Dark Matter in a 4 Higgs Doublets model with S3 symmetry.

Presenter: Humberto Reyes.

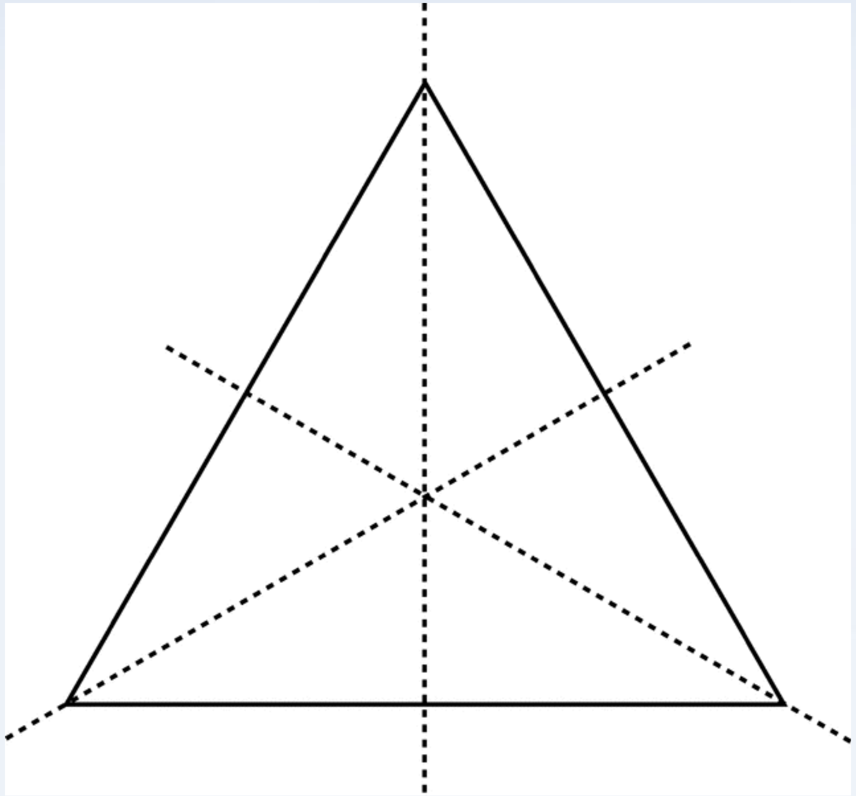
Authors:

- Catalina Espinoza (IF UNAM)
- Estela Garcés (CINVESTAV)
- Myriam Mondragón (IF UNAM)
- Humberto Reyes (IF UNAM)

Outline.

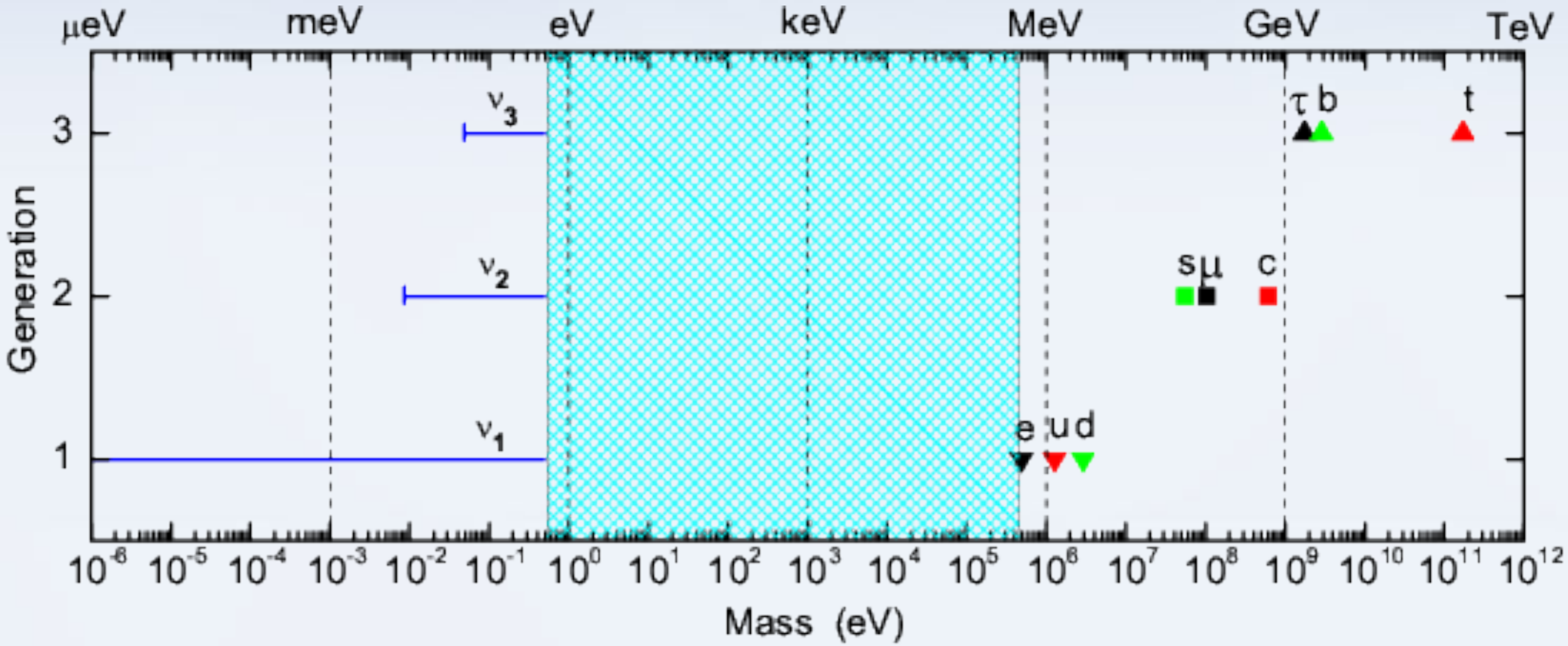
- Introduction. The S3 flavor symmetry and the S3-3HDM.
- The 4HDM with S3 symmetry (+Z2).
- Dark matter in a 4HDM with S3 symmetry (a first aproach).
- Conclusions and perspectives.

The S3 symmetry.



- Is the group of all permutations (6) of a three element set.
- Is the group of rotations and reflexions that leave an equilateral triangle invariant.
- Is the simplest non abelian group.
- **It has three irreducible representations: two singlets and one doublet**.

The Flavor Symmetry.



Shu Luo and Zhi-Zhong Xing. Theoretical Overview on the Flavor Issues of Massive Neutrinos. *Int. J. Mod. Phys.*, A27:1230031, 2012.

The Flavor Symmetry.

Three fermionic families are accommodated in one S3 doublet and one S3 singlet.

$$\psi_{D,(L,R)} \equiv \begin{pmatrix} \psi_{1,(L,R)} \\ \psi_{2,(L,R)} \end{pmatrix} \backsim \mathbf{2}$$

$$\psi_{S,(L,R)} \equiv \psi_{3,(L,R)} \backsim \mathbf{1}_S$$

$$\psi_{3,L} = (b_L, t_L), \;\; \psi_{3,R} = t_R \text{ ó } \psi_{3,R} = b_R$$

$$\begin{pmatrix} \psi_{1,L} \\ \psi_{2,L} \end{pmatrix} = \begin{pmatrix} (u_L, d_L) \\ (c_L, s_L) \end{pmatrix}, \;\; \begin{pmatrix} \psi_{1,R} \\ \psi_{2,R} \end{pmatrix}_{\psi=u} = \begin{pmatrix} u_R \\ c_R \end{pmatrix}$$

$$\begin{pmatrix} \psi_{1,R} \\ \psi_{2,R} \end{pmatrix}_{\psi=d} = \begin{pmatrix} d_R \\ s_R \end{pmatrix}$$

J. Kubo, A. Mondragon, M. Mondragon, and E. Rodriguez-Jauregui. The Flavor symmetry. *Prog. Theor. Phys.*, 109:795–807, 2003. [Erratum: Prog. Theor. Phys.114,287(2005)].

The Flavor Symmetry.

The S3 model with 3 Higgs Doublet has been extensively studied by (and many more):

F. González Canales, A. Mondragón, M. Mondragón, U. J. Saldaña Salazar, and L. Velasco-Sevilla. Quark sector of S3 models: classification and comparison with experimental data. *Phys. Rev.*, D88:096004, 2013.

E. Barradas-Guevara, O. Félix-Beltrán, and E. Rodríguez-Jáuregui. Trilinear self-couplings in an S(3) flavored Higgs model. *Phys. Rev.*, D90(9):095001, 2014.

Dipankar Das and Ujjal Kumar Dey. Analysis of an extended scalar sector with S_3 symmetry. *Phys. Rev.*, D89(9):095025, 2014. [Erratum: Phys. Rev.D91,no.3,039905(2015)].

Adriana Perez. Potencial de 3 dobletes de higgs bajo la simetría s3. 2017.

D. Emmanuel-Costa, O. M. Ogreid, P. Osland, and M. N. Rebelo. Spontaneous symmetry breaking in the S_3-symmetric scalar sector. *JHEP*, 02:154, 2016.

A. Mondragon and E. Rodriguez-Jauregui. Breaking of flavor permutational symmetry and the CKM matrix. *AIP Conf. Proc.*, 531:310–314, 2000. [AIP Conf. Proc.490,393(1999)].

F. Gonzalez Canales, A. Mondragon, and M. Mondragon. The S_3 Flavour Symmetry: Neutrino Masses and Mixings. *Fortsch. Phys.*, 61:546–570, 2013.

A. Mondragon, M. Mondragon, and E. Peinado. Lepton masses, mixings and FCNC in a minimal S(3)-invariant extension of the Standard Model. *Phys. Rev.*, D76:076003, 2007.

But to have dark matter we need 4 Higgs Doublets…

The S3 Model with 4 Higgs Doublets.

The Scalar Potential.

$$
\begin{aligned}
V_4 = {} & \mu_0^2 H_s^\dagger H_s + \mu_1^2 (H_1^\dagger H_1 + H_2^\dagger H_2) + \mu_2^2 H_a^\dagger H_a \\
& + \lambda_1 (H_1^\dagger H_1 + H_2^\dagger H_2)^2 + \lambda_2 (H_1^\dagger H_2 - H_2^\dagger H_1)^2 \\
& + \lambda_3 [(H_1^\dagger H_1 - H_2^\dagger H_2)^2 + (H_1^\dagger H_2 + H_2^\dagger H_1)^2] \\
& + \lambda_4 [(H_s^\dagger H_1)(H_1^\dagger H_2 + H_2^\dagger H_1) + (H_s^\dagger H_2)(H_1^\dagger H_1 - H_2^\dagger H_2) + \text{h.c.}] \\
& + \lambda_5 (H_s^\dagger H_s)(H_1^\dagger H_1 + H_2^\dagger H_2) \\
& + \lambda_6 [(H_s^\dagger H_1)(H_1^\dagger H_s) + (H_s^\dagger H_2)(H_2^\dagger H_s)] \\
& + \lambda_7 [(H_s^\dagger H_1)(H_s^\dagger H_1) + (H_s^\dagger H_2)(H_s^\dagger H_2) + \text{h.c.}] \\
& + \lambda_8 (H_s^\dagger H_s)^2 \\
& + \lambda_9 [(\mathbf{H}_a^\dagger \mathbf{H}_2)(\mathbf{H}_1^\dagger \mathbf{H}_2 + \mathbf{H}_2^\dagger \mathbf{H}_1) - (\mathbf{H}_a^\dagger \mathbf{H}_1)(\mathbf{H}_1^\dagger \mathbf{H}_1 - \mathbf{H}_2^\dagger \mathbf{H}_2) + \text{h.c.}] \\
& + \lambda_{10} (H_a^\dagger H_a)(H_1^\dagger H_1 + H_2^\dagger H_2) \\
& + \lambda_{11} [(H_a^\dagger H_1)(H_1^\dagger H_a) + (H_a^\dagger H_2)(H_2^\dagger H_a)] \\
& + \lambda_{12} [(H_a^\dagger H_1)(H_a^\dagger H_1) + (H_a^\dagger H_2)(H_a^\dagger H_2) + \text{h.c.}] \\
& + \lambda_{13} (H_a^\dagger H_a)^2 + \lambda_{14} (H_s^\dagger H_a H_a^\dagger H_s) \\
& + \lambda_{15} [(\mathbf{H}_1^\dagger \mathbf{H}_s)(\mathbf{H}_2^\dagger \mathbf{H}_a) + \text{h.c.}],
\end{aligned}
$$

where

$$H_D \equiv \begin{pmatrix} H_1 \\ H_2 \end{pmatrix} \backsim \mathbf{2} \quad H_s \backsim \mathbf{1}_s \quad H_a \backsim \mathbf{1}_a .$$

The S3 Model with 4 Higgs Doublets + Z2.

To ensure stability we need to impose a Z2 symmetry, where
Ha-> -Ha.

$$
\begin{aligned}
V_4 = {} & \mu_0^2 H_s^\dagger H_s + \mu_1^2(H_1^\dagger H_1 + H_2^\dagger H_2) + \mu_2^2 H_a^\dagger H_a \\
& + \lambda_1(H_1^\dagger H_1 + H_2^\dagger H_2)^2 + \lambda_2(H_1^\dagger H_2 - H_2^\dagger H_1)^2 \\
& + \lambda_3[(H_1^\dagger H_1 - H_2^\dagger H_2)^2 + (H_1^\dagger H_2 + H_2^\dagger H_1)^2] \\
& + \lambda_4[(H_s^\dagger H_1)(H_1^\dagger H_2 + H_2^\dagger H_1) + (H_s^\dagger H_2)(H_1^\dagger H_1 - H_2^\dagger H_2) + \mathrm{h.c.}] \\
& + \lambda_5(H_s^\dagger H_s)(H_1^\dagger H_1 + H_2^\dagger H_2) \\
& + \lambda_6[(H_s^\dagger H_1)(H_1^\dagger H_s) + (H_s^\dagger H_2)(H_2^\dagger H_s)] \\
& + \lambda_7[(H_s^\dagger H_1)(H_s^\dagger H_1) + (H_s^\dagger H_2)(H_s^\dagger H_2) + \mathrm{h.c.}] \\
& + \lambda_8(H_s^\dagger H_s)^2 \\
& + \lambda_{10}(H_a^\dagger H_a)(H_1^\dagger H_1 + H_2^\dagger H_2) \\
& + \lambda_{11}[(H_a^\dagger H_1)(H_1^\dagger H_a) + (H_a^\dagger H_2)(H_2^\dagger H_a)] \\
& + \lambda_{12}[(H_a^\dagger H_1)(H_a^\dagger H_1) + (H_a^\dagger H_2)(H_a^\dagger H_2) + \mathrm{h.c.}] \\
& + \lambda_{13}(H_a^\dagger H_a)^2 + \lambda_{14}(H_s^\dagger H_a H_a^\dagger H_s)
\end{aligned}
$$

The S3 Model with 4 Higgs Doublets + Z2.

Yukawa Lagrangian.

Ha doesn’t couple directly to fermions.

$$
\begin{aligned}
-\mathcal{L}_{Y_f} = {} & Y_1^f(\bar{\psi}^f_{S,L}\psi^f_{S,R}H_s) + \frac{1}{\sqrt{2}}Y_2^f(\bar{\psi}^f_{1,L}\psi^f_{1,R} + \bar{\psi}^f_{2,L}\psi^f_{2,R})H_s \\
& + \frac{1}{2}Y_3^f[(\bar{\psi}^f_{1,L}H_2 + \bar{\psi}_{2,L}H_1)\psi^f_{1,R} + (\bar{\psi}_{1,L}H_1 - \bar{\psi}^f_{2,L}H_2)\psi^f_{2,R}] \\
& + \frac{1}{\sqrt{2}}Y_5^f(\bar{\psi}^f_{1,L}H_1 + \bar{\psi}^f_{1,L}H_1 + \bar{\psi}^f_{2,L}H_2)\psi^f_{S,R} \\
& + \frac{1}{\sqrt{2}}Y_6^f(\bar{\psi}^f_{S,L}(H_1\psi^f_{1,R} + H_2\psi^f_{2,R})] + \text{h.c.}
\end{aligned}
$$

$$f = d, e$$

The S3 Model with 4 Higgs Doublets + Z2.

From the tadpole equations…

$$\frac{\partial V}{\partial v_0} = \frac{1}{2}(\lambda_4(-3v_1^2 v_2 + v_2^3) + v_0((2\lambda_7 + \lambda_5 + \lambda_6)(-v_1^2 - v_2^2)$$
$$- 2(\lambda_8 v_0^2 + \mu_0^2))) = 0$$
$$\frac{\partial V}{\partial v_1} = -\frac{1}{2}v_1(2((\lambda_1 + \lambda_3)(v_1^2 + v_2^2) + \mu_1^2) + (2\lambda_7 + \lambda_5 + \lambda_6)v_0^2$$
$$+ 6\lambda_4 v_0 v_2) = 0$$
$$\frac{\partial V}{\partial v_2} = \frac{1}{2}(-(2((\lambda_1 + \lambda_3)v_1^2 + \mu_1^2) + (2\lambda_7 + \lambda_5 + \lambda_6)v_0^2)v_2 -$$
$$2(\lambda_1 + \lambda_3)v_2^3 + 3\lambda_4 v_0(-v_1^2 + v_2^2)) = 0$$
$$\frac{\partial V}{\partial v_a} = 0.$$

we get the following conditions:

$$\mu_0^2 = -(\lambda_5 + \lambda_6 + 2\lambda_7)(v_1^2 + v_2^2) - 2\lambda_8 v_0^2 + \frac{\lambda_4(v_2^2 - 3v_1^2)v_2}{v_0}$$
$$\mu_1^2 = -(\lambda_5 + \lambda_6 + 2\lambda_7)v_0^2 - 2(\lambda_1 + \lambda_3)(v_1^2 + v_2^2) - 6\lambda_4 v_2 v_0$$
$$\mu_1^2 = -(\lambda_5 + \lambda_6 + 2\lambda_7)v_0^2 - 2(\lambda_1 + \lambda_3)(v_1^2 + v_2^2) + 3\lambda_4\frac{v_0(v_2^2 - v_1^2)}{v_2}.$$

$$v_1 = \sqrt{3}v_2. \longrightarrow$$ By self consistency.

The S3 Model with 4 Higgs Doublets + Z2.

The masses are found by diagonalizing the matrix:

$$(\mathcal{M}_H^2)_{ij} = \frac{1}{2}\frac{\partial^2 V}{\partial H_i \partial H_j}\Big|_{min} .$$

Which is block diagonal, and all the submatrices have the following form:

$$m_{H^S}^2 = \begin{pmatrix} m_{h_s^n h_s^n} & m_{h_1^n h_s^n} & m_{h_2^n h_s^n} & 0 \\ m_{h_s^n h_1^n} & m_{h_1^n h_1^n} & m_{h_2^n h_1^n} & 0 \\ m_{h_s^n h_2^n} & m_{h_1^n h_2^n} & m_{h_2^n h_2^n} & 0 \\ 0 & 0 & 0 & m_{h_a^n h_a^n} \end{pmatrix}$$

The fields corresponding to Ha are decoupled.

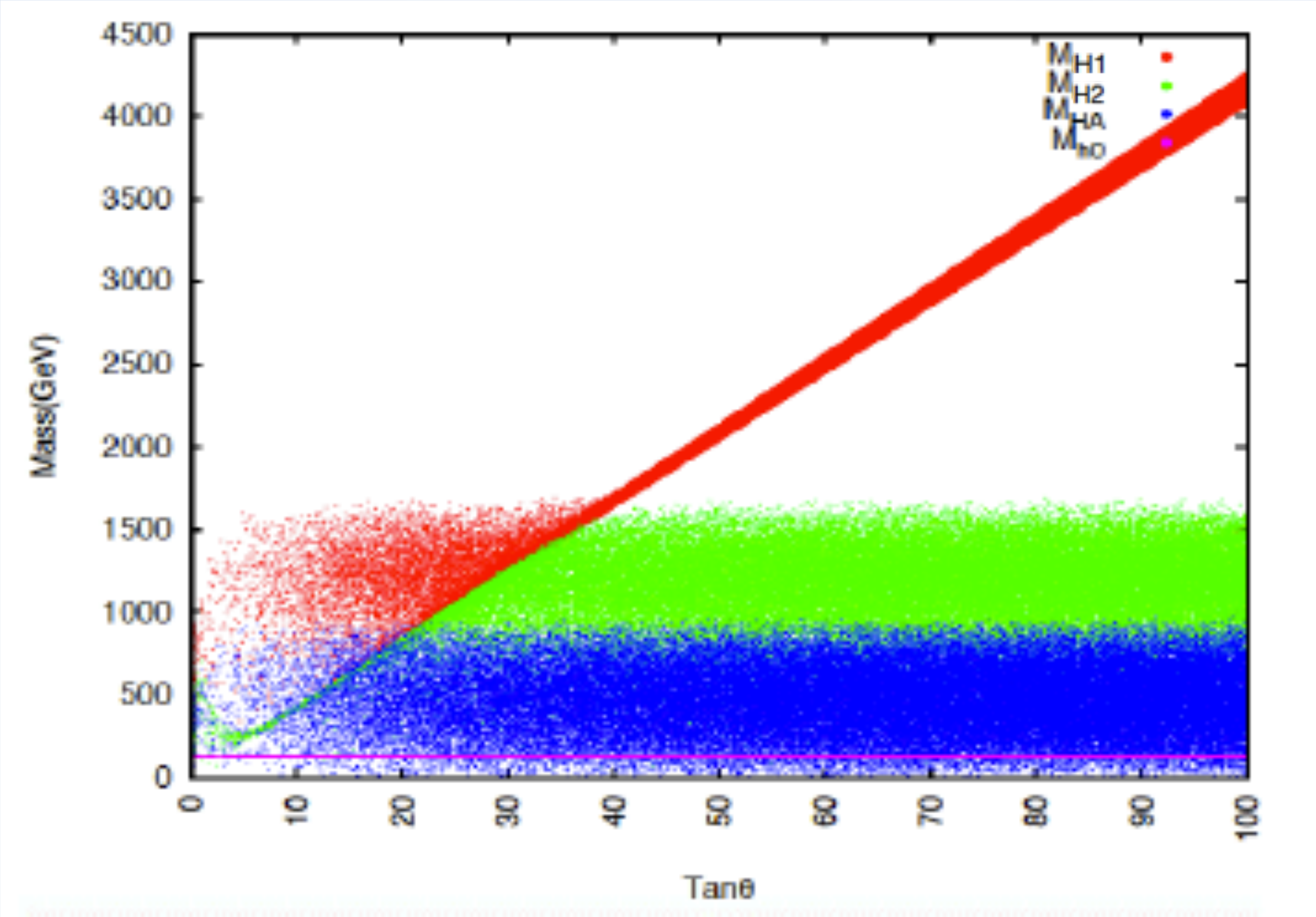
The S3 Model with 4 Higgs Doublets + Z2.

And the corresponding eigenvalues are:

$$m^2_{h^n_s} = -18\lambda_4 v_0 v_2$$
$$m^2_{h^n_a} = \mu_2^2 + \lambda_{14} v_0^2 + 4(\lambda_{10} + \lambda_{11} + 2\lambda_{12}) v_2^2$$
$$m^2_{h^n_1} = (\frac{1}{v_0})(2\lambda_8 v_0^3 + v_2(3\lambda_4 v_0^2 + 8(\lambda_1+\lambda_3)v_0 v_2 - 4\lambda_4 v_2^2) + ((4\lambda_8^2 v_0^6 - 12\lambda_4\lambda_8 v_0^5 v_2 + (9\lambda_4^2 + 16((\lambda_5+\lambda_6+2\lambda_7)^2 - 2(\lambda_1+\lambda_3)\lambda 8)) v_0^4 v_2^2 + 16\lambda_4(3(\lambda_1+\lambda_3+2(\lambda_5+\lambda_6+2\lambda_7)) - \lambda_8) v_0^3 v_2^3 + 8(8(\lambda_1+\lambda_3)^2 + 21\lambda_4^2) v_0^2 v_2^4 + 64(\lambda_1+\lambda_3)\lambda_4 v_0 v_2^5 + 16\lambda_4^2 v_2^6))^{1/2}$$
$$m^2_{h^n_2} = (\frac{1}{v_0})(2\lambda_8 v_0^3 + v_2(3\lambda_4 v_0^2 + 8(\lambda_1+\lambda_3)v_0 v_2 - 4\lambda_4 v_2^2) - (4\lambda_8^2 v_0^6 - 12\lambda_4\lambda_8 v_0^5 v_2 + (9\lambda_4^2 + 16((\lambda_5+\lambda_6+2\lambda_7)^2 - 2(\lambda_1+\lambda_3)\lambda_8)) v_0^4 v_2^2 + 16\lambda_4(3(\lambda_1+\lambda_3+2(\lambda_5+\lambda_6+2\lambda_7)) - \lambda_8) v_0^3 v_2^3 + 8(8(\lambda_1+\lambda_3)^2 + 21\lambda_4^2) v_0^2 v_2^4 + 64(\lambda_1+\lambda_3)\lambda_4 v_0 v_2^5 + 16\lambda_4^2 v_2^6))^{1/2}.$$

$$m^2_{h^p_s} = 0$$
$$m^2_{h^p_a} = \mu_2^2 + \lambda_{14} v_0^2 + 4(\lambda_{10} + \lambda_{11} - 2\lambda_{12}) v_2^2$$
$$m^2_{h^p_1} = -\frac{2(2\lambda_7 v_0^3 + 5\lambda_4 v_0^2 v_2 + 8\lambda_2 v_0 v_2^2 + 8\lambda_3 v_0 v_2^2)}{v_0}$$
$$m^2_{h^p_2} = -\frac{2(2\lambda_7 v_0 + \lambda_4 v_2)(v_0^2 + 4v_2^2)}{v_0}.$$
$$m_{h^\pm_s} = 0$$
$$m_{h^\pm_a} = \mu_2^2 + 4\lambda_{10} v_2^2$$
$$m_{h^\pm_1} = -(\lambda_6 + 2\lambda_7) v_0^2 - 10\lambda_4 v_0 v_2 - 16\lambda_3 v_2^2$$
$$m_{h^\pm_2} = -\frac{(\lambda_6 v_0 + 2\lambda_7 v_0 + 2\lambda_4 v_2)(v_0^2 + 4v_2^2)}{v_0}.$$

The S3 Model with 4 Higgs Doublets + Z2.



Neutral scalar Higgses mass range, with Hs as the SM Higgs.

Reparametrizing the vevs as

$$v_0 = v\cos\theta$$
$$v_1 = v\sin\theta\cos\phi$$
$$v_2 = v\sin\theta\sin\phi$$

We get

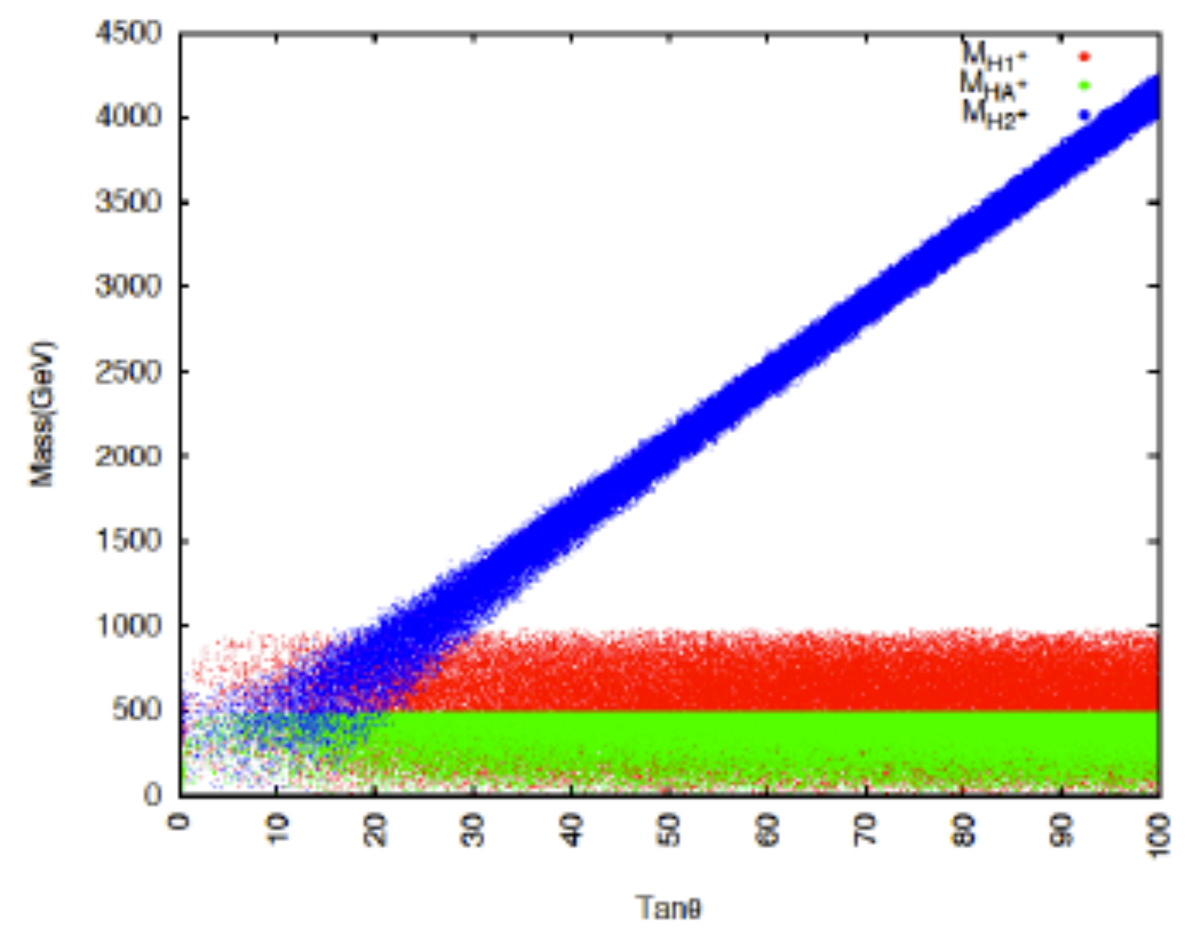
$$tan^2\phi = \frac{1}{3}$$

and thus

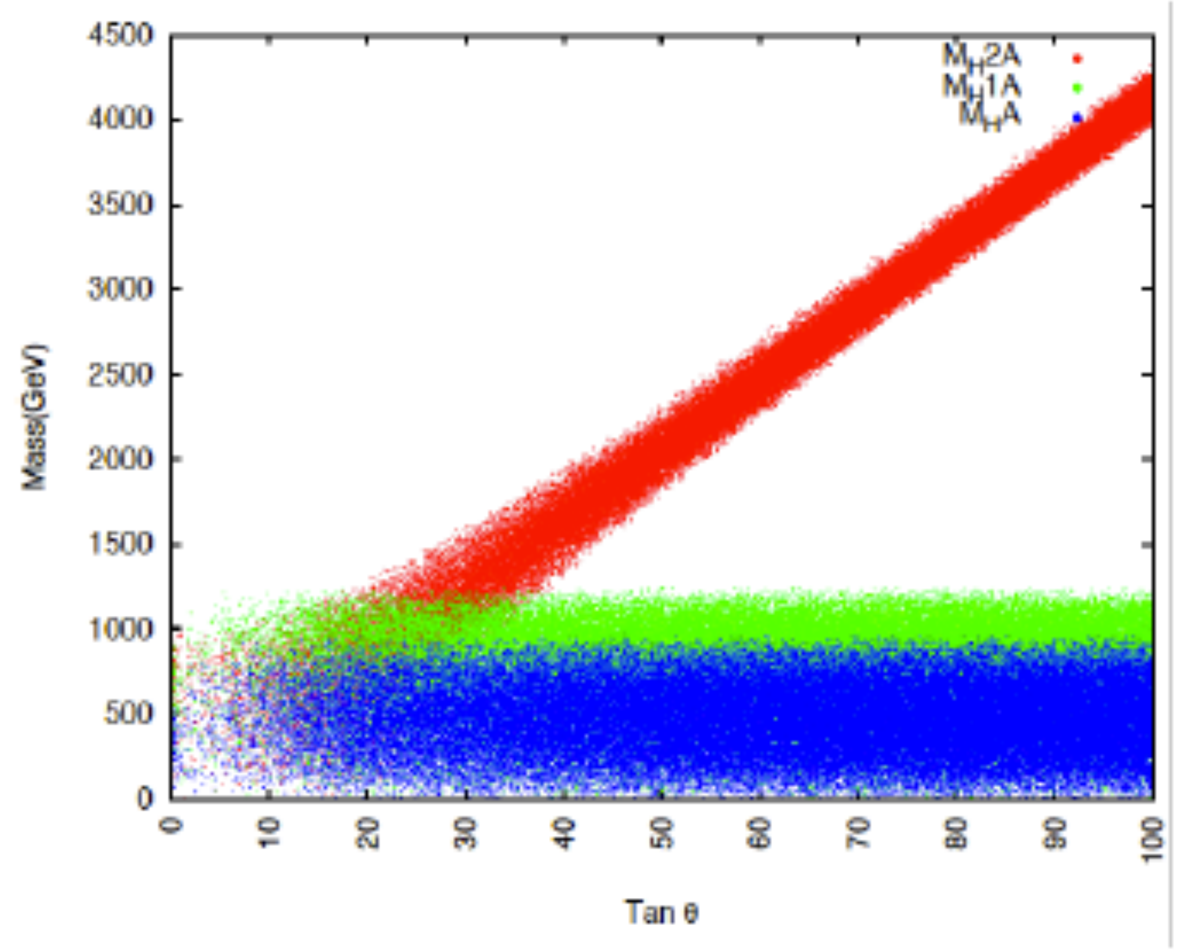
$$v_2 = \frac{1}{2}v\sin\theta \text{ and } v_3 = v\cos\theta$$

***Unitarity** and **stability** conditions were taken into account for the mass scan.

The S3 Model with 4 Higgs Doublets + Z2.

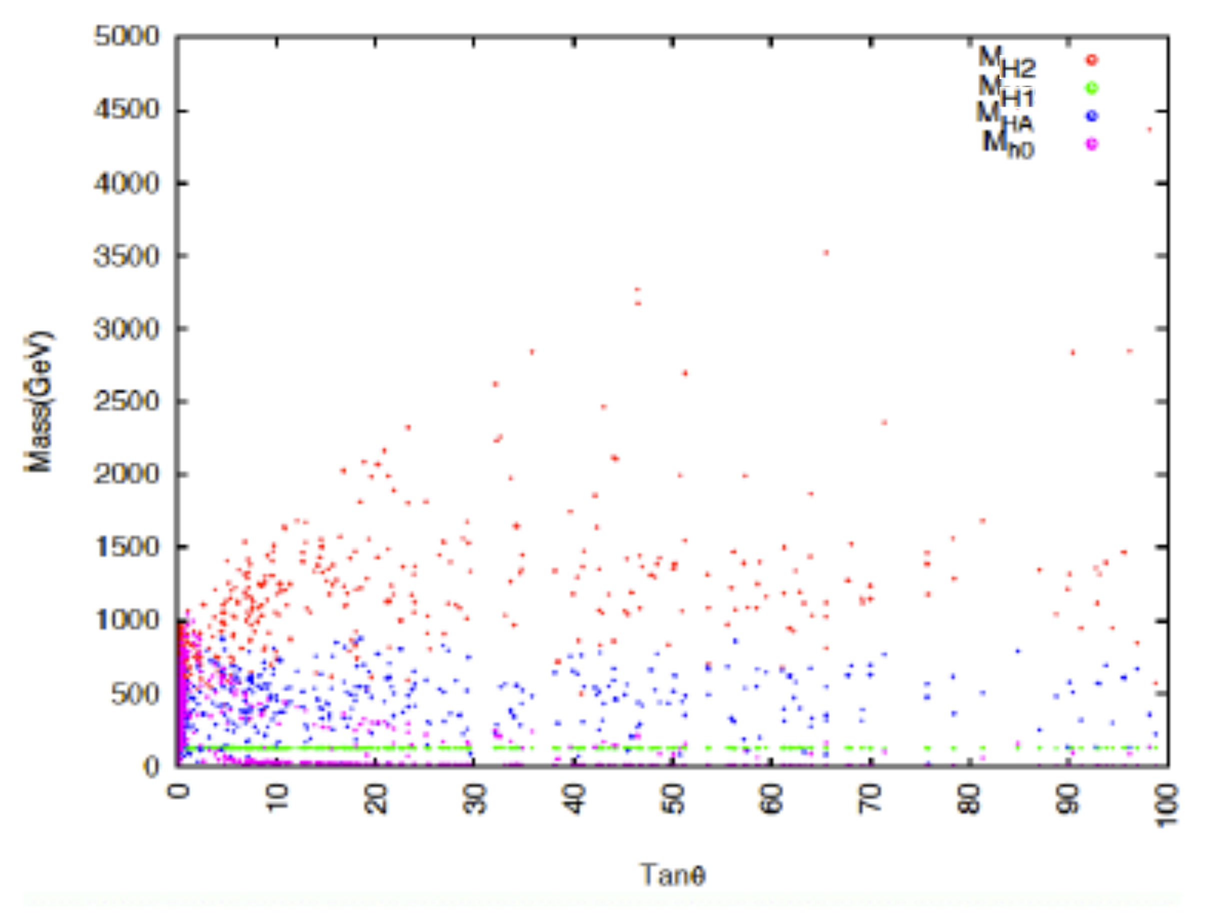


Charged Higgses mass range, with the scalar Hs as the SM Higgs.



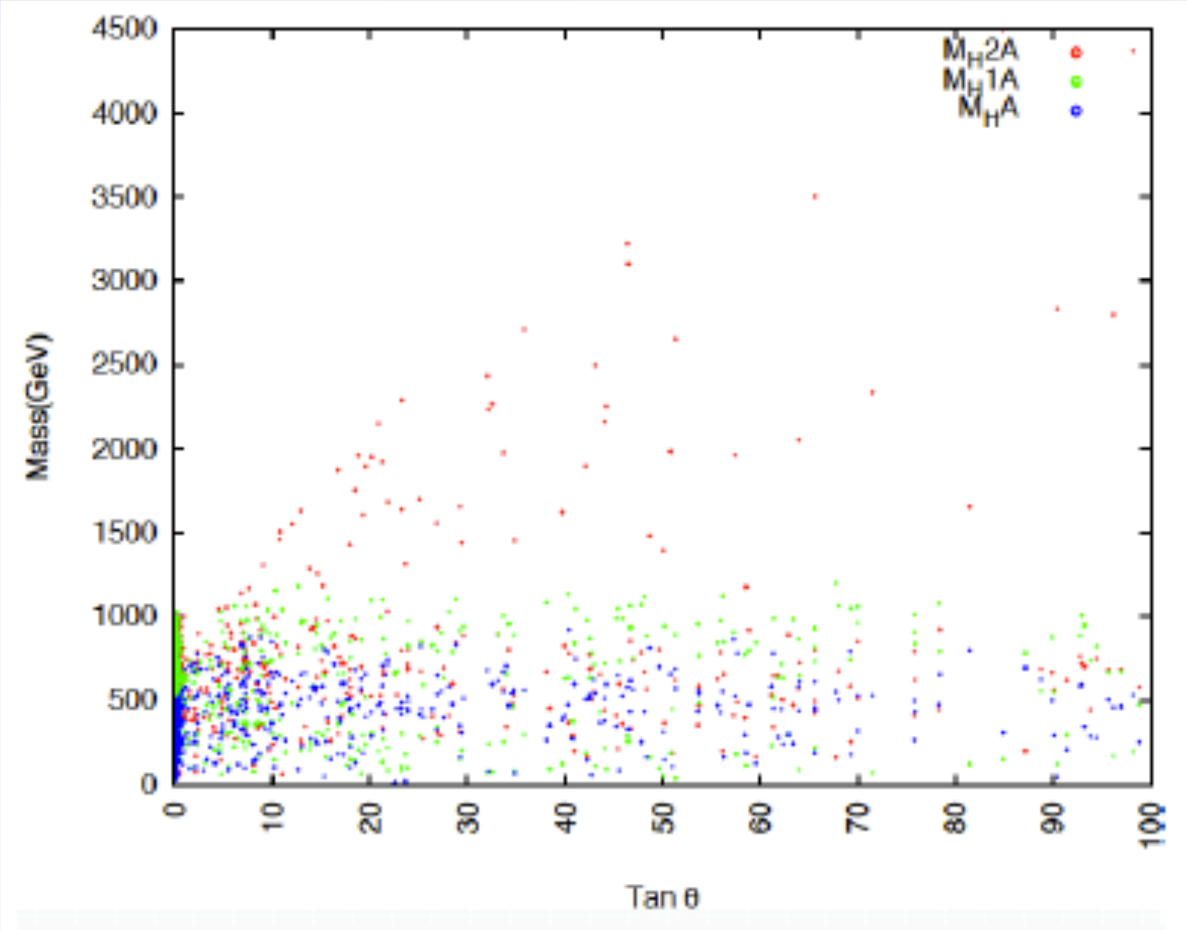
Pseudo scalar Higgses mass range, with the scalar Hs as the SM Higgs.

The S3 Model with 4 Higgs Doublets + Z2.

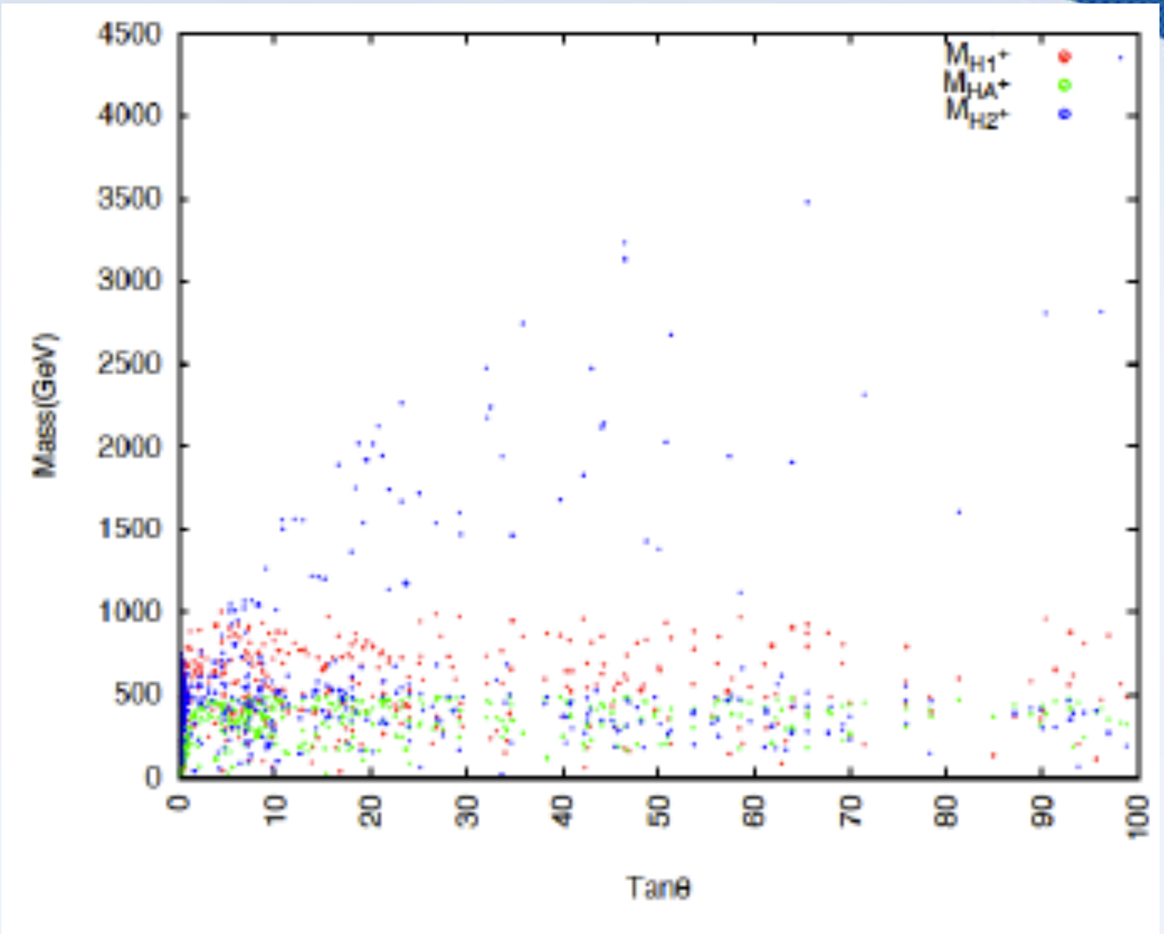


Neutral scalar Higgses mass range,
with the scalar H2 as the SM Higgs.

The S3 Model with 4 Higgs Doublets + Z2.



Pseudo scalar Higgses mass range,
with the scalar H2 as the SM Higgs.



Charged scalar Higgses mass range,
with the scalar H2 as the SM Higgs.

Dark Matter in the S3 Model with 4 Higgs Doublets + Z2.

Viable dark matter candidates.

$$m^2_{h^p_a} = \mu_2^2 + \lambda_{14} v_0^2 + 4(\lambda_{10} + \lambda_{11} - 2\lambda_{12}) v_2^2$$
$$m^2_{h^n_a} = \mu_2^2 + \lambda_{14} v_0^2 + 4(\lambda_{10} + \lambda_{11} + 2\lambda_{12}) v_2^2.$$

Theoretical restrictions.

- The potential must have a finite vacuum.
- The dispersion matrix must be unitary.
- Quartic couplings must be perturbative.

Experimental restrictions.

- The mass of the SM Higgs is

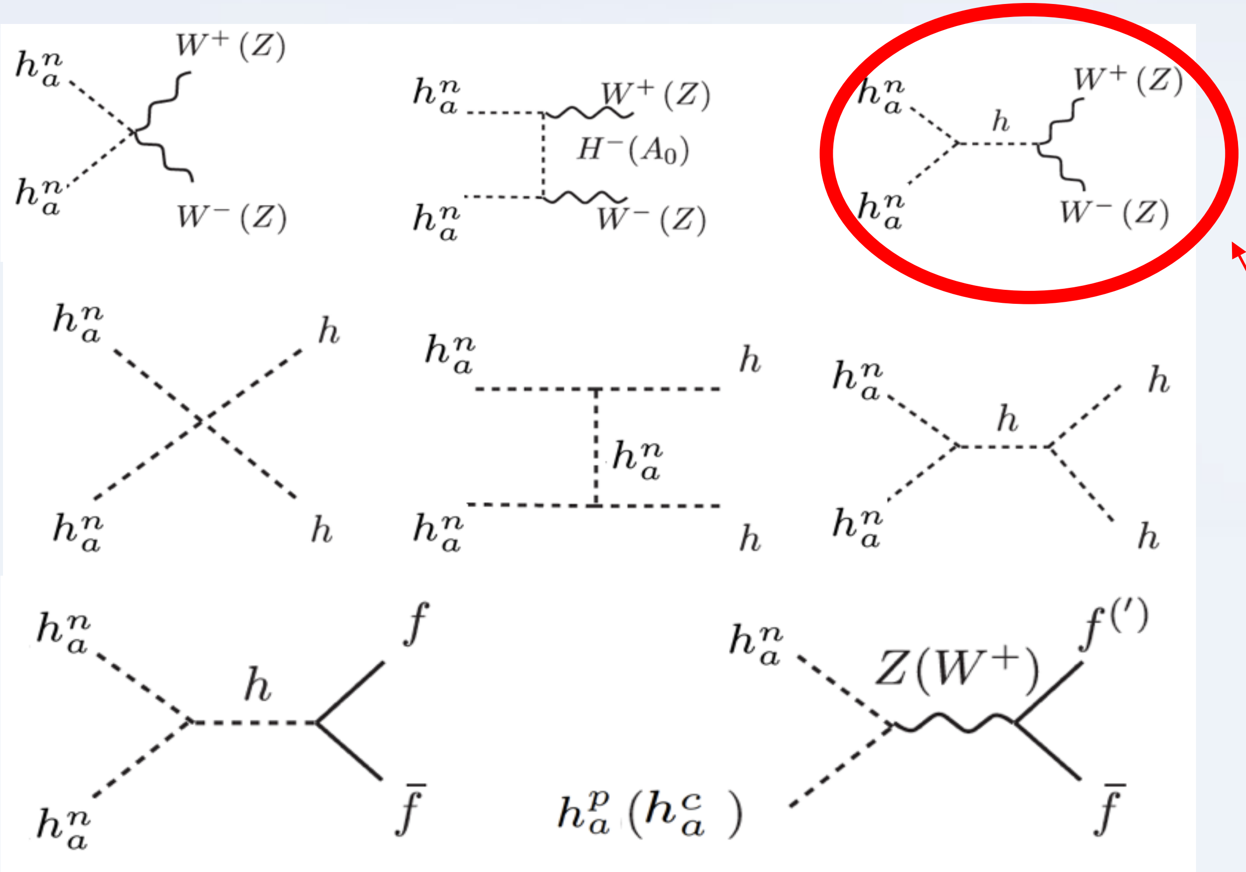
$$m_{h^n_s} = 125.09 \pm .21 \mathrm{GeV}.$$

- The measured relic density is

$$\Omega_{\mathrm{nbm}} h^2 = 0.1186 \pm 0.0020$$

Dark Matter in the S3 Model with 4 Higgs Doublets + Z2.

Allowed Feynman diiagrams for the DM candidate.



To find the relic density we need to calculate all the annihilation cross sections of the model.

As a first test, we only used the circled Feynman Diagram.

Dark Matter in the S3 Model with 4 Higgs Doublets + Z2.

To calculate the cross sections, first we need to find the **trillinear self-couplings**.

$$\lambda_{ijk} = \frac{-i\partial^3 V}{\partial H_i \partial H_j \partial H_k}.$$

And can be rewritten as:

$$\lambda_{ijk} = \sum_{m\leq n\leq o=1,2,3}^{*} R_{mi'} R_{nj'} R_{ok'} (-i) a_{mno},$$

where

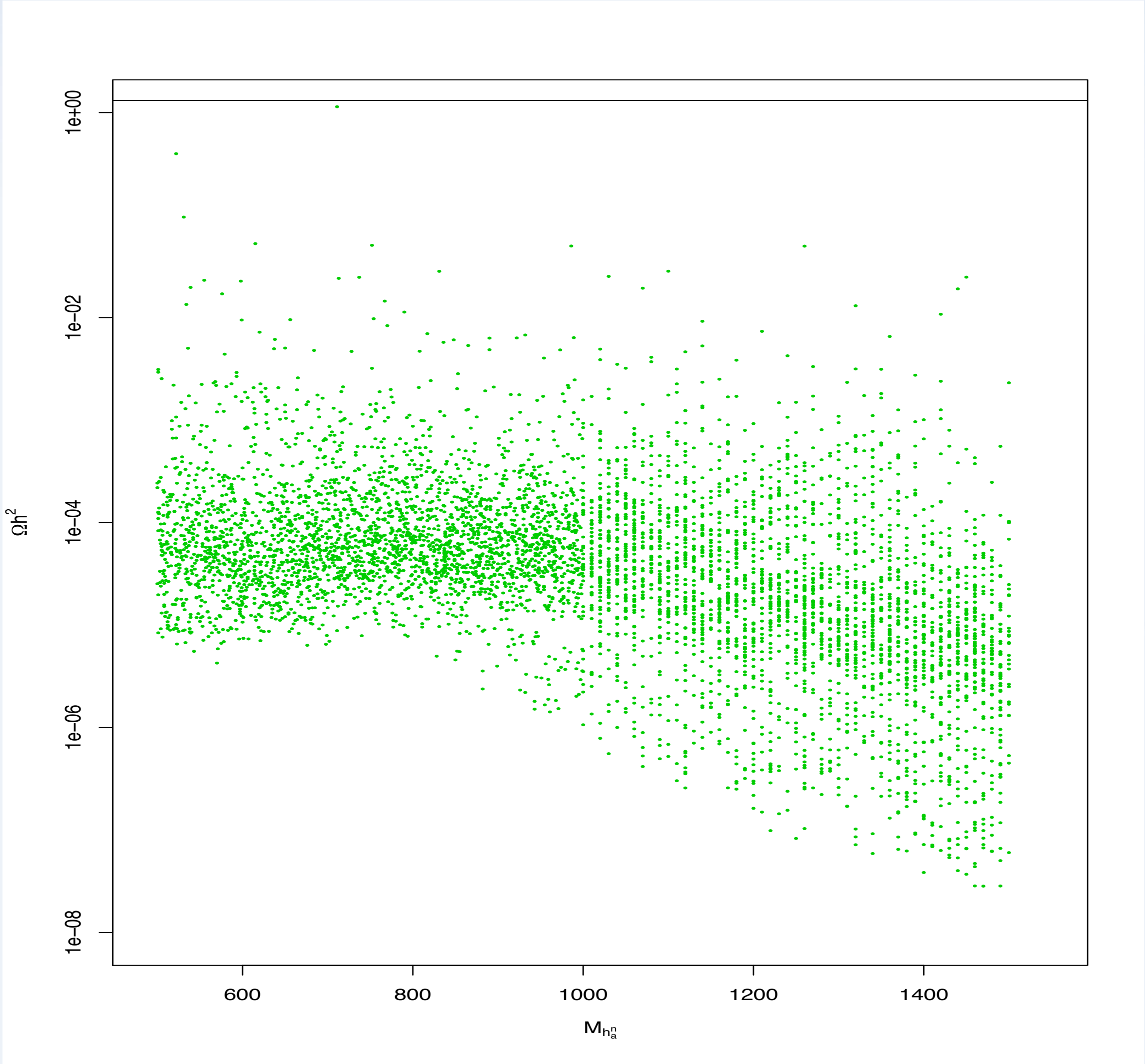
$$a_{mno} = \frac{\partial^3 V}{\partial \phi_m \partial \phi_n \partial \phi_o}.$$

We took the neutral scalar boson from the H2 doublet as the SM Higgs.

$$\lambda_{h_2^n h_a^n h_a^n} = -\frac{8iv}{\gamma\sqrt{4+\kappa^-}}((\alpha - \sqrt{\beta})\lambda_{14}\cos\theta + 2\gamma(\lambda_{10} + \lambda_{11} + 2\lambda_{12})\sin\theta)$$

Dark Matter in the S3 Model with 4 Higgs Doublets + Z2.

Relic Density scan, using MICROMEGAS.



The black line represents the measured relic density, approximately 0.118. All points in this graph are below this value.

This scan was performed for a simplified model, nevertheless, this is a promising result. There are other contributions yet to be added, so a complete scan could show us results with the measured RD. Still, the fact that all this values are below 0.118 is a good thing, it means that our candidate could account as a percentage of the total DM.

G. Belanger, F. Boudjema, and A. Pukhov. micrOMEGAs : a code for the calculation of Dark Matter properties in generic models of particle interaction. In *The Dark Secrets of the Terascale*, pages 739–790, 2013.

Conclusions and perspectives.

- Higgs models provide viable dark matter candidates: Cold, neutral and with a suitable mass range.
- There is a lot of work yet to be done: Calculate the relic density for the complete model and further constrain the free parameters.
- So far, it seems that there is at least one viable dark matter candidate in the model.
- Detect and characterize dark matter remains one of the biggest challenges of the upcoming years.

BACK UP

The S3 Model with 4 Higgs Doublets + Z2.

The Higgs Doublets are defined as:

$$H_s = \begin{pmatrix} h_s^c \\ h_s^s + v_0 + i h_s^p \end{pmatrix} \qquad H_a = \begin{pmatrix} h_a^c \\ h_a^s + i h_a^p \end{pmatrix}$$

$$H_1 = \begin{pmatrix} h_1^c \\ h_1^s + v_1 + i h_1^p \end{pmatrix} \qquad H_2 = \begin{pmatrix} h_2^c \\ h_2^s + v_2 + i h_2^p \end{pmatrix}$$

The S3 Model with 4 Higgs Doublets + Z2.

We need conditions that constrain the free parameters of the model. For that matter we obtained the **stability** and **unitarity** conditions.

$$
\begin{aligned}
\lambda_8 &> 0 \\
\lambda_1 + \lambda_3 &> 0 \\
\lambda_5 &> -2\sqrt{(\lambda_1 + \lambda_3)\lambda_8} \\
\lambda_5 + \lambda_6 - 2|\lambda_7| &> \sqrt{(\lambda_1 + \lambda_3)\lambda_8} \\
\lambda_1 - \lambda_2 &> 0 \\
\lambda_1 + \lambda_3 + |\, 2\lambda_4 \,| + \lambda_5 + 2\lambda_7 + \lambda_8 &> 0 \\
\lambda_{13} &> 0 \\
\lambda_{10} &> -2\sqrt{(\lambda_1 + \lambda_3)\lambda_{13}} \\
\lambda_{10} + \lambda_{11} - 2|\lambda_{12}| &> \sqrt{(\lambda_1 + \lambda_3)\lambda_{13}} \\
\lambda_{14} &> -2\sqrt{\lambda_8\lambda_{13}}.
\end{aligned}
$$

Stability conditions.

Found by requiring that the cuartic sector of the potencial remains positive when the fields tend to infinity.

The S3 Model with 4 Higgs Doublets + Z2.

$$a_1^{\pm} = (\lambda_1 - \lambda_2 + \frac{\lambda_5 + \lambda_6}{2}) \pm \sqrt{(\lambda_1 - \lambda_2 + \frac{\lambda_5 + \lambda_6}{2})^2 - 4[(\lambda_1 - \lambda_2)(\frac{\lambda_5 + \lambda_6}{2}) - \lambda_4^2]}$$

$$a_2^{\pm} = (\lambda_1 + \lambda_2 + 2\lambda_3 + \lambda_8) \pm \sqrt{(\lambda_1 + \lambda_2 + 2\lambda_3 + \lambda_8)^2 - 4[\lambda_8(\lambda_1 + \lambda_2 + 2\lambda_3) - 2\lambda_7^2]}$$

$$a_3^{\pm} = (\lambda_1 - \lambda_2 + 2\lambda_3 + \lambda_8) \pm \sqrt{(\lambda_1 - \lambda_2 + 2\lambda_3 + \lambda_8)^2 - 4[\lambda_8(\lambda_1 + \lambda_2 + 2\lambda_3) - \frac{\lambda_6^2}{2}]}$$

$$a_4^{\pm} = (\lambda_1 + \lambda_2 + \frac{\lambda_5}{2} + \lambda_7) \pm \sqrt{(\lambda_1 + \lambda_2 + \frac{\lambda_5}{2} + \lambda_7)^2 - 4[(\lambda_1 - \lambda_2)(\frac{\lambda_5}{2} + \lambda_7) - \lambda_4^2]}$$

$$a_5^{\pm} = (5\lambda_1 - \lambda_2 + 2\lambda_3 + 3\lambda_8) \pm \sqrt{(5\lambda_1 - \lambda_2 + 2\lambda_3 + 3\lambda_8)^2 - 4[3\lambda_8(5\lambda_1 - \lambda_2 + 2\lambda_3) - \frac{1}{2}(2\lambda_5 + \lambda_6)^2]}$$

$$a_6^{\pm} = (\lambda_1 + \lambda_2 + 4\lambda_3 + \frac{\lambda_5}{2} + \lambda_6 + 3\lambda_7) \pm ((\lambda_1 + \lambda_2 + 4\lambda_3 + \frac{\lambda_5}{2} + \lambda_6 + 3\lambda_7)^2 - 4[(\lambda_1 + \lambda_2 + 4\lambda_3)(\frac{\lambda_5}{2} + \lambda_6 + 3\lambda_7) - 9\lambda_4^2])^{1/2}$$

Unitarity Conditions.

The eigenvalues of the dispersion matrix must be

$$|\, a_i^{\pm} \,|,\ |\, b_i \,| \leqslant 16\pi,$$

$$b_4 = 2(\lambda_1 - \lambda_1 - 2\lambda_3) \qquad b_1 = \lambda_5 + 2\lambda_6 - \lambda_7$$
$$b_5 = 2(\lambda_1 + \lambda_1 - 2\lambda_3) \qquad b_2 = \lambda_5 - 2\lambda_7$$
$$b_6 = \lambda_5 - \lambda_6. \qquad b_3 = 2(\lambda_1 - 5\lambda_1 - 2\lambda_3)$$

*The ones in this slide correspond to the 3HDM-S3 found by Das and Dey

Dipankar Das and Ujjal Kumar Dey. Analysis of an extended scalar sector with S_3 symmetry. *Phys. Rev.*, D89(9):095025, 2014. [Erratum: Phys. Rev.D91,no.3,039905(2015)].

$$b_7 = \lambda_{10} + \lambda_{11}$$
$$b_8 = \lambda_{10} + 2\lambda_{12}$$
$$b_9 = \lambda_{10} + 2\lambda_{11} + 6\lambda_{12}$$
$$a_7 = \frac{1}{2}\sqrt{(2\lambda_{10} + 3(\lambda_{11} - 4\lambda_{12}))(2\lambda_{10} + \lambda_{11} - 4\lambda_{12})}.$$
$$a_8 = \frac{1}{2}\left(\sqrt{2} \pm 2\right)\lambda_{14}$$
$$a_9 = \pm\frac{i\lambda_{14}}{2}$$
$$b_{10} = \lambda_{14}$$
$$a_{10} = \lambda_{10} \pm \lambda_{11}$$
$$a_{11} = \lambda_{10} \pm 2\lambda_{12}$$

Unitarity conditions from the decoupled Ha dispersion chanels.

Dark Matter in the S3 Model with 4 Higgs Doublets + Z2.

Trillinear self coupling of two neutral antisymmetric and one neutral symmetric (the SM Higgs) Higgses

$$
\begin{aligned}
\lambda_{h_s^n h_a^n h_a^n} = \lambda_{91212} = \\
& -2i \sum_{m \leq n \leq o = 1,2,3} [(R^n_{m9} R^n_{n12} R^n_{n12} + R^n_{m12} R^n_{n9} R^n_{o12} \\
& + R^n_{m12} R^n_{n12} R^n_{o12}) a_{mno}] \\
& = -i2[R^n_{109} R^n_{1212} R^n_{1212} a_{101212} + R^n_{119} R^n_{1212} R^n_{1212} a_{111212}] \\
& = -2i[-\frac{1}{2}(4\sqrt{3}\lambda_{10} v \sin\theta + 4\sqrt{3}\lambda_{11} v \sin\theta + 8\sqrt{3}\lambda_{12} v \sin\theta) \\
& + \frac{\sqrt{3}}{2}(4\lambda_{10} v \sin\theta + 4\lambda_{11} v \sin\theta + 8\lambda_{12} v \sin\theta)] \\
& = 0.
\end{aligned}
$$

Equals zero!

The S3 Model with 4 Higgs Doublets.

The Yukawa Lagrangian.

$$
\begin{aligned}
-\mathcal{L}_{Y_f} = {} & Y_1^f(\bar{\psi}^f_{S,L}\psi^f_{S,R}H_s) + \frac{1}{\sqrt{2}}Y_2^f(\bar{\psi}^f_{1,L}\psi^f_{1,R} + \bar{\psi}^f_{2,L}\psi^f_{2,R})H_s \\
& + \frac{1}{2}Y_3^f[(\bar{\psi}^f_{1,L}H_2 + \bar{\psi}_{2,L}H_1)\psi^f_{1,R} + (\bar{\psi}_{1,L}H_1 - \bar{\psi}^f_{2,L}H_2)\psi^f_{2,R}] \\
& + \frac{1}{\sqrt{2}}Y_4^f(\bar{\psi}^f_{1,L}\psi_{2,R} - \bar{\psi}^f_{2,L}\psi^f_{1,R})H_a \\
& + \frac{1}{\sqrt{2}}Y_5^f(\bar{\psi}^f_{1,L}H_1 + \bar{\psi}^f_{1,L}H_1 + \bar{\psi}^f_{2,L}H_2)\psi^f_{S,R} \\
& + \frac{1}{\sqrt{2}}Y_6^f(\bar{\psi}^f_{S,L}(H_1\psi^f_{1,R} + H_2\psi^f_{2,R})] + \text{h.c.} \\
& f = d, e.
\end{aligned}
$$