-q)^2)(p-q)^2 D^0_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu \times \frac{\lambda^a}{2}\gamma_\nu$$

- This corresponds to the ladder truncation. Together with the rainbow truncation, it is called Rainbow-ladder truncation (RL).

Bethe-Salpeter equation



- The Bethe-Salpeter equation in the RL approximation is written as:

$$\Gamma_M(p;P) = -\int \frac{d^4q}{(2\pi)^4} G((p-q)^2)(p-q)^2 D^0_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu[S(q^+)\Gamma_M(q;P)S(q^-)]\frac{\lambda^a}{2}\gamma_\nu$$

- For a pseudoscalar meson, the Dirac structure of the Bethe-Salpeter amplitude (BSA) is written as:

$$\Gamma_M(p;P) = \gamma_5(iE_M(p;P) + \gamma\cdot P F_M(p;P) + \gamma\cdot p\; p\cdot P G_M(p;P) + p_\alpha\sigma_{\alpha\beta}P_\beta H_M(q;P))$$

- In the case of the pion, the axial vector Ward-Takahashi identity (axWTI) relates the dominant amplitude, E_π(p;P), with the quark propagator as follows:

$$f_\pi E_\pi(p; P^2 = 0) = B(p^2)$$ **"Goldstone's theorem"**

- The relationship above is exact in the chiral limit, and it implies that the two-body problem is solved (almost) completely, once solution of one body problem is known.

Mixing states

- For η-η', the deal is a bit different. There is mixing between the two states:

$$|\eta\rangle = \cos\theta\,|\eta_8\rangle - \sin\theta\,|\eta_0\rangle\ , \quad |\eta_8\rangle = \frac{1}{\sqrt{6}}(|u\bar{u}\rangle + |d\bar{d}\rangle - 2|s\bar{s}\rangle)\ ,$$

$$|\eta'\rangle = \sin\theta\,|\eta_8\rangle + \cos\theta\,|\eta_0\rangle\ , \quad |\eta_0\rangle = \frac{1}{\sqrt{3}}(|u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle)\ .$$

- Or, in the S-NS basis (which is more convenient for our numerical treatment):

$$|\eta\rangle = \cos\phi_P|\eta_{NS}\rangle - \sin\phi_P|\eta_S\rangle\ , \quad |\eta_{NS}\rangle = \frac{1}{\sqrt{2}}(|u\bar{u}\rangle + |d\bar{d}\rangle) = \frac{1}{\sqrt{3}}|\eta_8\rangle + \sqrt{\frac{2}{3}}|\eta_0\rangle\ ,$$

$$|\eta'\rangle = \sin\phi_P|\eta_{NS}\rangle + \cos\phi_P|\eta_S\rangle\ . \quad |\eta_S\rangle = |s\bar{s}\rangle = -\sqrt{\frac{2}{3}}|\eta_8\rangle + \frac{1}{\sqrt{3}}|\eta_0\rangle\ .$$

- The Dirac structure is contained in $\eta_{0,8}$ and $\eta_{S,NS}$. Also:

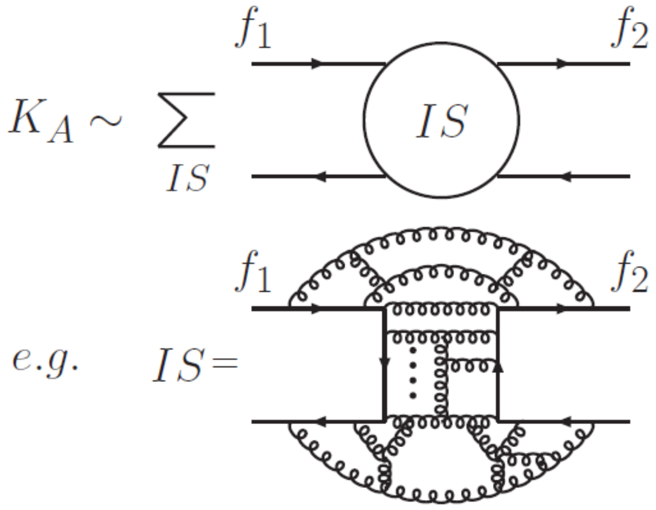
$$\theta = \phi - \arctan\sqrt{2} = \phi - 54.74°.$$

Non-Abelian anomaly

- To include the non-abelian anomaly, we need to go beyond Rainbow-Ladder. We exted the discussion of **Phys.Rev. C76 (2007) 045203** by Bhagwat et al., for a realistic momentum dependent interaction:

$$(K_A)^{tu}_{rs}(q,p;P) = -G_A((q-p)^2)\left\{\cos^2\phi_A[\gamma_5]_{rs}[\gamma_5]_{tu} + \sin^2\phi_A[\zeta\gamma\cdot P\gamma_5]_{rs}[\zeta\gamma\cdot P\gamma_5]_{tu}\right\}$$

$$\zeta = \mathrm{diag}\left[1/M_{u/d}(0), 1/M_{u/d}(0), 1/M_s(0)\right]$$



- $G_A(k^2)$ is a finite-width delta function. Together with ϕ_A,

it fixes the masses and mixing angles.

- In our case, the strength of $G_A(k^2)$, is 8 times smaller to that of

the RL effective coupling in the IR regime (and quickly dampens in the UV).

Perturbation theory integral representations (PTIRs)

- The quark propagator may be expressed as:

$$S(p;\zeta) = -i\gamma\cdot p\,\sigma_v(p^2;\zeta) + \sigma_s(p^2;\zeta)$$

- The numerical solutions are parametrized in terms of N pairs of complex conjugate poles:

$$\sigma_v(q) = \sum_{k=1}^{N}\left(\frac{z_k}{q^2+m_k^2} + \frac{z_k^*}{q^2+m_k^{*2}}\right)\ ,\ \sigma_s(q) = \sum_{k=1}^{N}\left(\frac{z_k m_k}{q^2+m_k^2} + \frac{z_k^* m_k^*}{q^2+m_k^{*2}}\right)\ .$$

- Constrained to the UV conditions of the free quark propagator form. For our computations, we found that N=2 is adequate.

Phys.Rev. D67 (2003) 054019. “Confinement phenomenology in the Bethe-Salpeter equation”

M. S. Bhagwat, M. A. Pichowsky, and P. C. Tandy

Perturbation theory integral representations (PTIRs)

- On the other hand, BSAs may be written as in the Nakanishi representation. We split the amplitude in IR and UV:

$$A(q,P) = \int_{-1}^{1} dz \int_{0}^{\infty} d\Lambda \left[\frac{\rho^i(z,\Lambda)}{(q^2 + zq\cdot P + \Lambda^2)^{m+n}} + \frac{\rho^u(z,\Lambda)}{(q^2 + zq\cdot P + \Lambda^2)^{n}} \right] .$$

- In principle, one should plug into the BSE the above expression for the BSA and solve for $\rho(z,\Lambda)$, as described in Nakanishi's work, **Phys. Rev. 130 1230-1235 (1963).**

- However, what we do, is to solve directly for the BSA, and match the Nakanishi-like representation to the numerical solution.

Perturbation theory integral representations (PTIRs)

- Our particular choice was first described in **Phys.Rev.Lett. 110 (2013) no.13, 132001** (Chang et al.), and refined in **Phys.Rev. D93 (2016) no.7, 074017** (KR et al.):

$$E^u(k;P) = c_E^u \int_{-1}^{1} dz\ \rho_{\nu_E^u}(z) \hat{\Delta}^{1+\alpha}_{\Lambda_E^u}(k_z^2) \qquad F^u(k,P) = c_F^u \int_{-1}^{1} dz \rho_{\nu_F^u}(z) k^2 \Lambda_F^u \Delta^{2+\alpha}_{\Lambda_F^u}(k_z^2)$$

$$G^u(k,P) = c_G^u \int_{-1}^{1} dz \rho_{\nu_G^u}(z) \Lambda_G^u \Delta^{2+\alpha}_{\Lambda_G^u}(k_z^2) \qquad A^i(k,P) = c_A^i \int_{-1}^{1} dz \rho_{\nu_A^i}(z) [b_A \hat{\Delta}^4_{\Lambda_A^i}(k_z^2) + \bar{b}_A \hat{\Delta}^5_{\Lambda_A^i}(k_z^2)]\ .$$

- Where Λ, ν, a, b, are parameters fitted to the numerical data. The following definitions apply:

$$\hat{\Delta}_\Lambda(s) = \Lambda\ \Delta_\Lambda(s)\ ,\ \Delta_\Lambda(s) = (s+\Lambda^2)^{-1}\ ,\ k_z^2 = k^2 + z\, k \cdot P\ . \qquad \rho_\nu(z) \sim (1-z^2)^\nu$$

- H(k,P) is negligible for pion and η_c; G(k,P) and H(k,P) are negligible for η_b.

Parton distribution amplitudes

- M. Ding's talk -

- Valence-quark distribution amplitude (PDA) is the probability density of having a quark-antiquark bound state, with momentum fraction x and 1-x, respectively.

- The PDA is a projection of the system's Bethe-Salpeter wave-function onto the light-front. It is therefore process independent and hence plays a crucial role in explaining and understanding a wide range of a given meson's properties and interactions.

- Given a pseudoscalar meson with total momentum P, a resolution scale ζ and a light-cone four-vector n ($n^2 = 0, n.P = -m_\pi$), the PDA reads as:

$$f_M \phi_M(x;\zeta) = Z_2(\Lambda;\zeta) \int_q^\Lambda \delta(n \cdot q^+ - x n \cdot P) \gamma_5 \gamma \cdot n \chi_M(q;P), \ \chi_M(q;P) = S(q^+) \Gamma_M(q;P) S(q^-)$$

- The moments of the distribution are given by:

$$< x^m >= \int_0^1 dx x^m \phi_M(x;\zeta), \ f_M (n \cdot P)^{m+1} < x^m >= \mathrm{tr}_{CD} Z_2 \int_q^\Lambda (n \cdot q^+)^m \gamma_5 \gamma \cdot n \chi_M(q;P)$$

Parton distribution amplitudes

- According to **Phys. Rev. D22, 2157 (1980**) by G. Peter Lepage, Stanley J. Brodsky, in the neighborhood of the conformal limit, it is written in terms of 3/2-Gegenbauer polynomials.

- PDA should evolve with the resolution scale ζ^2=Q^2 through the ERBL evolution equations (see **Phys. Lett. B87, 359(1979)** and **Phys. Lett. B94, 245 (1980)**).

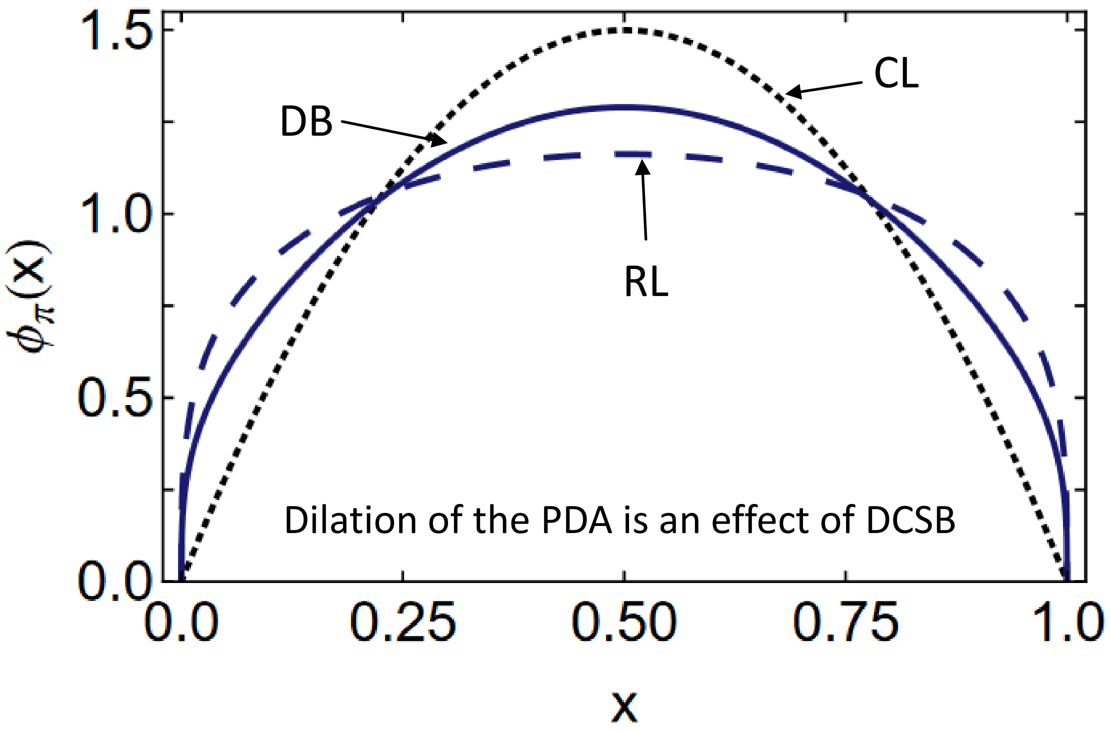
- Evolution enables the dressed-quark and antiquark degrees of freedom, to split into less well-dressed partons via the addition of gluons and sea quarks in the manner prescribed by QCD dynamics.

- The asymptotic form (conformal limit) of the PDA is the well known result:

$$\phi^{cl}(x) = 6x(1-x)$$

Pion PDA

Pion PDA at hadronic scale (ζ=2 GeV), pion PDA is a broad concave function of x.



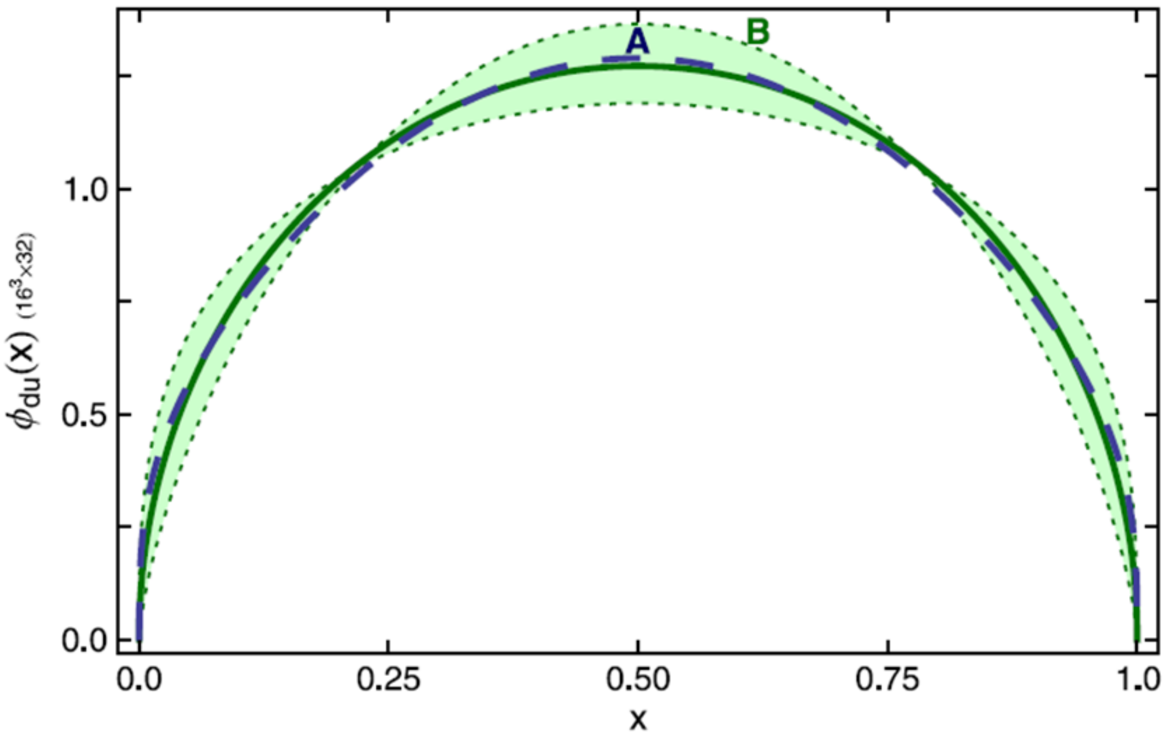
Phys.Rev.Lett. 110 (2013) no.13, 132001

“Imaging dynamical chiral symmetry breaking: pion wave function on the light front”

Lei Chang, Ian C. Cloët, J. Javier Cobos-Martinez, Craig D. Roberts, Sebastian. M. Schmidt, Peter. C. Tandy

Pion PDA

Precise agreement of DSE with lQCD result (R. Arthur et al., **Phys.Rev. D83 (2011) 074505**).



A: DSE prediction

Phys.Rev.Lett. 110 (2013) no.13, 132001.

$$< (2x-1)^2 >_{DSE} = 0.25$$

B: Infered PDA from lattice

Phys.Lett. B731 (2014) 13-18.

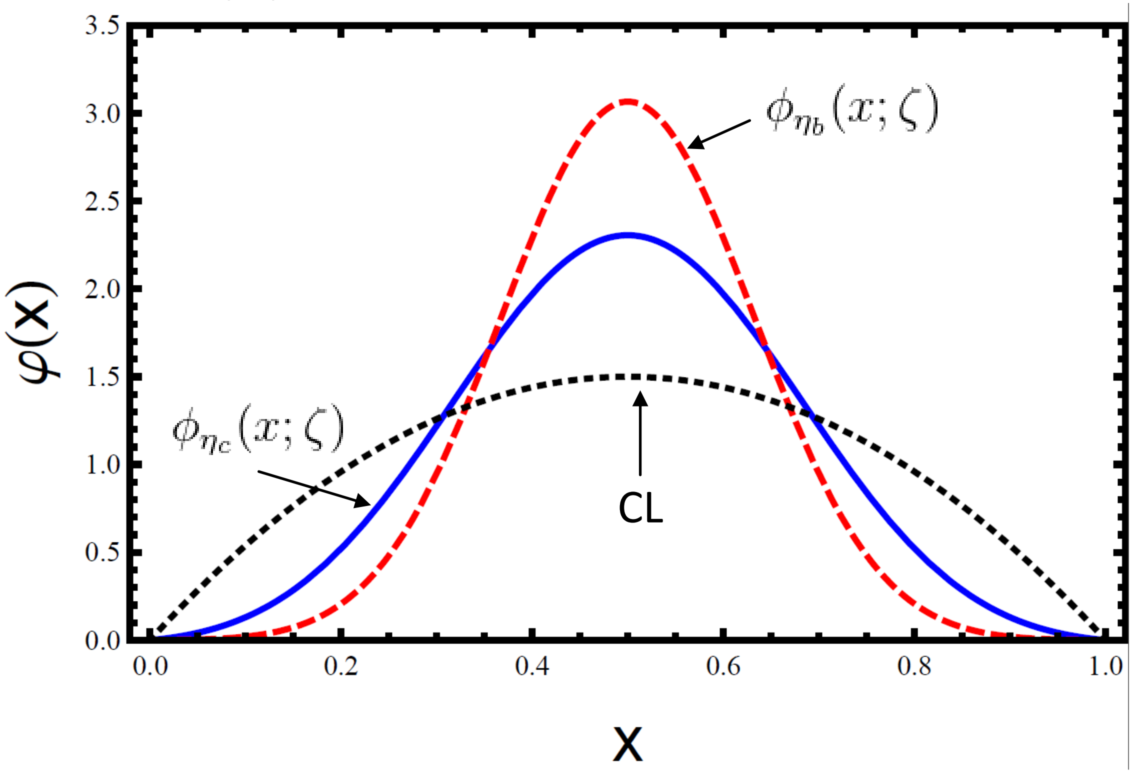
$$< (2x-1)^2 >_{lQCD} = 0.25(1)(2)$$

Phys.Lett. B731 (2014) 13-18. *"Distribution amplitudes of light-quark mesons from lattice QCD"*

Jorge Segovia, Lei Chang, Ian C. Cloët, Craig D. Roberts, Sebastian M. Schmidt, Hong-shi Zong

Heavy quarkonia PDAs

Unlike pion PDA, heavy quarkonia PDAs are narrow at real-life scales (ζ=2 GeV).



Phys.Lett. B753 (2016) 330-335

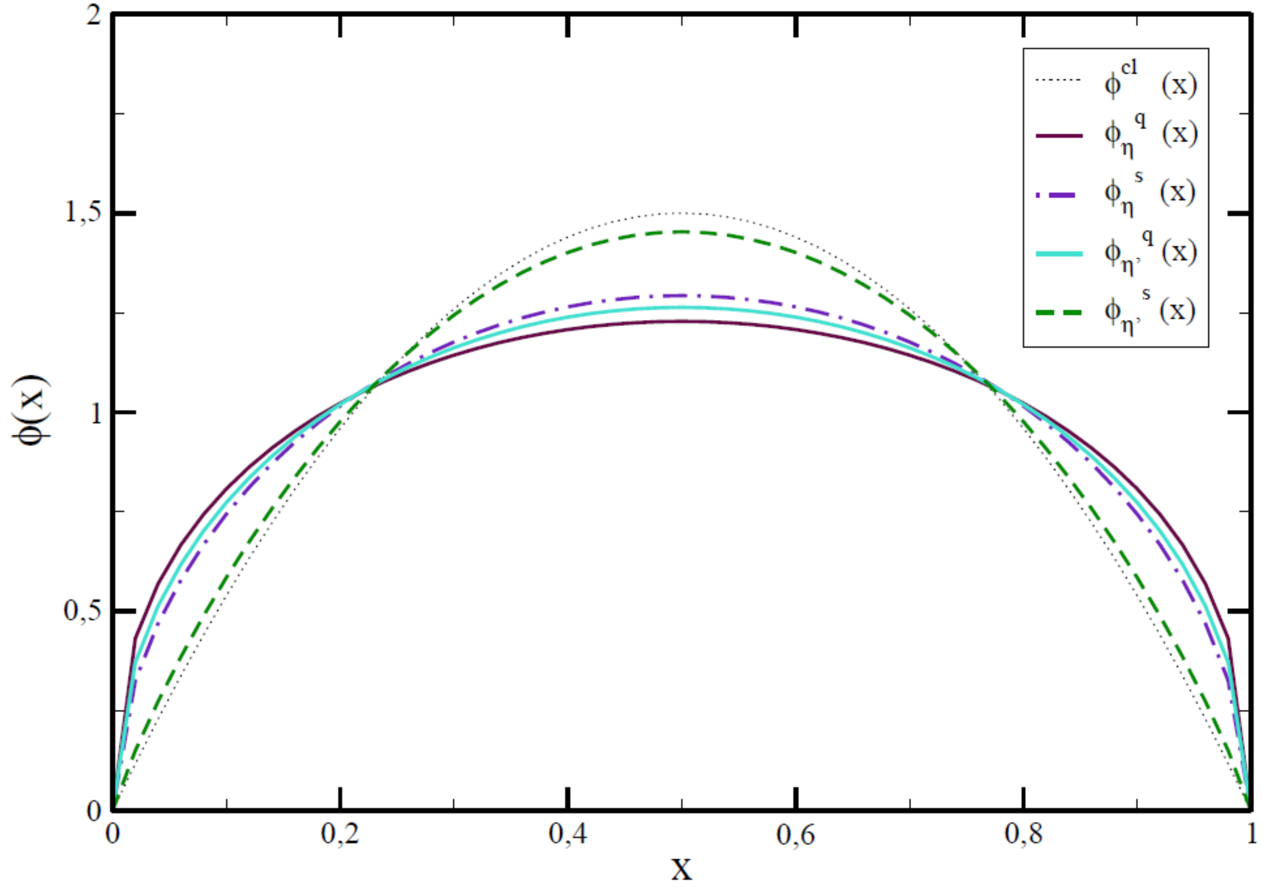
"Leading-twist parton distribution amplitudes of S-wave heavy-quarkonia"

Minghui Ding, Fei Gao, Lei Chang, Yu-Xin Liu, Craig D. Roberts

η,η' Valence quarks PDAs

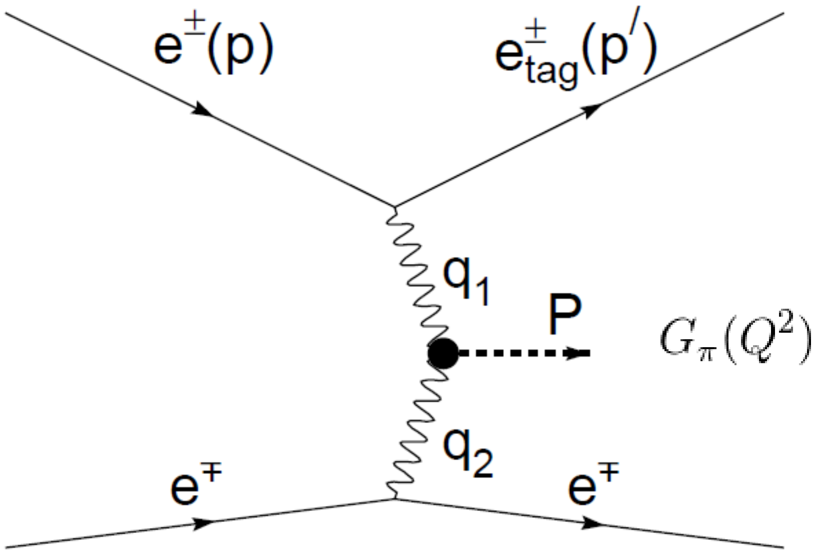
Eta and eta' PDA are concave at real-life scales (ζ=2 GeV).

s quark PDA is close to the CL PDA, as expected from **Phys.Lett. B753 (2016) 330-335**



Transition form factors

- The hadron structure is probed with a photon.



$\gamma\gamma^{*}\rightarrow$ Pseudoscalar transition form factor (TFF)

- **Transition form factor:** In electron-positron scattering, neutral pseudoscalar is produced via two-photon fusion. Studied at Babar and Belle.

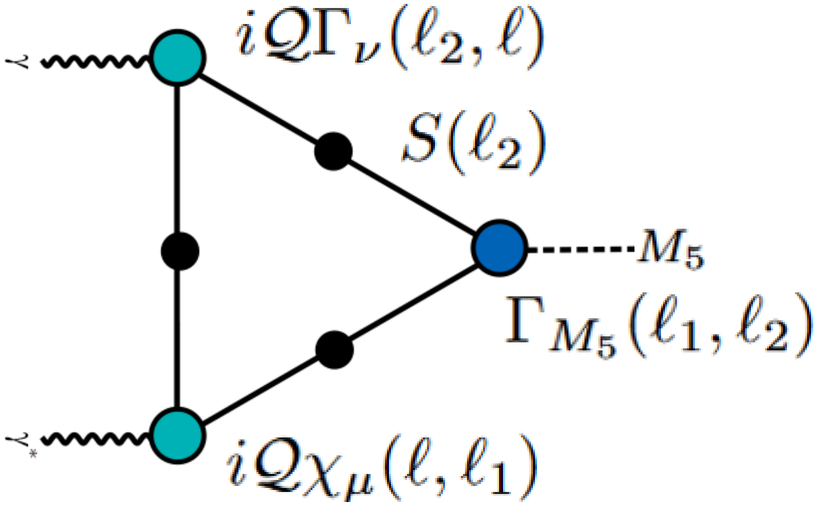
Pion transition form factor

- For a pseudoscalar meson M_5, the $\gamma\gamma^* \to M_5$ transition is written as:

$$T_{\mu\nu}(k_1, k_2) = \epsilon_{\mu\nu\alpha\beta} k_{1\alpha} k_{2\beta} G_{M_5}(k_1^2, k_1 \cdot k_2, k_2^2) \ ,$$

$$T_{\mu\nu}(k_1, k_2) = \mathrm{tr} \int \frac{d^4 l}{(2\pi)^4} i\mathcal{Q}\chi_\mu(l, l_1)\Gamma_{M_5}(l_1, l_2)S(l_2)i\mathcal{Q}\Gamma_\nu(l_2, l)$$

- We will construct a fully consistent quark-photon vertex, which at the same time, expedites the computation of both elastic and transition form factors.



Quark-photon vertex

- We employ the ansätz explained in **Phys.Rev.Lett. 111 (2013) no.14, 141802**, **Phys.Rev. D93 (2016) no.7, 074017** and **Phys.Rev. D95 (2017) no.7, 074014**.

- With the following definitions (m = meson mass):

$$\Delta_F = [F(k_f^2) - F(k_i^2)]/[k_f^2 - k_i^2] \qquad \mathcal{E} = \sqrt{Q^2/4 + m^2} - m$$

$$s = 1 + s_0 \mathrm{Exp}[-\mathcal{E}/M_E] \qquad M_E = \{p | p^2 = M^2(p^2), p^2 > 0\}$$

- The vertex ansätz is:

$$\chi_\mu(k_f, k_i) = \gamma_\mu \Delta_{k^2\sigma_v} + [s\gamma \cdot k_f \gamma_\mu \gamma \cdot k_i + \bar{s}\gamma \cdot k_i \gamma_\mu \gamma \cdot k_f]\Delta_{\sigma_v}$$
$$+ [s(\gamma \cdot k_f \gamma_\mu + \gamma_\mu \gamma \cdot k_i) + \bar{s}(\gamma \cdot k_i \gamma_\mu + \gamma_\mu \gamma \cdot k_f)] i\Delta_{\sigma_s}$$

Elastic and transition form factors

- Recalling the expression for the elastic and transition form factors:

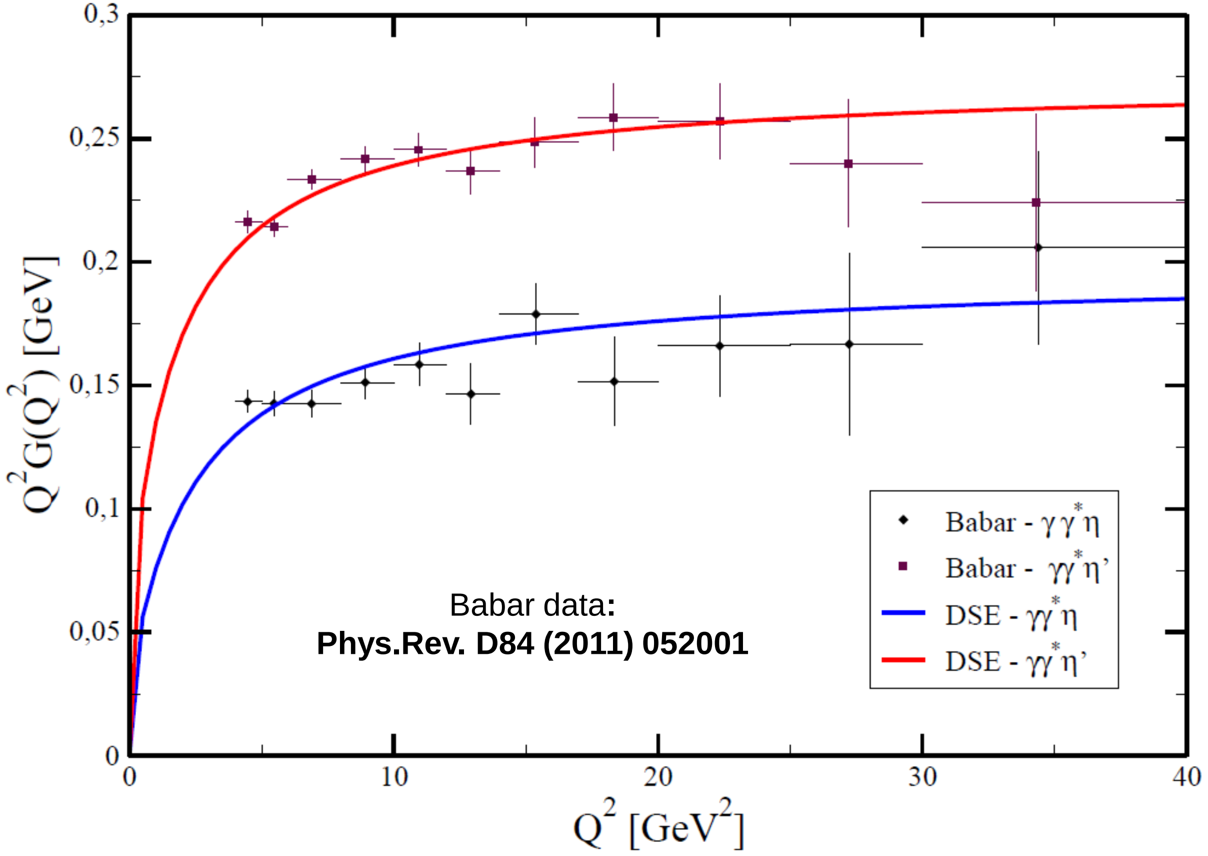
$$\boxed{\begin{aligned} T_{\mu\nu}(k_1,k_2) &= \epsilon_{\mu\nu\alpha\beta}k_{1\alpha}k_{2\beta}G_{M_5}(k_1^2, k_1\cdot k_2, k_2^2)\ , \\ T_{\mu\nu}(k_1,k_2) &= \mathrm{tr}\int\frac{d^4l}{(2\pi)^4}i\mathcal{Q}\chi_\mu(l,l_1)\Gamma_{M_5}(l_1,l_2)S(l_2)i\mathcal{Q}\Gamma_\nu(l_2,l) \end{aligned}}$$

- Computation of the form factors reduces to the task of summing a series of terms, all of which involve a single four-momentum integral. The denominator of the integrand in every term is a product of ℓ-quadratic forms.

- One uses a Feynman parametrisation in order to combine the denominators into a single quadratic form. It enables straightforward evaluation of the four momentum integration.

- After calculation of the four-momentum integration, we integrate numerically over the Feynman parameters and the spectral integrals. The complete result follows after summing the series.

η,η' transition form factors



- TFF DSE **computation:**

$\Gamma[\eta\to\gamma\gamma]$=0.515 keV.

$\Gamma[\eta'\to\gamma\gamma]$=4.35 keV.

S-NS Basis (ideal mixing = 0):

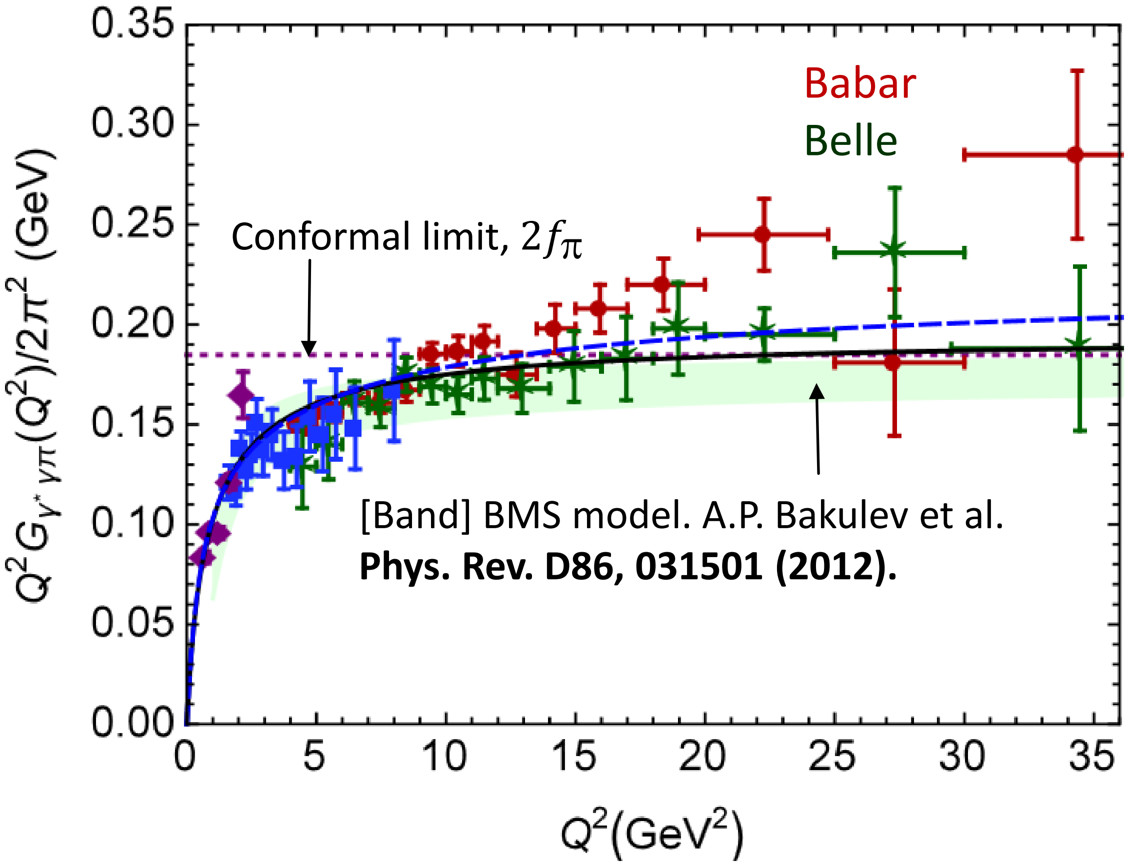
$$\phi_{\eta\eta'}(m_\eta^2) = 42.3^o,\ \phi_{\eta\eta'}(m_{\eta'}^2) = 42.8^o$$

$\eta_0 - \eta_8$ Basis (ideal mixing = -54.7):

$$\theta_{\eta\eta'}(m_\eta^2) = -12.4^o,\ \theta_{\eta\eta'}(m_{\eta'}^2) = -11.9^o$$

PRELIMINARY RESULTS

Pion transition form factor



DSE frozen-computation
($\zeta = 2\ GeV, r = 0.68\ fm$)

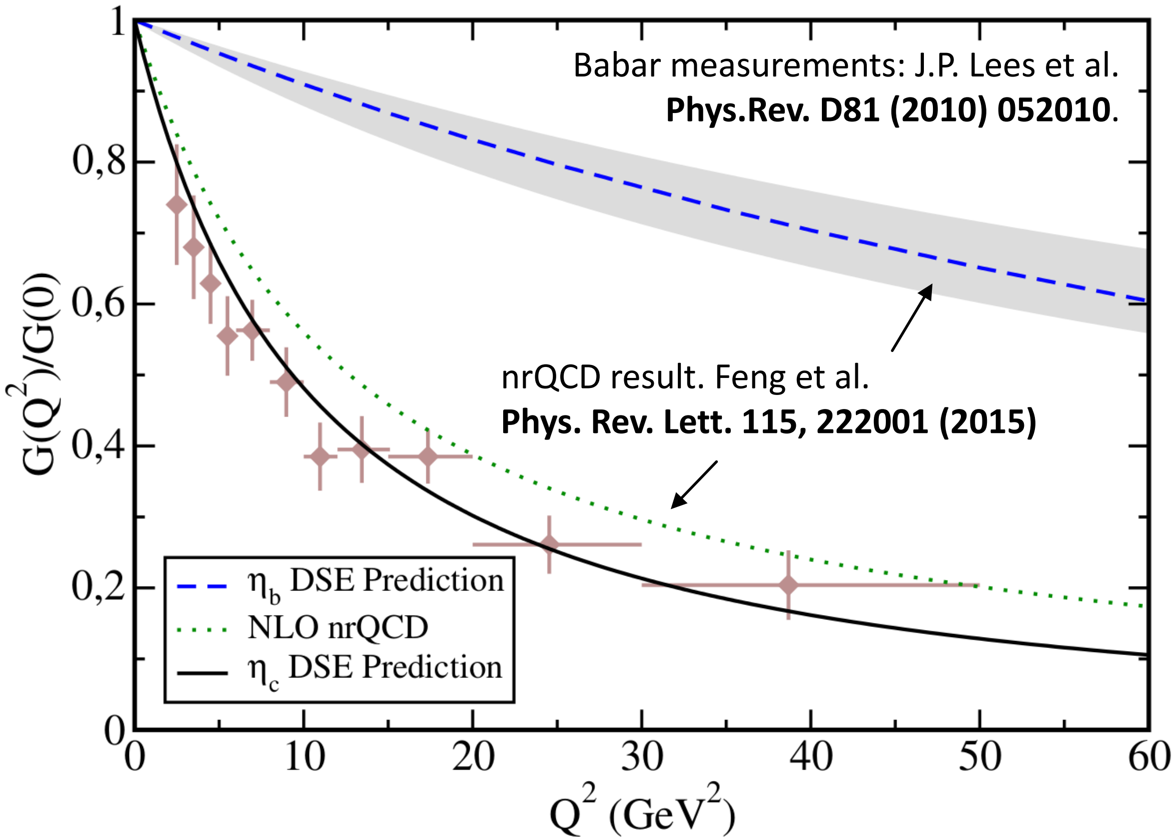
DSE **prediction**
($\zeta^2 = Q^2, r = 0.68\ fm$)

Phys.Rev. D93 (2016) no.7, 074017.

"Structure of the neutral pion and its electromagnetic transition form factor"

K. R., L. Chang, A. Bashir, J.J. Cobos-Martinez, L.X. Gutiérrez-Guerrero, C.D. Roberts, P.C. Tandy

η_c, η_b transition form factors



- η_c TFF DSE **prediction:**

$\Gamma[\eta_c \to \gamma\gamma]$=6.10 keV, r=0.16 fm.
$\Gamma[\eta_b \to \gamma\gamma]$=0.52 keV, r=0.04 fm.

Our η_c result matches the available data and the empirical value of the interaction radius (r=0.17 fm).

NNLO result of nrQCD is vastly different from the data. Our agreement with data tells that nrQCD is not a reliable effective field theory for exclusive processes involving charmonia. However, the agreement of our prediction with nrQCD for bottomonia, shows reliability in both approaches.

Phys.Rev. D95 (2017) no.7, 074014

"Partonic structure of neutral pseudoscalars via two photon transition form factors"

K. R., M. Ding, A. Bashir, L. Chang, C.D. Roberts

Phys.Rev. D95 (2017) no.7, 074014. KR et al.

Phys.Rev. D93 (2016) no.7, 074017. KR et al.

Conclusions

- We described a computation of $\gamma\gamma^*$- pseudoscalars TFFs, in which all elements employed are determined by solutions of QCD's Dyson-Schwinger equations. The novel analysis techniques we employed made it possible to compute $G(Q^2)$, on the entire domain of space-like momenta, for the first time in a framework with a direct connection to QCD.

- Our QCD based theoretical computation resolves the Babar puzzle, conclusively demonstrating that the results of asymptotic QCD are faithfully reproduced, while also successfully agreeing with experimental data for low and intermediate values of momentum transfer. Belle data supports this conclusion. The η_c result agrees with the Babar data. We have proven that nrQCD approaches are not adequate for charmonia, but they are for bottomonia.

- Results for η,η' are **preliminary** and should be checked.

- Within a single systematic and consistent approach, we unified the description of those form factors with the valence-quark distribution amplitudes, masses, decay constants (**Phys.Rev.Lett. 110 (2013) no.13, 132001** and **Phys.Lett. B753 (2016) 330-335**) and with that of the charged pion elastic form factor (**Phys.Rev.Lett. 111 (2013) no.14, 141802**).

Scope

- We will extend our $\gamma\gamma^* \rightarrow$ pseudoscalar meson TFFs analysis, to the fully off-Shell case (for both photons and pion). This is important to estimate the HLbL contribution to muon's g-2. (P. Roig)
- In continuation with the study of form factors, the computation of $\gamma^* N \rightarrow N^*(1535)$ is underway. In the quark-diquark picture, we have computed all the intermediate diquark transitions, the overall transition is underway.
- Besides form factors, following the same novel analysis techniques we have employed, other non perturbative objects have been computed: PDFs (**Phys.Lett. B737 (2014) 23-29**), GPDs (**Phys.Lett. B741 (2015) 190-196**), for example.

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